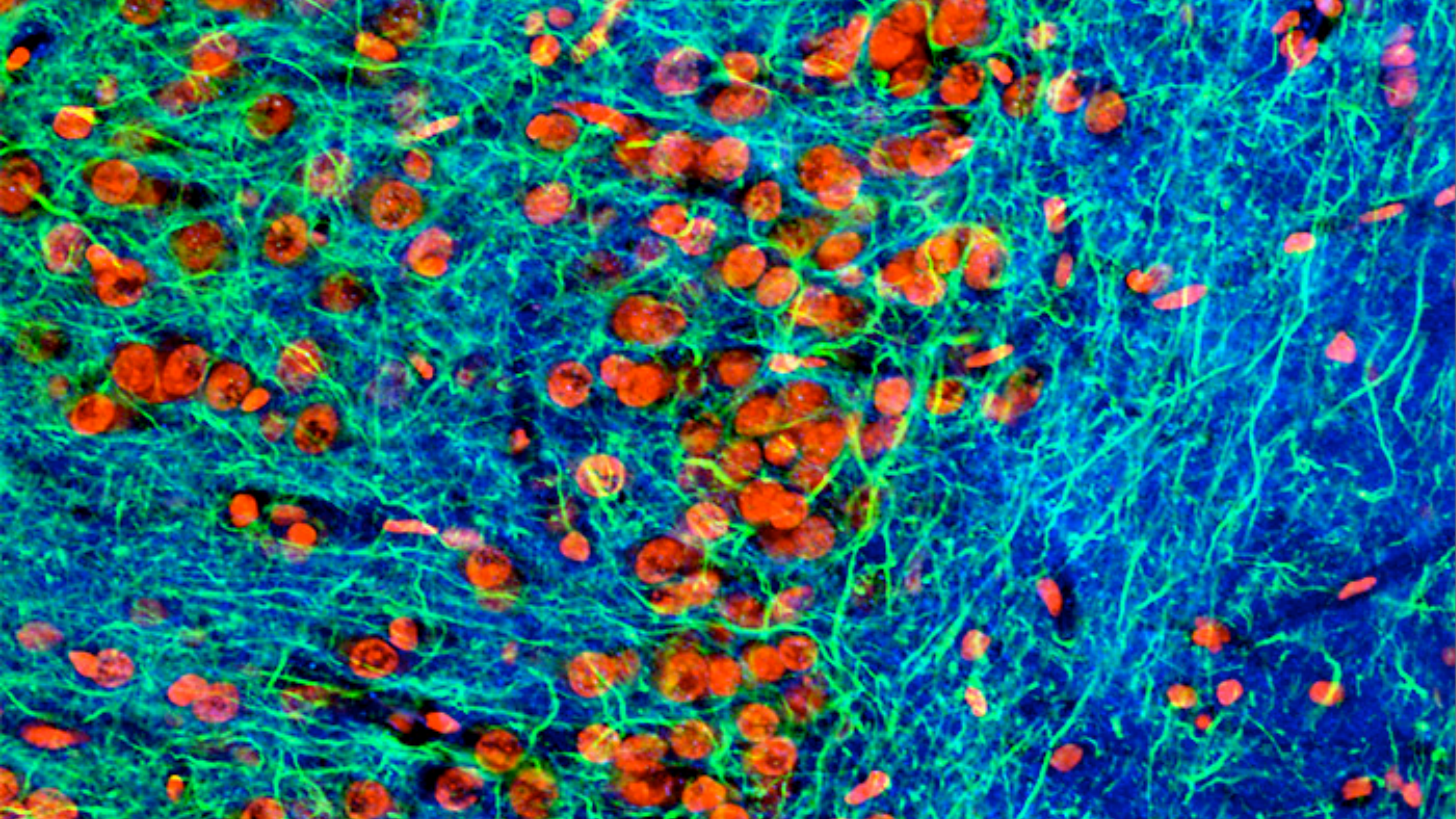
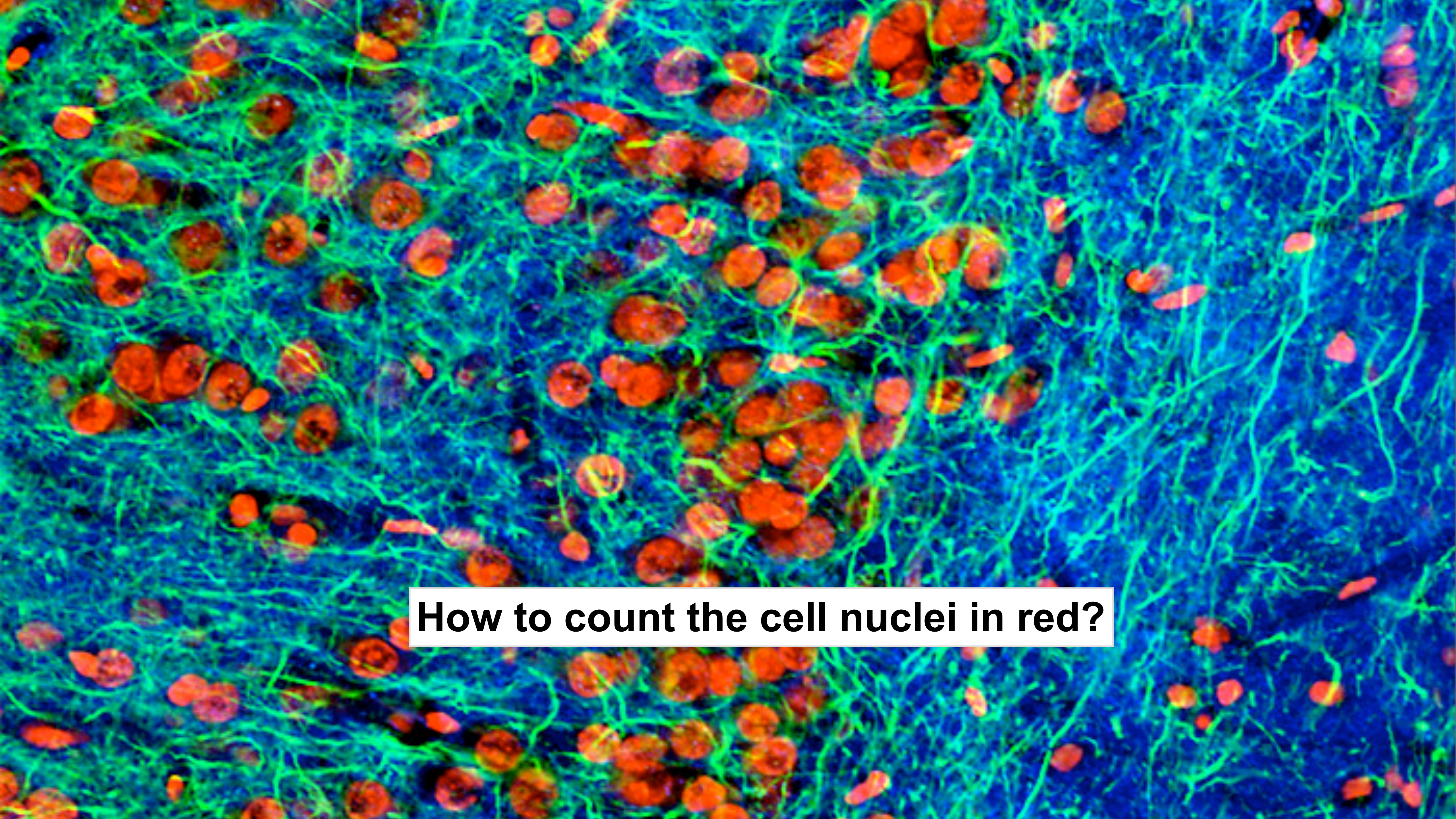
# Feature Extractors

FYS-2010-1 25V Image Analysis
by Elisabeth Wetzer

# Overview

- What is feature extraction?
- Invariance vs Equivariance
- Color Features
- Boundary Features
- Region Descriptors
- Corner Descriptors
- Hough Transforms
- Applications



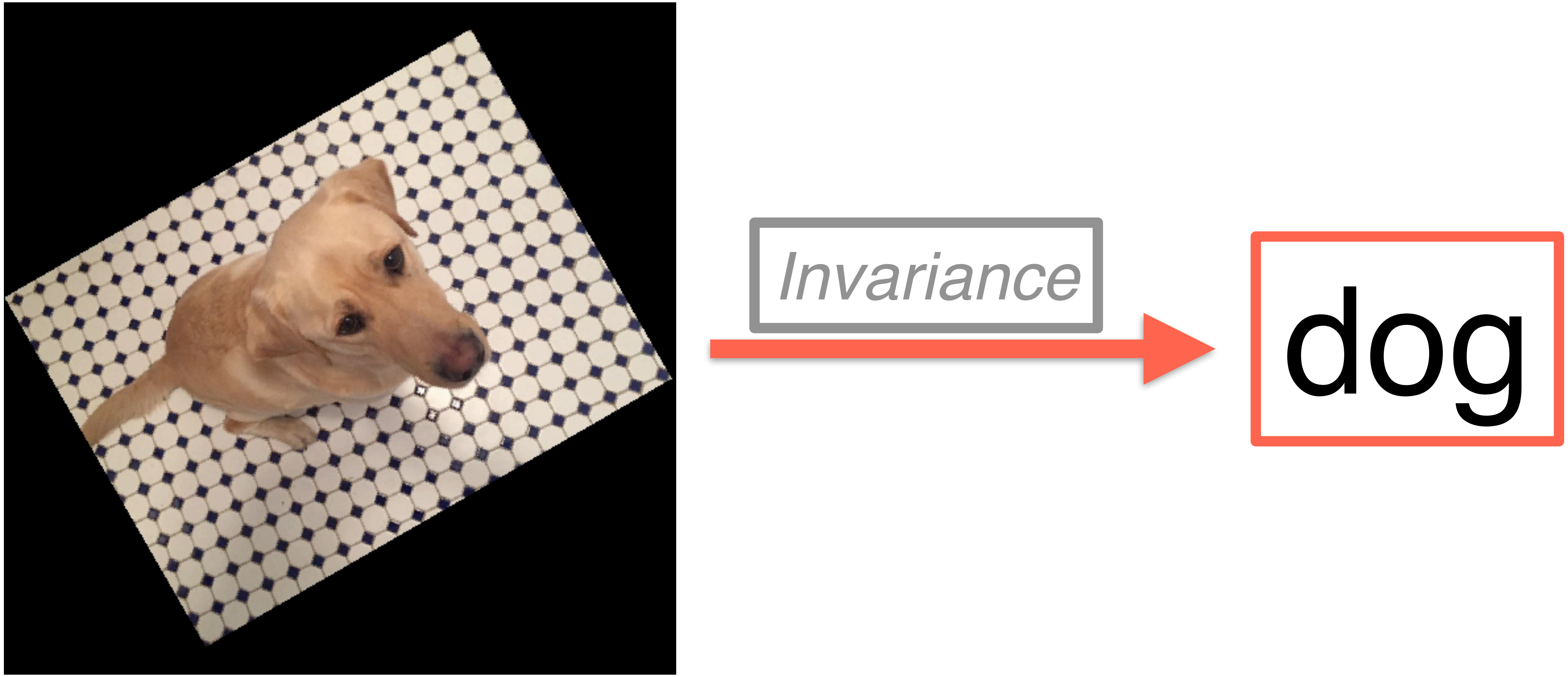

How to count the cell nuclei in red?

# Feature Extraction

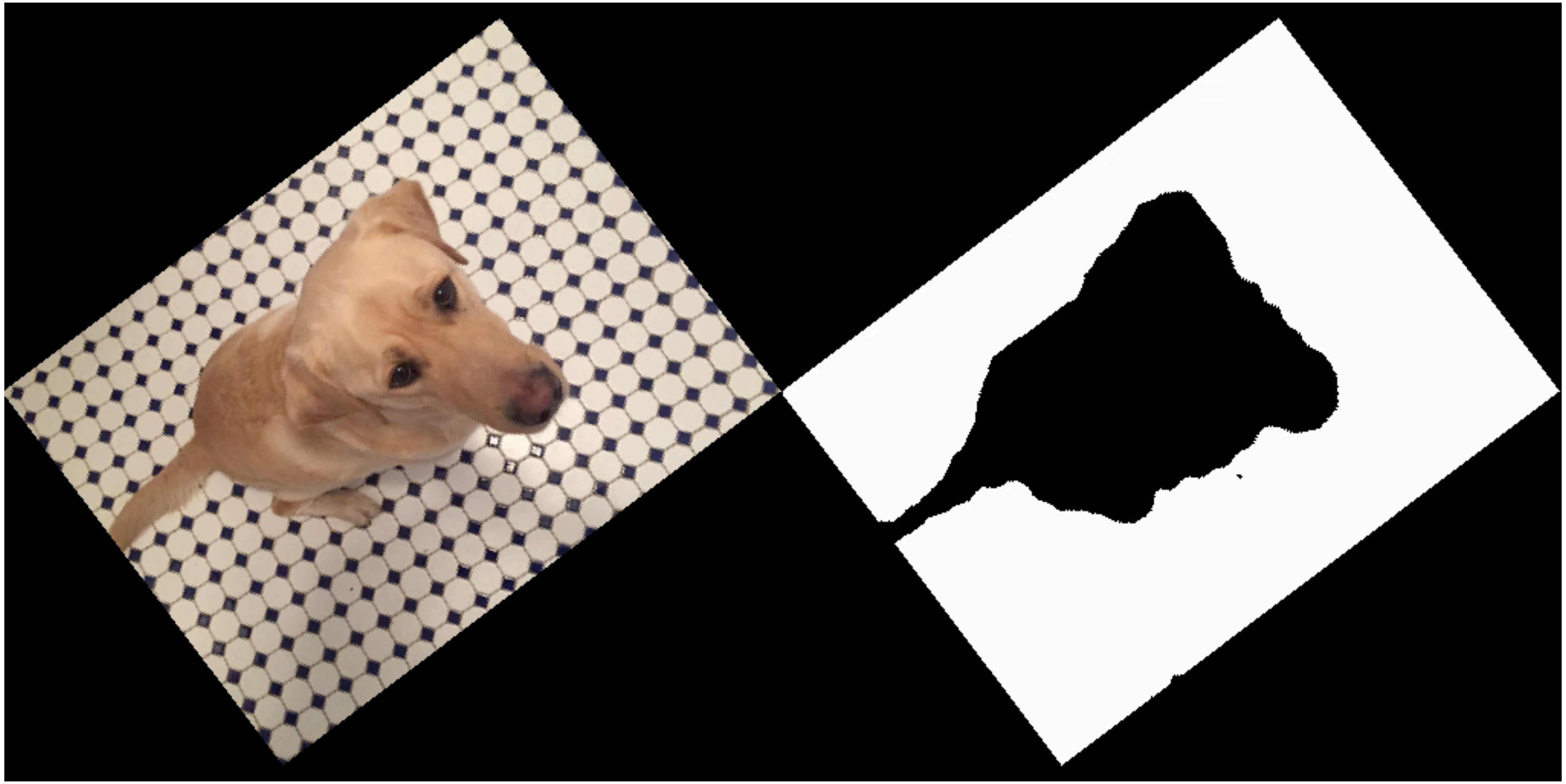
- **Feature Extraction**
  - **Feature Detection**
    - Finding certain features in an image (e.g. corners)
  - **Feature Description**
    - Assigns quantitative attributes to the detected features (e.g. orientation and location)

- **Invariances**
  - Usually favorable to be invariant to scale, illumination, translation, rotation…
- **Purpose**
  - Image Registration, Object Detection, Finding Patterns, Classification,…

