Texture

One of the simplest approaches for describing texture is to use statistical moments of the intensity histogram of an image or region. Let z be a random variable denoting intensity, and let $p(z_i)$, $i = 0,1,2,\ldots,L-1$, be the corresponding normalized histogram, where L is the number of distinct intensity levels. From Eq. (3-24), the nth moment of z about the mean is

$$\mu_n(z) = \sum_{i=0}^{L-1} (z_i - m)^n p(z_i)$$

where m is the mean value of z (i.e., the average intensity of the image or region):

$$m = \sum_{i=0}^{L-1} z_i p(z_i)$$

Note from Eq. (11-27) that $\mu_0 = 1$ and $\mu_1 = 0$. The second moment [the variance $\sigma^2(z) = \mu_2(z)$] is particularly important in texture description. It is a measure of intensity contrast that can be used to establish descriptors of relative intensity smoothness. For example, the measure

$$R(z) = 1 - \frac{1}{1 + \sigma^2(z)}$$

is 0 for areas of constant intensity (the variance is zero there) and approaches 1 for large values of $\sigma^2(z)$. Because variance values tend to be large for grayscale images with values, for example, in the range 0 to 255, it is a good idea to normalize the variance to the interval $[0,1]$ for use in Eq. (11-29). This is done simply by dividing $\sigma^2(z)$ by $(L-1)^2$ in Eq. (11-29). The standard deviation, $\sigma(z)$, also is used frequently as a measure of texture because its values are more intuitive.

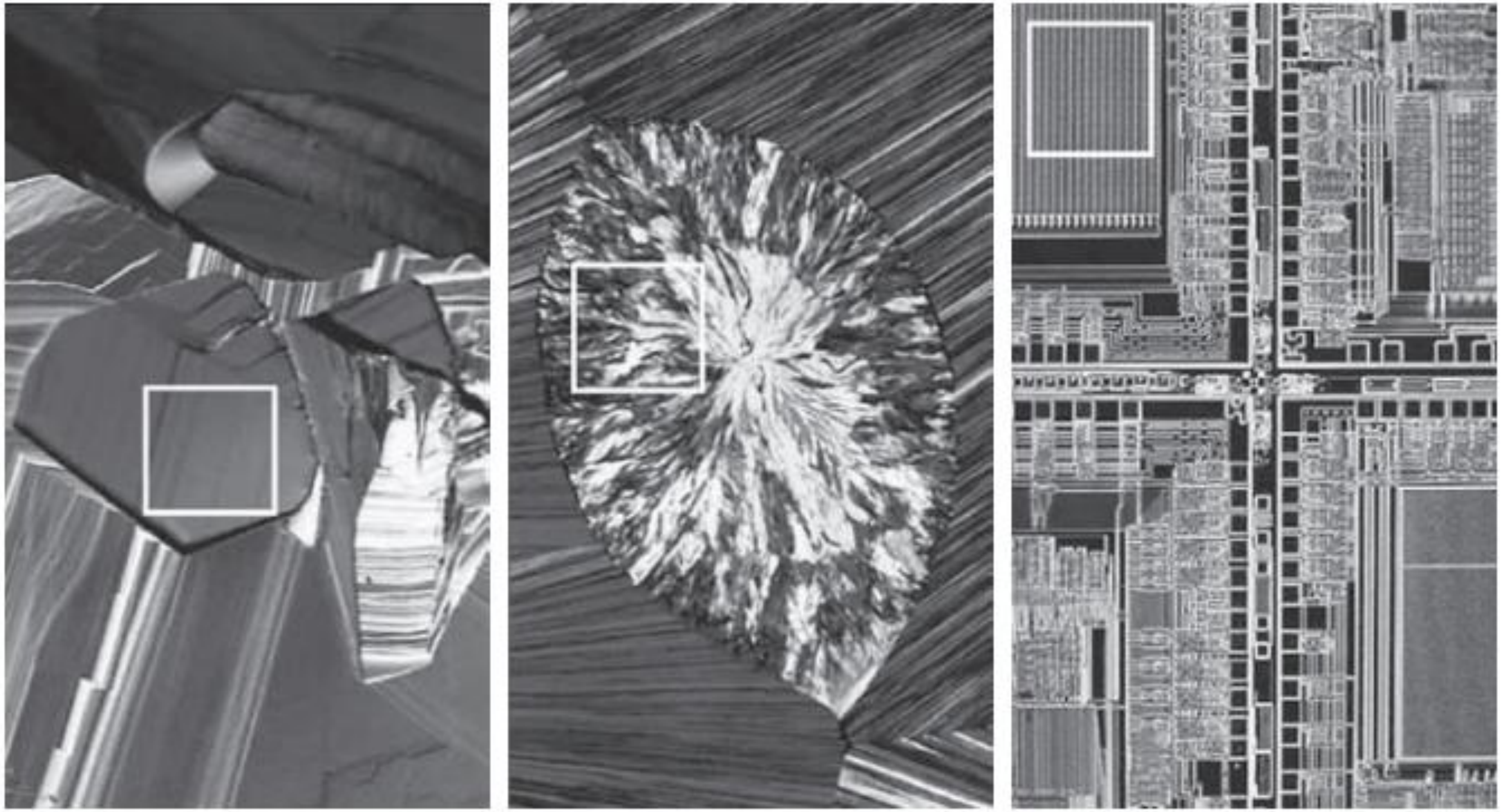
As discussed in Section 2.6, the third moment, $\mu_3(z)$, is a measure of the skewness of the histogram while the fourth moment, $\mu_4(z)$, is a measure of its relative flatness. The fifth and higher moments are not so easily related to histogram shape, but they do provide further quantitative discrimination of texture content. Some useful additional texture measures based on histograms include a measure of *uniformity*, defined as

$$U(z) = \sum_{i=0}^{L-1} p^2(z_i)$$

and a measure of *average entropy* that, as you may recall from information theory, is defined as

$$e(z) = -\sum_{i=0}^{L-1} p(z_i) \log_2 p(z_i)$$

Because values of p are in the range [0, 1] and their sum equals 1, the value of descriptor U is maximum for an image in which all intensity levels are equal (maximally uniform), and decreases from there. Entropy is a measure of variability, and is 0 for a constant image.



a b c

FIGURE 11.29
The white squares mark, from left to right, smooth, coarse, and regular textures. These are optical microscope images of a superconductor, human cholesterol, and a microprocessor. (Courtesy of Dr. Michael W. Davidson, Florida State University.)

Statistical texture measures for the subimages in Fig.

