Approximating waveforms via additive synthesis

Additive synthesis is the process of approximating waveforms by adding sine waves together. In this article, I provide some examples of how specific waveforms can be composed using this method.

Sawtooth

Equation

A sawtooth waveform can be represented by the following equation:

$$x_{\text{sawtooth}}(t) = -\frac{A}{\pi}\sum_{k=1}^{\infty}\frac{\sin(2\pi k f t)}{k}$$

Where A is the amplitude, f is the frequency of the desired waveform, k is the *order*, or number of harmonics to use for the approximation, and t is time.

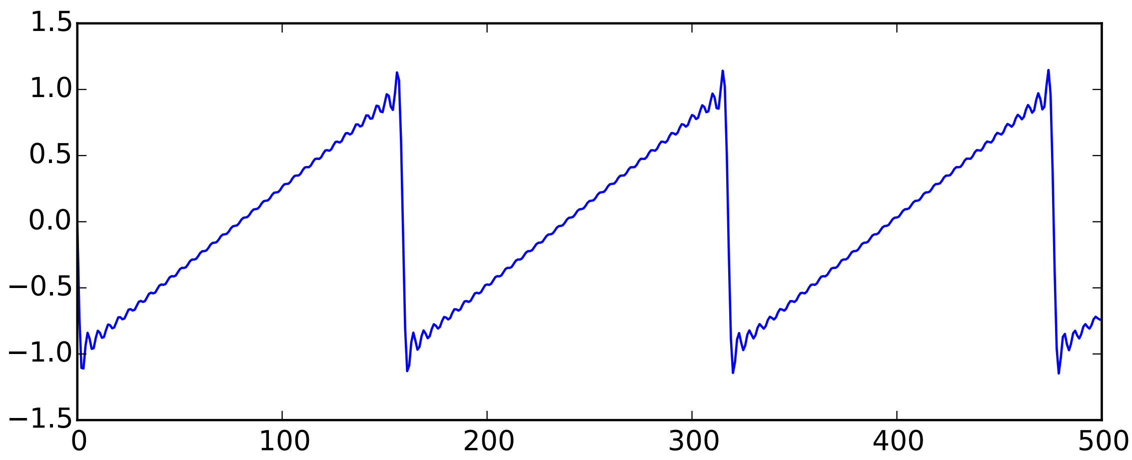
Code

Using NumPy we can perform a relatively straightforward translation of the above equation into the following Python code:

```
>>> from numpy import pi, sin, linspace
>>> order = 30
>>> t = linspace(0, pi, 500)
>>> waveform = -(2/pi) * sum([
...     sin(2 * pi * k * t)/k
...     for k in range(1, (order+2))
... ])
>>>
```