# Invariances vs Equivariances

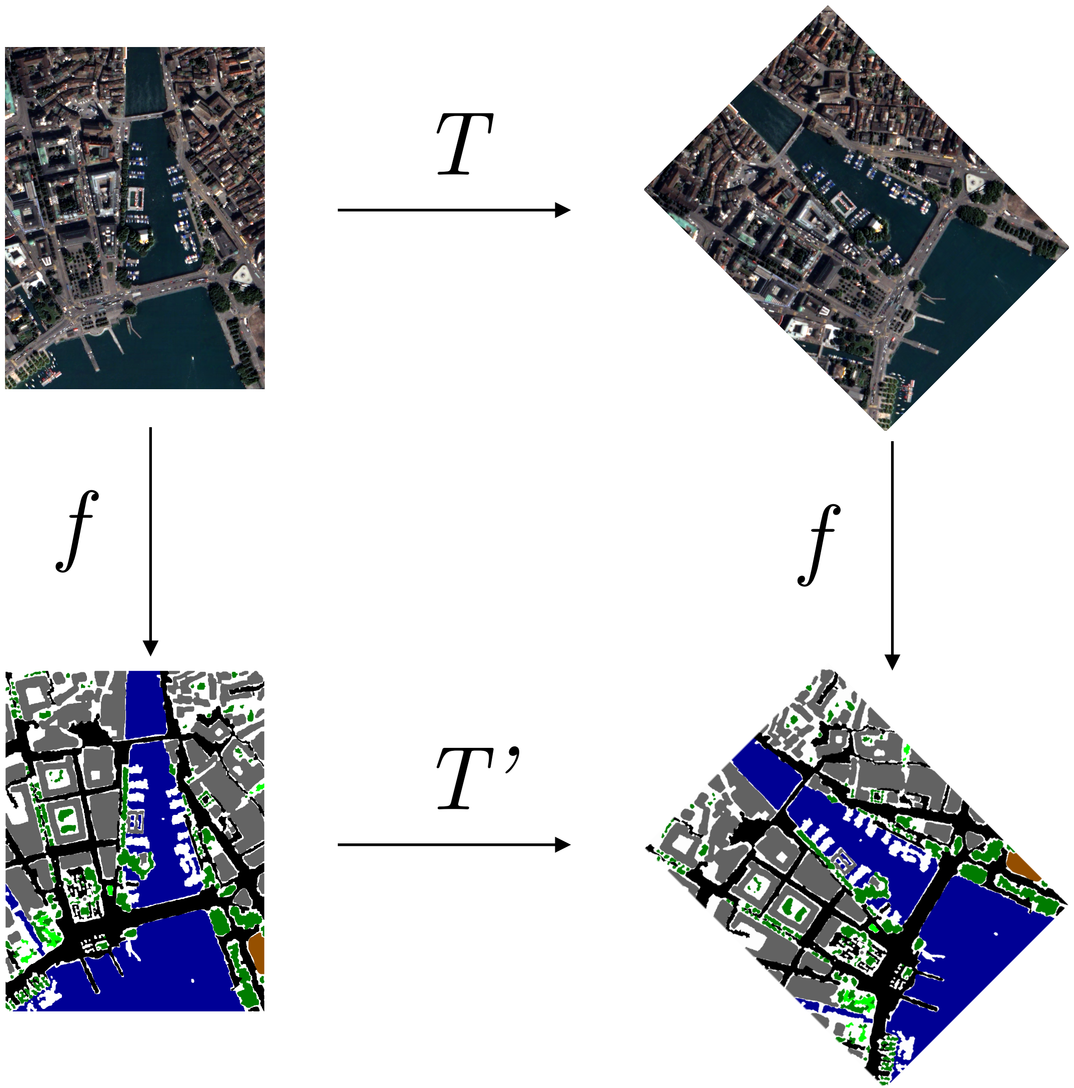


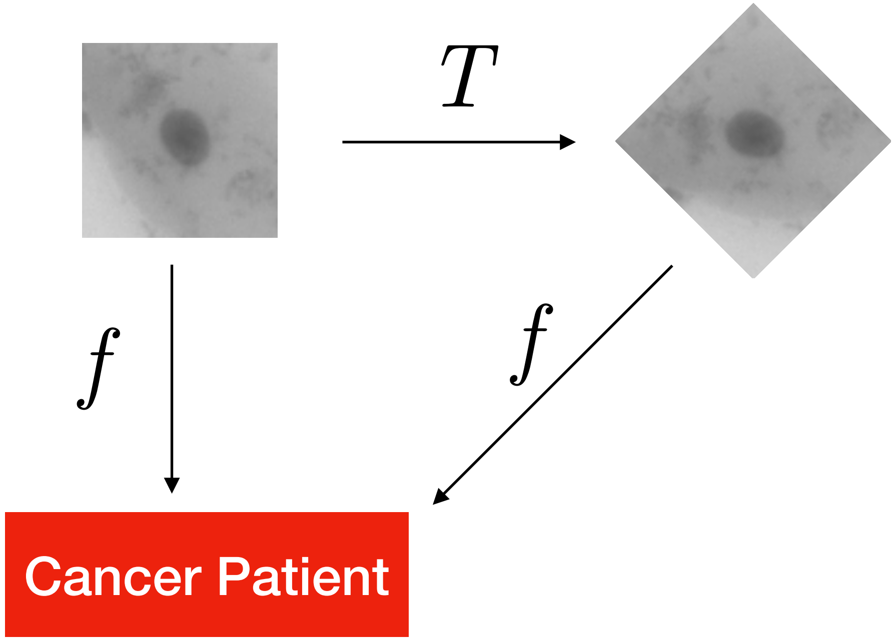
# Invariances vs Equivariances

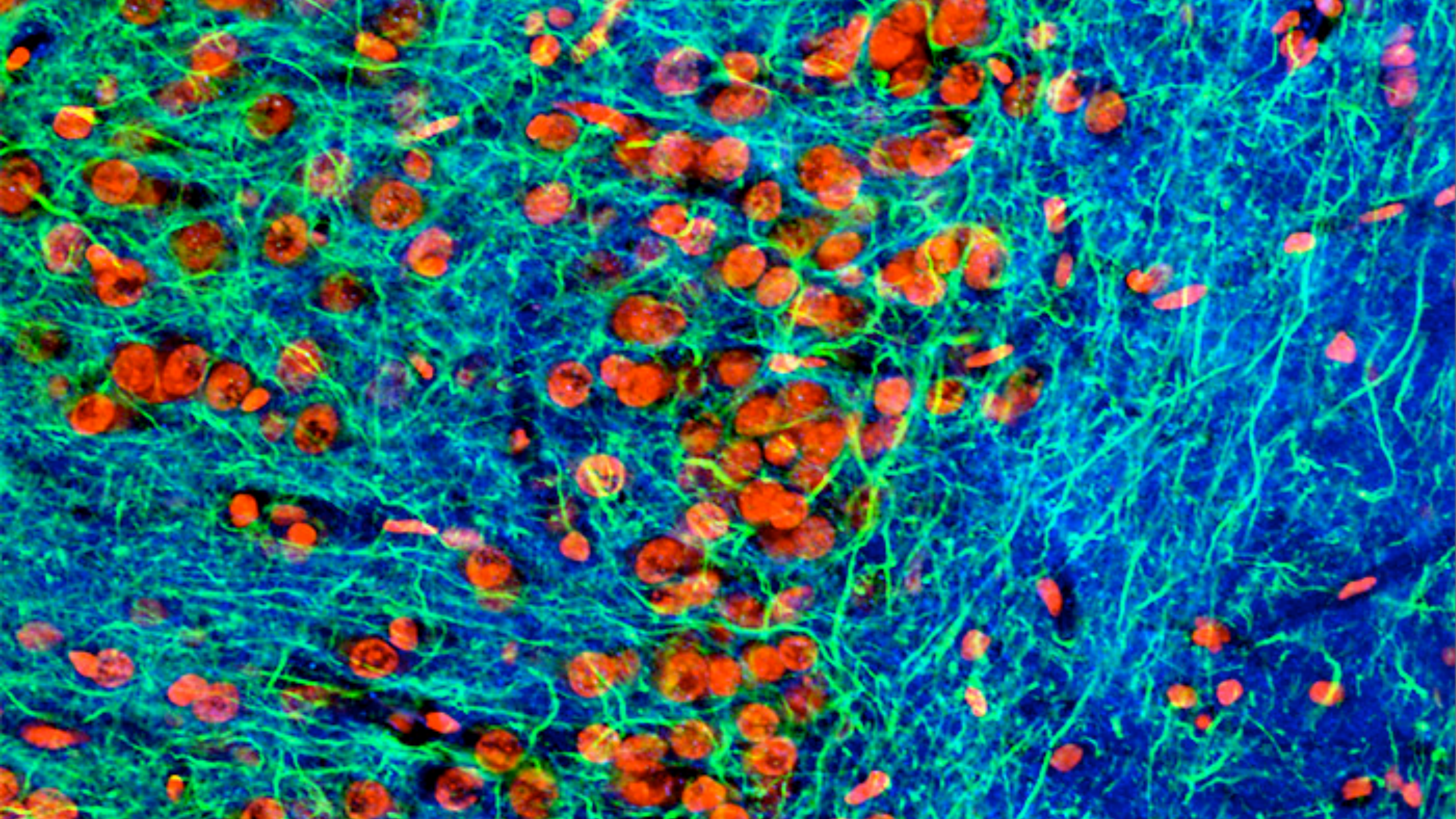


Equivariance

# Invariances vs Equivariances



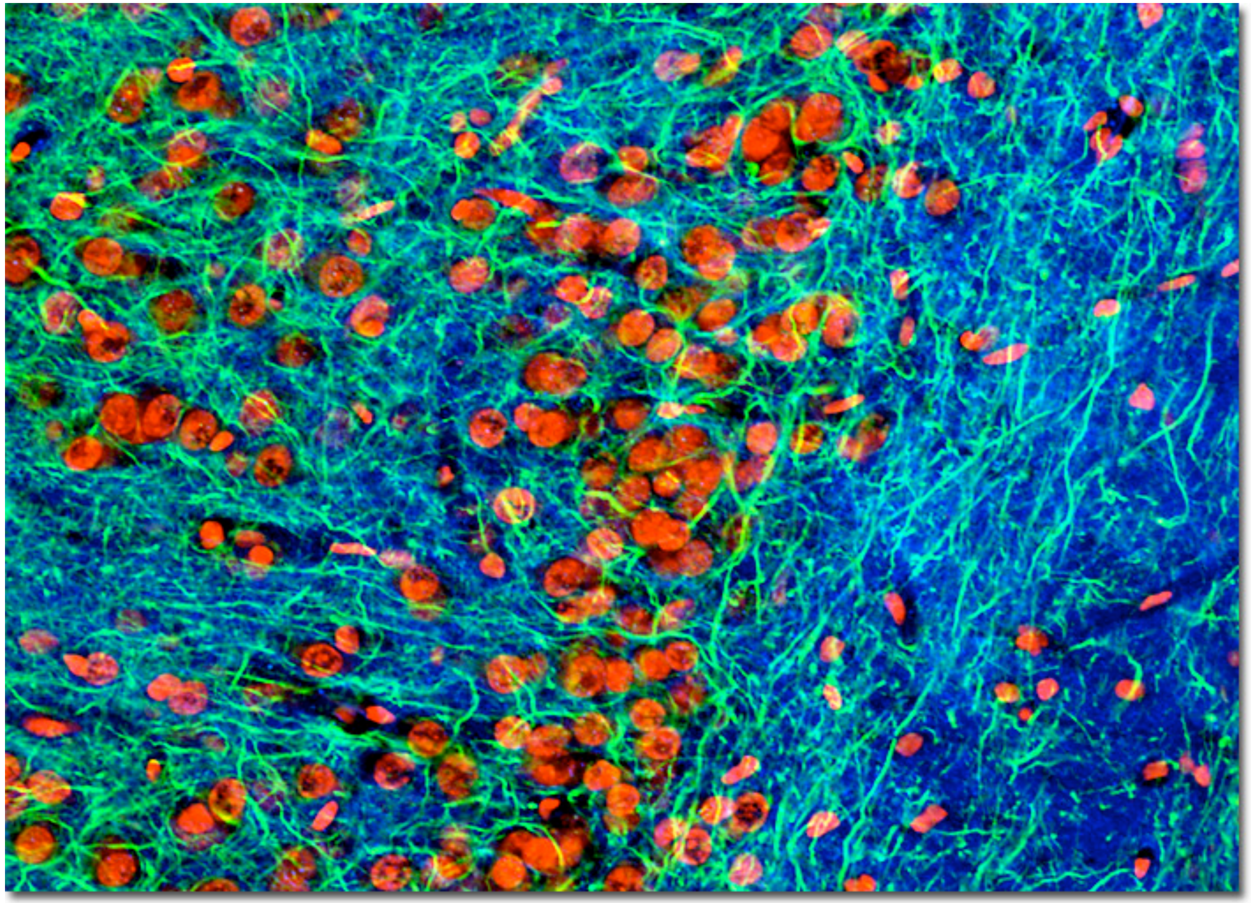


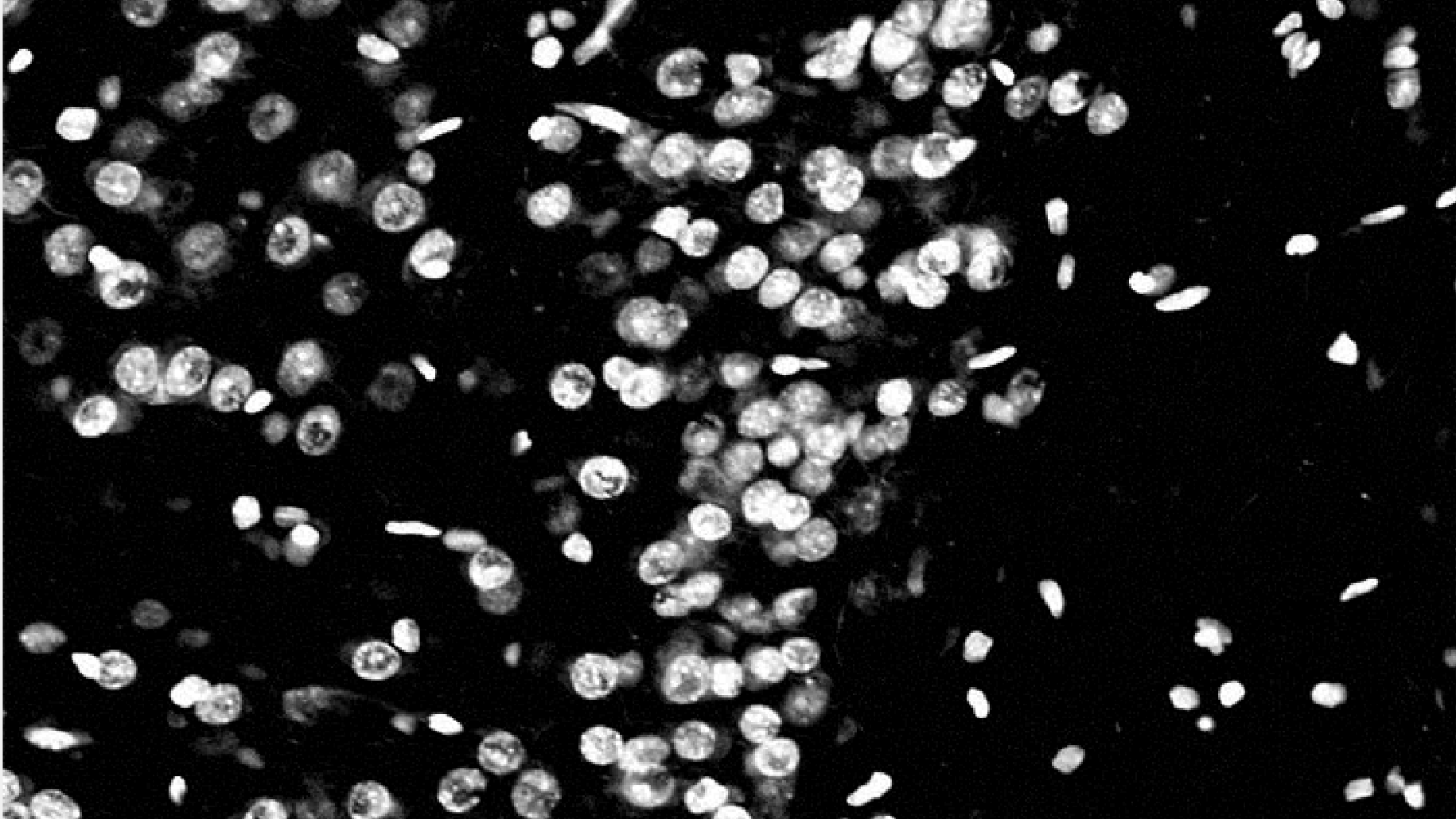
# Features

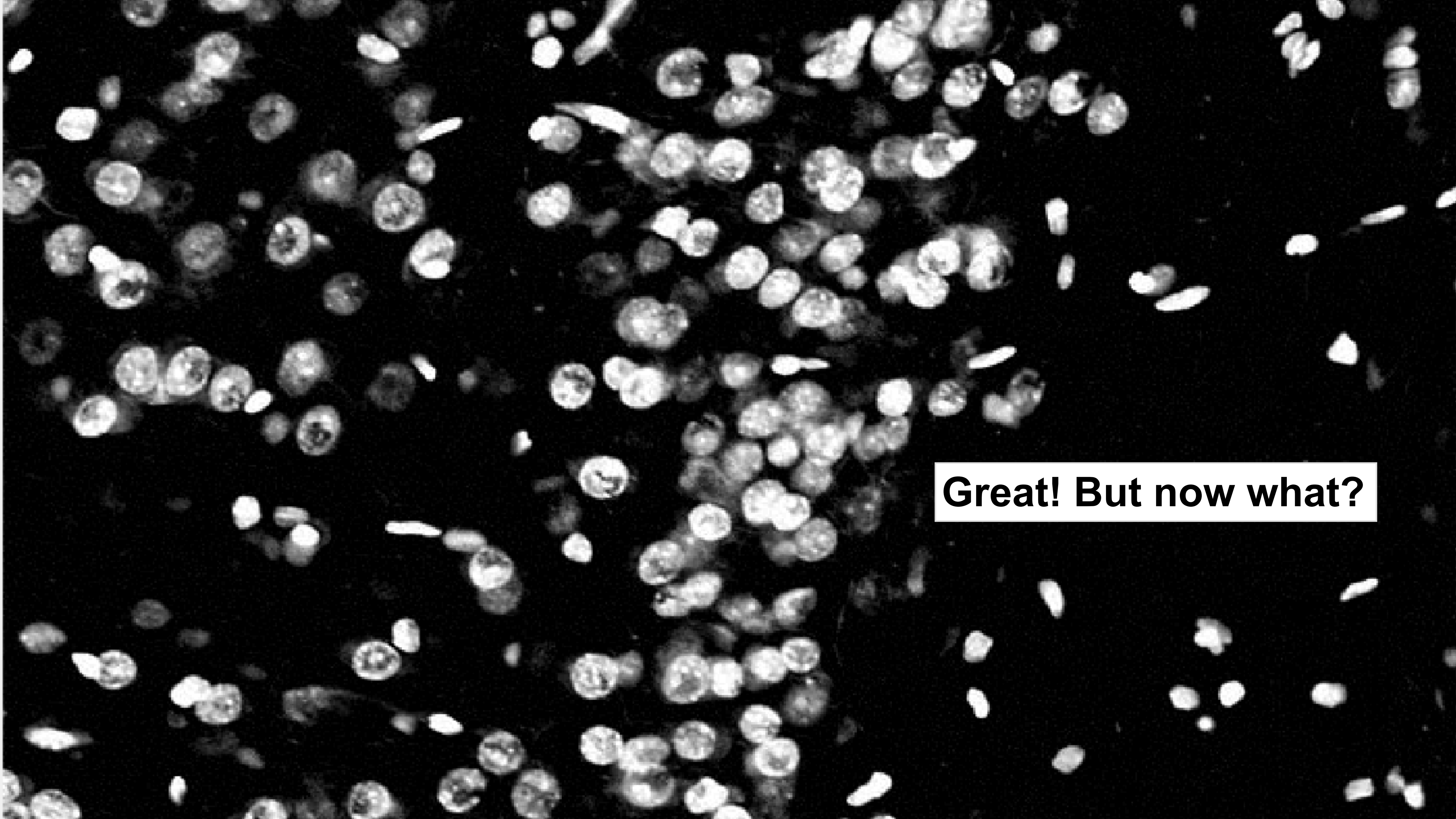
- **Simple Choice**
  - RGB

    every pixel is given by $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

  - Objects of interest all red





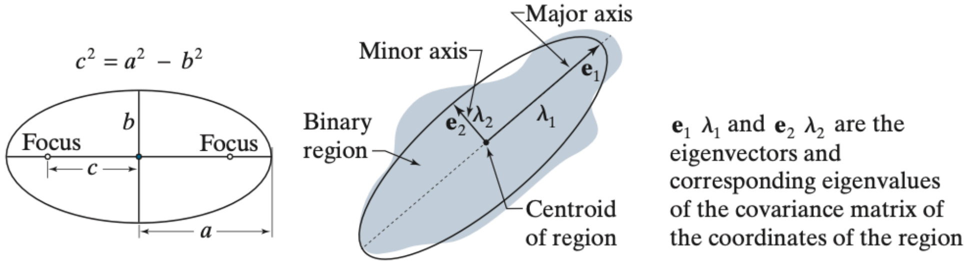

Great! But now what?

