# TDT4137 – Exercise 4

Fuzzy Reasoning

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## Task a)

### Step 1 - Fuzzification

Crisp input x1:

*μ(Distance = Small) = 0.6*
*μ(Distance = Perfect) = 0.1*

Crisp input y1:

*μ(Delta = Stable) = 0.3*
*μ(Delta = Growing) = 0.4*

### Step 2 – Rule evaluation

**Rule 1:** IF Distance is Small (0.6)
AND Delta is Growing (0.4)
THEN Action is None (0.4)

**Rule 2:** IF Distance is Small (0.6)
AND Delta is Stable (0.3)
THEN Action is SlowDown (0.3)

**Rule 3:** IF Distance is Perfect (0.1)
AND Delta is Growing (0.4)
THEN Action is SpeedUp (0.1)

**Rule 4:** IF Distance is VeryBig (0.0)
AND (Delta is NOT Growing (1 – 0.4 = 0.6) OR
(Delta is NOT GrowingFast (0.0))
THEN Action is FloorIt (0.0)

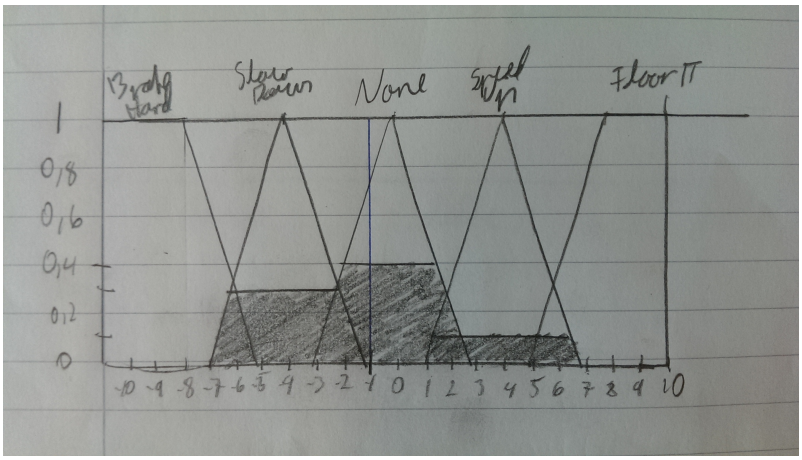
**Rule 5:** IF Distance is VerySmall (0.0)
THEN Action is BrakeHard (0.0)

### Step 3 – Aggregation of the rule outputs

Action is None (0.4)
Action is SlowDown (0.3)
Action is SpeedUp (0.1)
Action is FloorIt (0.0)
Action is BrakeHard (0.0)

### Step 4 - Defuzzification

Based on the fuzzy set created in step 3, I drew the following graph:




Using the values from this graph with the mathematical expression for finding Centre of Gravity, I got this equation:

*COG =*
*(-10-9-8-7-6-5) * 0.0 + (-7-6-5-4-3-2-1) * 0.3 + (-3-2-1+0+1+2+3) * 0.4 + (1+2+3+4+5+6+7) * 0.1 + (5+6+7+8+9+10) * 0.0 /*
*0.0 * 6 + 0.3 * 7 + 0.4 * 7 + 0.1 * 7 + 0.0 * 6 =*

*-5.6/5.6 = -1*

The centre of gravity is -1, which seems to be quite fitting.

# Task b)

Running the same values as in a), I get very similar results in my Mamdani Reasoner.
Here are some sample outputs, where the first one is with similar values as in a).

```
COG: 1.09

CRISP_X: 3.9
CRISP_Y: 1.4

DISTANCE: {'verysmall': 0.0, 'small': 0.4000000000000001, 'verybig': 0.0, 'perfect': 0.2666666666666666, 'big': 0.0}

DELTA: {'shrinkingfast': 0.0, 'growing': 0.6, 'shrinking': 0.0, 'growingfast': 0.0, 'stable': 0.06666666666666672}

ACTIONS: {'none': 0.4000000000000001, 'slowdown': 0.06666666666666672, 'floorit': 0.0, 'speedup': 0.2666666666666666, 'brakehard': 0.0}
```

```
COG: 4.00

CRISP_X: 4.8
CRISP_Y: 2.6

DISTANCE: {'verysmall': 0.0, 'small': 0.0, 'big': 0.0, 'perfect': 0.8666666666666666, 'verybig': 0.0}

DELTA: {'stable': 0.0, 'shrinkingfast': 0.0, 'growingfast': 0.06666666666666672, 'growing': 0.6, 'shrinking': 0.0}

ACTIONS: {'speedup': 0.6, 'floorit': 0.0, 'slowdown': 0.0, 'brakehard': 0.0, 'none': 0.0}
```

```
COG: -2.80

CRISP_X: 2.6
CRISP_Y: 0.8

DISTANCE: {'verybig': 0.0, 'verysmall': 0.0, 'perfect': 0.0, 'big': 0.0, 'small': 0.7333333333333334}

DELTA: {'stable': 0.4666666666666666, 'growingfast': 0.0, 'shrinking': 0.0, 'shrinkingfast': 0.0, 'growing': 0.20000000000000004}

ACTIONS: {'slowdown': 0.4666666666666666, 'none': 0.20000000000000004, 'floorit': 0.0, 'speedup': 0.0, 'brakehard': 0.0}
```

I suspect the slight difference is due reading the numbers of the x- and y axis of the graphs in the task in an uneven matter. Anyway, you can see that the centre of gravity appears in the right position of the actions, according the their 'mass'.

