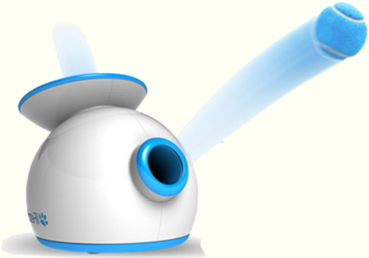
E-10

Simulation of a tennis ball throwing machine

A competition announced by the E-10 Simulation GmbH

by Lars Schiller



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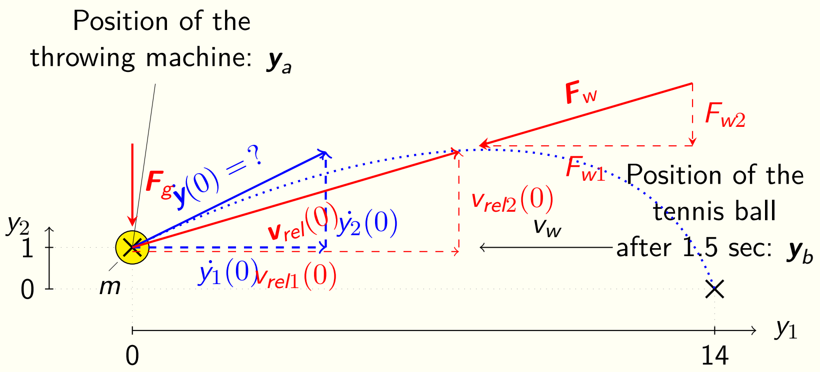
Task description



- We are looking for $\boldsymbol{\eta} := [\dot{y}_1(0) \quad \dot{y}_2(0)]^T$, the dropping velocity of the ball.

Mathematical model

By defining $y_3 = \dot{y}_1,\ y_4 = \dot{y}_2$ and the state vector $\boldsymbol{y} = [y_1 \quad y_2 \quad y_3 \quad y_4]^T$ we can set up a system of 1st order (non linear) differential equations:

$$\dot{\boldsymbol{y}} := f(\boldsymbol{y}) = \begin{bmatrix} y_3 \\ y_4 \\ -\frac{k}{m}|v_{rel}|\,(y_3 - v_w) \\ -\frac{k}{m}|v_{rel}|y_4 - g \end{bmatrix}, \tag{1}$$

with $|v_{rel}| = \sqrt{(\dot{y}_1 - v_w)^2 + \dot{y}_2^2}$.
Set up the Residual function:

$$F(\boldsymbol{\eta}) := \boldsymbol{y}_\eta(t_f) - \boldsymbol{y}_b = 0, \tag{2}$$

where $\boldsymbol{y}_\eta(t_f)$ is the solution of the initial value problem given in eq. 1 with the initial condition $\boldsymbol{y}_0 = [\boldsymbol{y}_a \quad \boldsymbol{\eta}]^T$ evaluated at $t = 1.5$.

Choice of suitable solvers

- We need a method to find the zero of the residual function F given in eq. 2:
- The *Newton*-method
  - calculates the Jacobian of F in each iteration step,
  - needs $n+1$ evaluations of F per step.
- The *Broyden*-method
  - gets along with an approximation of the Jacobian,
  - needs only 1 evaluation of F per step.
- For more detailed information look up in the technical report.

Evaluation of the results

- *Newton* converges a bit faster,
- *Broyden* needs less function evaluations and computational time.
- Therefore its reasonable to prefer the *Broyden*-method.

method	iteration steps	total evals of f	comp. time [sec]
Newton	3	1010	0.0444
Broyden	4	509	0.0238