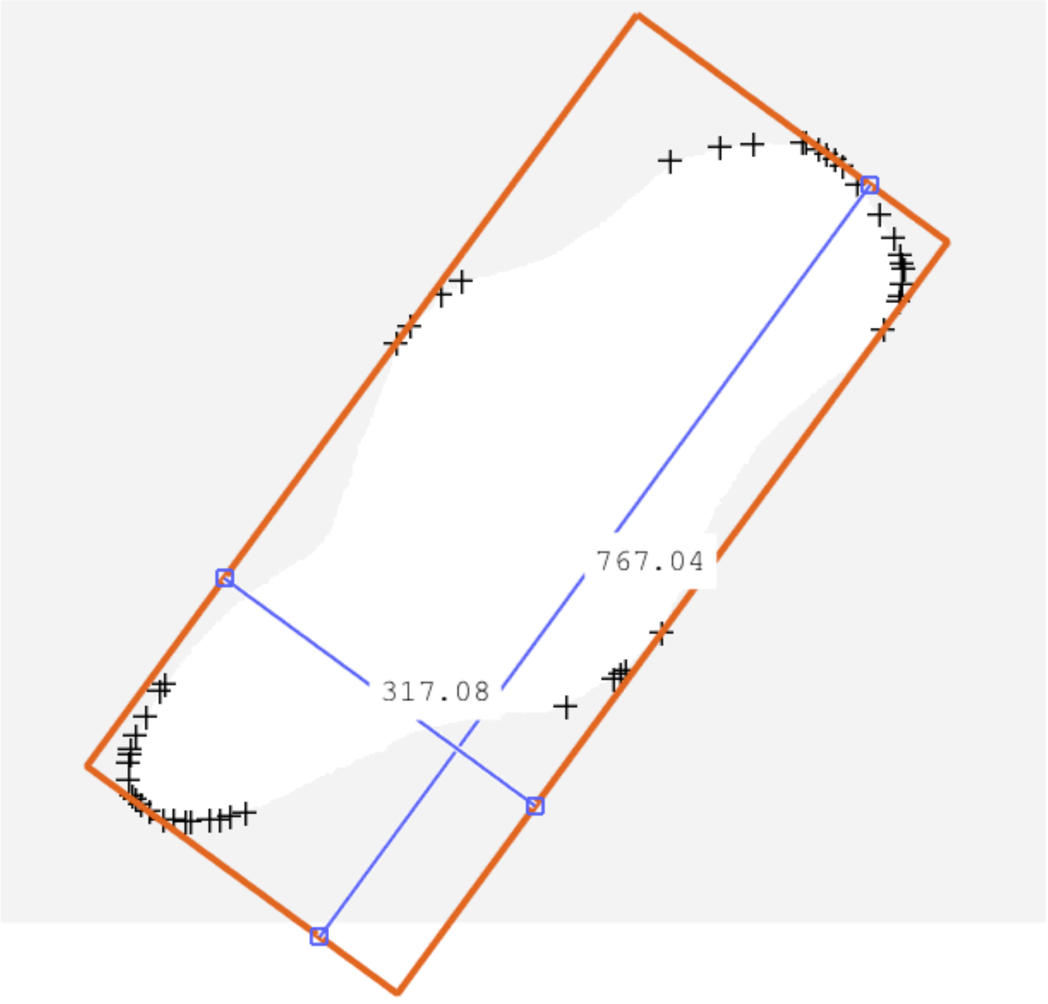
# Boundary Features

- **Features**
  - length (number of pixels along the boundary)
  - diameter (major axis) $$diameter(B) = \max_{i,j}\left[D\left(p_i, p_j\right)\right]$$
  - minor axis
  - curvature (rate of change of slope)
  - convex, concave

a b

**FIGURE 11.21**
(a) An ellipse in standard form.
(b) An ellipse approximating a region in arbitrary orientation.



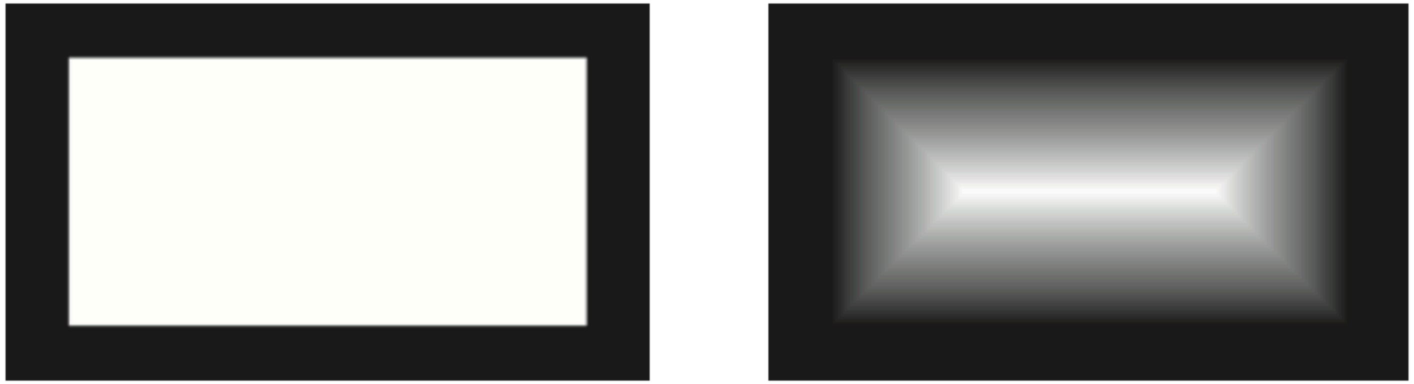

Source: https://blogs.mathworks.com/steve/tag/regionprops/

# Distance Transforms

- **Metrics**
  - Euclidean
  - City-Block
  - L1
  - many more…

a b
c d
e f

**FIGURE 11.13**
(a) A small image and (b) its distance transform. Note that all 1-valued pixels in (a) have corresponding 0's in (b). (c) A small image, and (d) the distance transform of its *complement*. (e) A larger image, and (f) the distance transform of its complement. The Euclidian distance was used throughout.



# Distance Transforms

- **Metrics**
  - Euclidean
  - City-Block
  - L1
  - many more…

| a | b |
|---|---|
| c | d |
| e | f |

**FIGURE 11.13** (a) A small image and (b) its distance transform. Note that all 1-valued pixels in (a) have corresponding 0's in (b). (c) A small image, and (d) the distance transform of its *complement*. (e) A larger image, and (f) the distance transform of its complement. The Euclidian distance was used throughout.

(a)

| | | | | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 |

(b)

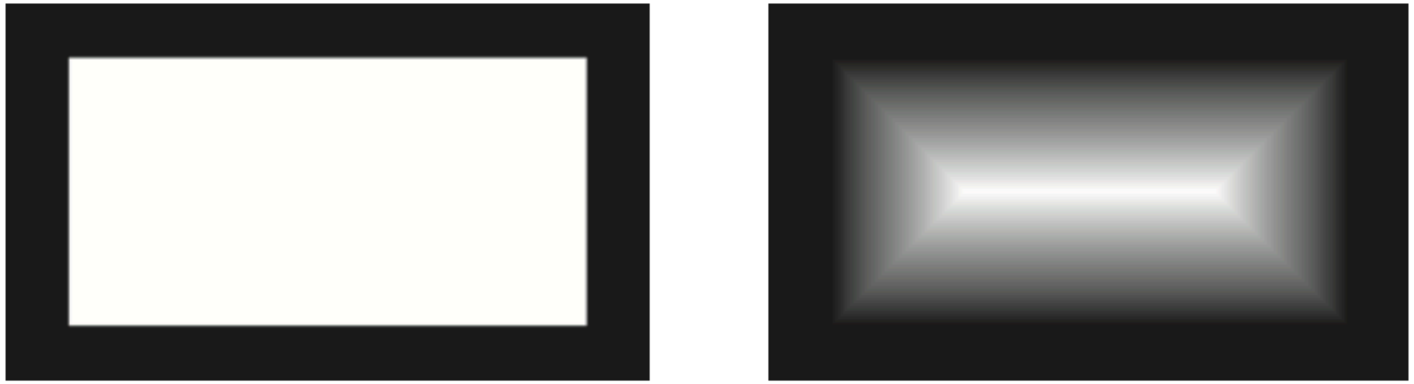
| | | | | |
|---|---|---|---|---|
| 1.41 | 1 | 1 | 1 | 1.41 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 |
| 1.41 | 1 | 1 | 1 | 1.41 |

(c)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

(d)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | **1** | 1 | 1 | 1 | 1 | 1 | **1** | 0 |
| 0 | 1 | **2** | 2 | 2 | 2 | **2** | 1 | 0 |
| 0 | 1 | 2 | **3** | **3** | **3** | 2 | 1 | 0 |
| 0 | 1 | **2** | 2 | 2 | 2 | **2** | 1 | 0 |
| 0 | **1** | 1 | 1 | 1 | 1 | 1 | **1** | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |



# Skeletons

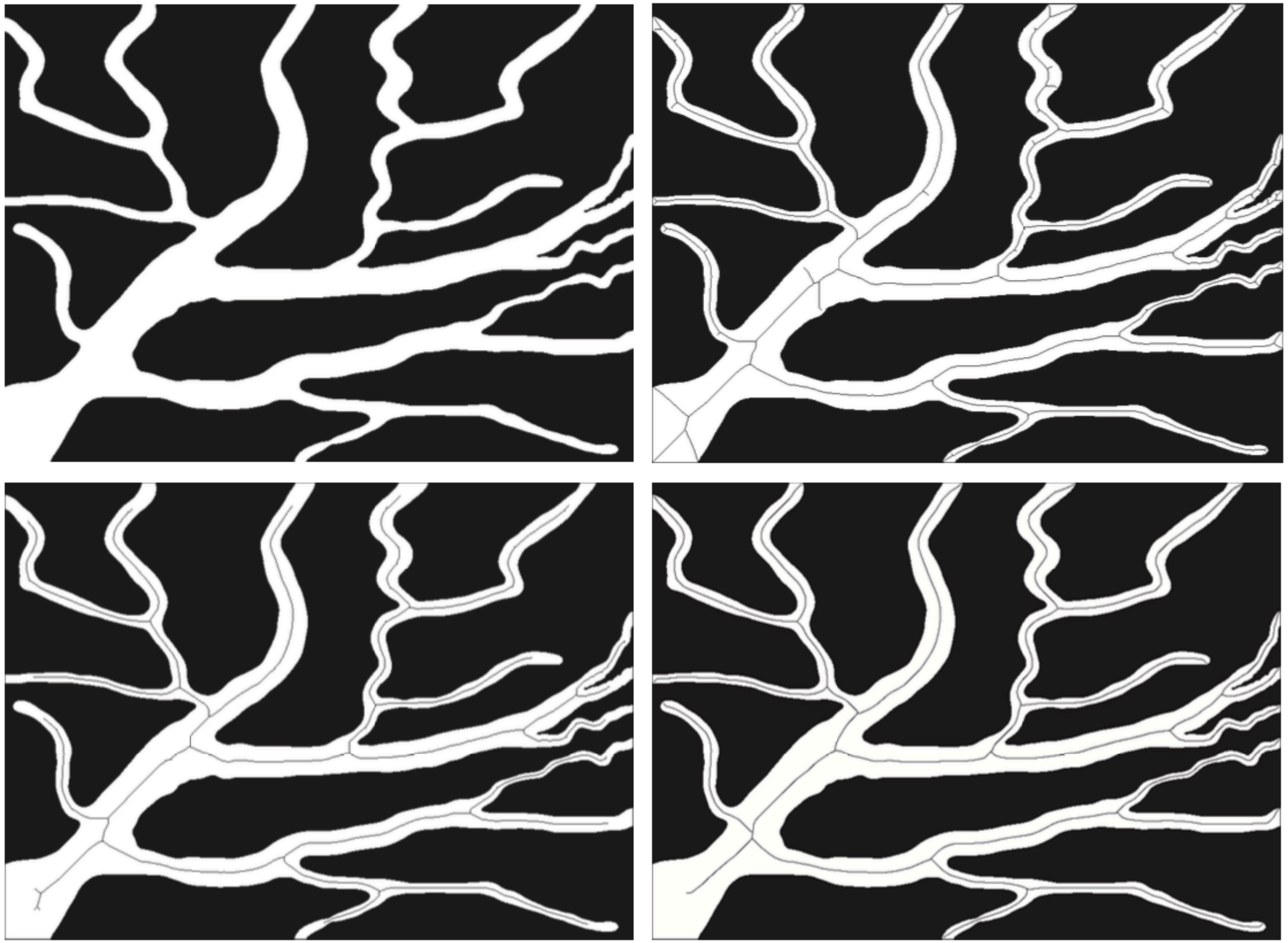
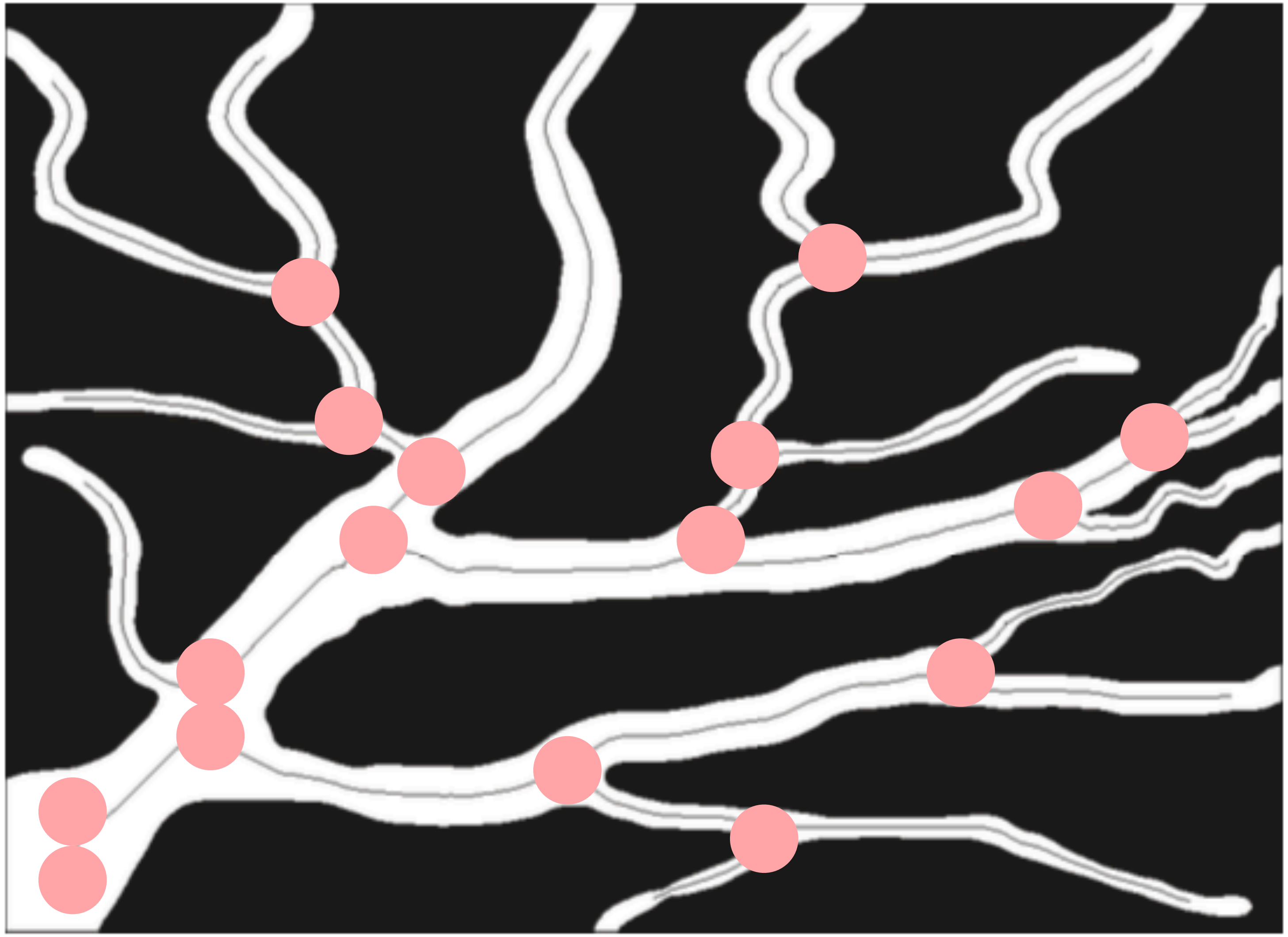



**FIGURE 11.14**
(a) Thresholded image of blood vessels.
(b) Skeleton obtained by thinning, shown superimposed on the image (note the spurs).
(c) Result of 40 passes of spur removal.
(d) Skeleton obtained using the distance transform.

# Skeletons

- **Descriptors**
  - number of branchpoints
  - total length of skeleton
  - lengths of sections
  - distribution of branch points, proximity of branchpoints,…



# Region Descriptors

- **Region Descriptors**
  - area
  - perimeter (length of boundary)
  - compactness
    $$\text{compactness} = \frac{p^2}{A}$$
  - circularity
    $$\text{circularity} = \frac{4\pi A}{p^2}$$
  - eccentricity (major axis/minor axis)

# Feature Descriptors

a b c d

**FIGURE 11.22** Compactness, circularity, and eccentricity of some simple binary regions.

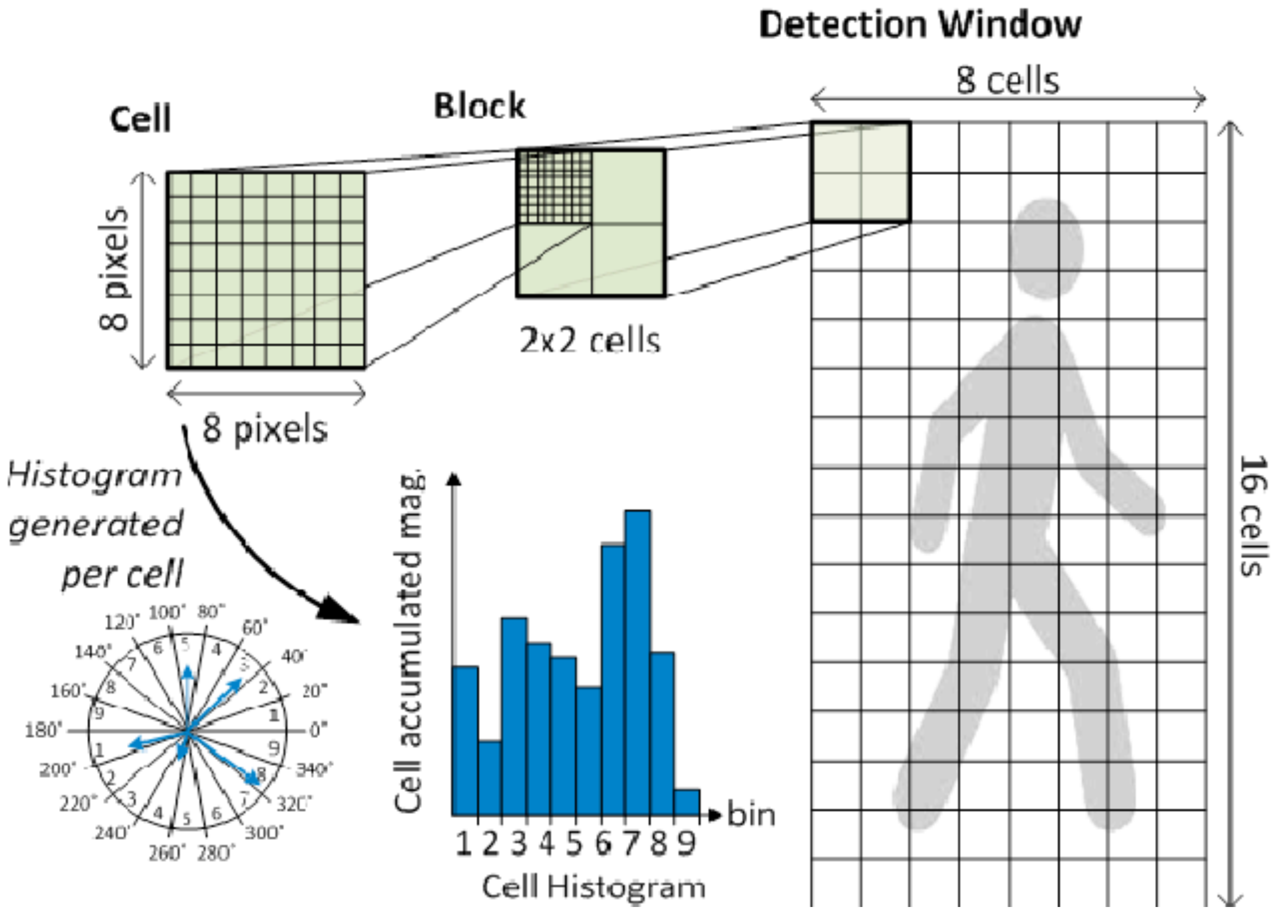
| **Descriptor** |  |  |  |  |
|---|---|---|---|---|
| *Compactness* | 10.1701 | 42.2442 | 15.9836 | 13.2308 |
| *Circularity* | 1.2356 | 0.2975 | 0.7862 | 0.9478 |
| *Eccentricity* | 0.0411 | 0.0636 | 0 | 0.8117 |

