ELE204 Control Systems Engineering 1
Homework Assignment 1

Distributed:	21.09.23
Deadline:	03.10.23

Problem 1

Given the following function $Y(s)$ that represents the unit step response of a dynamical system

$$Y(s) = \frac{s+12}{s(s+3)} = \frac{c_1}{s} + \frac{c_2}{s+3}$$

a) Use partial fraction expansion (*delbrøkoppløsning*) and find the coefficients c_1 and c_2;
b) Use the inverse Laplace transform from the Laplace-table and determine $y(t)$;
c) What is the final value of the output $y(\infty)$ (when $t \to \infty$)?

Problem 2

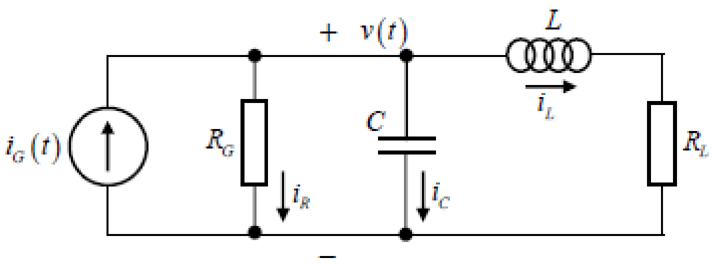
A dynamical system having the output $h(t)$, the input $g(t)$ and all the initial values negligible is described by the following differential equation

$$\frac{d^4h(t)}{dt^4} + 3\frac{d^3h(t)}{dt^3} + 7\frac{d^2h(t)}{dt^2} + 4h(t) = 2\frac{d^3g(t)}{dt^3} + 11\frac{d^2g(t)}{dt^2} + 4\frac{dg(t)}{dt} + 12g(t)$$

a) Find the transfer function of the system $W(s) = H(s)/G(s)$;
b) What is the final value of the output $h(\infty)$ when the input is a step with magnitude 150?

Problem 3

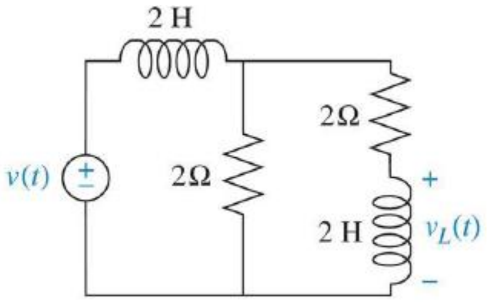
Consider the electrical circuit shown below with the output $i_L(t)$ and the input $i_G(t)$. The differential equations describing the reactive elements are $v_L(t) = Ldi_L(t)/dt = Li_L'(t)$, $i_C(t) = Cdv_C(t)/dt = Cv_C'(t)$. The initial values are $i_L(t) = 0\,\text{A}$, $i_L'(t) = 0\,\text{A/s}$.



a) Find the differential equation of the circuit;
b) Find the transfer function of the circuit;
c) What is the output when $t \to \infty$ and the input is a step with amplitude I_G A?

Problem 4

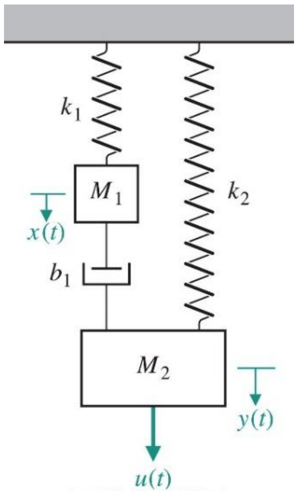
The electrical circuit shown in the figure on the right having $v_L(t)$ as output and $v(t)$ as input is given.



a) Find the differential equation;
b) Find the transfer function of the circuit $V_L(s)/V(s)$;

Problem 5

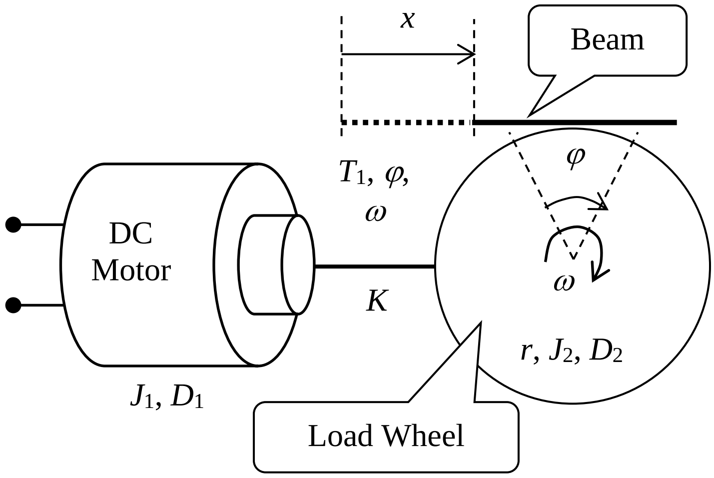
Given the two-mass mechanical system shown:



a) Find the set of the differential equations describing the system;
b) Find the transfer function $G(s) = X(s)/U(s)$.

Problem 6

The rotational mechanical system shown below transforms the rotational movement of the load wheel caused by the DC motor into the translational movement of the beam. The mass of the beam is zero (m_{beam} = 0 Kg). If the wheel rotates the angle φ [rad], the beam translates a corresponding displacement (forflytning) $x(t)$.



$$\omega(t) = \frac{d\varphi(t)}{dt} = \varphi'(t)\left[\frac{\text{rad}}{\text{sek}}\right],$$
$$T_{fjær}(t) = K\varphi(t)\ [\text{Nm}],$$
$$T_{demp}(t) = D\varphi'(t) = D\omega(t)\ [\text{Nm}]$$

a) Find the differential equation that describes the translational displacement $x(t)$ as a function of the applied motor torque $T_1(t)$;
b) Find the transfer function between the applied motor torque $T_1(s)$ and the position $X(s)$;
c) Sketch a block diagram or a signal-flow graph of the system;

Problem 7

The block diagram of a system is given in figure below. Find the transfer function of the overall system $T(s) = C(s)/R(s)$ **either** by:

a) Using block diagram reduction;
b) Drawing the signal-flow graph of the system and applying Mason's rule.