## When is it Worthwhile to buy Powerball Tickets?

#### September 6, 2025

## 1 Rules

Powerball is played by selecting five distinct numbers  $w_1, w_2, w_3, w_4, w_5 \in \{1, ..., 69\}$  and one number  $r \in \{1, ..., 26\}$ , where r is independent of the  $w_i$ .

During drawing, five distinctly-numbered white balls are selected from  $\{1, \ldots 69\}$  and one independent red ball is selected from  $\{1, \ldots, 26\}$ . A random power play multiplier m is also selected from  $\{2, 3, 4, 5, 10\}$ .

The player is awarded prizes according to the following scheme:

Match	Prize	Power Play 2X	Power Play 3X	Power Play 4X	Power Play 5X	Power Play 10X
+ GALL	Grand Prize	Grand Prize	Grand Prize	Grand Prize	Grand Prize	Grand Prize
	\$1 Million	\$2 Million	\$2 Million	\$2 Million	\$2 Million	\$2 Million
+ GALL	\$50,000	\$100,000	\$150,000	\$200,000	\$250,000	\$500,000
	\$100	\$200	\$300	\$400	\$500	\$1,000
+ GALL	\$100	\$200	\$300	\$400	\$500	\$1,000
	\$7	\$14	\$21	\$28	\$35	\$70
+ BALL	\$7	\$14	\$21	\$28	\$35	\$70
+ BALL	\$4	\$8	\$12	\$16	\$20	\$40
BALL	\$4	\$8	\$12	\$16	\$20	\$40

Figure 1: Powerball Prizes

## 2 Probabilities

The Official Powerball Website lists the probabilities of each prize, but just to make sure they're being honest, let's check the probabilities of the large prizes.

Grand Prize There are a total of  $\binom{69}{5}$  possible white ball drawings and 26 possible red ball drawings, so there are  $26\binom{69}{5}$  possible Powerball drawings. Of these, exactly one possible ticket matches. Thus, the probability of winning the grand prize is  $3.42229781(10^{-9})$  or 1 out of 292201338.

\$1 Million This is the same as the grand prize, but without matching the red ball. Thus, there are 25 equally likely tickets that win this prize. So the probability of winning the million dollar prize is  $25/\left(26\binom{69}{5}\right) = 8.55574453(10^{-8})$  or 1 out of 11688053.52.

\$50 Thousand To win the 50 thousand dollar prize, you must match exactly 4 of the 5 white balls along with the red ball. Given 5 fixed white balls, the number of ways we can match four of them is  $\binom{5}{4} = 5$ , and the number of ways we can *not* match the last white ball is 64 (since it cannot match any of the 5 whites drawn). There is only one way to match the red ball. Thus, the probability of winning the 50 thousand dollar prize is  $5(64)/(26\binom{69}{5}) = 1.09513(10^{-6})$  or 1 out of 913129.18125.

All of these calculations match the official Powerball odds, and the remaining prizes are relatively small, so we will trust the probabilities they provide. The official probabilities are listed in the table below:

Prize	Odds		
Grand Prize	1 in 292,201,338.00		
\$1 Million	1 in 11,688,053.52		
\$50,000	1 in 913,129.18		
\$100	1 in 36,525.17		
\$100	1 in 14,494.11		
\$7	1 in 579.76		
\$7	1 in 701.33		
\$4	1 in 91.98		
\$4	1 in 38.32		

Figure 2: Official Powerball Odds

This gives us some duplicates (multiple ways to win \$100, for example), so let's reorganize this by outcome:

Prize	Probability
Grand Prize	$3.42229781(10^{-9})$
\$1 Million	$8.55574453(10^{-8})$
\$50 Thousand	$1.09513(10^{-6})$
\$100	$9.637192(10^{-5})$
\$7	$3.15071395(10^{-3})$
\$4	$3.696796208(10^{-2})$

Table 1: Probabilities of all Possible Rewards

# 3 Expectation

Let's consider the random variable f(g) representing the net profit of a particular ticket, in dollars, where f is a function of the grand prize g. We wish to find  $\mathbb{E}f(g)$  as a function of g. The expectation of f(g) will be the sum of the expectations of each of the individual prizes.

First, we calculate the expected value of each individual prize under some Power Play multiplier m. Note we do not consider the 10x Power Play multiplier because it is only active when g < \$150 million, which is below the grand prizes we are interested in. The expected value of a prize without any multipliers is just the probability of winning it multiplied by the value.

Prize	Expected Value ( $\$$ ) ( $m \times$ Power Play)
Grand Prize	$g \cdot 3.42229781(10^{-9})$
\$1 Million	$\min(2, m) \cdot 0.0855574453$
\$50 Thousand	$m \cdot 0.0547565$
\$100	$m \cdot 0.009637192$
\$7	$m \cdot 0.02205499765$
\$4	$m \cdot 0.14787184832$

Table 2: Probabilities of all Possible Rewards. Power Play does not affect the grand prize, and the \$1\$ million prize is capped at a  $2\times$  multiplier.

To choose the power play multiplier, one ball is chosen from a set containing 24 two× balls, 13 three× balls, 3 four× balls, and 2 five× balls (we don't consider ten× for large grand prizes). Thus, the expected multiplier is about 2.595 Since m is independent of the other balls and independent of q, we can write:

$$\mathbb{E}[f(g)] = 3.4222(10^{-9})g + 0.1711 + 2.595(0.2343)$$

When Power Play is active (we factored out m and set it equal to 3.5), and

$$\mathbb{E}[f(g)] = 3.4222(10^{-9})g + 0.3198$$

When Power Play is inactive.

#### 4 Powerball Thresholds

Regular Power Ball tickets cost \$2 and do not benefit from the Power Play multiplier. For a regular ticket to be profitable in the long run, we need:

$$0 < \mathbb{E}f(g) - 2$$
$$2 < 3.4222(10^{-9})g + 0.3198$$
$$490, 970, 720.58 < g$$

Of course, this is the amount you would need to receive after taxes. Taxes vary from state to state, but if the grand prize is more than  $1.5 \times$  the g threshold (about \$763 million), then the post-tax winnings will safely pass the threshold of being long-term profitable.

Power play tickets cost 3 and allow you to apply the m multiplier to your prizes as described above. If you live in Idaho or Montana, you can only buy Power Play tickets. For a power play ticket to be profitable in the long run, we need:

$$0 < \mathbb{E}f(g) - 3$$
 
$$3 < 3.4222(10^{-9})g + 0.1711 + 2.595(0.2343)$$
 
$$648, 966, 016.01 < g$$

Or if you do a liberal tax estimate as done above, somewhere around \$973 million.

If you really want to make sure your tickets are long run profitable, you should check your state's specific tax code.