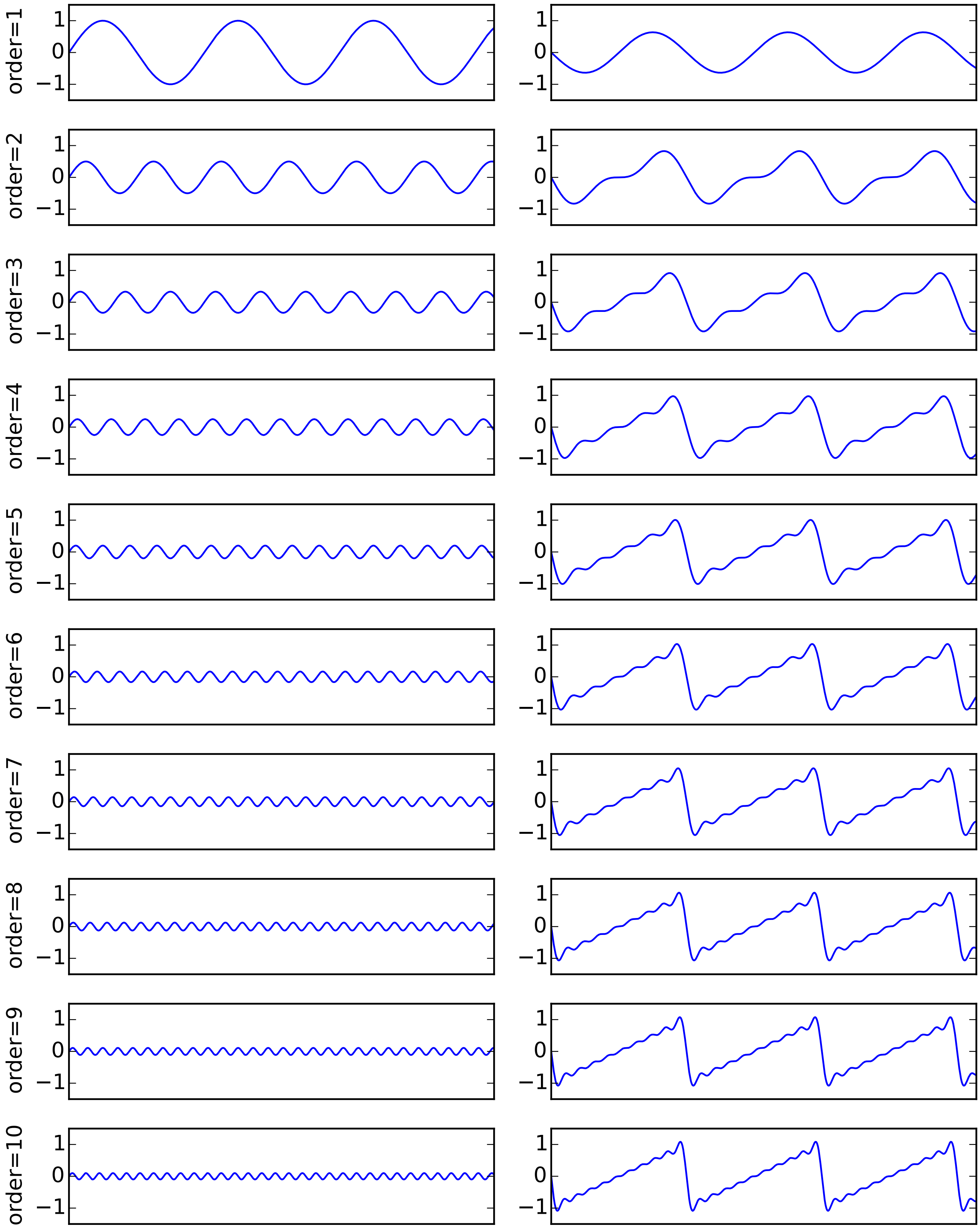
Result

The above code results in:



Steps

The following figure shows the result of each successive step as k iterates from 1 to 10.


order=1
order=2
order=3
order=4
order=5
order=6
order=7
order=8
order=9
order=10
1
0
−1