# Region Descriptors

- **More advanced region descriptors**
  - HOG (Histogram of Oriented Gradients)
  - Haar-like features
  - Topological Descriptors
  - Fourier Descriptors
  - Texture descriptors (e.g. local binary patterns)

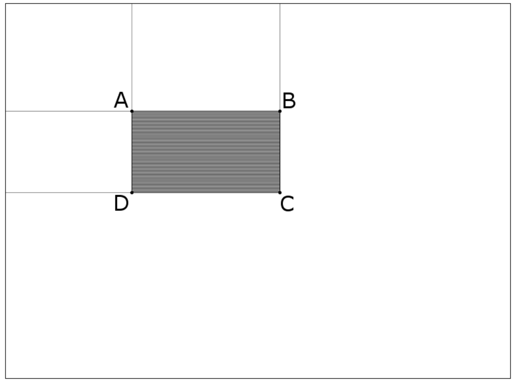
# Region Descriptors: HOG

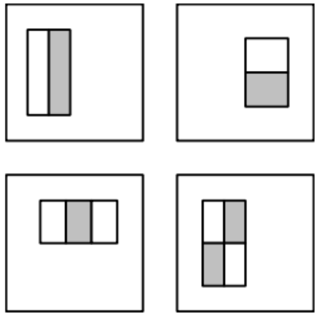
- **Histogram of Gradients**
  - Compute gradients in image using $[-1\ 0\ 1]$ and $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ masks, divide into cells
  - Compute histogram of gradient orientations on each cell
  - Group cells into overlapping blocks, normalize vector of histogram values
  - Slide over all relevant windows/regions
  - Concatenate all histograms into one region descriptor

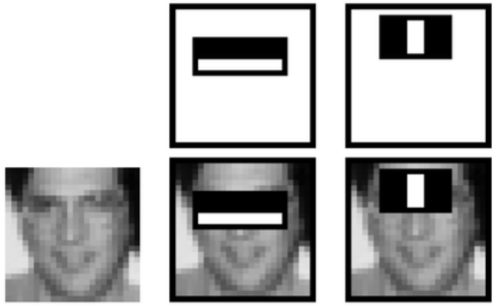


# Region Descriptors: Haar-like

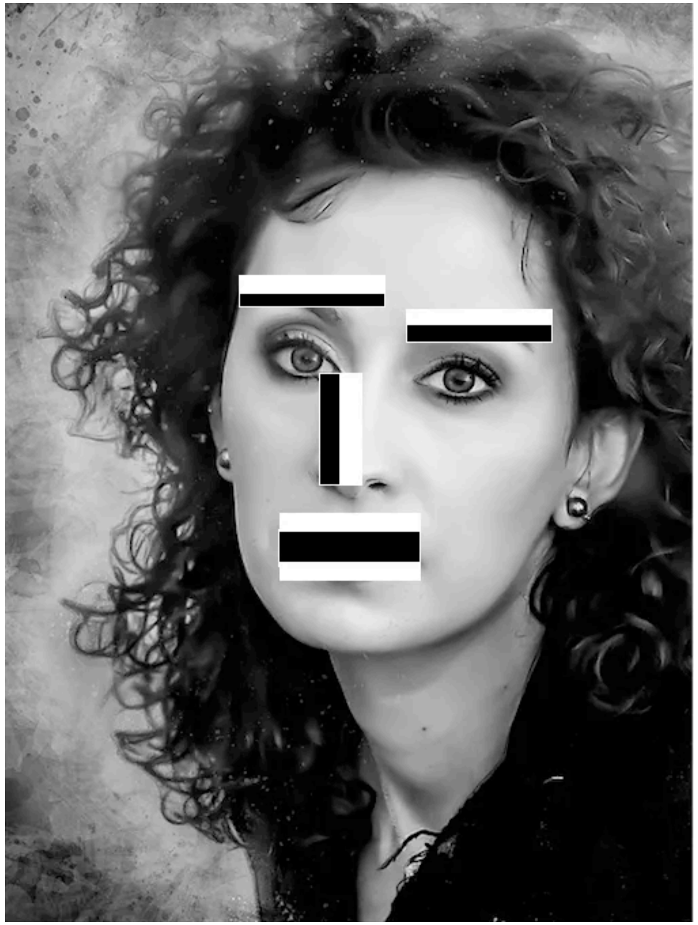
- **Haar-Like**
  - Approximations to Gaussian derivative filters
  - Uses integral image for fast extraction
  - Compute sum in white and black and subtract them
  - Shift and scale all 4 rectangles across window
  - for 24x24 window → 180k features





# Region Descriptors: Haar-like

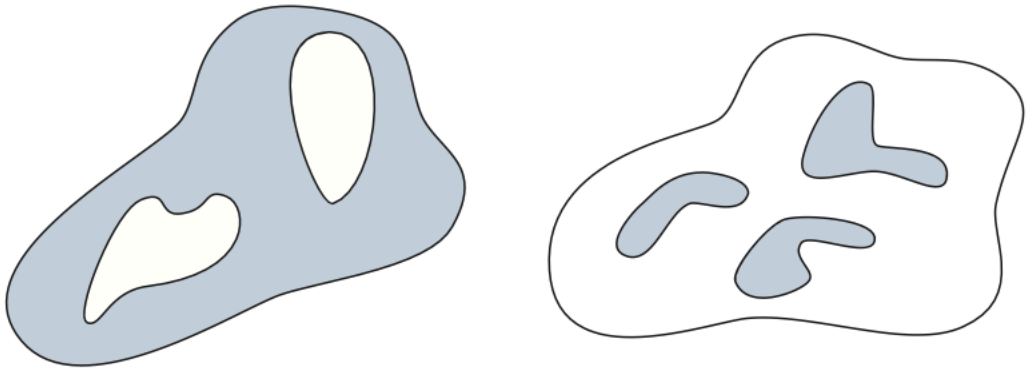


- Popular in Face Detection
- Viola-Jones Algorithm

# Topological Descriptors

- **Features**
  - Number of connected components of an image or region
  - Number of holes H and connected components C defines Euler number, E

$$E = C - H$$



a b

**FIGURE 11.25**
(a) A region with two holes.
(b) A region with three connected components.

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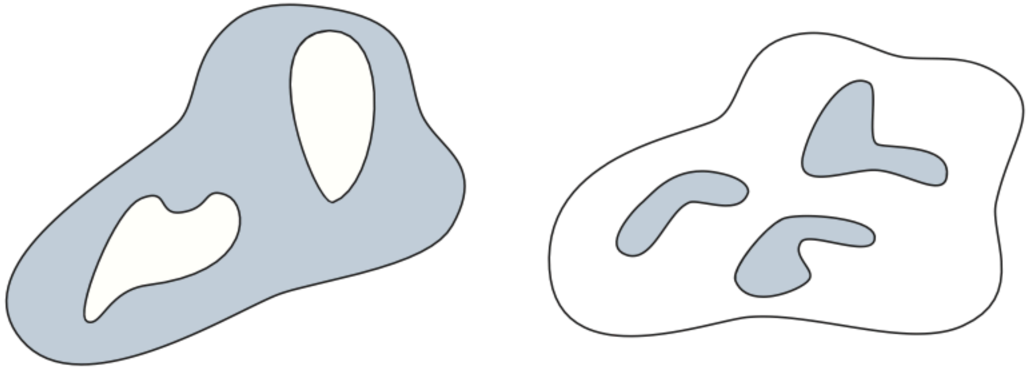
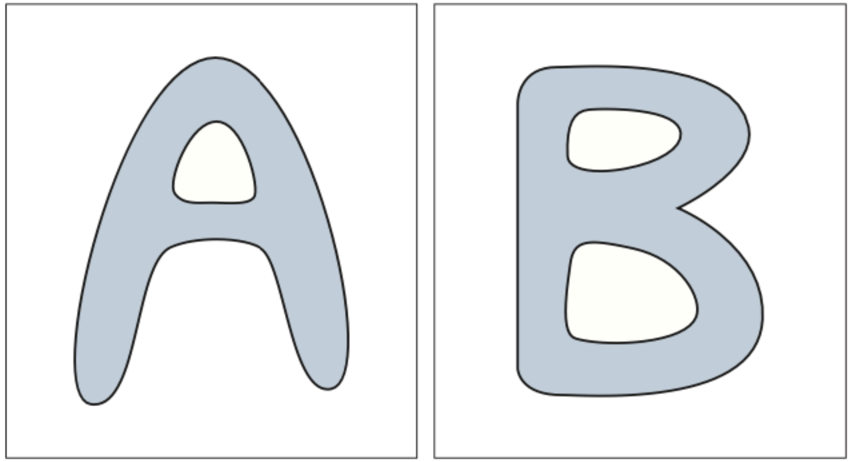
a b

**FIGURE 11.25**
(a) A region with two holes.
(b) A region with three connected components.



$E = -1$ $\qquad$ $E = -2$

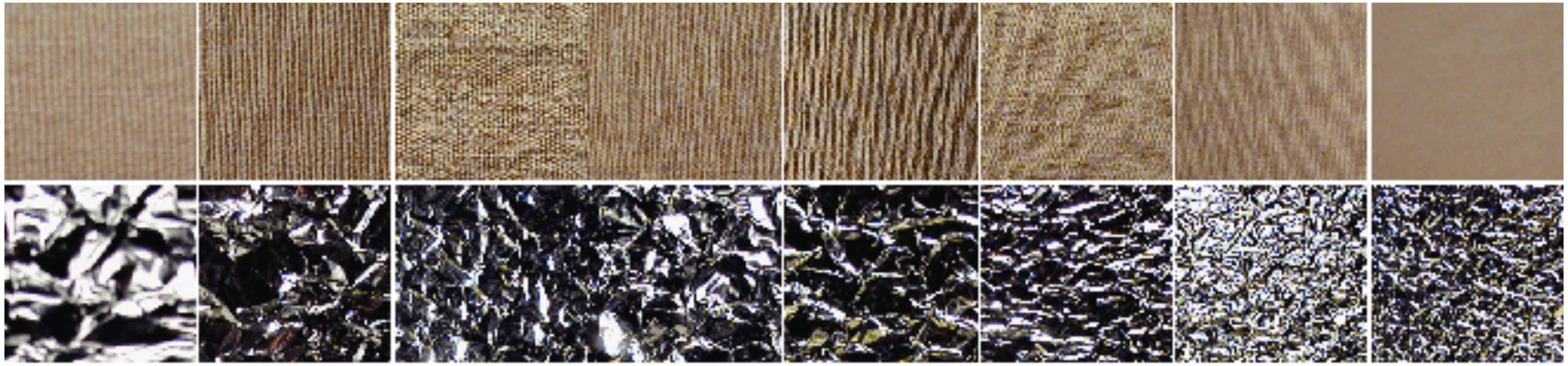
# Topological Descriptors



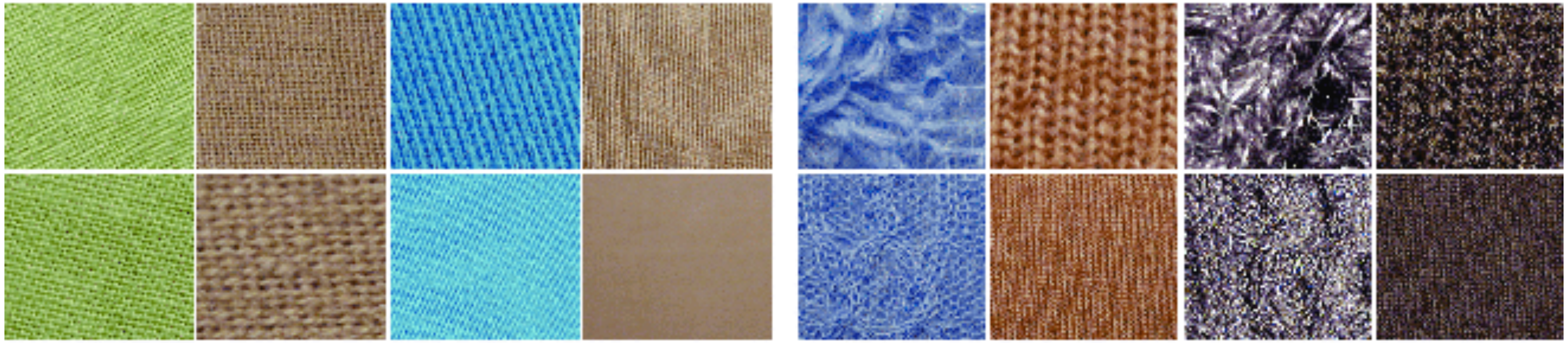
Euler Numbers?

# Texture

- **Region Descriptor**
- **No formal definition**
- **Properties**
  - smooth
  - coarse
  - regular



(a) KTH-TIPS



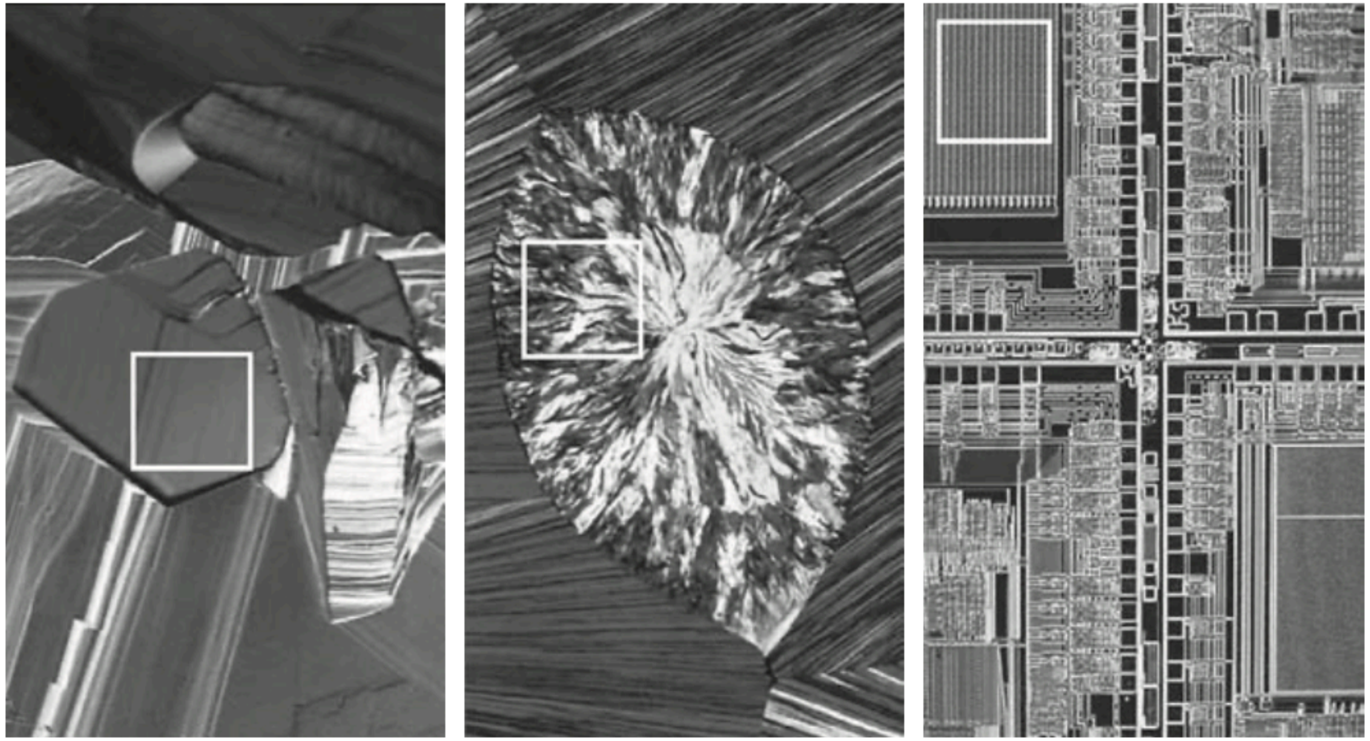
(b) KTH-TIPS2a

# Texture

- **Region Descriptor**
- **No formal definition**
- **Properties**
  - smooth
  - coarse
  - regular

a b c

**FIGURE 11.29** The white squares mark, from left to right, smooth, coarse, and regular textures. These are optical microscope images of a superconductor, human cholesterol, and a microprocessor. (Courtesy of Dr. Michael W. Davidson, Florida State University.)