Texture	Mean	Standard deviation	R (normalized)	3rd moment	Uniformity	Entropy
Smooth	82.64	11.79	0.002	−0.105	0.026	5.434
Coarse	143.56	74.63	0.079	−0.151	0.005	7.783
Regular	99.72	33.73	0.017	0.750	0.013	6.674

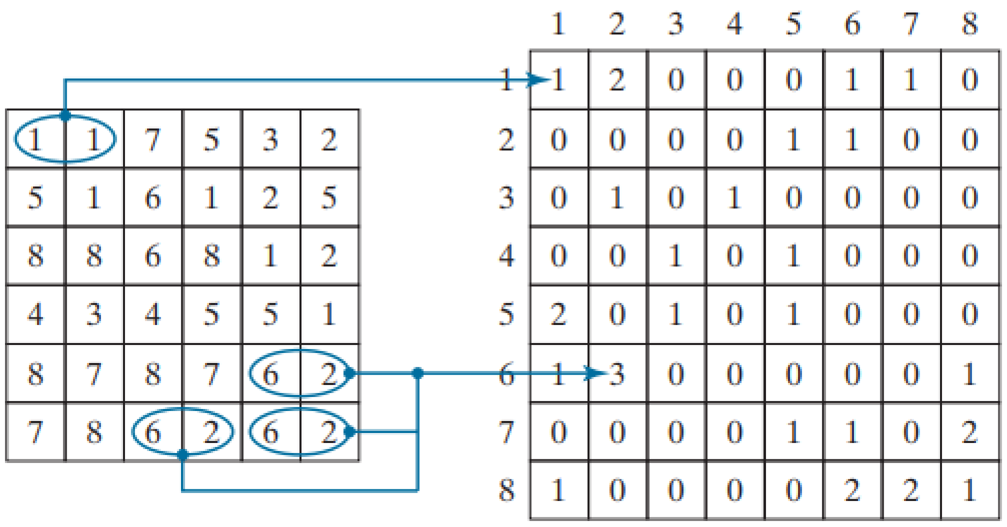


Image f Co-occurrence matrix $\mathbf{G}$

Descriptor	Explanation	Formula
Maximum probability	Measures the strongest response of **G**. The range of values is [0, 1].	$\max_{i,j}(p_{ij})$
Correlation	A measure of how correlated a pixel is to its neighbor over the entire image. The range of values is 1 to −1 corresponding to perfect positive and perfect negative correlations. This measure is not defined if either standard deviation is zero.	$\sum_{i=1}^{K}\sum_{j=1}^{K}\frac{(i-m_r)(j-m_c)p_{ij}}{\sigma_r\,\sigma_c}$ $\sigma_r \neq 0;\ \sigma_c \neq 0$
Contrast	A measure of intensity contrast between a pixel and its neighbor over the entire image. The range of values is 0 (when **G** is constant) to $(K-1)^2$.	$\sum_{i=1}^{K}\sum_{j=1}^{K}(i-j)^2\,p_{ij}$
Uniformity (also called Energy)	A measure of uniformity in the range [0, 1]. Uniformity is 1 for a constant image.	$\sum_{i=1}^{K}\sum_{j=1}^{K}p_{ij}^2$
Homogeneity	Measures the spatial closeness to the diagonal of the distribution of elements in **G**. The range of values is [0, 1], with the maximum being achieved when **G** is a diagonal matrix.	$\sum_{i=1}^{K}\sum_{j=1}^{K}\frac{p_{ij}}{1+\lvert i-j\rvert}$
Entropy	Measures the randomness of the elements of **G**. The entropy is 0 when all p_{ij}'s are 0, and is maximum when the p_{ij}'s are uniformly distributed. The maximum value is thus $2\log_2 K$.	$-\sum_{i=1}^{K}\sum_{j=1}^{K}p_{ij}\log_2 p_{ij}$

Multiresolution Analysis

- Blur and subsample

$$
\begin{aligned}
f &= Tf + (1\text{-}T)\,f \\
&= TTf + (1-T)Tf + (1-T)f \\
&= \cdots
\end{aligned}
$$

- T: Laplacian filter, convolution with a Gaussian, lowpass ...

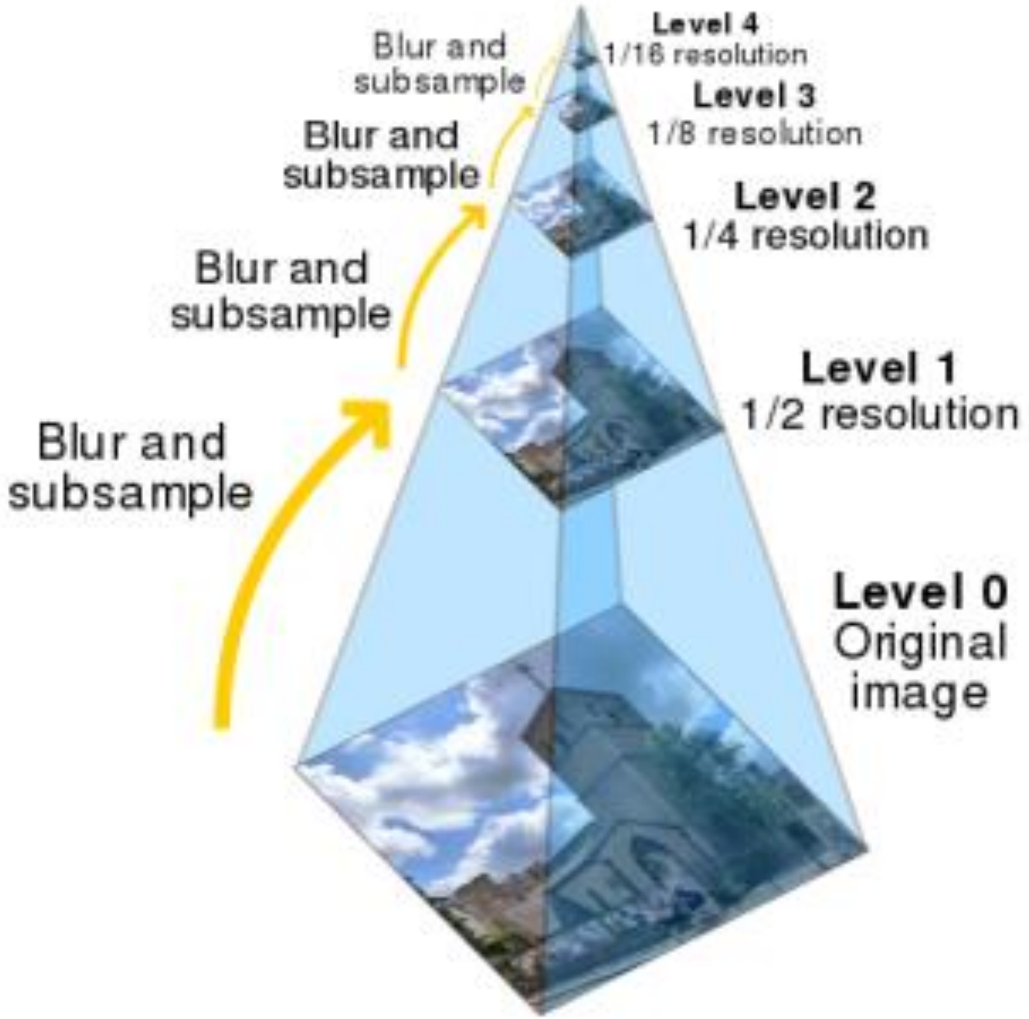


Figure: Wikipedia: Pyramid (image processing)