Square

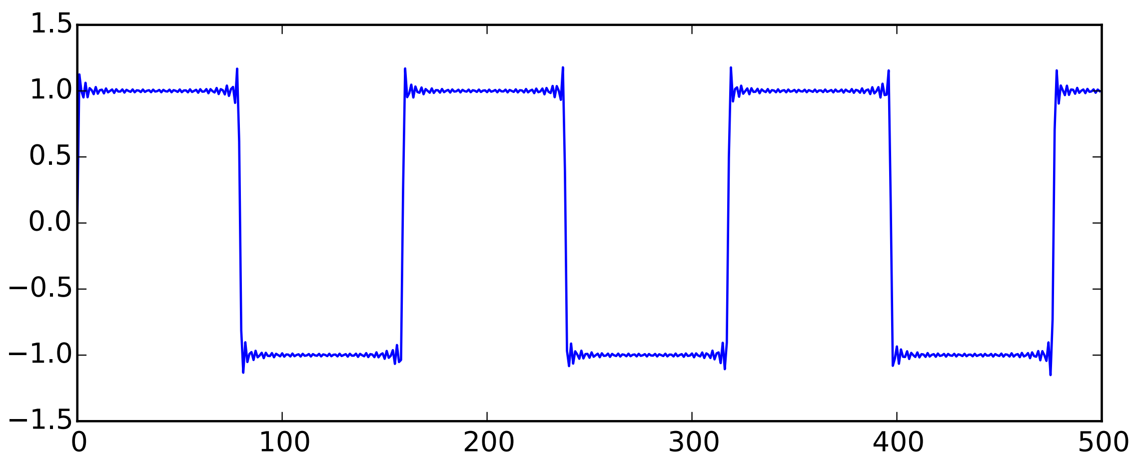
Equation

$$x_{\text{square}}(t) = \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{\sin\left(2\pi(2k-1)ft\right)}{(2k-1)}$$

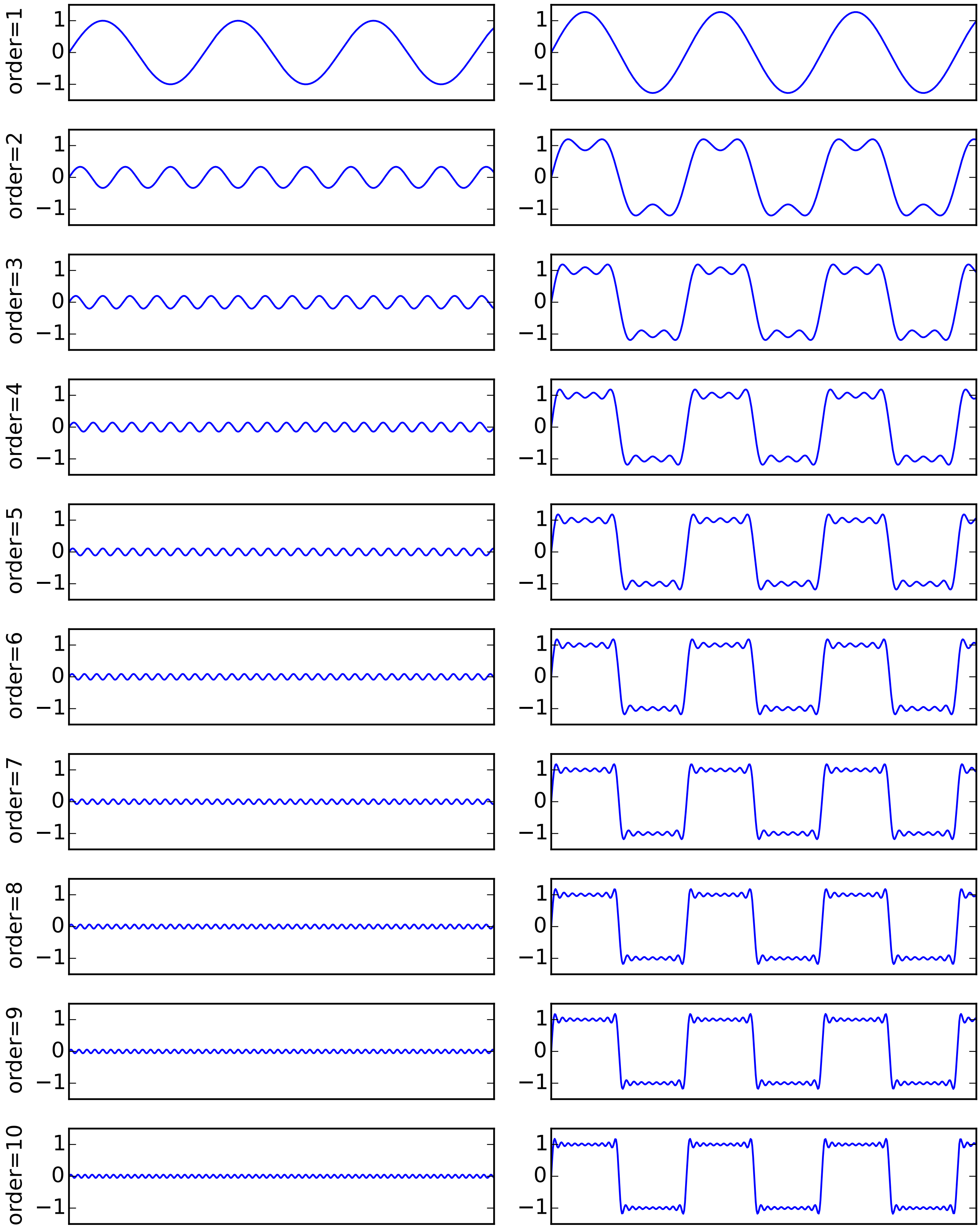
Code

```
>>> from numpy import pi, sin, linspace
>>> order = 30
>>> t = linspace(0, pi, 500)
>>> waveform = (4/pi) * sum([
...   sin(2 * pi * (2 * k - 1) * t)/(2 * k - 1)
...   for k in range(1, (order+2))
... ])
>>>
```