# Texture: Statistical Descriptors

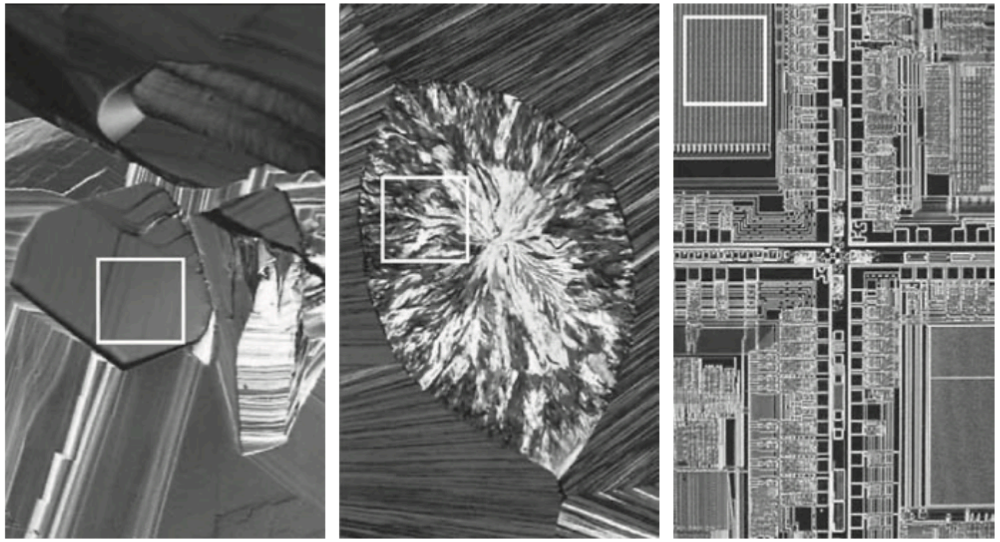


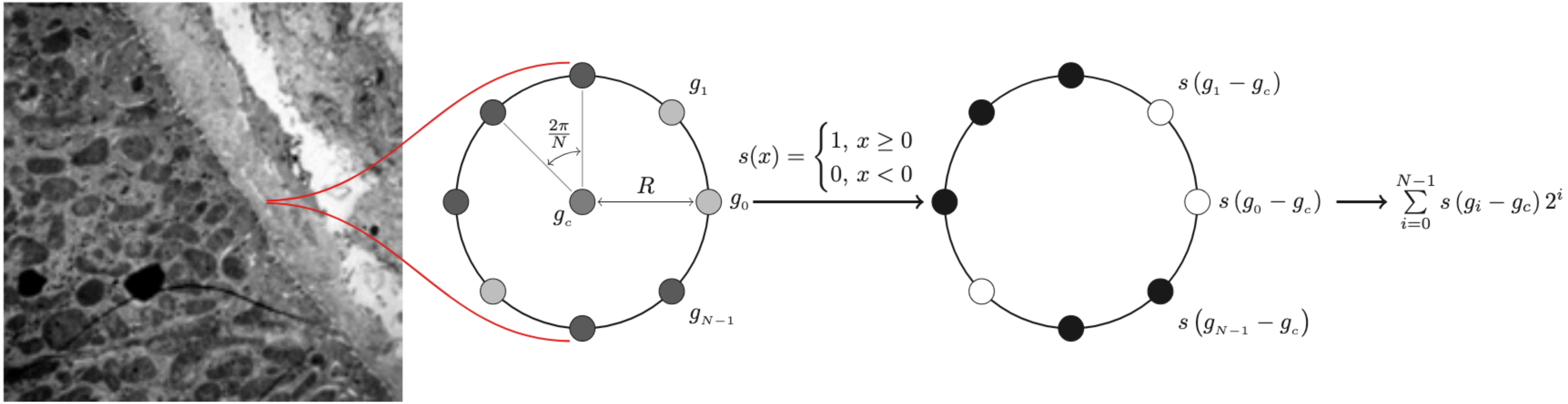
**TABLE 11.2**
Statistical texture measures for the subimages in Fig. 11.29.

| Texture | Mean | Standard deviation | *R* (normalized) | 3rd moment | Uniformity | Entropy |
|---|---|---|---|---|---|---|
| Smooth | 82.64 | 11.79 | 0.002 | –0.105 | 0.026 | 5.434 |
| Coarse | 143.56 | 74.63 | 0.079 | –0.151 | 0.005 | 7.783 |
| Regular | 99.72 | 33.73 | 0.017 | 0.750 | 0.013 | 6.674 |

$$m = \sum_{i=0}^{L-1} z_i p(z_i) \qquad \sigma^2(z) = \mu_2(z) \qquad R(z) = 1 - \frac{1}{1+\sigma^2(z)} \qquad \mu_n(z) = \sum_{i=0}^{L-1} (z_i - m)^n p(z_i) \qquad U(z) = \sum_{i=0}^{L-1} p^2(z_i) \qquad e(z) = -\sum_{i=0}^{L-1} p(z_i) \log_2 p(z_i)$$

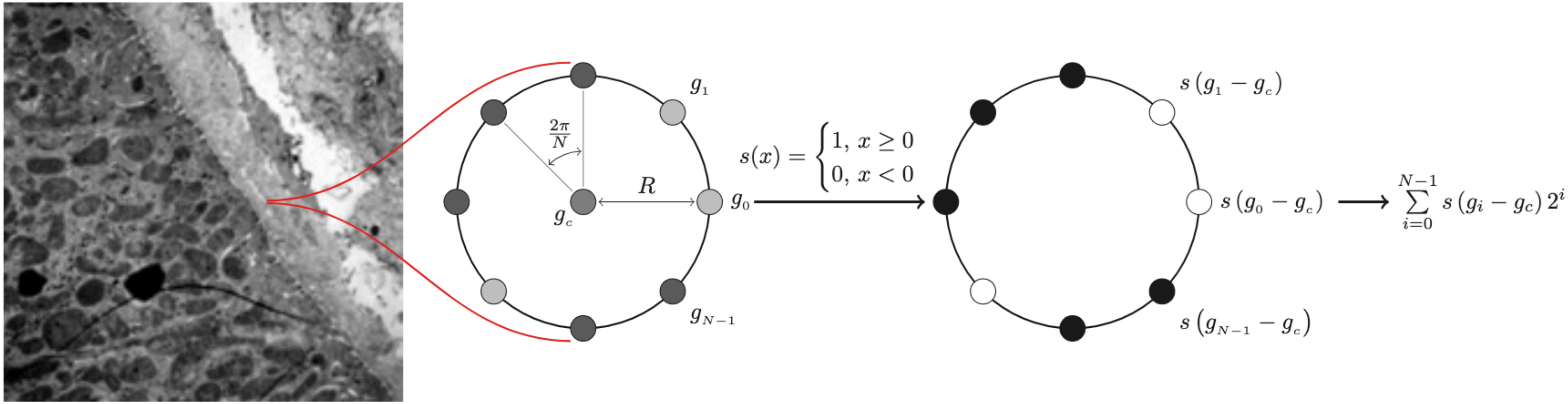
# Texture Descriptors: LBP

- **Local Binary Patters (LBP)**



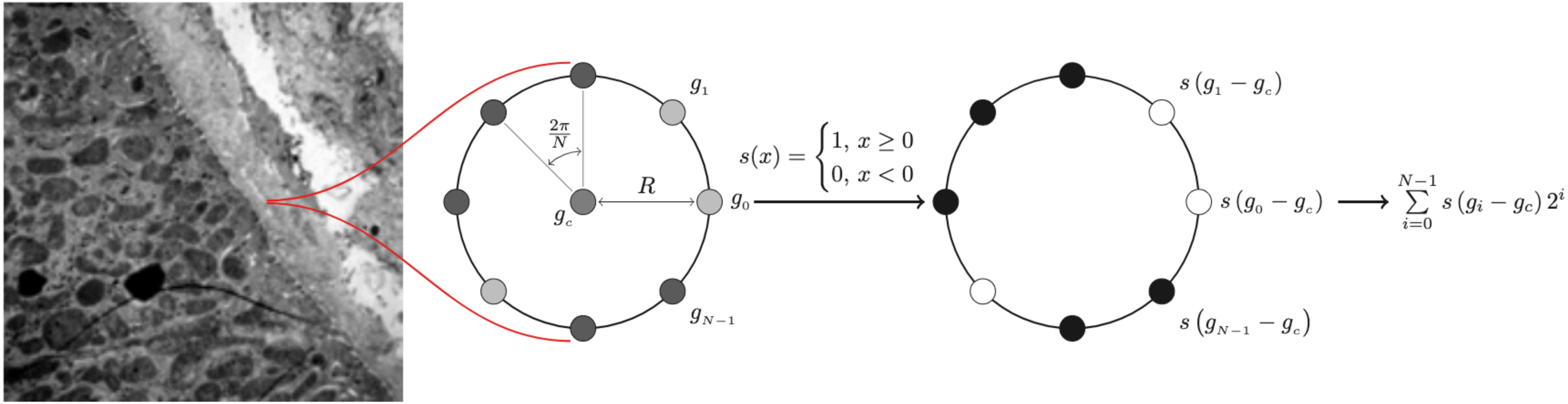
# Texture Descriptors: LBP

- **Local Binary Patters (LBP)**
  - [1,1,0,0,0,1,0,0]
  - One binary code per pixel

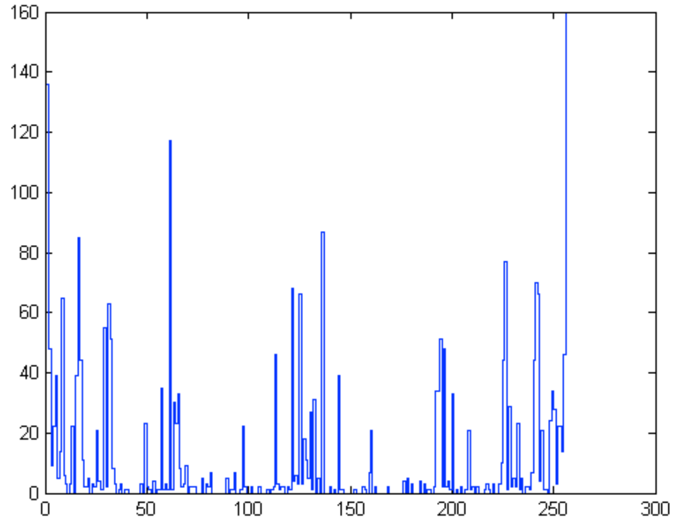


# Texture Descriptors: LBP

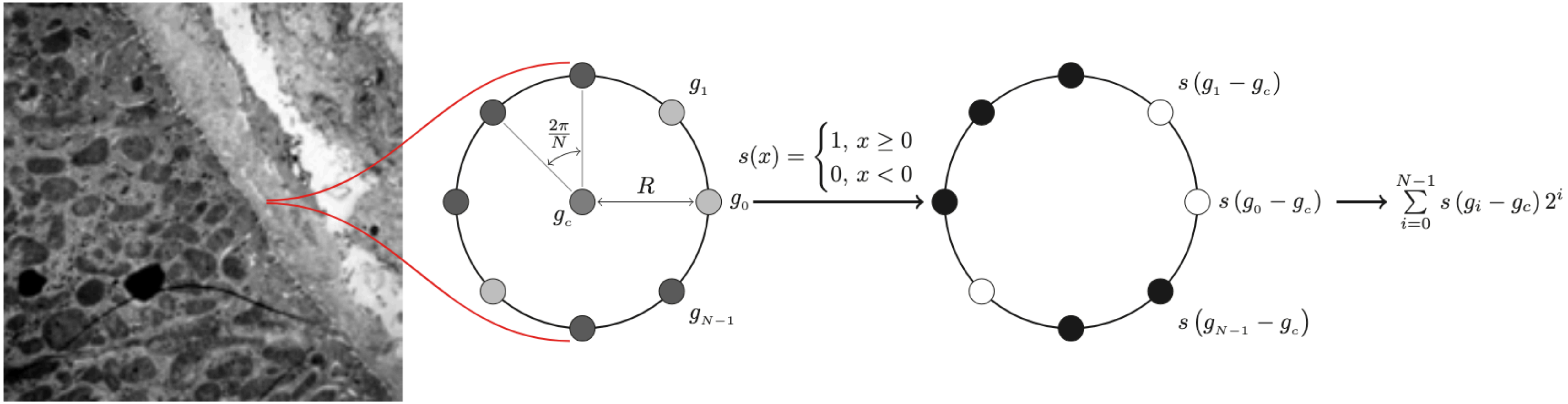
- **Local Binary Patters (LBP)**
  - $[1,1,0,0,0,1,0,0] \triangleq 196$
  - One binary code per pixel



# Texture Descriptors: LBP



- **Local Binary Patters (LBP)**
  - $[1,1,0,0,0,1,0,0] \triangleq 196$
  - One binary code per pixel -> histogram over all codes as global descriptor on image level



# Texture: (Graylevel) Co-occurance Matrix

- **Size**
  - for image with intensity range of $[0,1,2,\ldots,255]$ the co-occurance matrix will be $256 \times 256$

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 7 | 5 | 3 | 2 |
| 5 | 1 | 6 | 1 | 2 | 5 |
| 8 | 8 | 6 | 8 | 1 | 2 |
| 4 | 3 | 4 | 5 | 5 | 1 |
| 8 | 7 | 8 | 7 | 6 | 2 |
| 7 | 8 | 6 | 2 | 6 | 2 |

Image $f$

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 0 | 0 | 0 | 1 | 1 | 0 |
| 2 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| 3 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| 5 | 2 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| 6 | 1 | 3 | 0 | 0 | 0 | 0 | 0 | 1 |
| 7 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 2 |
| 8 | 1 | 0 | 0 | 0 | 0 | 2 | 2 | 1 |

Co-occurrence matrix **G**

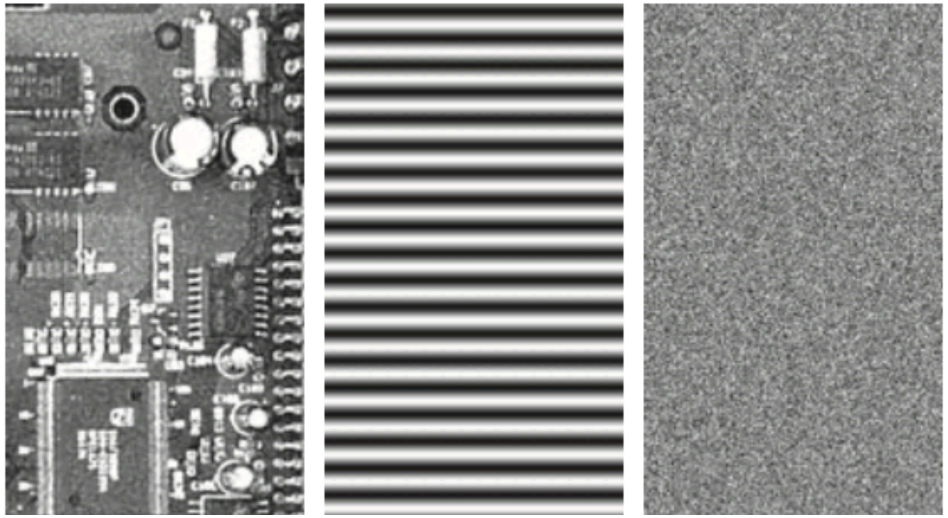
# Texture: (Graylevel) Co-occurance Matrix

**TABLE 11.3** Descriptors used for characterizing co-occurrence matrices of size $K \times K$. The term $p_{ij}$ is the $ij$-th term of **G** divided by the sum of the elements of **G**.

| Descriptor | Explanation | Formula |
|---|---|---|
| Maximum probability | Measures the strongest response of **G**. The range of values is [0, 1]. | $\max_{i,j}(p_{ij})$ |
| Correlation | A measure of how correlated a pixel is to its neighbor over the entire image. The range of values is 1 to −1 corresponding to perfect positive and perfect negative correlations. This measure is not defined if either standard deviation is zero. | $\sum_{i=1}^{K}\sum_{j=1}^{K}\frac{(i-m_r)(j-m_c)p_{ij}}{\sigma_r\sigma_c}$ <br> $\sigma_r \neq 0;\ \sigma_c \neq 0$ |
| Contrast | A measure of intensity contrast between a pixel and its neighbor over the entire image. The range of values is 0 (when **G** is constant) to $(K-1)^2$. | $\sum_{i=1}^{K}\sum_{j=1}^{K}(i-j)^2 p_{ij}$ |
| Uniformity (also called Energy) | A measure of uniformity in the range [0, 1]. Uniformity is 1 for a constant image. | $\sum_{i=1}^{K}\sum_{j=1}^{K}p_{ij}^2$ |
| Homogeneity | Measures the spatial closeness to the diagonal of the distribution of elements in **G**. The range of values is [0, 1], with the maximum being achieved when **G** is a diagonal matrix. | $\sum_{i=1}^{K}\sum_{j=1}^{K}\frac{p_{ij}}{1+\lvert i-j\rvert}$ |
| Entropy | Measures the randomness of the elements of **G**. The entropy is 0 when all $p_{ij}$'s are 0, and is maximum when the $p_{ij}$'s are uniformly distributed. The maximum value is thus $2\log_2 K$. | $-\sum_{i=1}^{K}\sum_{j=1}^{K}p_{ij}\log_2 p_{ij}$ |

# Texture: (Graylevel) Co-occurance Matrix

**FIGURE 11.31** Images whose pixels have (a) random, (b) periodic, and (c) mixed texture patterns. Each image is of size $263 \times 800$ pixels.



a b c

**FIGURE 11.32** $256 \times 256$ co-occurrence matrices $\mathbf{G}_1$, $\mathbf{G}_2$, and $\mathbf{G}_3$, corresponding from left to right to the images in Fig. 11.31.