- Separating low and high frequencies

- Low frequency part: No high frequency, can be subsampled without loss of information
- Generalization to images using 2D filters

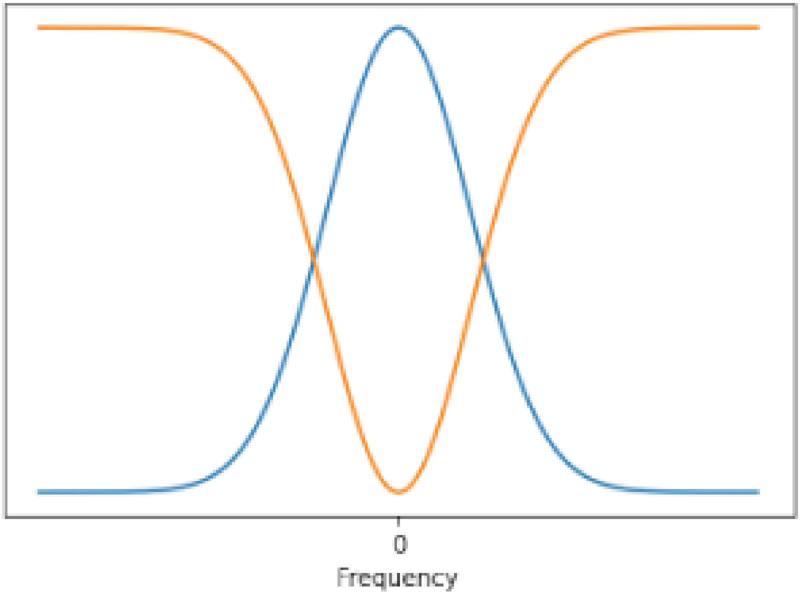
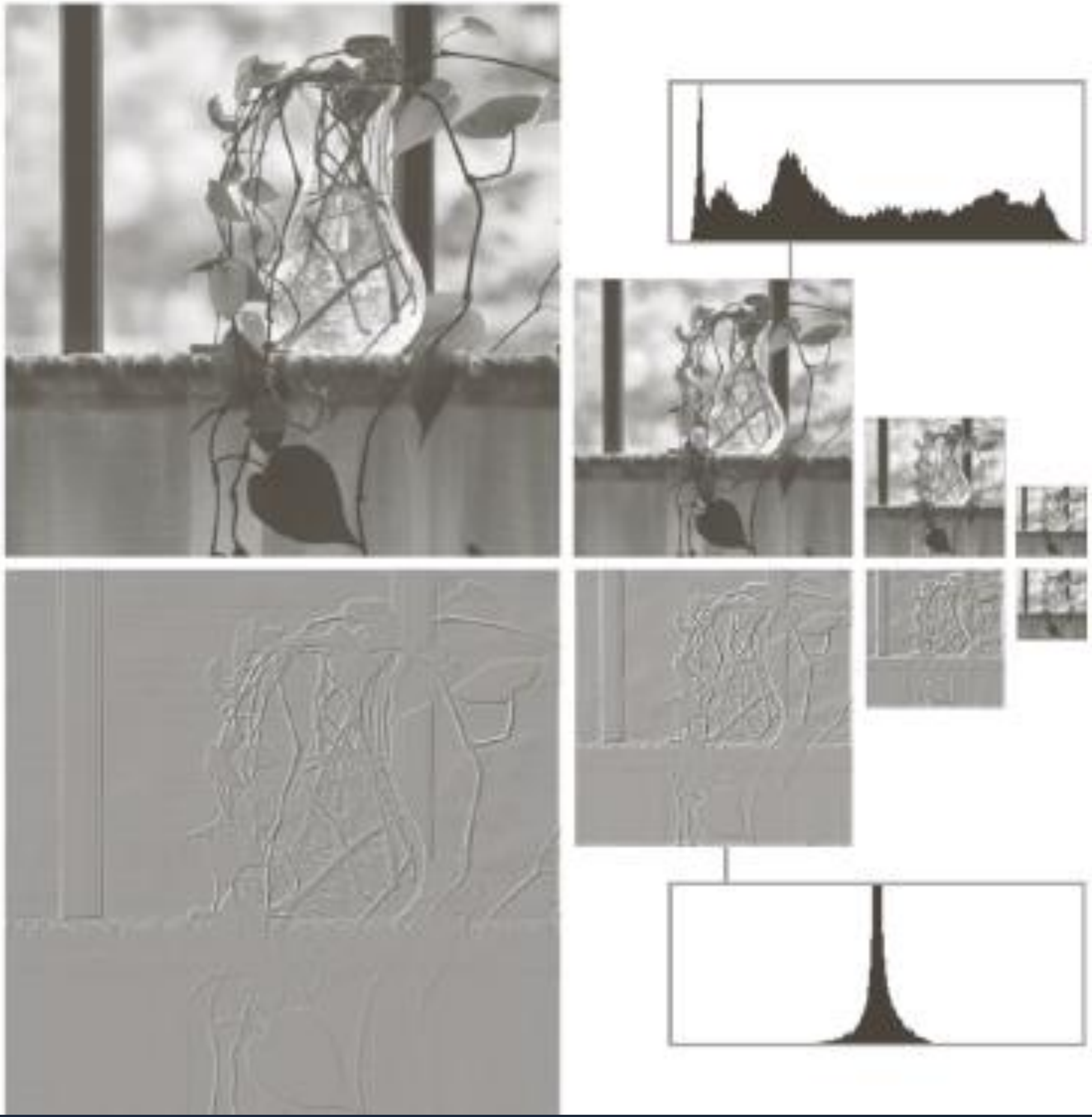


