SICP Ex 1.15

$$p(x) = 3x - 4x^3$$

$$\text{sine}(x) = \begin{cases} x, & \text{if } |x|_1 \leq 0.1 \\ p(\text{sine}(x/3)), & \text{else} \end{cases}$$

$$\text{sine}(12.15) = p(\text{sine}(4.05)) = p(p(\text{sine}(1.35)))$$

$$= p(p(p(\text{sine}(0.45)))) = p(p(p(p(\text{sine}(0.15)))))$$

$$= p(p(p(p(p(\text{sine}(0.05))))))$$

$$= p(p(p(p(p(0.05))))) \Rightarrow p \text{ is applied } \underline{5 \text{ times}}$$

In general, the process spawned by sine(a) terminates after n steps so that n is the smallest integer satisfying

$$a/3^n \leq 0.1 \Leftrightarrow \log a - n \log 3 \leq \log 0.1 \Leftrightarrow n \geq \frac{\log a - \log 0.1}{\log 3}$$

Since each step takes a constant amount of compute, the order of growth for number of calculations is $O(\log a)$.

Same applies for space since each step leads to one more delayed call of p.