Result



Steps


order=1
order=2
order=3
order=4
order=5
order=6
order=7
order=8
order=9
order=10
1
0
−1

Triangle

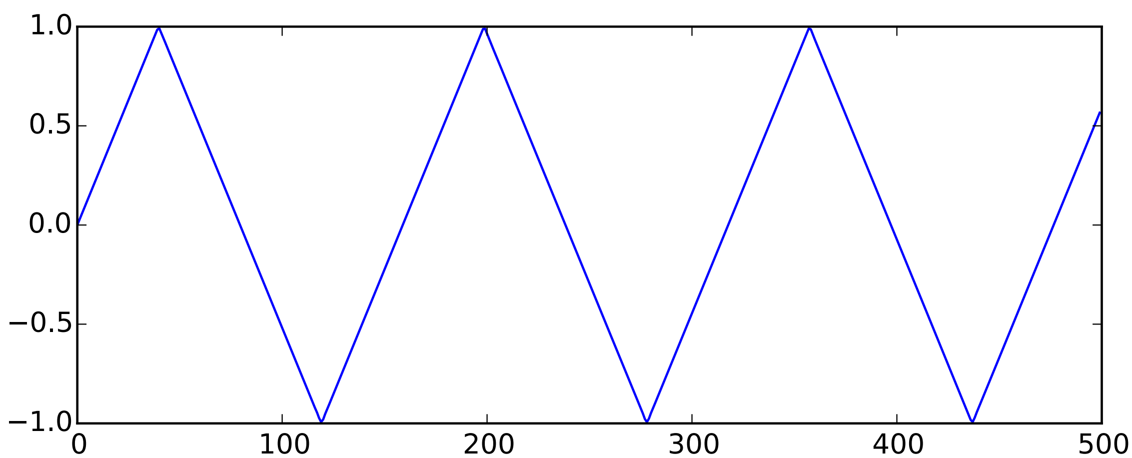
Equation

$$x_{\mathrm{triangle}}(t) = \frac{8}{\pi^2} \sum_{k=0}^{\infty} (-1)^k \frac{\sin(2\pi(2k+1)ft)}{(2k+1)^2}$$