Figure: 2 filters with sum of filters = one



Difference between smoothings lead to bandpass filters

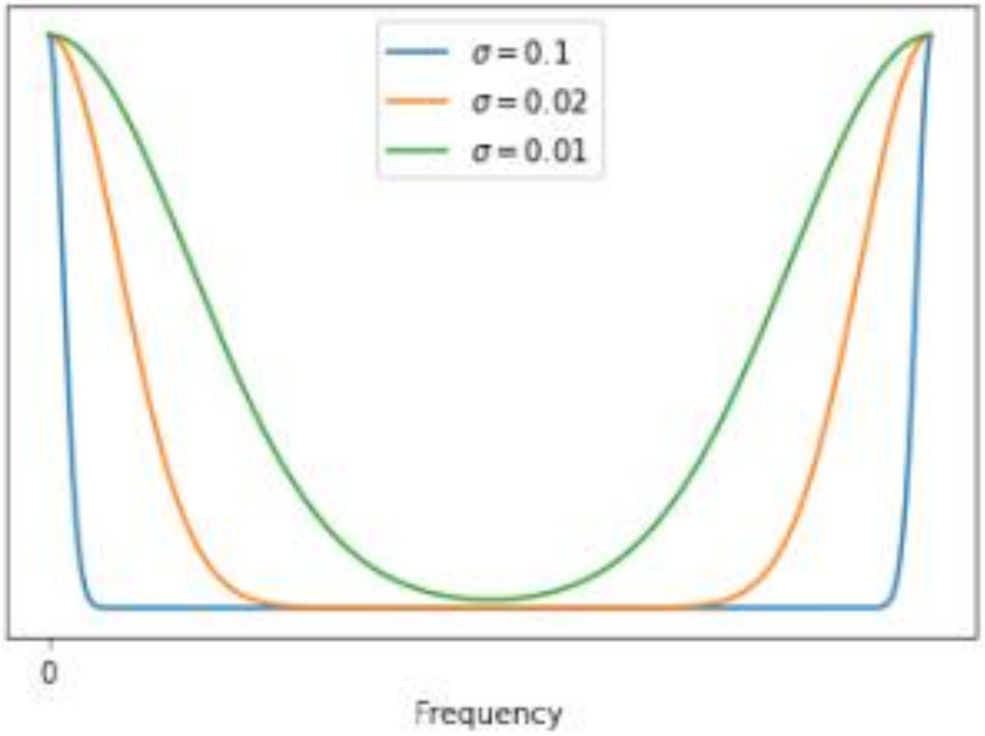


Figure: Spectrum of different Gaussians

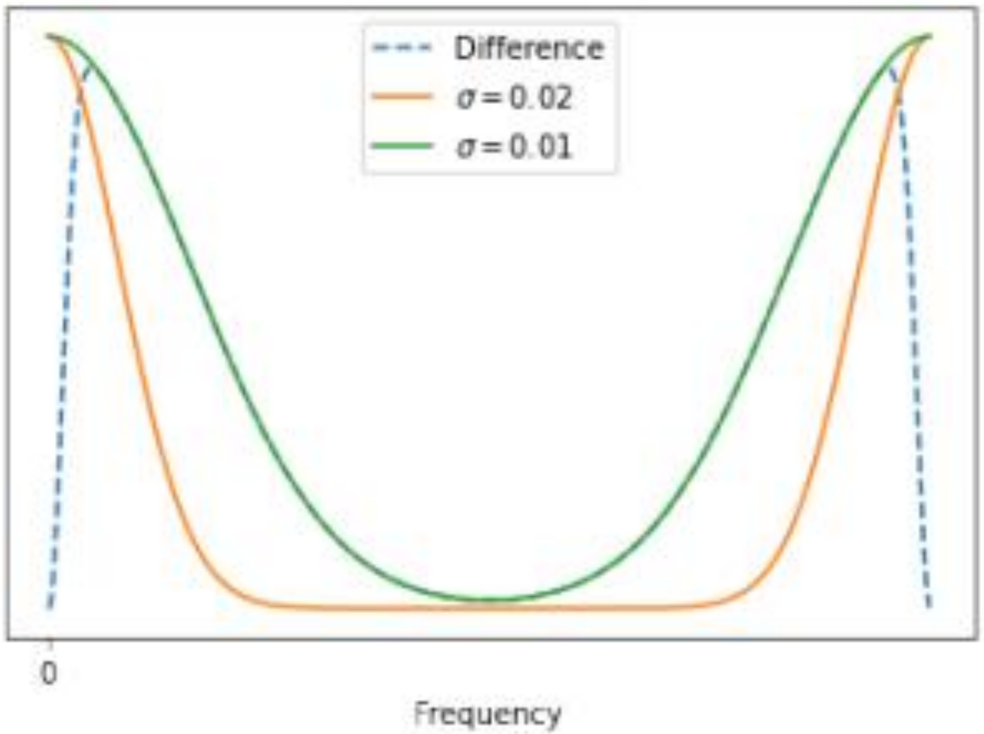


Figure: Difference of Gaussians in Fourier: bandpass filter

- Separating low and high frequencies
- $G2 - G1$ and $1 - (G2 - G1)$

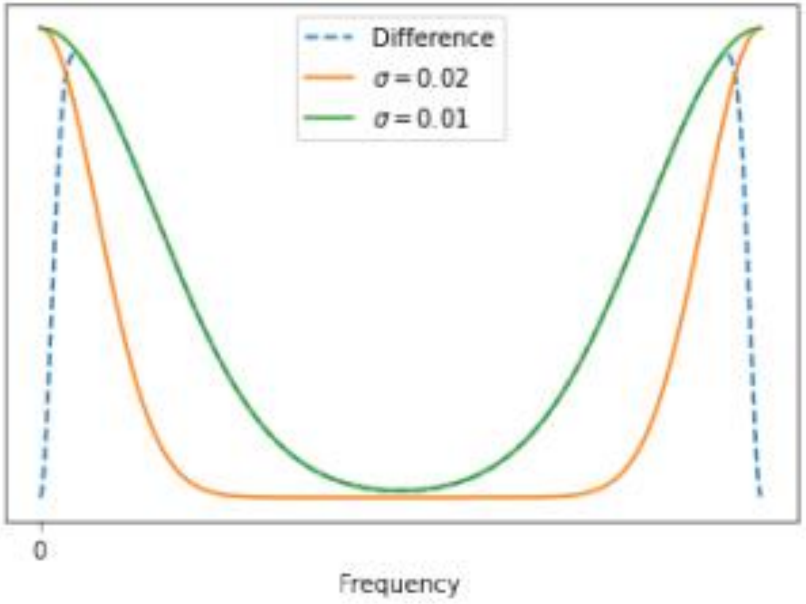


Figure: Spectrum of Gaussians and their difference

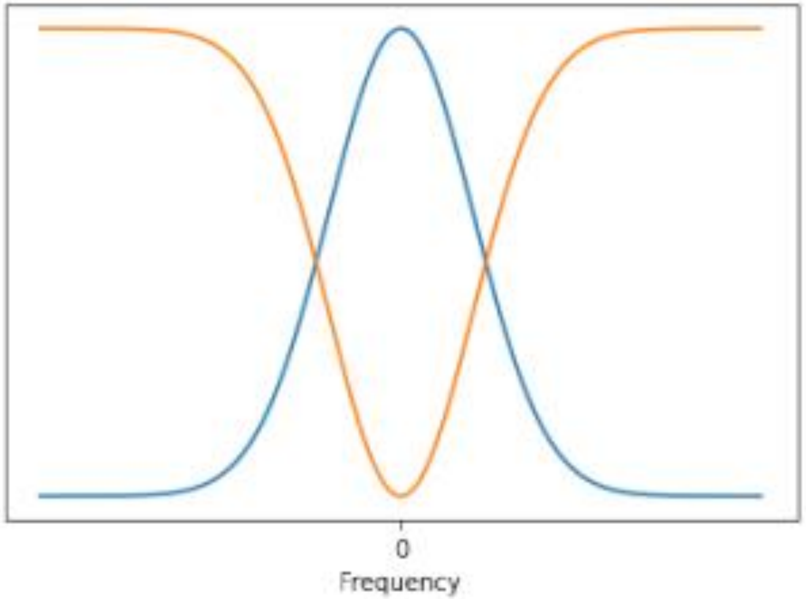


Figure: 2 filters with sum of filters = one

Filterbanks

- Information separated in different frequency bands,
- Note the logarithmic scale!

