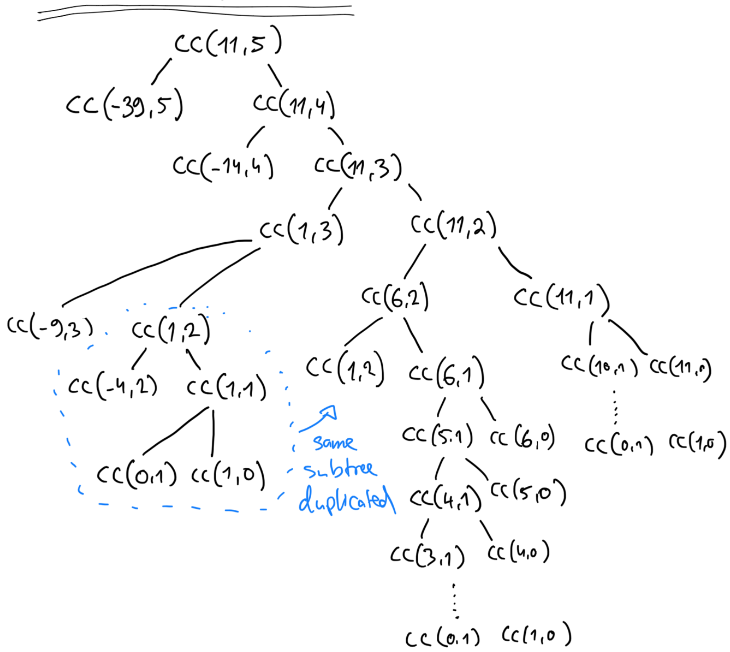
SICP Ex 1.14



Let $T(n,k)$ be time complexity for n amount and k kinds of coins.

We want to know $T(n,5) = f(n)$

$T(n,1) = n$, just look at graph above

$$
\begin{aligned}
T(n,2) &= T(n,1) + T(n-c_2,2)\\
&= n + T(n-5,2)\\
&= n + T(n-5,1) + T(n-10,2)\\
&= n + (n-5) + T(n-10,2)\\
&= n + (n-5) + (n-10) + \dots + \overbrace{T(0,2)}^{=\text{constant}}\\
&\approx \sum_{i=0}^{n/5} (n-5i) = \frac{1}{5}n^2 + \sum_{i=0}^{n} i\\
&= \frac{1}{5}n^2 + n\cdot\frac{n}{2} = \frac{1}{5}n^2 + \frac{1}{2}n^2 \approx n^2
\end{aligned}
$$

$$
\begin{aligned}
T(n,3) &= T(n,2) + T(n-10,3)\\
&= n^2 + (n-10)^2 + T(n-20,3)\\
&= \dots \text{ as above}\\
&\approx n^3
\end{aligned}
$$

One can see where this leads: we'll eventually get to

$T(n,5) \sim \underline{O(n^5)}$ (time complexity)

Space complexity is the length of longest path in the recursion. This path is taken by reducing n "one at a time", and the length of this path is $O(n)$

$\Rightarrow$ space complexity is $\underline{\underline{O(n)}}$