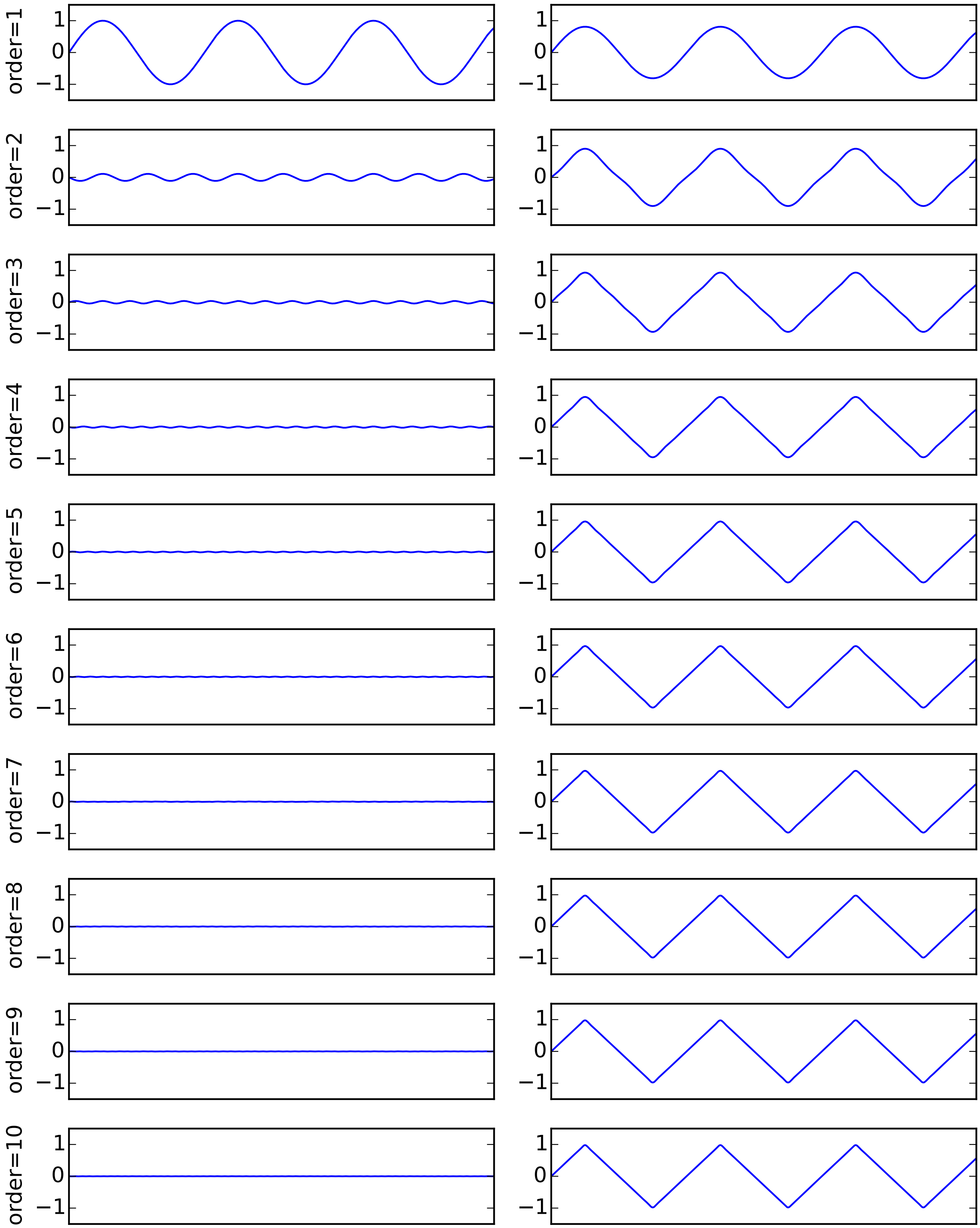
Code

```
>>> from numpy import pi, sin, linspace
>>> order = 30
>>> t = linspace(0, pi, 500)
>>> waveform = (8/pi**2) * sum([
...   (-1)**k *
...   sin(2 * pi * (2 * k + 1) * t)/(2 * k + 1)**2
...   for k in range(0, (order+1))
... ])
>>>
```

Result



Steps



order=1
order=2
order=3
order=4
order=5
order=6
order=7
order=8
order=9
order=10
1
0
−1