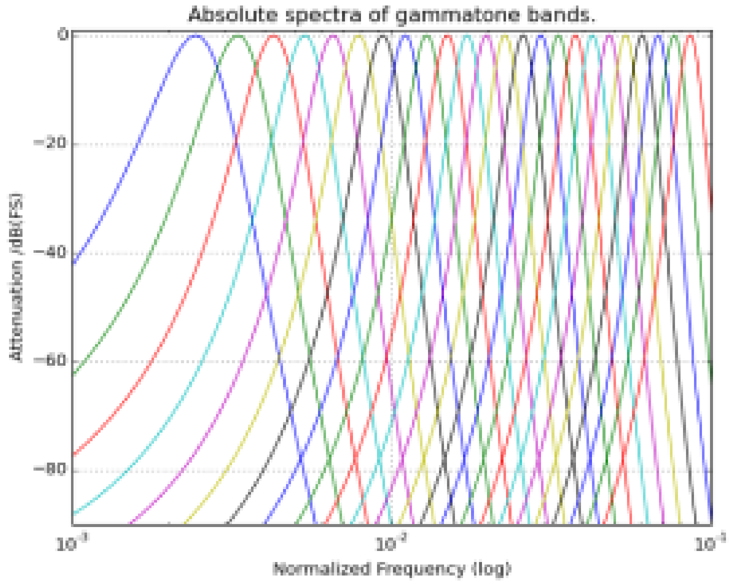


Figure: Filterbank (image from pyfilterbank website)

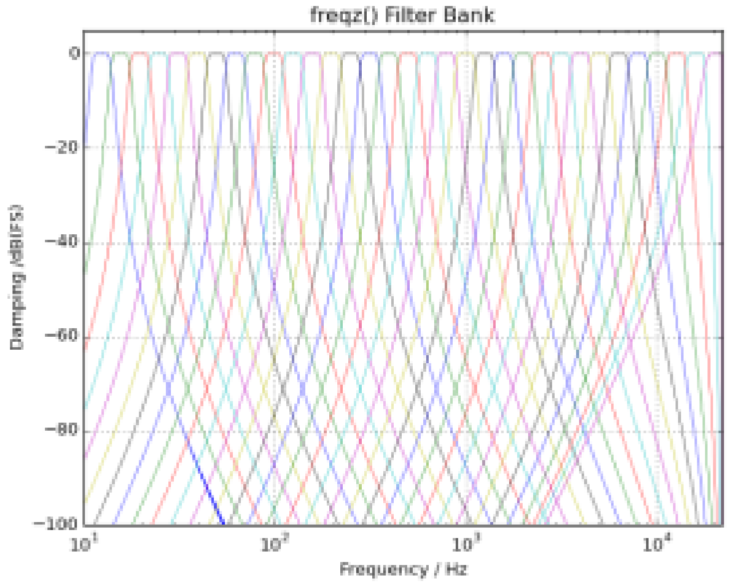
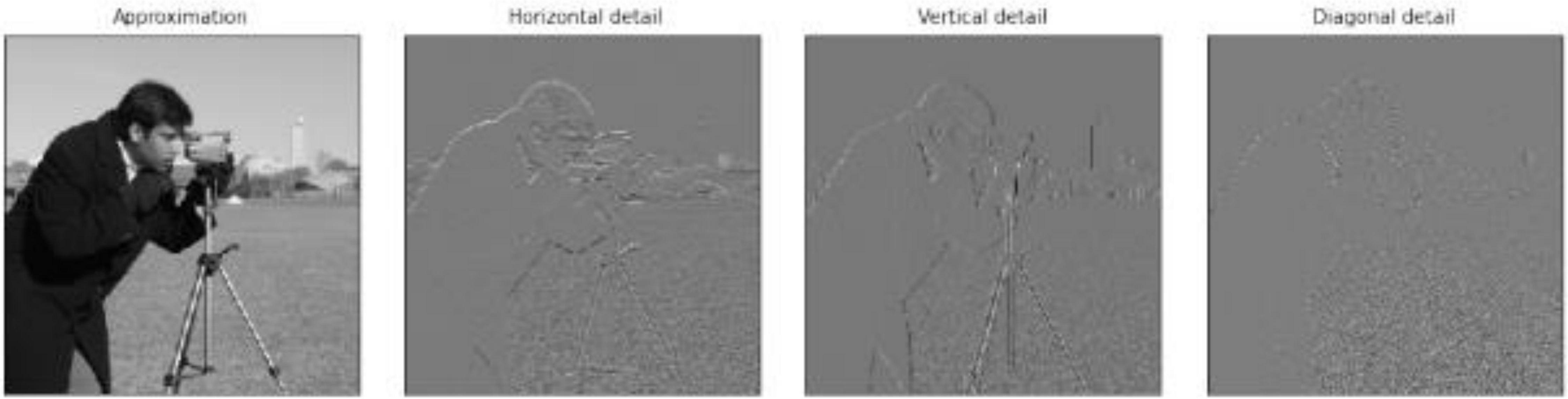
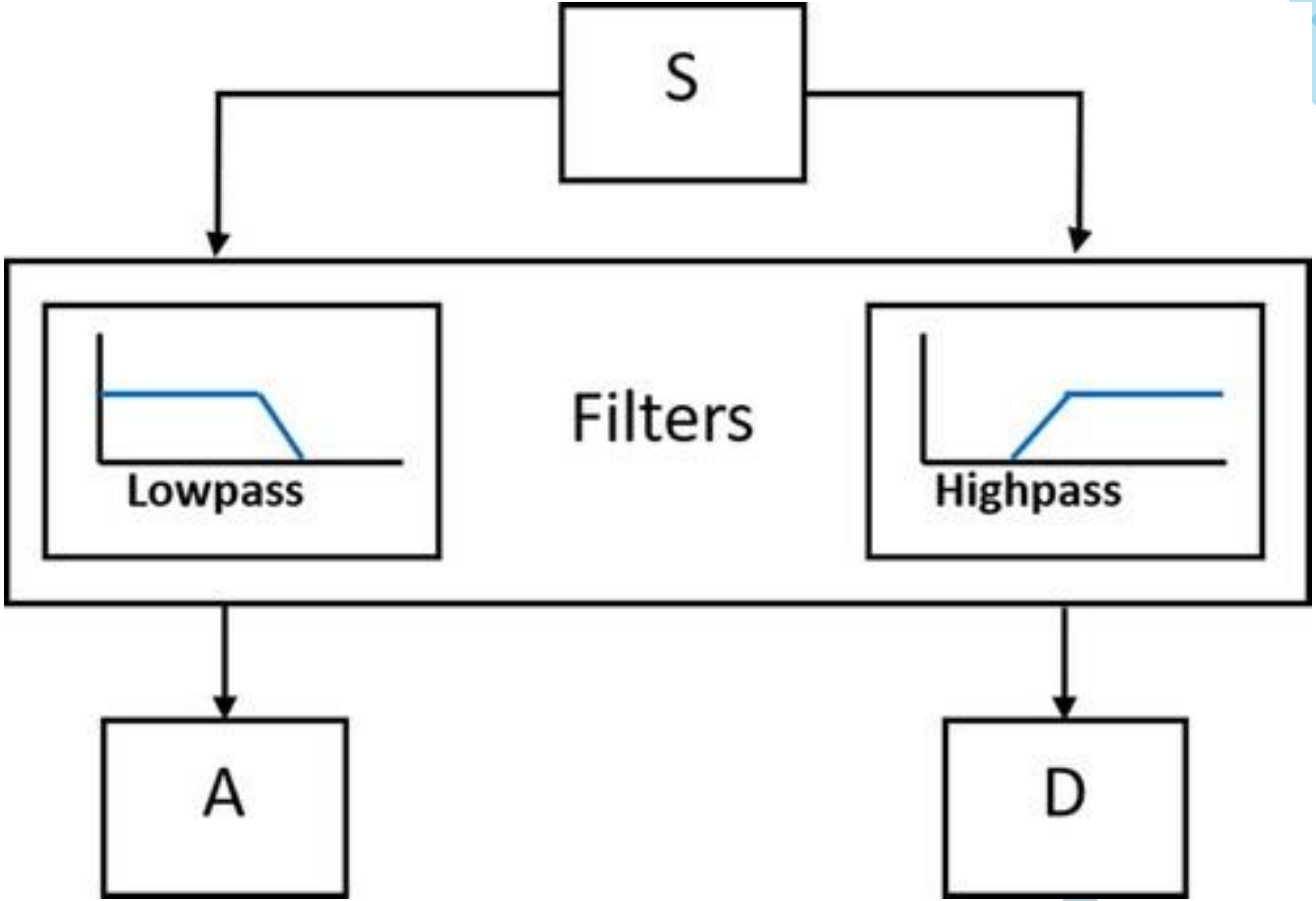