Following is the code for my Mamdani Reasoner, implemented in Python: (Step 3 is disregarded, as step 2 (rule-evaluation) does both steps at the same time in the program)

```
class Reasoner:
    def __init__(self, crisp_x, crisp_y):
        self.crisp_x = crisp_x
        self.crisp_y = crisp_y
        # Dictionary to keep track of the intersection value of the distance graphs and the crisp x
        self.distance = {'verysmall': 0, 'small': 0, 'perfect': 0, 'big': 0, 'verybig': 0}
        # Dictionary to keep track of the intersection value of the delta graphs and crisp y
        self.delta = {'shrinkingfast': 0, 'shrinking': 0, 'stable': 0, 'growing': 0, 'growingfast': 0}
        # Dictionary to keep track of the outputs of each action
        self.actions = {'none': 0, 'slowdown': 0, 'speedup': 0, 'floorit': 0, 'brakehard': 0}

    def __repr__(self):
        return '\n' + "CRISP_X: " + str(self.crisp_x) + \
               '\n' + "CRISP_Y: " + str(self.crisp_y) + \
               '\n'*2 + "DISTANCE: " + str(self.distance) + \
               '\n'*2 + "DELTA: " + str(self.delta) + '\n'*2 \
               + "ACTIONS: " + str(self.actions) + '\n'

    def fuzzification(self):
        self.distance['verysmall'] = self.reverse_grade(self.crisp_x, 0.0, 2.5, 1.0)
        self.distance['small'] = self.triangle(self.crisp_x, 1.5, 3.0, 4.5, 1.0)
        self.distance['perfect'] = self.triangle(self.crisp_x, 3.5, 5.0, 6.5, 1.0)
        self.distance['big'] = self.triangle(self.crisp_x, 5.5, 7.0, 8.5, 1.0)
        self.distance['verybig'] = self.grade(self.crisp_x, 7.5, 10.0, 1.0)
        self.delta['shrinkingfast'] = self.reverse_grade(self.crisp_y, -4.0, -2.5, 1.0)
        self.delta['shrinking'] = self.triangle(self.crisp_y, -3.5, -2, -0.5, 1.0)
        self.delta['stable'] = self.triangle(self.crisp_y, -1.5, 0, 1.5, 1.0)
        self.delta['growing'] = self.triangle(self.crisp_y, 0.5, 2, 3.5, 1.0)
        self.delta['growingfast'] = self.grade(self.crisp_y, 2.5, 4, 1.0)

    def rule_evaluation(self):
        self.actions['none'] = min(self.distance['small'], self.delta['growing'])        # Rule 1
        self.actions['slowdown'] = min(self.distance['small'], self.delta['stable'])     # Rule 2
        self.actions['speedup'] = min(self.distance['perfect'], self.delta['growing'])   # Rule 3
        self.actions['floorit'] = min(self.distance['verybig'],                          # Rule 4
                                      max(1 - self.delta['growing'], 1- self.delta['growingfast']))
        self.actions['brakehard'] = self.distance['verysmall']                           # Rule 5

    def defuzzification(self):
        above = (((-10 - 9 - 8 - 7 - 6 - 5) * self.actions['brakehard'])
                 + ((-7 - 6 - 5 - 4 - 3 - 2 - 1) * self.actions['slowdown'])
                 + ((-3 - 2 - 1 + 0 + 1 + 2 + 3) * self.actions['none'])
                 + ((1 + 2 + 3 + 4 + 5 + 6 + 7) * self.actions['speedup'])
                 + ((5 + 6 + 7 + 8 + 9 + 10) * self.actions['floorit']))
        below = (self.actions['brakehard'] * 6
                 + self.actions['slowdown'] * 7
                 + self.actions['none'] * 7
                 + self.actions['speedup'] * 7
                 + self.actions['floorit'] * 6)
        print("COG: " + '%.2f' % (above / below))

    def triangle(self, pos, x0, x1, x2, clip):
        value = 0.0
        if pos >= x0 and pos <= x1:
            value = (pos - x0) / (x1 - x0)
        elif pos  >= x1 and pos <= x2:
            value = (x2 - pos) / (x1 - x0)
        if value > clip:
            value = clip
        return value

    def grade(self, position, x0, x1, clip):
        value = 0.0
        if position >= x1:
            value = 1.0
        elif position <= x0:
            value = 0.0
        else:
            value = (position-x0)/(x1-x0)
        if value > clip:
            value = clip
        return value

    def reverse_grade(self, position, x0, x1, clip):
        value = 0.0
        if position <= x0:
            value = 1.0
        elif position >= x1:
            value = 0.0
        else:
            value = (x1-position)/(x1-x0)
        if value > clip:
            value = clip
        return value

if __name__ == '__main__':
    crisp_x_value = 2.6
    crisp_y_value = 0.8
    reasoner = Reasoner(crisp_x_value, crisp_y_value)
    # Step 1
    reasoner.fuzzification()
    # Step 2
    reasoner.rule_evaluation()
    # Step 4
    reasoner.defuzzification()
    # Printing rest of values in the reasoner
    print(reasoner)
```