NTNU
Kunnskap for en bedre verden

TDT4136 – Introduction to Artificial Intelligence

Assignment no. 5

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October, 2023

1 Models and entailment in propositional logic

1.1 Validity and Soundness

a) *Generate the vocabulary of the following argument.*

b) *Translate the argument into propositional logic statements.*

c) *Add a premise (P4) to make the conclusion of the argument valid.*

P1 to P3 are the premises, C is the conclusion:

- *(P1) If Peter's argument is valid and all the premises of Peter's argument are true, then Peter's argument is sound.*
- *(P2) If the premises of Peter's argument entail the conclusion of Peter's argument, then Peter's argument is valid.*
- *(P3) The premises of Peter's argument entail the conclusion of Peter's argument.*
- *(C) Peter's argument is sound.*

a) Vocabulary:

- V: Peter's argument is valid
- P: The premises of Peter's argument
- S: Peter's argument is sound
- C: The conclusion of Peter's argument

b)

- (P1): $V \land P \implies S$
- (P2): $P \models C \implies V$
- (P3): $P \models C$
- (C): S

c)

To make the conclusion of the argument valid, one can add the premise:

(P4): The Premises of Peter's argument are true.

1.2 Modelling

For each of the following statements, determine if they are satisfiable by building the complete model (truth table) and mark tautologies.

I have given steps in the truth tables alternative lettering to save space in the tables.

a) $(p \iff q) \implies ((p \implies r) \implies (q \implies r))$

p	q	r	$(p \iff q)$ a	$(p \implies r)$ b	$(q \implies r)$ c	$(b \implies c)$	$a \implies (b \implies c)$
1	1	1	1	1	1	1	1
1	1	0	1	0	0	1	1
1	0	0	0	0	1	1	1
1	0	1	0	1	1	1	1
0	1	0	0	1	0	0	1
0	1	1	0	1	1	1	1
0	0	1	1	1	1	1	1
0	0	0	1	1	1	1	1

As we can see this is a tautology as the expression is always true and thus it is also satisfiable.

b) $(p \lor (\neg q \implies r)) \implies (q \lor (\neg p \implies r))$

p	q	r	$(\neg q \implies r)$ a	$(\neg p \implies r)$ b	$(p \lor a)$	$(q \lor b)$	$(p \lor a) \implies (q \lor b)$
1	1	1	1	1	1	1	1
1	1	0	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	1	1	1	1	1
0	1	0	1	0	1	0	0
0	1	1	1	1	1	1	1
0	0	1	1	1	1	1	1
0	0	0	0	0	0	0	1

The statement is satisfiable.

c) $(\neg(p \land (q \implies \neg r))) \implies ((p \implies q) \land (p \implies r))$

p	q	r	$(q \implies \neg r)$ a	$(p \implies q)$ b	$(p \implies r)$ c	$(p \land a)$	$(b \land c)$	$(\neg(p \land a)) \implies (b \land c)$
1	1	1	0	1	1	0	1	1
1	1	0	1	1	0	1	0	1
1	0	0	1	0	0	1	1	1
1	0	1	0	0	1	0	0	0
0	1	0	1	1	1	0	1	1
0	1	1	1	1	1	0	1	1
0	0	1	1	1	1	0	1	1
0	0	0	0	1	1	1	1	1

The statement is satisfiable.

d) $(\neg(\neg p \implies (q \land r))) \implies (\neg(p \lor q) \land r)$

p	q	r	$(q \land r)$ a	$(p \lor q)$ b	$(\neg p \implies a)$ c	$(\neg b \land r)$ d	$(\neg c \implies d)$
1	1	1	1	1	1	1	1
1	1	0	0	1	1	1	1
1	0	0	1	1	1	1	1
1	0	1	0	1	1	1	1
0	1	0	0	1	0	1	1
0	1	1	1	1	1	1	1
0	0	1	0	0	0	1	1
0	0	0	1	0	1	0	1

The statement is a tautology and is thus also satisfiable.