Figure: Another filterbank separating musical octaves

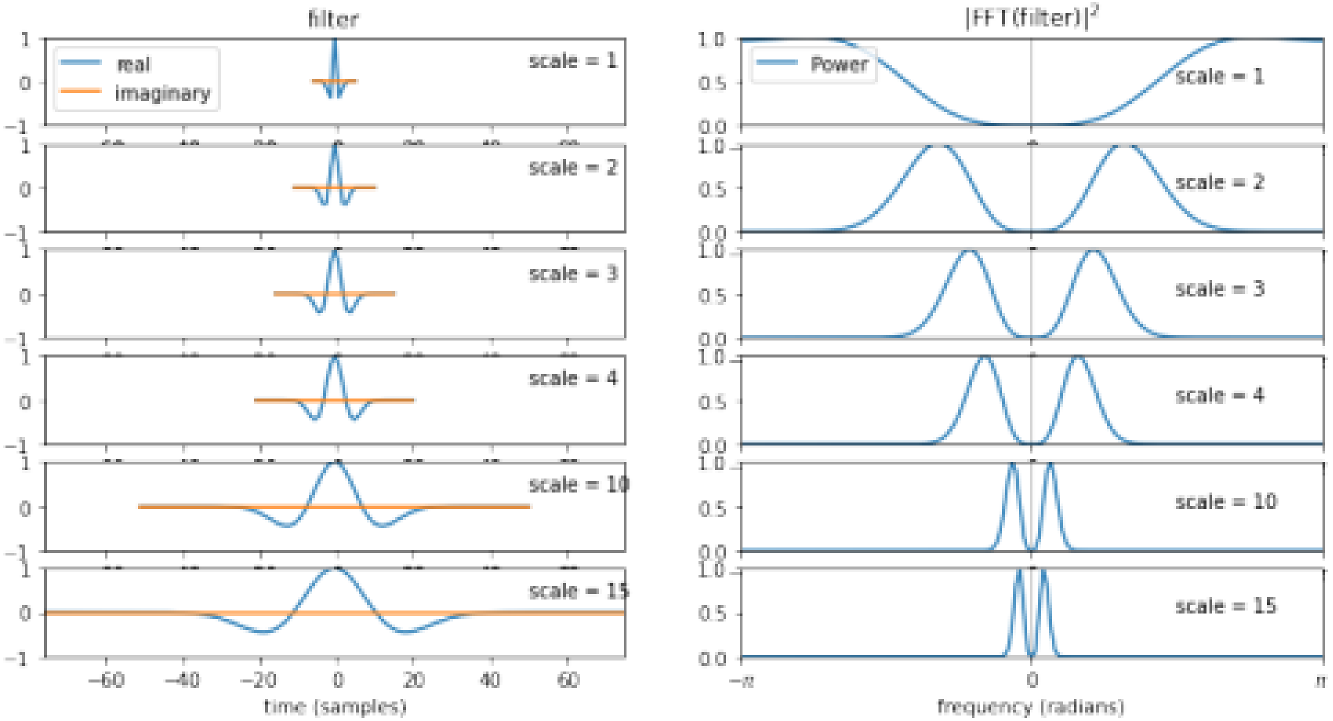
- The width, cutoff and shape depends on the wavelet type or the design method.



- Approximation: low-pass filter (blurred image)
- wavelets captures details, sharp transitions
- In 2 dimensions $\rightarrow$ 2 scales parameters, dilation along x and y


S
Filters
Lowpass
Highpass
A
D

Multi-scale or multiresolution analysis



- different sizes capture different patterns
- different frequency bands capture different patterns

The Haar wavelet

- Orthonormal basis of $\ell^2(\mathbb{R}^4)$

$$W = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ \sqrt{2} & -\sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{2} & -\sqrt{2} \end{bmatrix}$$

- Each row is a filter
- 4*4=16 filters in 2D
- Large scales are possible by increasing the matrix size

The Haar wavelet

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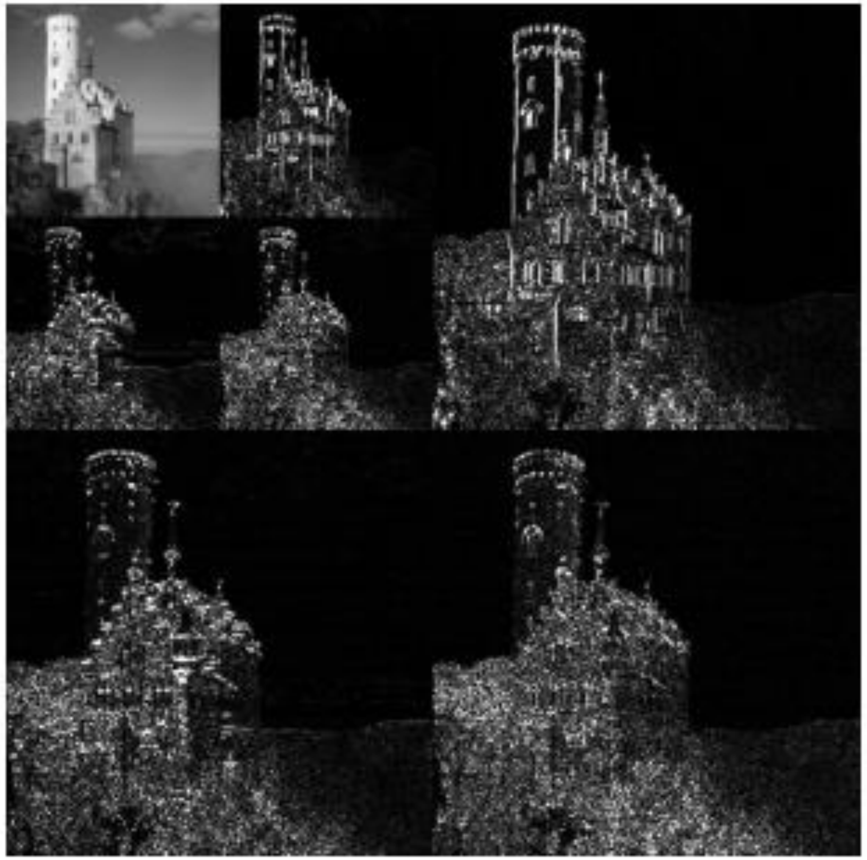
$$W = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ \sqrt{2} & -\sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{2} & -\sqrt{2} \end{bmatrix}$$

