This article surveys the theory of compressive sampling, also known as compressed sensing or CS, a novel sensing/sampling paradigm that goes against the common wisdom in data acquisition. CS theory asserts that one can recover certain signals and images from far fewer samples or measurements than traditional methods use. To make this possible, CS relies on two principles: *sparsity*, which pertains to the signals of interest, and *incoherence*, which pertains to the sensing modality.

- Sparsity expresses the idea that the "information rate" of a continuous time signal may be much smaller than suggested by its bandwidth, or that a discrete-time signal depends on a number of degrees of freedom which is comparably much smaller than its (finite) length. More precisely, CS exploits the fact that many natural signals are sparse or compressible in the sense that they have concise representations when expressed in the proper basis Ψ.
- Incoherence extends the duality between time and frequency and expresses the idea that objects having a sparse representation in Ψ must be spread out in the domain in which they are acquired, just as a Dirac or a spike in the time domain is spread out in the frequency domain. Put differently, incoherence says that unlike the signal of interest, the sampling/sensing waveforms have an extremely dense representation in Ψ.

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The crucial observation is that one can design efficient sensing or sampling protocols that capture the useful information content embedded in a sparse signal and condense it into a small amount of data. What is most remarkable about these sampling protocols is that they allow a sensor to very efficiently capture the information in a sparse signal without trying to comprehend that signal. In other words, CS is a very simple and efficient signal acquisition protocol which samples—in a signal independent fashion—at a low rate and later uses computational power for reconstruction from what appears to be an incomplete set of measurements.

Our intent in this article is to overview the basic CS theory that emerged in the works [1]–[3], present the key mathematical ideas underlying this theory, and survey a couple of important results in the field. Our goal is to explain CS as plainly as possible, and so our article is mainly of a tutorial nature. One of the charms of this theory is that it draws from various subdisciplines within the applied mathematical sciences, most notably probability theory. In this review, we have decided to highlight this aspect and especially the fact that randomness can—perhaps surprisingly—lead to very effective sensing mechanisms. We will also discuss significant implications, explain why CS is a concrete protocol for sensing and compressing data simultaneously (thus the name), and conclude our tour by reviewing important applications.

THE SENSING PROBLEM

In this article, we discuss sensing mechanisms in which information about a signal $f(t)$ is obtained by linear functionals recording the values

$$y_k = \langle f, \varphi_k \rangle, \qquad k = 1, \ldots, m. \tag{1}$$

That is, we simply correlate the object we wish to acquire with the waveforms $\varphi_k(t)$. This is a standard setup. If the sensing waveforms are Dirac delta functions (spikes), for example, then y is a vector of sampled values of f in the time or space domain. If the sensing waveforms are indicator functions of pixels, then y is the image data typically collected by sensors in a digital camera. If the sensing waveforms are sinusoids, then y is a vector of Fourier coefficients; this is the sensing modality used in magnetic resonance imaging (MRI). Other examples abound.

Although one could develop a CS theory of continuous time/space signals, we restrict our attention to discrete signals $f \in \mathbb{R}^n$. The reason is essentially twofold: first, this is conceptually simpler and second, the available discrete CS theory is far more developed (yet clearly paves the way for a continuous theory—see also "Applications"). Having said this, we are then interested in *undersampled* situations in which the number m of available measurements is much smaller than the dimension n of the signal f. Such problems are extremely common for a variety of reasons. For instance, the number of sensors may be limited. Or the measurements may be extremely expensive as in certain imaging processes via neutron scattering. Or the sensing process may be slow so that one can only measure the object a few times as in MRI. And so on.

These circumstances raise important questions. Is accurate reconstruction possible from $m \ll n$ measurements only? Is it possible to design $m \ll n$ sensing waveforms to capture almost all the information about f? And how can one approximate f from this information? Admittedly, this state of affairs looks rather daunting, as one would need to solve an underdetermined linear system of equations. Letting A denote the $m \times n$ sensing matrix with the vectors $\varphi_1^*, \ldots, \varphi_m^*$ as rows (a^* is the complex transpose of a), the process of recovering $f \in \mathbb{R}^n$ from $y = Af \in \mathbb{R}^m$ is ill-posed in general when $m < n$: there are infinitely many candidate signals $\tilde{f}$ for which $A\tilde{f} = y$. But one could perhaps imagine a way out by relying on realistic models of objects f which naturally exist. The Shannon