1.3 Modelling 2

Let ϕ be a sentence that contains three atomic constituents and let the truth conditions of ϕ be defined by the truth table below. Write a propositional logic statement that contains p, q, and r as constituents, and that is equivalent to ϕ.

p	q	r	ϕ
1	1	1	1
1	1	0	1
1	0	1	0
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	0
0	0	0	1

A statement that contains p, q, and r and is equivalent to ϕ is

$$((p \implies r) \implies (p \implies q)) \land ((r \implies p) \lor q)$$

2 Resolution in propositional logic

2.1 Conjuctive Normal Form

Convert each of the following sentences to their Conjuctive Normal Form (CNF).

a) $p \iff q$

$$\begin{aligned} &\Rightarrow (p \implies q) \land (q \implies p) \\ &\Rightarrow (\neg p \lor q) \land (\neg q \lor p) \end{aligned}$$

b) $\neg((p \implies q) \land r)$

$$\begin{aligned} &\Rightarrow \neg((\neg p \lor q) \land r) \\ &\Rightarrow (p \land \neg q) \lor (\neg r) \\ &\Rightarrow (p \lor \neg r) \land (\neg q \lor \neg r) \end{aligned}$$

c) $((p \lor q) \lor (r \land (\neg(q \implies r))))$

$$\begin{aligned} &\Rightarrow (p \lor q \lor (r \land (\neg(\neg q \lor r)))) \\ &\Rightarrow (p \lor q \lor (r \land (q \land \neg r))) \\ &\Rightarrow (p \lor (r \land q \land \neg r)) \land (q \lor (r \land q \land \neg r)) \\ &\Rightarrow (p \lor r) \land (p \lor q) \land (p \lor \neg r) \land (q \lor r) \land (\lor p) \land (q \lor \neg r) \\ &\Rightarrow (p \lor q \lor r) \land (p \lor q \lor \neg r) \\ &\Rightarrow (p \lor q) \end{aligned}$$

d) *Is the solution to c) really a CNF?*

As the solution to c) can be simplified to $(p \vee q)$ the solution is not a true CNF, but rather a disjunction of literals. It can still be written as a CNF with r and $\neg r$, but it is in practice $(p \vee q)$.

2.2 Inference in propositional logic

Use resolution to conclude r from the following statements.

a) $(p \implies q) \implies q$

$$\begin{aligned}
&(\neg p \vee q) \implies q \\
&\neg(\neg p \vee q) \vee q \\
&p \wedge \neg q \vee q \\
&p
\end{aligned}$$

b) $p \implies r$

$$\neg p \vee r$$

c) $(r \implies s) \implies (\neg(s \implies q))$

$$\begin{aligned}
&(\neg r \vee s) \implies (\neg(\neg s \vee q)) \\
&\neg(\neg r \wedge s) \vee (s \wedge \neg q) \\
&r \wedge (s \wedge \neg q)
\end{aligned}$$

We can see that from a) and b) we get the clauses p and $(\neg p \vee r)$ from these two we can infer r. Thus r can be concluded from the statements.

$$\frac{p, \quad \neg p \vee r}{r}$$

3 Representation in First-Order Logic (FOL)

Consider the following baseball vocabulary:

1. *Pitcher(p)* is a predicate where person p is a pitcher.
2. *flies*(p_1, p_2) is a predicative where person p_1 flies out to person p_2.
3. *Centerfielder(p)* is a predicate where person p is a centerfielder.
4. *scores(p)* is a predicate where person p scores.
5. *friend*(p_1, p_2) is a predicate where person p_1 is the friend of person p_2 (but not vice versa).
6. *Robinson, Crabb, Samson, Jones* are constants denoting persons.

Now look at the following translations of natural language into first order logic statements describing a baseball game. ***Using the provided vocabulary, translate the conclusion of each of the following arguments into an FOL statement.***

a) **Argument A**

Only pitchers fly out to Robinson. Crabb scores only if Samson flies out to Robinson and Robinson is a centerfielder. Crabb scores.