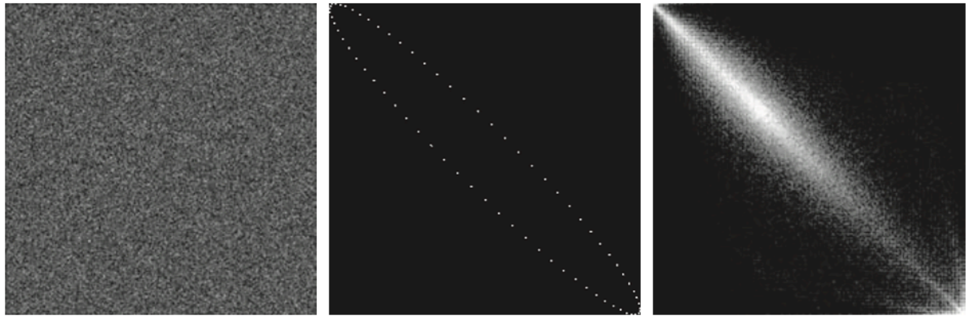


**TABLE 11.4**
Descriptors evaluated using the co-occurrence matrices displayed as images in Fig. 11.32.

| Normalized Co-occurrence Matrix | Maximum Probability | Correlation | Contrast | Uniformity | Homogeneity | Entropy |
|---|---|---|---|---|---|---|
| $\mathbf{G}_1/n_1$ | 0.00006 | −0.0005 | 10838 | 0.00002 | 0.0366 | 15.75 |
| $\mathbf{G}_2/n_2$ | 0.01500 | 0.9650 | 00570 | 0.01230 | 0.0824 | 06.43 |
| $\mathbf{G}_3/n_3$ | 0.06860 | 0.8798 | 01356 | 0.00480 | 0.2048 | 13.58 |

# Texture: Moment-Invariants

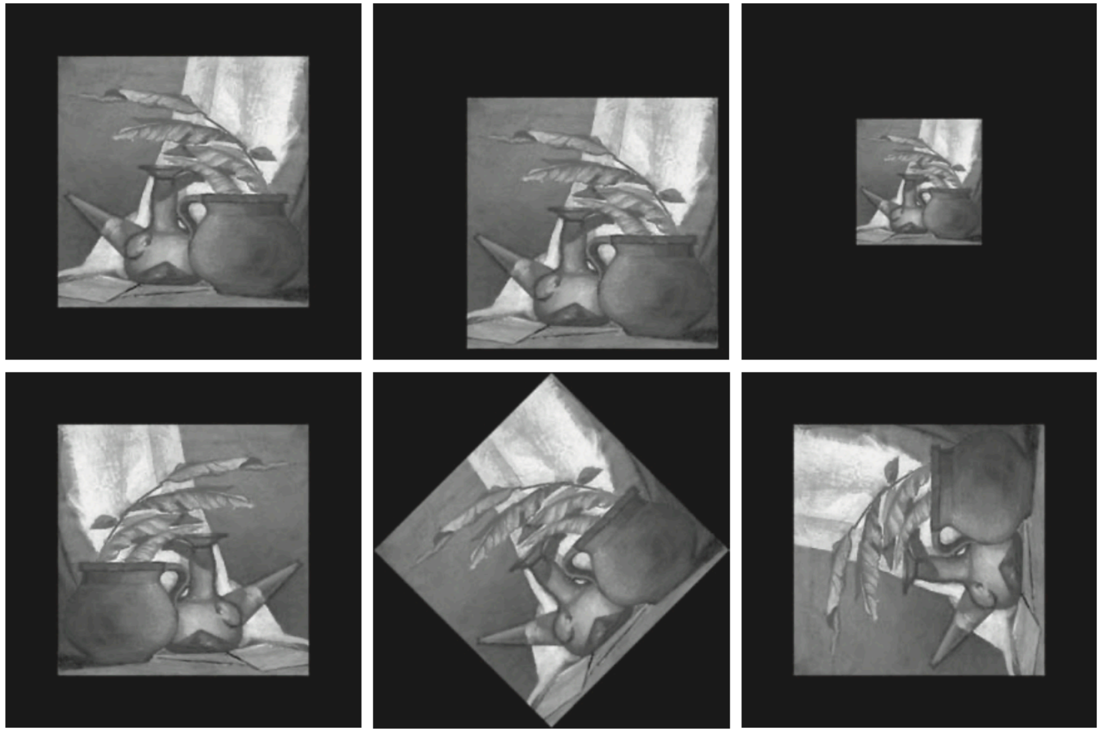


**TABLE 11.5**
Moment invariants for the images in Fig. 11.37.

| Moment Invariant | Original Image | Translated | Half Size | Mirrored | Rotated 45° | Rotated 90° |
|---|---|---|---|---|---|---|
| $\phi_1$ | 2.8662 | 2.8662 | 2.8664 | 2.8662 | 2.8661 | 2.8662 |
| $\phi_2$ | 7.1265 | 7.1265 | 7.1257 | 7.1265 | 7.1266 | 7.1265 |
| $\phi_3$ | 10.4109 | 10.4109 | 10.4047 | 10.4109 | 10.4115 | 10.4109 |
| $\phi_4$ | 10.3742 | 10.3742 | 10.3719 | 10.3742 | 10.3742 | 10.3742 |

## MOMENT INVARIANTS

The 2-D *moment* of order $(p+q)$ of an $M \times N$ digital image, $f(x,y)$, is defined as

$$m_{pq} = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} x^p y^q f(x,y) \qquad (11\text{-}34)$$

where $p = 0,1,2,\ldots$ and $q = 0,1,2,\ldots$ are integers. The corresponding *central moment* of order $(p+q)$ is defined as

$$\mu_{pq} = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} (x-\bar{x})^p (y-\bar{y})^q f(x,y) \qquad (11\text{-}35)$$

for $p = 0,1,2,\ldots$ and $q = 0,1,2,\ldots$, where

$$\bar{x} = \frac{m_{10}}{m_{00}} \text{ and } \bar{y} = \frac{m_{01}}{m_{00}} \qquad (11\text{-}36)$$

The *normalized central moment* of order $(p+q)$, denoted $\eta_{pq}$, is defined as

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{\gamma}} \qquad (11\text{-}37)$$

where

$$\gamma = \frac{p+q}{2} + 1 \qquad (11\text{-}38)$$

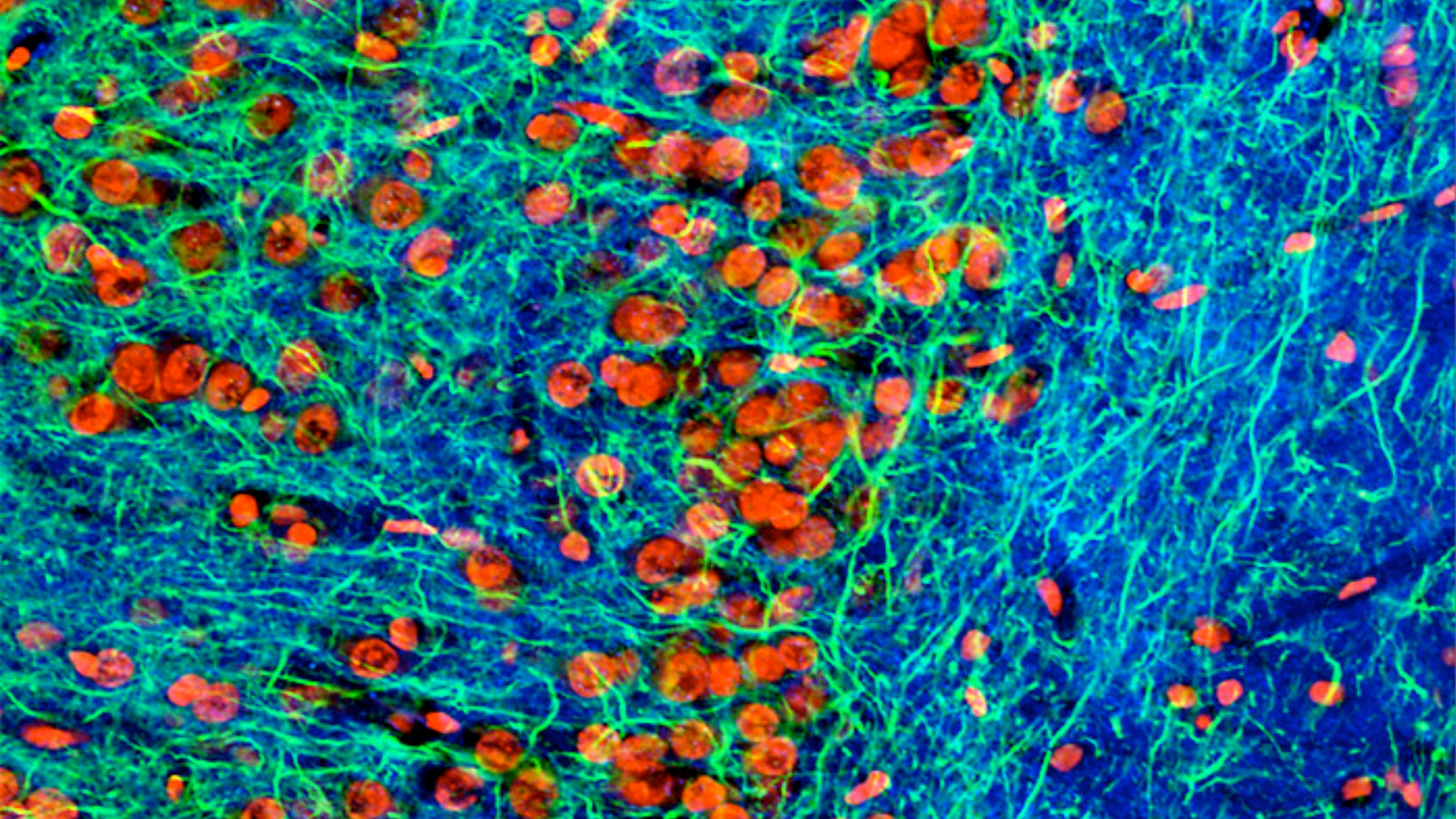
for $p+q = 2,3,\ldots$. A set of seven, 2-D *moment invariants* can be derived from the second and third normalized central moments:†

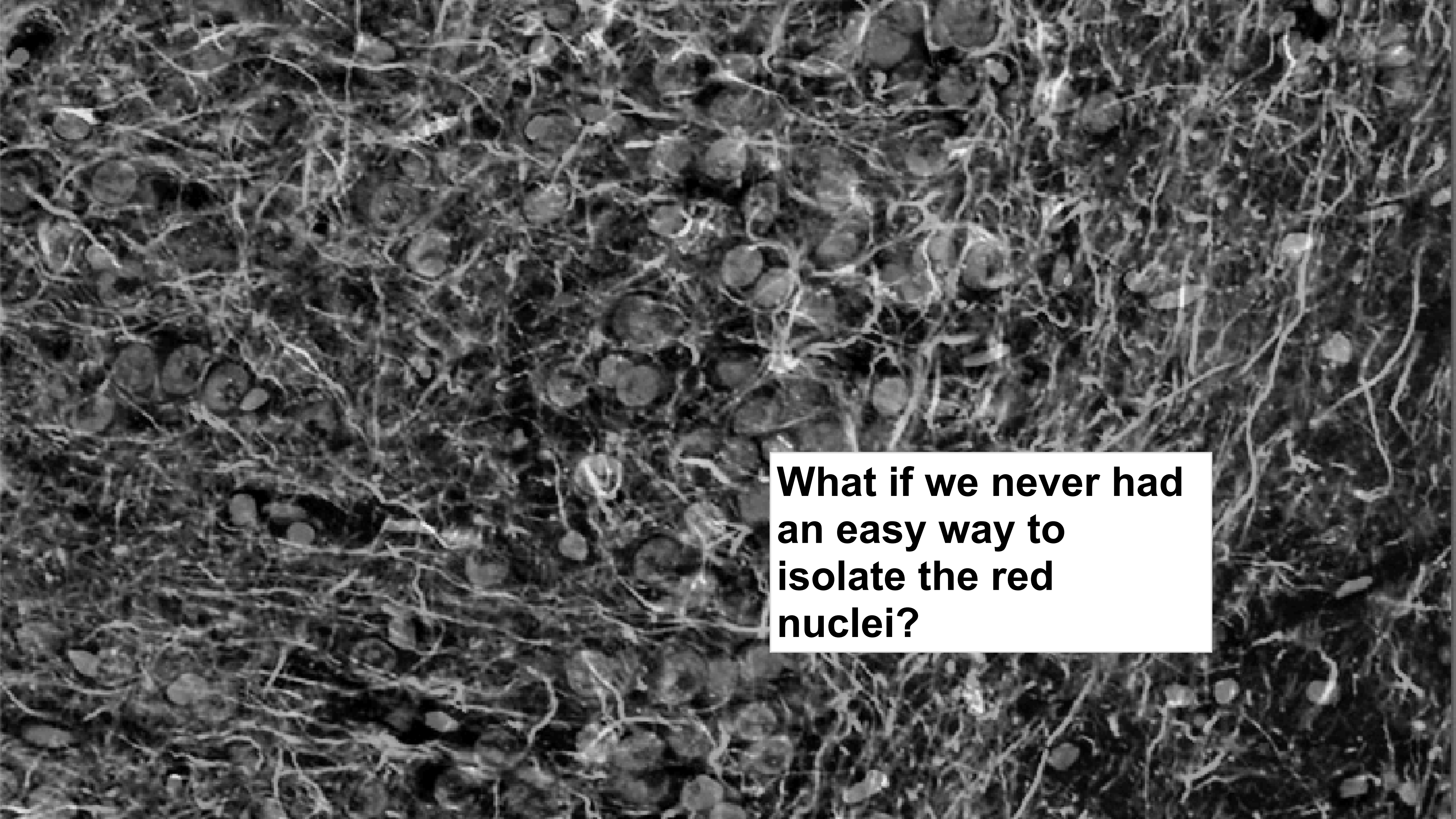
$$\phi_1 = \eta_{20} + \eta_{02} \qquad (11\text{-}39)$$

$$\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2 \qquad (11\text{-}40)$$

$$\phi_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2 \qquad (11\text{-}41)$$

$$\phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2 \qquad (11\text{-}42)$$




What if we never had an easy way to isolate the red nuclei?

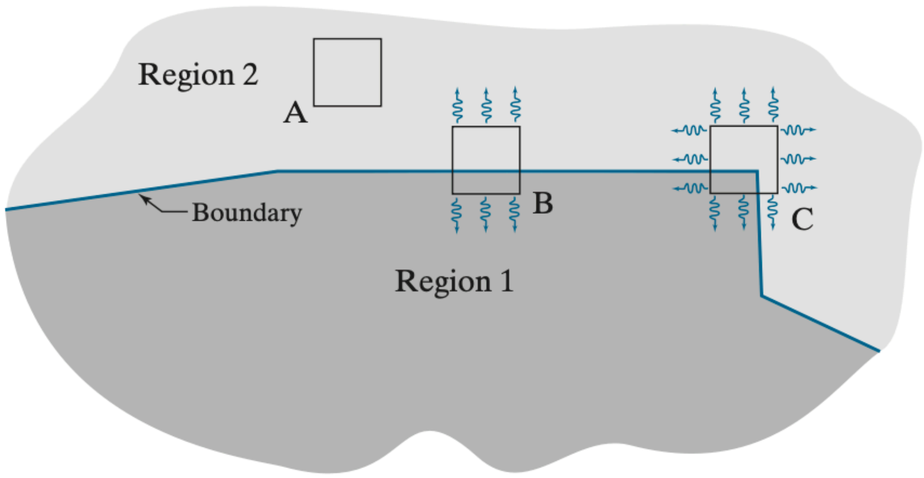
# Corner Descriptors

- **Features**
  - Harris Stephens
  - SIFT
  - SURF

# Corner Descriptors: Harris Stephens

- **Compute Intensity Changes**
  - Scenario A: Areas of zero (or small) intensity changes in all directions
  - Scenario B: areas of changes in one direction but no (or small) changes in the orthogonal direction
  - Scenario C: areas of significant changes in all directions

**FIGURE 11.45** Illustration of how the Harris-Stephens corner detector operates in the three types of subregions indicated by A (flat), B (edge), and C (corner). The wiggly arrows indicate graphically a directional response in the detector as it moves in the three areas shown.



# Corner Descriptors: Harris Stephens

- **Compare shifted patches**

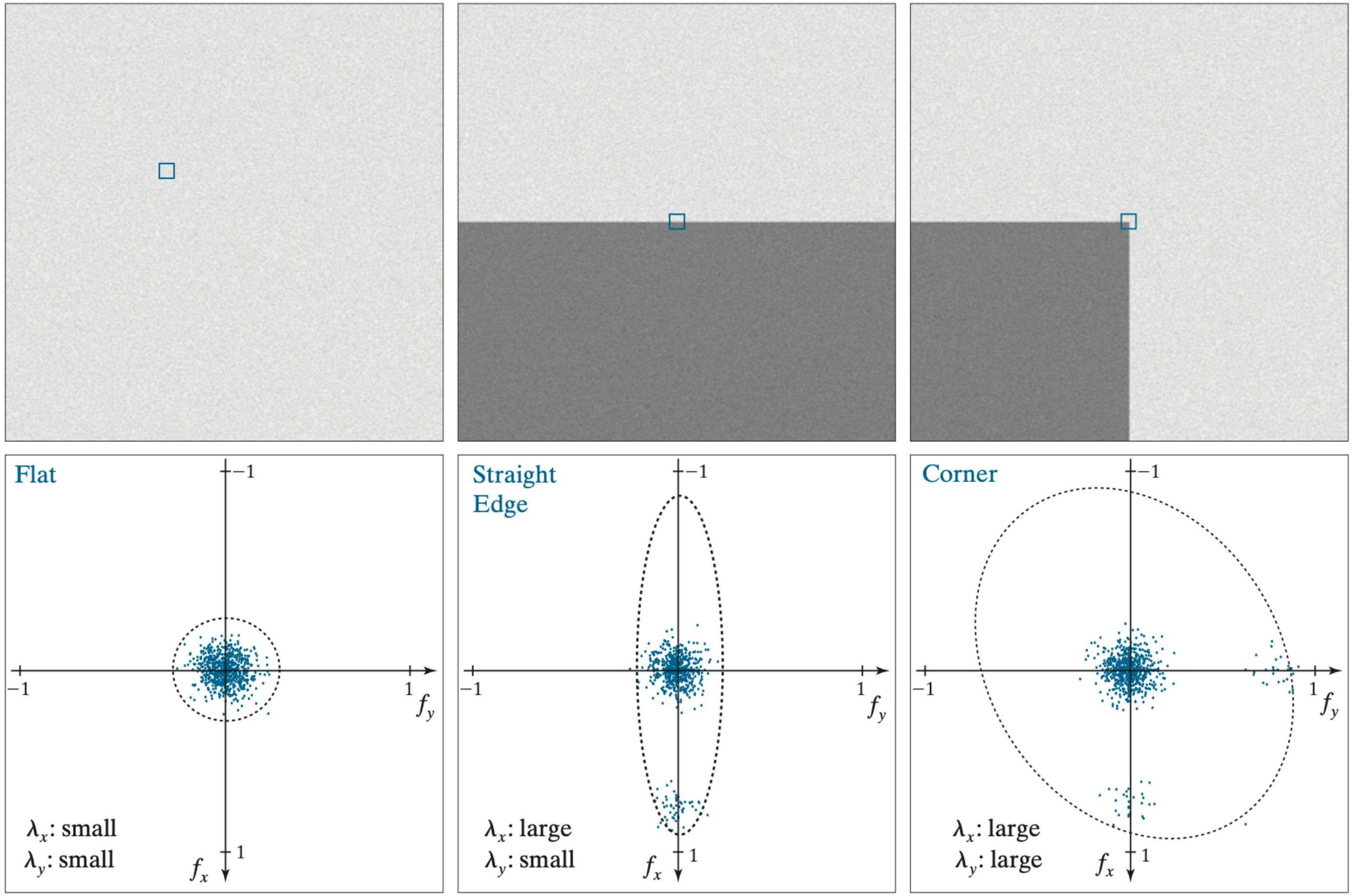
$$C(x,y) = \sum_s \sum_t w(s,t)\left[f(s+x,t+y) - f(s,t)\right]^2$$

- **Rewrite using Taylor Expansion**

$$C(x,y) = [x \ \ y]\,\mathbf{M}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\mathbf{M} = \sum_s \sum_t w(s,t)\,\mathbf{A}$$

