Fys-STK4155

Exercise week 35 . show $\frac{\partial(\vec{b}^T\vec{a})}{\partial\vec{a}} = \vec{b}$

Ex 1

$$\frac{\partial}{\partial\vec{a}} = \left[\frac{\partial}{\partial a_1}, \frac{\partial}{\partial a_2}, \frac{\partial}{\partial a_3}, \dots \frac{\partial}{\partial a_n}\right]^T$$

$$\vec{b}^T\vec{a} = [b_1, b_2, b_3, \dots, b_n]\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{bmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3 + \dots + a_n b_n$$

$$\frac{\partial}{\partial\vec{a}}(\vec{b}^T\vec{a}) = \frac{\partial}{\partial\vec{a}}(a_1 b_1 + a_2 b_2 + a_3 b_3 + \dots + a_n b_n)$$

$$= \left[\frac{\partial}{\partial a_1}(a_1 b_1 + a_2 b_2 + \dots a_n b_n), \frac{\partial}{\partial a_2}(a_1 b_1 + a_2 b_2 + \dots a_n b_n), \frac{\partial}{\partial a_3}(\dots\right]^T$$

$$= [b_1, b_2, b_3, \dots b_n]^T = \vec{b}$$



Lars Johan Brodtkorb

Show that $\frac{\partial}{\partial \vec{a}}(\vec{a}^T A \vec{a}) = \vec{a}^T (A + A^T)$

$$\frac{\partial}{\partial \vec{a}} = \left[\frac{\partial}{\partial a_1}, \frac{\partial}{\partial a_2}, \frac{\partial}{\partial a_3}, \ldots, \frac{\partial}{\partial a_n}\right]^T$$

$$\vec{a}^T A \vec{a} = \sum_{i=1}^{n} \sum_{j=1}^{n} a_i A_{ij} a_j$$

$$\frac{\partial}{\partial a_K}(\vec{a}^T A \vec{a}) = \frac{\partial}{\partial a_K}\Big( \sum_{i \neq K} a_i A_{iK} a_K + \sum_{j \neq K} a_K A_{Kj} a_j + a_K^2 A_{KK} + \underbrace{\sum_{i\neq K} \sum_{j \neq K} a_i A_{ij} a_j}_{=0} \Big)$$

$$= \sum_{i \neq K} a_i A_{iK} + \sum_{j \neq K} A_{Kj} a_j + 2 a_K A_{KK}$$

$$= \sum_{S \neq K} a_s (A_{sK} + A_{Ks}) + 2 a_K A_{KK}$$

$$= \sum_{s=1}^{n} a_s (A_{sK} + A_{Ks})$$

$$\frac{\partial}{\partial \vec{a}}(\vec{a}^T A \vec{a}) = \left[ \sum_{s=1}^{n} a_s (A_{s1} + A_{1s}), \sum_{s=1}^{n} a_s (A_{s2} + A_{2s}), \ldots, \sum_{s=1}^{n} a_s (A_{sn} + A_{ns}) \right]^T$$

$$A + A^T = \begin{bmatrix} 2A_{11} & A_{12}+A_{21} & A_{13}+A_{31} \ldots & A_{1n}+A_{n1} \\ A_{21}+A_{12} & 2A_{22} & \ddots & \\ \vdots & & & \\ A_{n1}+A_{1n} & & & 2A_{nn} \end{bmatrix}$$

$$\vec{a}^T(A+A^T) = \Big[ \underbrace{a_1 \cdot 2A_{11} + a_2(A_{21}+A_{12}) + \ldots + a_n(A_{n1}+A_{1n})}_{= \sum_{s=1}^{n} a_s (A_{s1}+A_{1s})}, \ldots, \underbrace{a_1(A_{1n}+A_{n1}) + a_2(A_{2n}+A_{n2}) + \ldots + a_n(2A_{nn})}_{\sum_{s=1}^{n} a_s (A_{sn}+A_{ns})} \Big]^T$$

$$= \underline{\frac{\partial}{\partial \vec{a}}(\vec{a}^T A \vec{a})}$$



Lars Johan Brodtkorb

Show $\frac{\partial \left( (\vec{x} - A\vec{s})^T (\vec{x} - A\vec{s}) \right)}{\partial s} = -2(\vec{x} - A\vec{s})^T A$

$$f(s) = (\vec{x} - A\vec{s})^T(\vec{x} - A\vec{s}) = \left(\vec{x}^T - (A\vec{s})^T\right)(\vec{x} - A\vec{s})$$

$$= \vec{x}^T\vec{x} - (A\vec{s})^T\vec{x} - \vec{x}^T A\vec{s} + (A\vec{s})^T \cdot A\vec{s}$$

$$= \vec{x}^T\vec{x} - \vec{s}^T A^T \vec{x} - \vec{s}^T A^T \vec{x} + \vec{s}^T A^T A \vec{s}$$

$$= \vec{x}^T\vec{x} - 2\vec{x}^T A\vec{s} + \vec{s}^T \underbrace{A^T A}_{A^*} \vec{s}$$

$$\frac{\partial (f(s))}{\partial \vec{s}} = -2\vec{x}^T A + \vec{s}^T (A^* + A^{*T})$$

$$= -2\vec{x}^T A + \vec{s}^T (A^T A + (A^T A)^T)$$

$$= -2\vec{x}^T A + \vec{s}^T (2A^T A)$$

$$= -2\left((\vec{x}^T A) - (A\vec{s})^T A\right)$$

$$\underline{= -2(\vec{x} - A\vec{s})^T A}$$



Lars Johan Brodtkorb

$$\frac{\partial}{\partial \vec{s}}\left(-2(\vec{x}-A\vec{s})^T A\right) = \frac{\partial}{\partial \vec{s}}\left(\underbrace{-2\vec{x}^T A}_{\text{Const.}} + 2(A\vec{s})^T A\right)$$

$$= 0 + \frac{\partial}{\partial \vec{s}}\left(2\vec{s}^T A^T A\right)$$

$$= \underline{\underline{2A^TA}}$$

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