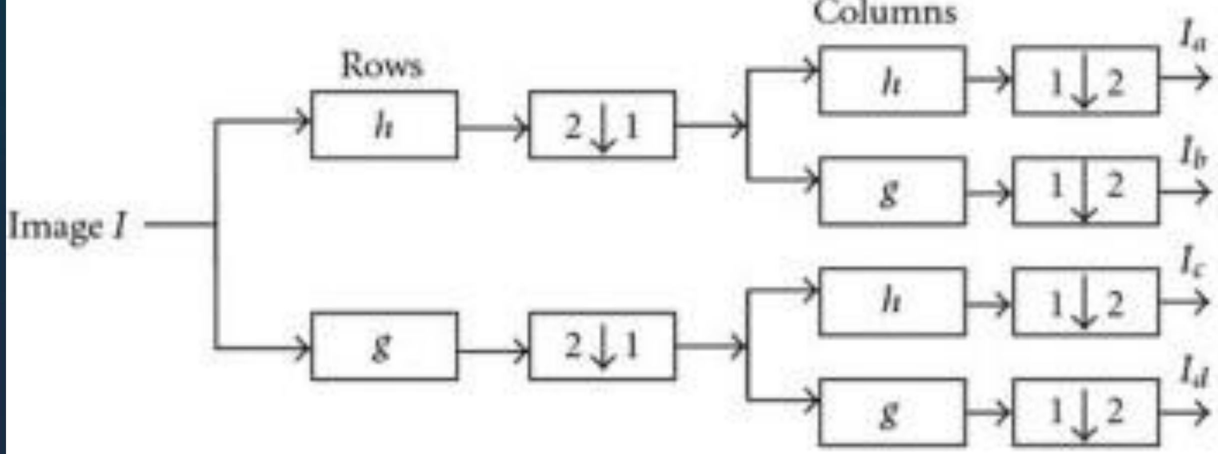
- Not orthogonal when convolution is performed pixelwise, redundant
- Orthogonal if translated by 4 pixels $F(n) = \sum_i f(i)w(i-4n)$
- $\Rightarrow$ capture all the information with less samples in space

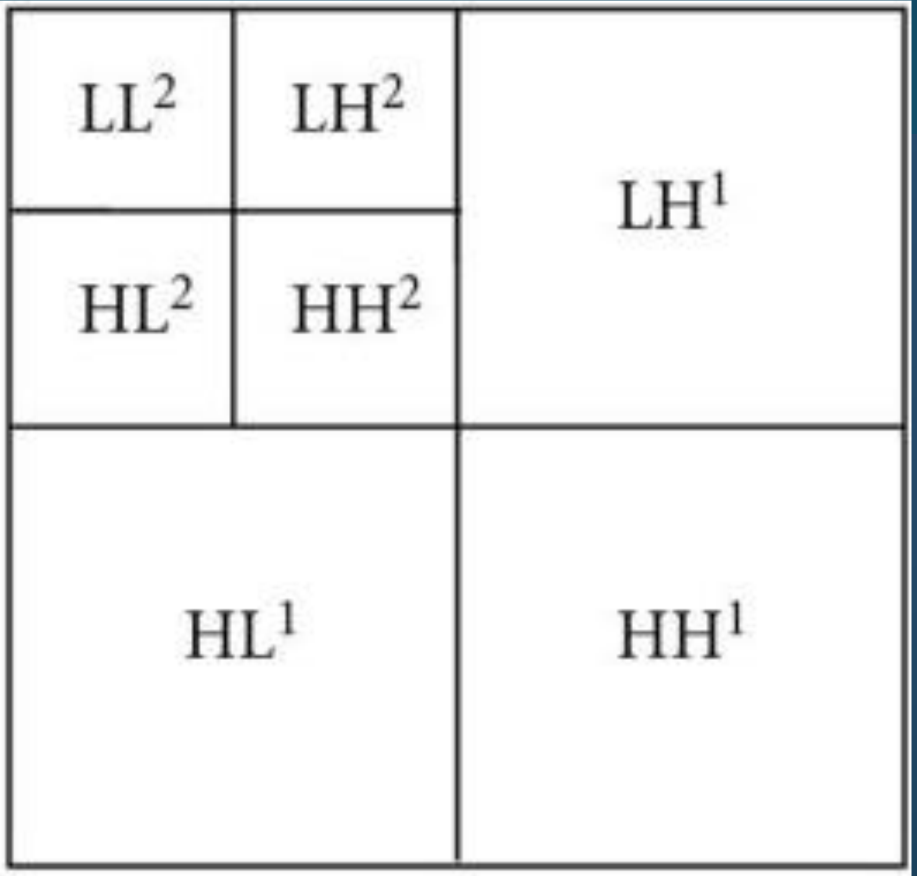
Sparse representation of an image.