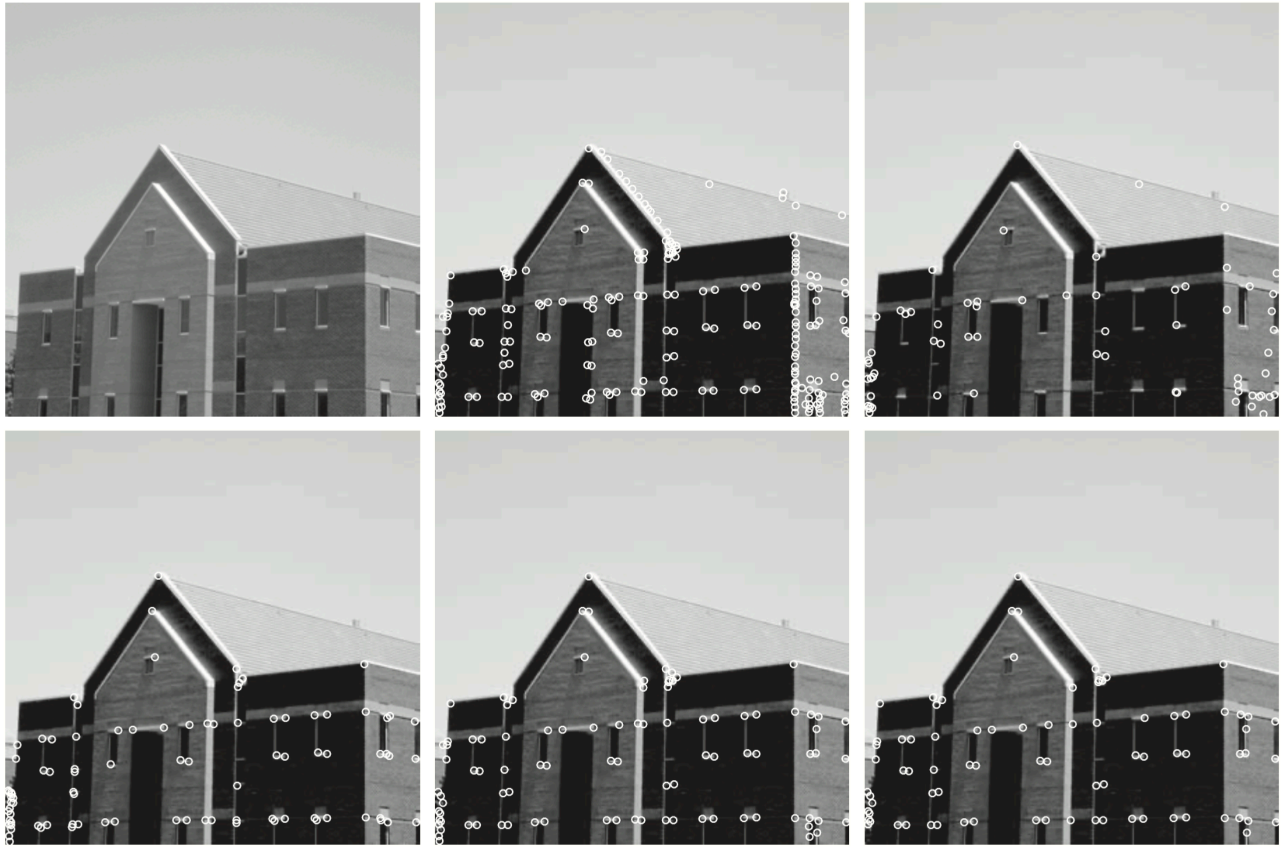
$$\mathbf{A} = \begin{bmatrix} f_x^2 & f_x f_y \\ f_x f_y & f_y^2 \end{bmatrix}$$




a b c
d e f

**FIGURE 11.46** (a)–(c) Noisy images and image patches (small squares) encompassing image regions similar in content to those in Fig. 11.45. (d)–(f) Plots of value pairs $(f_x, f_y)$ showing the characteristics of the eigenvalues of **M** that are useful for detecting the presence of a corner in an image patch.

# Corner Descriptors: Harris Stephens



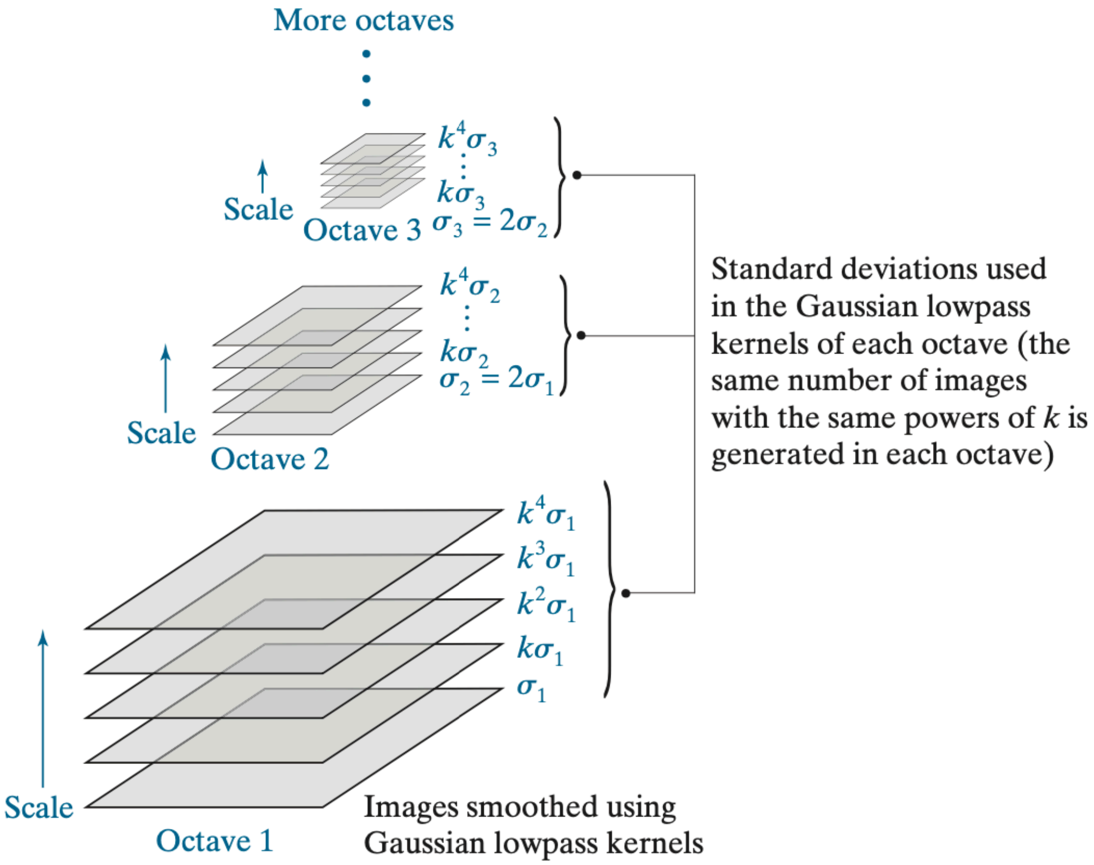
| a | b | c |
|---|---|---|
| d | e | f |

**FIGURE 11.49** $600 \times 600$ image of a building. (b) Result of applying the HS corner detector with $k = 0.04$ and $T = 0.01$ (the default values in our implementation). Numerous irrelevant corners were detected. (c) Result using $k = 0.249$ and the default value for $T$. (d) Result using $k = 0.17$ and $T = 0.05$. (e) Result using the default value for $k$ and $T = 0.05$. (f) Result using the default value of $k$ and $T = 0.07$.

# Corner Descriptors: SIFT

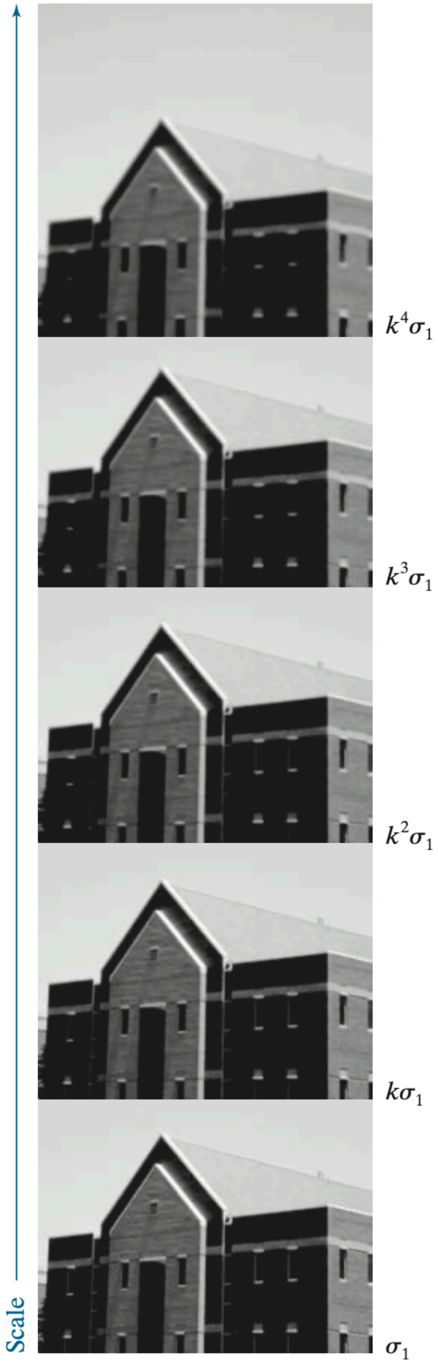
- **SCALE-INVARIANT FEATURE TRANSFORM (SIFT)**
  - invariant to image scale and rotation
  - robust across a range of affine distortions, changes in 3-D viewpoint, noise, and changes of illumination
  - very fast implementations
  - used often

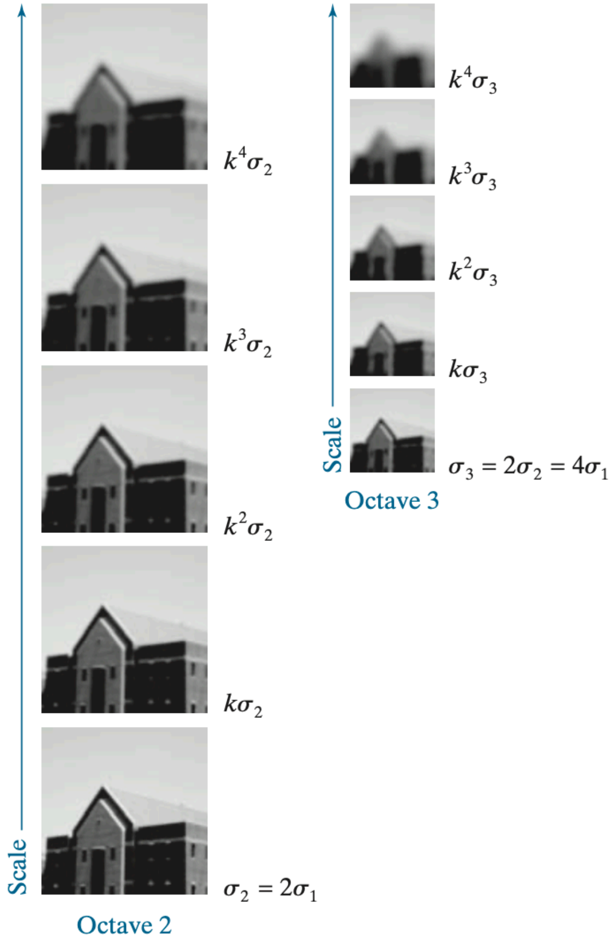
**FIGURE 11.56** Scale space, showing three octaves. Because $s = 2$ in this case, each octave has five smoothed images. A Gaussian kernel was used for smoothing, so the space parameter is $\sigma$.



# Corner Descriptors: SIFT

**FIGURE 11.57** Illustration using images of the first three octaves of scale space in SIFT. The entries in the table are values of standard deviation used at each scale of each octave. For example the standard deviation used in scale 2 of octave 1 is $k\sigma_1$, which is equal to 1.0. (The images of octave 1 are shown slightly overlapped to fit in the figure space.)




$\sigma_1 = \sqrt{2}/2 = 0.707 \quad k = \sqrt{2} = 1.414$

| Octave | Scale 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **1** | 0.707 | 1.000 | 1.414 | 2.000 | 2.828 |
| **2** | 1.414 | 2.000 | 2.828 | 4.000 | 5.657 |
| **3** | 2.828 | 4.000 | 5.657 | 8.000 | 11.314 |

# Corner Descriptors: SIFT

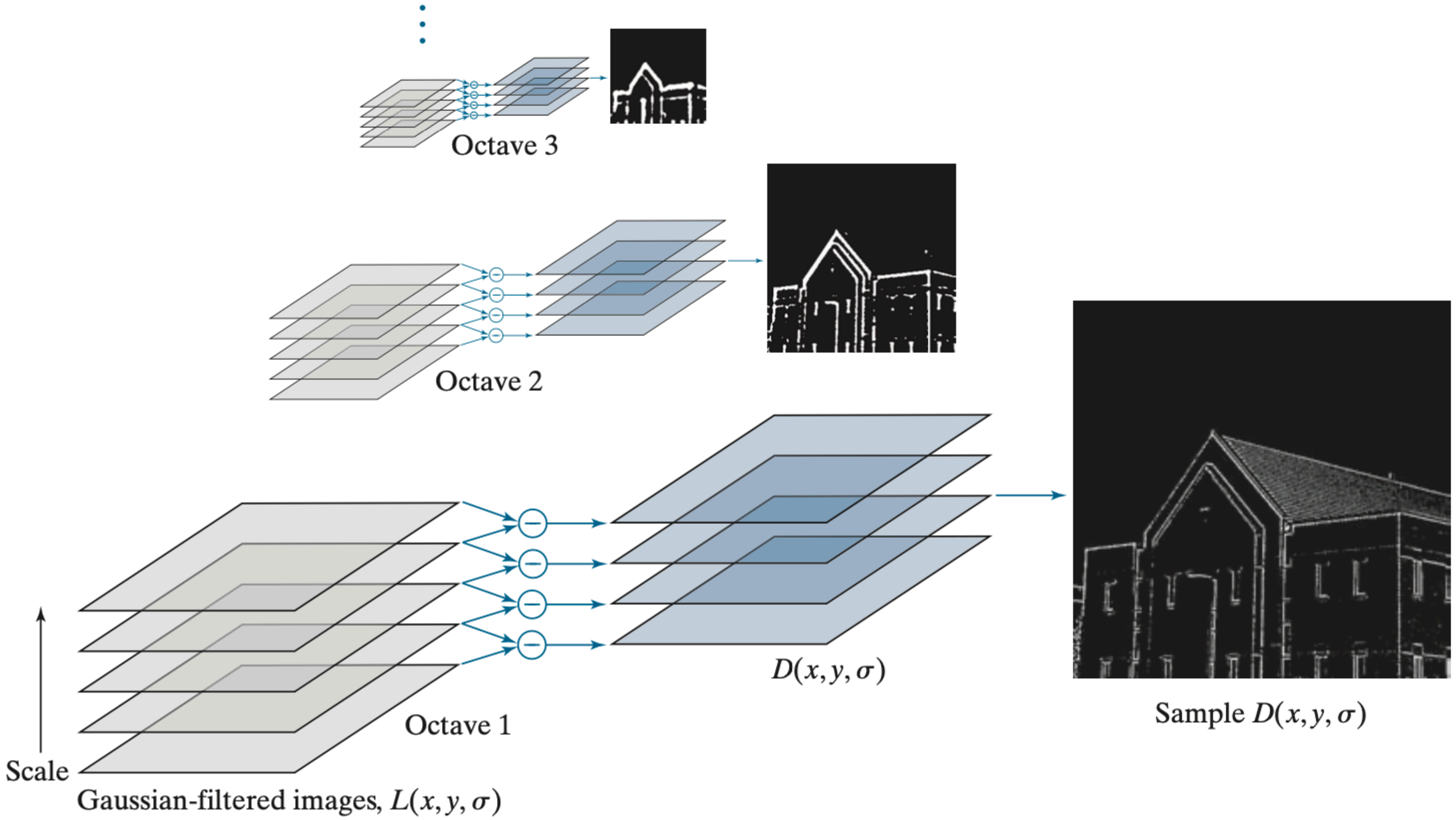


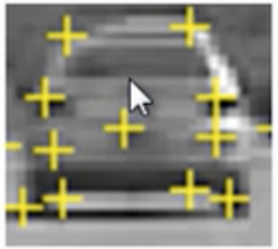

**FIGURE 11.58** How Eq. (11-69) is implemented in scale space. There are $s + 3$ $L(x, y, \sigma)$ images and $s + 2$ corresponding $D(x, y, \sigma)$ images in each octave.