Conclusion: *Samson is a pitcher.*

- $\forall x : [flies(x,\ Robinson) \implies Pitcher(x)]$
- $scores(Crabb) \implies (flies(Samson,\ Robinson) \wedge Centerfielder(Robinson))$
- $scores(Crabb)$

Conclusion as FOL statement: $Pitcher(Samson)$

b) **Argument B**

No centerfielder who does not score has any friends. Robinson and Jones are both centerfielders. Any centerfielder who flies out to Jones does not score. Robinson flies out to Jones.

Conclusion: *Jones is not a friend of Robinson*

- $\forall x : [(Centerfielder(x) \wedge \neg scores(x)) \implies \neg \exists y : [friend(y,\ x)]]$
- $Centerfielder(Robinson) \wedge Centerfielder(Jones)$
- $\forall x : [(Centerfielder(x) \wedge flies(x,\ Jones)) \implies \neg scores(x)]$
- $flies(Robinson,\ Jones)$

Conclusion as FOL statement: $\neg friend(Jones, Robinson)$

4 Resolution in FOL

*Using resolution, prove the conclusions from **Arguments A** and **B** from exercise 3*

Argument A

First, we need to write the clauses of the arguments on CNF:

- $\forall x : [flies(x, Robinson) \implies Pitcher(x)]$

$$\forall x : [\neg flies(x, Robinson) \vee Pitcher(x)]$$
$$\neg flies(x, Robinson) \vee Pitcher(x)$$

- $scores(Crabb) \implies (flies(Samson, Robinson) \wedge Centerfielder(Robinson))$

  $\neg scores(Crabb) \vee (flies(Samson, Robinson) \wedge Centerfielder(Robinson))$
  $(\neg scores(Crabb) \vee flies(Samson, Robinson)) \wedge (\neg scores(Crabb) \vee Centerfielder(Robinson))$

- $scores(Crabb)$
- $Pitcher(Samson)$

Thus we get the clauses:

1.1 $\neg flies(x, Robinson) \vee Pitcher(x)$

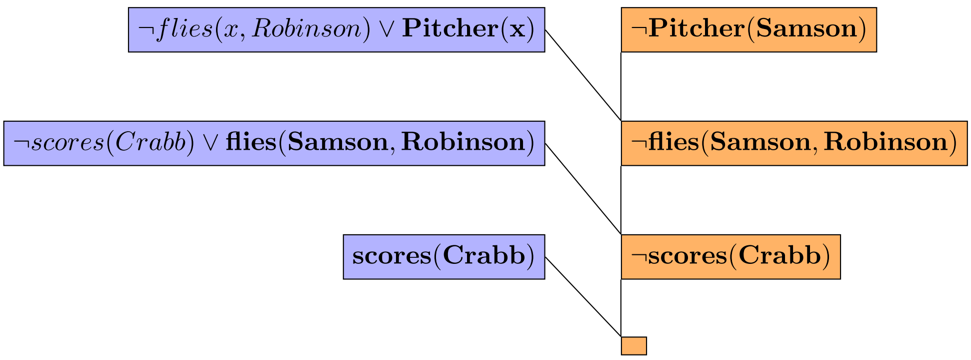
1.2 $\neg scores(Crabb) \vee flies(Samson, Robinson)$

2 $\neg scores(Crabb) \vee Centerfielder(Robinson)$

3 $scores(Crabb)$

¬G $\neg Pitcher(Samson)$

which by resolution yields



as $\neg Pitcher(Samson)$ is unsatisfiable by resolution the conclusion is proven.