# Corner Descriptors: SIFT

# Corner Descriptors

- **Task: Track car**
  - Edges - Significant gradient change in one direction
  - Corners - Significant gradient change in two directions
  - SURF - Speeded Up Robust Features, patented local feature detector, similar to SIFT
  - MSER - Maximally Stable Extremal Regions

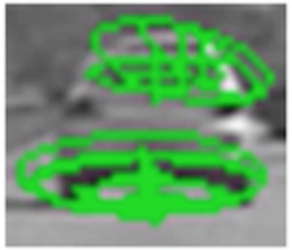
Edges



Corners

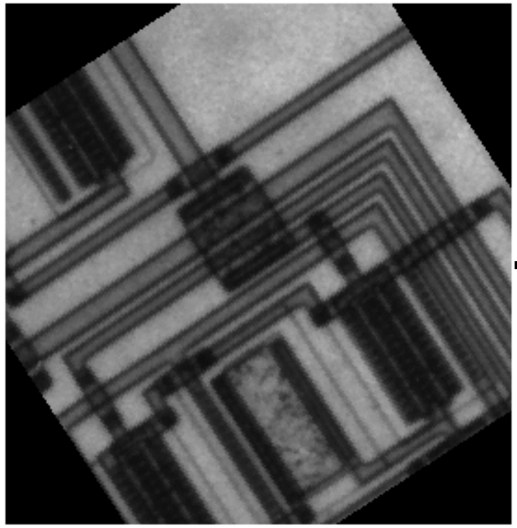
Template

SURF

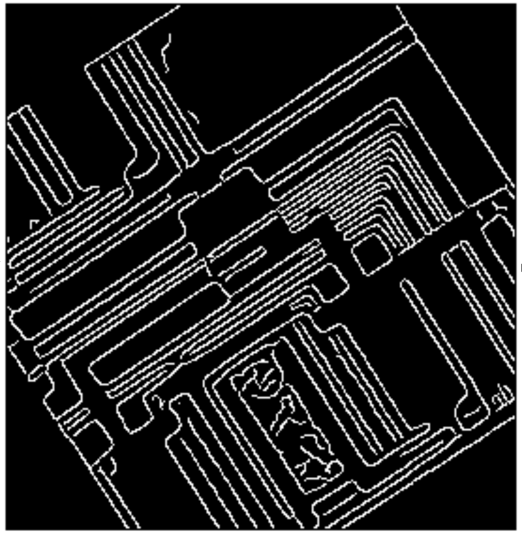


MSER

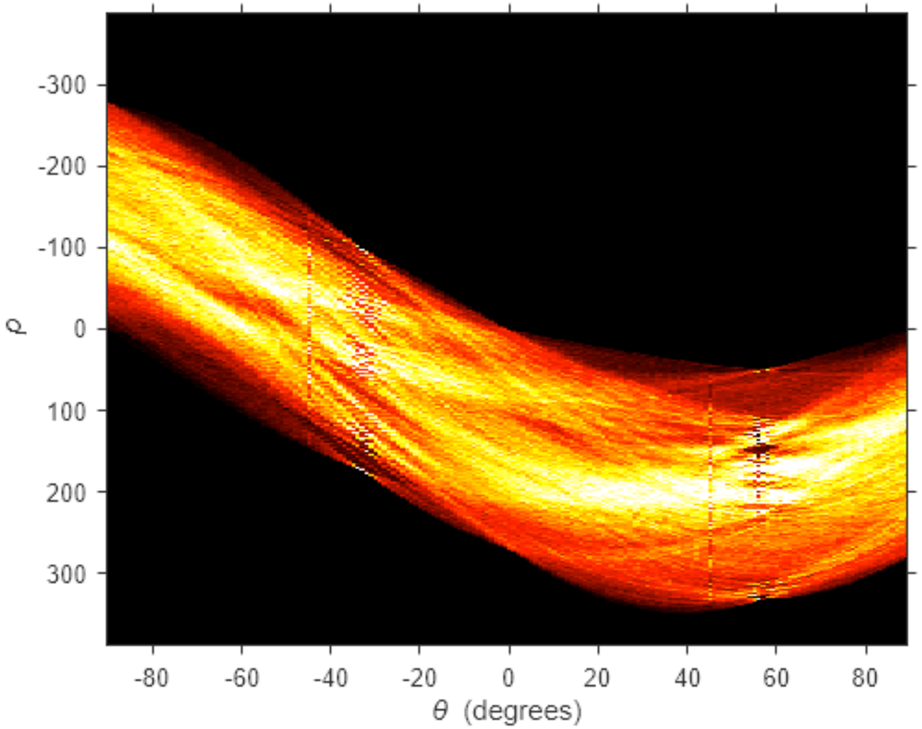
# Straight Line objects: Hough Transform



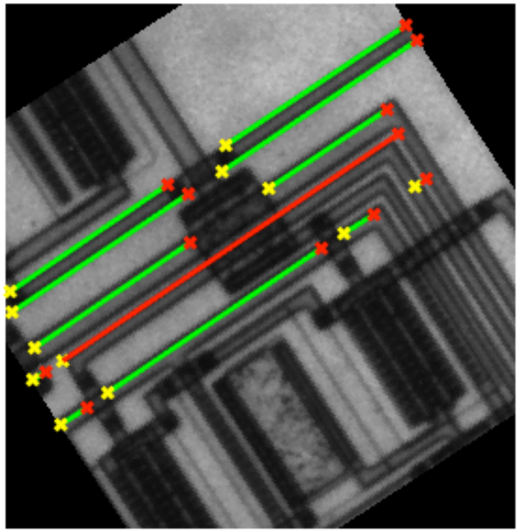
Edge Detection



Hough Transform




Find Peaks in HT

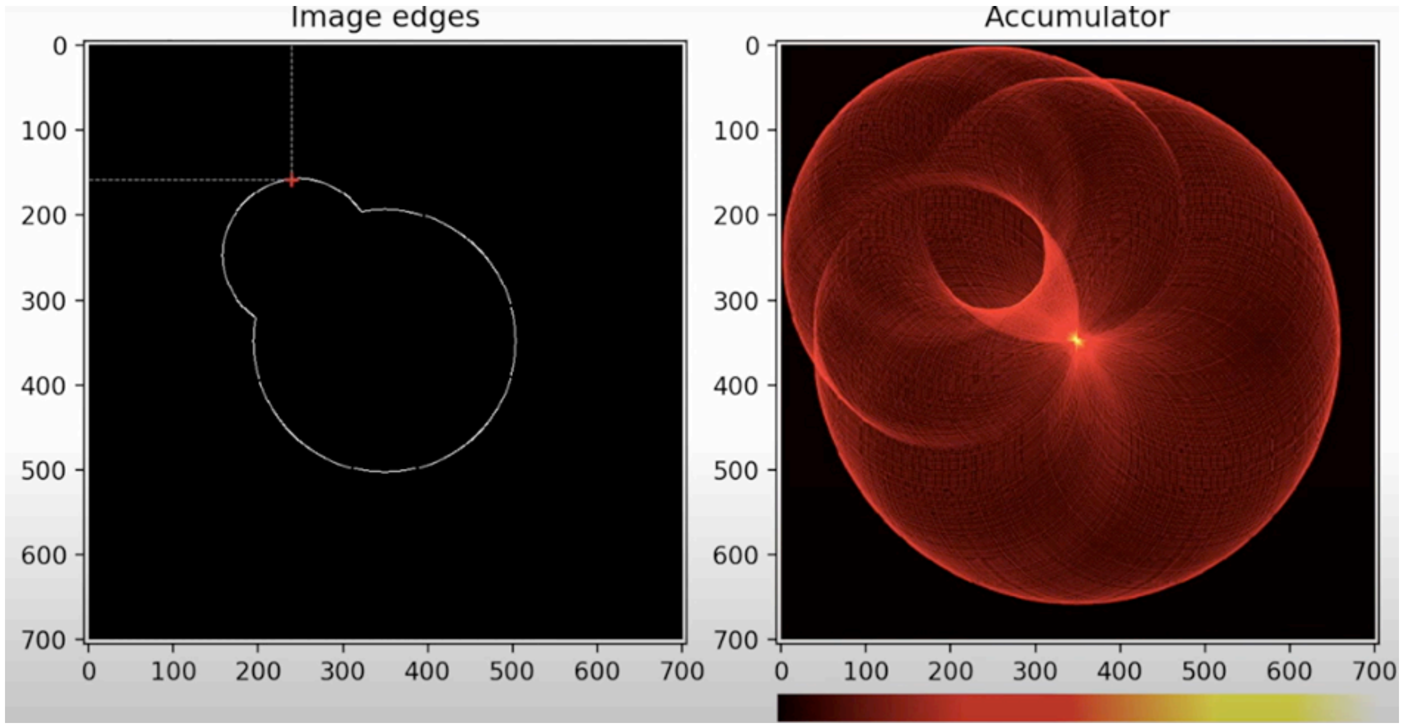


$$x\cos(\theta) + y\sin(\theta) = \rho$$

Nice Visualization:
https://www.youtube.com/watch?v=X1DxCPS1iwA

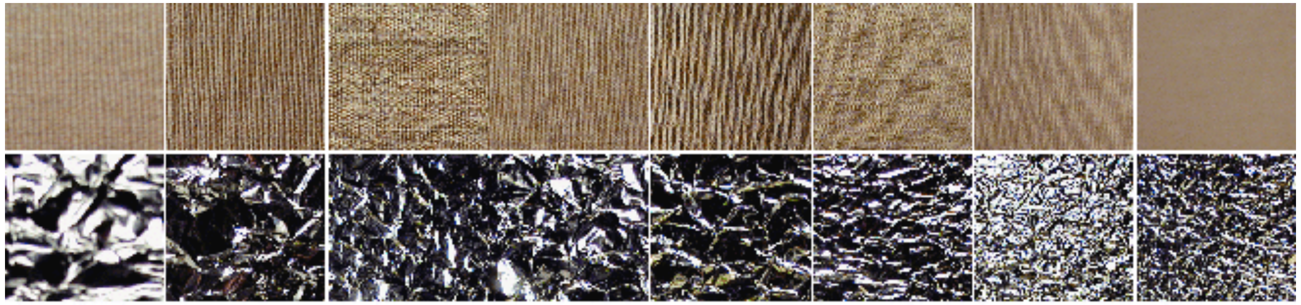
# Round objects: Circular Hough Transform

- **Circle**
  - $(x - x_0)^2 + (y - y_0)^2 = r^2$

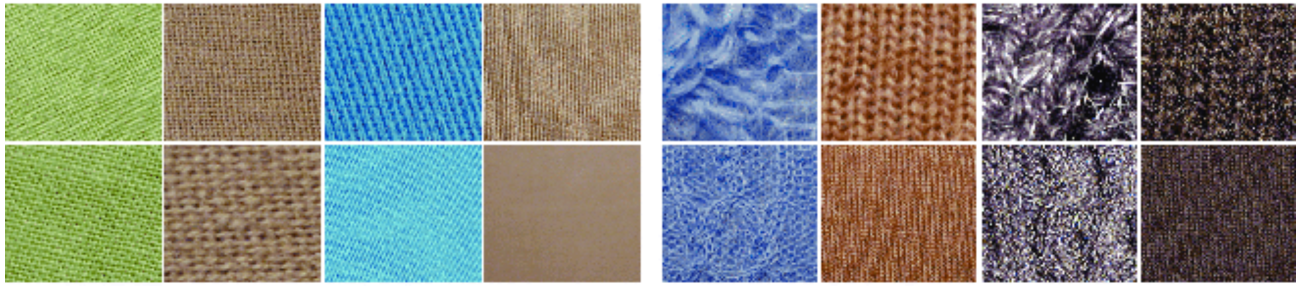


# Applications

- **Classification of Images**



(a) KTH-TIPS



(b) KTH-TIPS2a

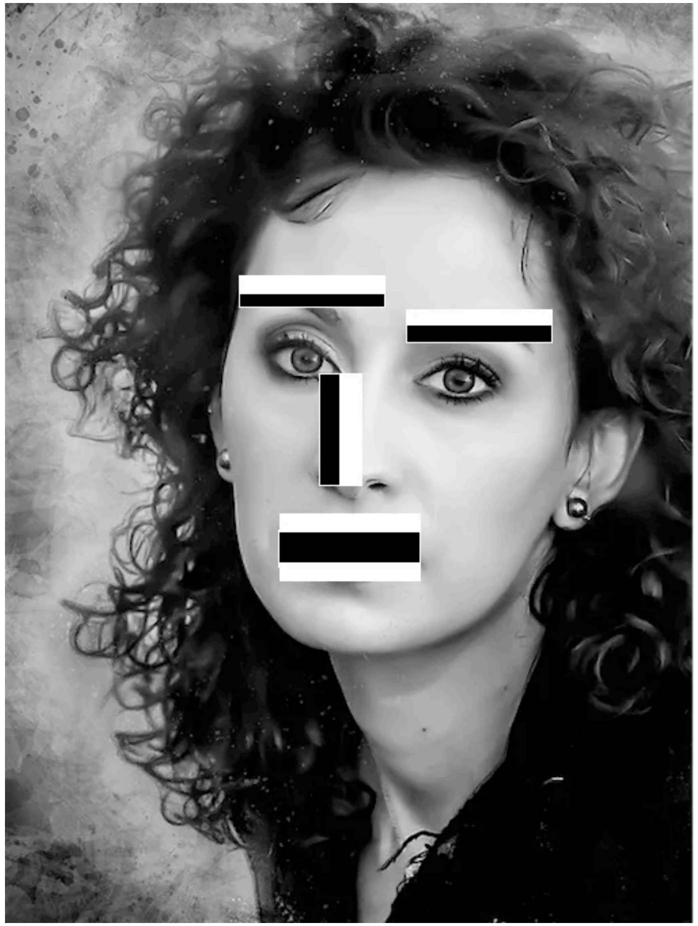
# Applications

- **Classification of Images**
- **Segmentation of Images**

Source: https://medium.com/analytics-vidhya/what-is-haar-features-used-in-face-detection-a7e531c8332b

# Applications

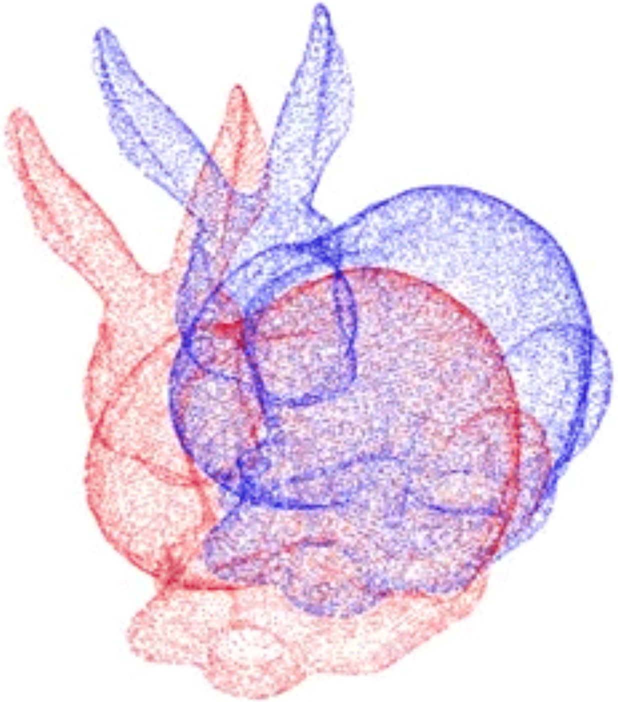
- **Classification of Images**
- **Segmentation of Images**
- **Object Detection**

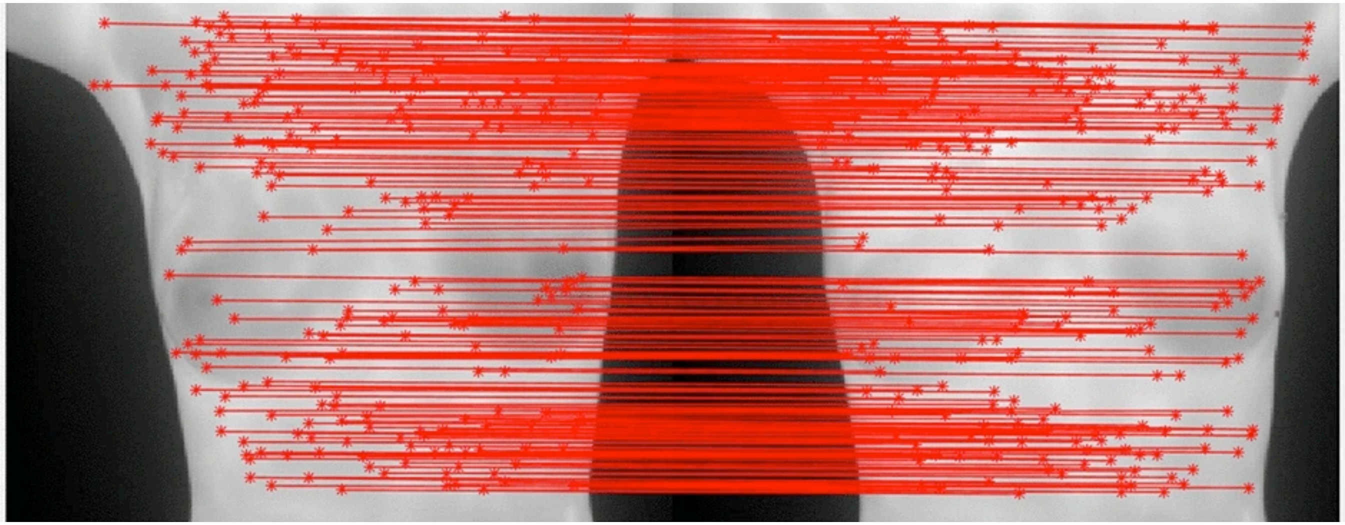


Source: https://medium.com/pymedix/a-brief-history-of-medical-image-registration-part-2-aecd188855fc

# Applications

- Classification of Images
- Segmentation of Images
- Object Detection
- Registration



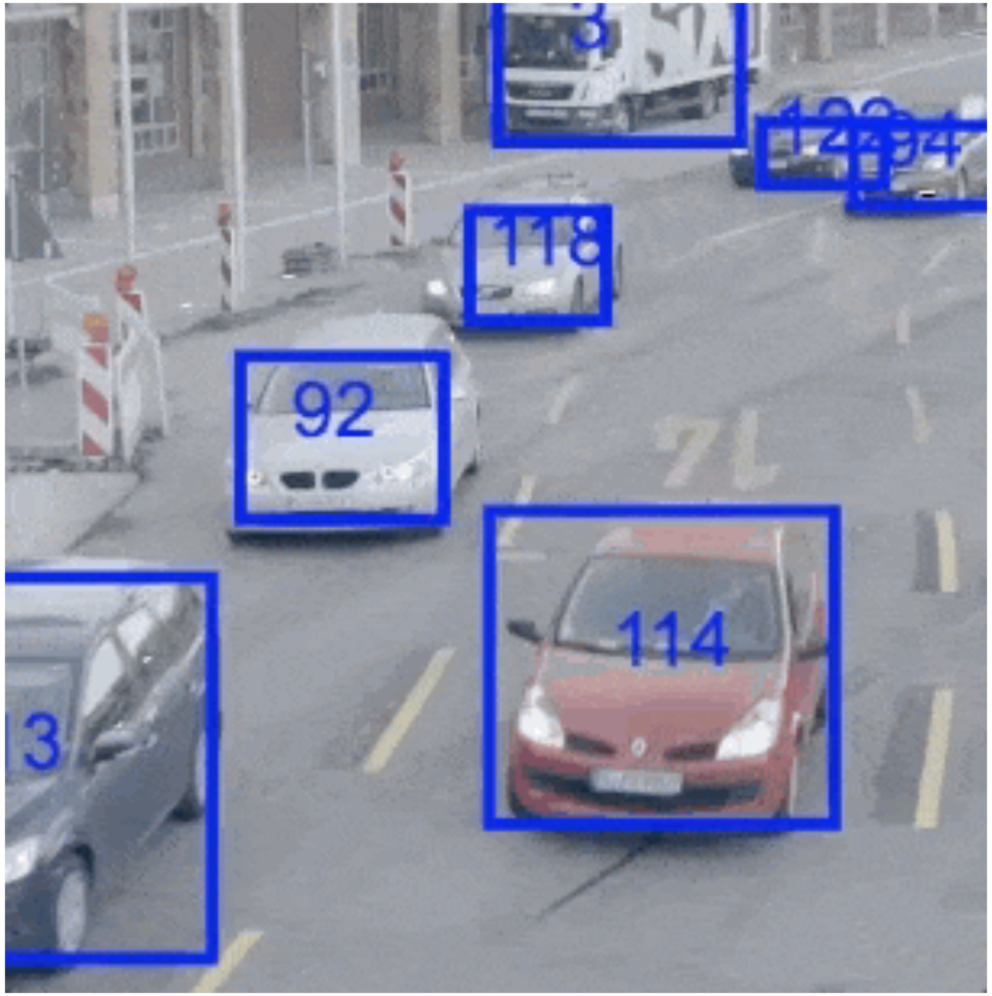


Source: https://www.nature.com/articles/s41598-022-11379-2

Source: https://vcs1994.medium.com/roadmap-of-object-tracking-fc520a0bae2a

# Applications

- Classification of Images
- Segmentation of Images
- Object Detection
- Registration
- Tracking



# Summary

- **Choice of feature descriptor entirely dependent on**
  - Your downstream task
  - The images
- **Descriptions**
  - boundaries
  - region shapes
  - textures in regions
  - significant features like corners, lines, circles
- **Invariance**
  - ask yourself what the descriptor should be invariant and robust against