Argument B

- $\forall x : [(Centerfielder(x) \land \neg scores(x)) \implies \neg \exists y : [friend(y,\, x)]]$

$$\forall x : [\neg(Centerfielder(x) \land \neg scores(x)) \lor \forall y : [\neg friend(y,\, x)]]$$
$$(\neg Centerfielder(x) \land scores(x)) \lor \neg friend(F(x),\, x)$$
$$(\neg Centerfielder(x) \lor \neg friend(F(x),\, x)) \land (scores(x) \lor \neg friend(F(x),\, x))$$

- $Centerfielder(Robinson) \land Centerfielder(Jones)$
- $\forall x : [(Centerfielder(x) \land flies(x,\, Jones)) \implies \neg scores(x)]$

$$\forall x : [\neg(Centerfielder(x) \land flies(x,\, Jones)) \lor \neg scores(x)]$$
$$(\neg Centerfielder(x) \land \neg flies(x,\, Jones)) \lor \neg scores(x)$$
$$(\neg Centerfielder(x) \lor \neg scores(x)) \land (\neg flies(x,\, Jones) \lor \neg scores(x))$$

- $flies(Robinson,\, Jones)$
- $\neg friend(Jones, Robinson)$

The clauses are then:

1.1 $\neg Centerfielder(x) \lor \neg friend(F(x),\, x)$

1.2 $scores(x) \lor \neg friend(F(x),\, x)$

2.1 $Centerfielder(Robinson)$

2.2 $Centerfielder(Jones)$

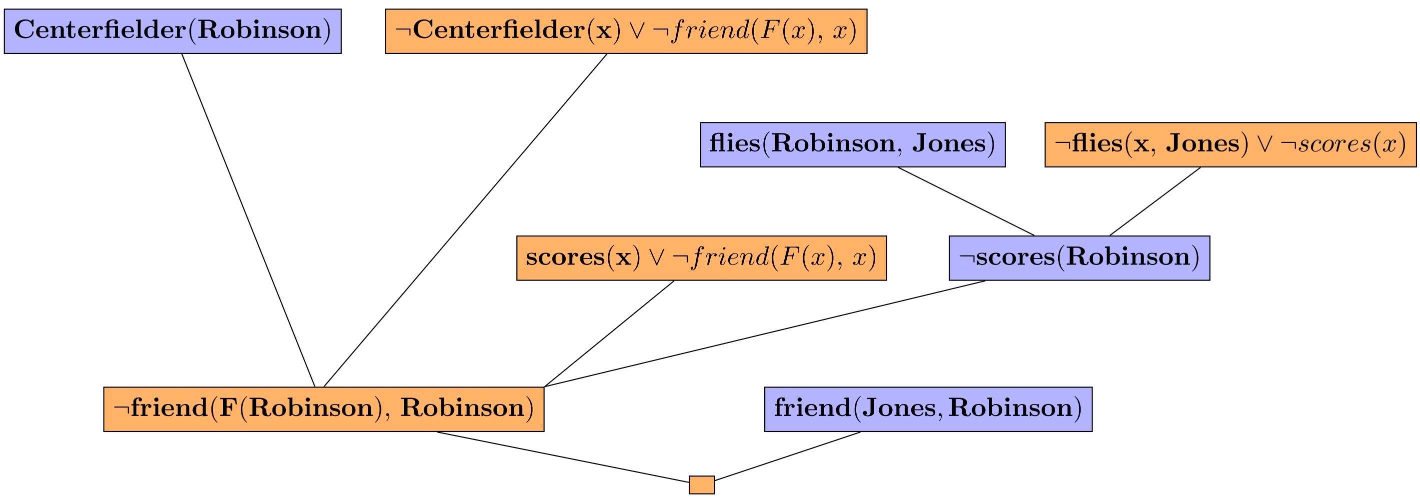
3.1 $\neg Centerfielder(x) \lor \neg scores(x)$

3.2 $\neg flies(x,\, Jones) \lor \neg scores(x)$

4 $flies(Robinson,\, Jones)$

¬G $friend(Jones, Robinson)$

which yields:



given $F(Robinson) = Jones$, $friend(Jones, Robinson)$ is unsatisfiable by resolution and the conclusion is proven.