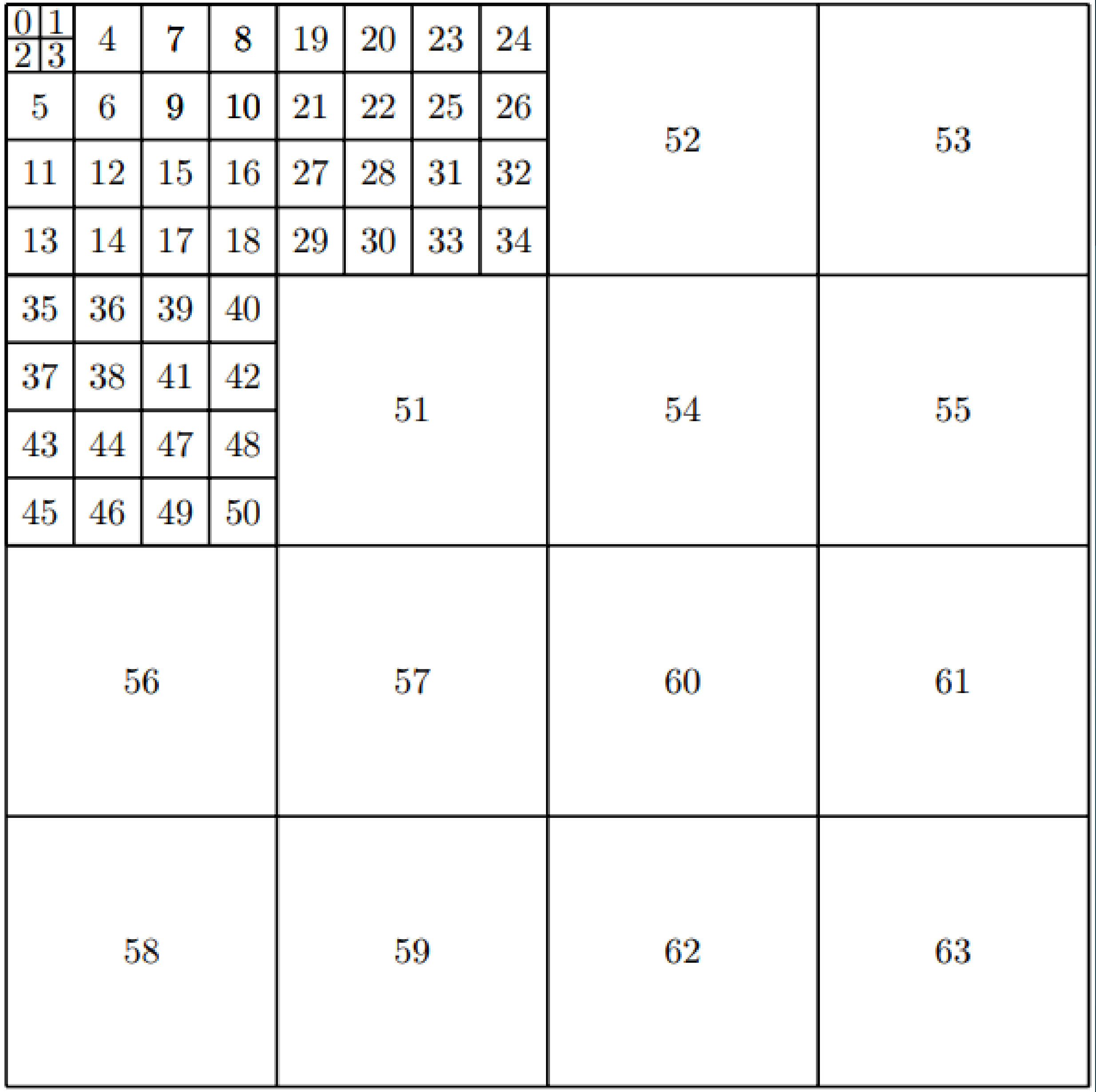
- Approximation: smoothed smaller image
- Sharp transitions and a lot of zeros in the scales



- Noise well separated, wavelet denoising (thresholding)
- Select the resolution, display without all the info
- Good compression


Columns
Rows
Image I
h
g
$2 \downarrow 1$
$1 \downarrow 2$
I_a
I_b
I_c
I_d

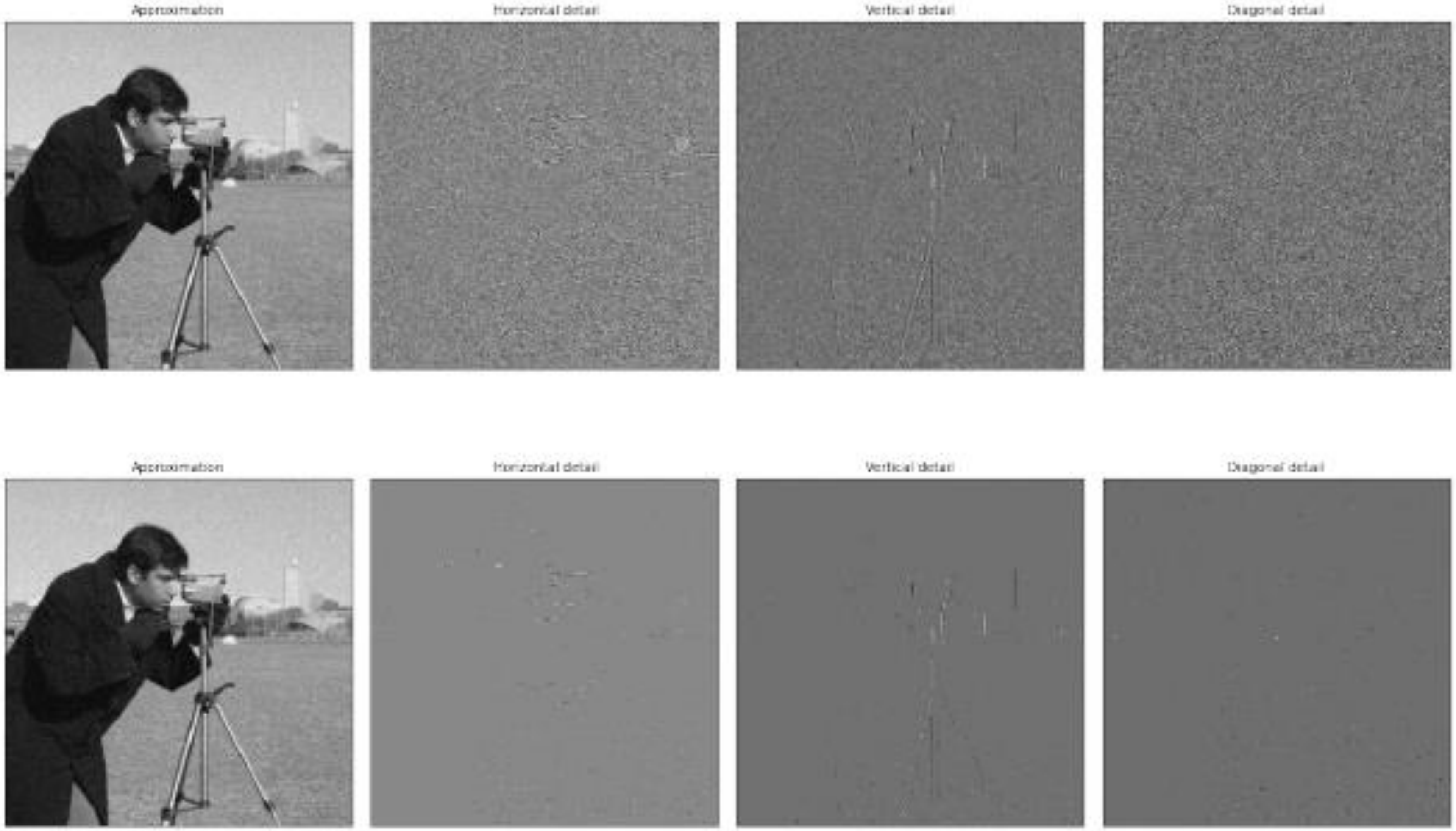

LL^2
LH^2
HL^2
HH^2
LH^1
HL^1
HH^1

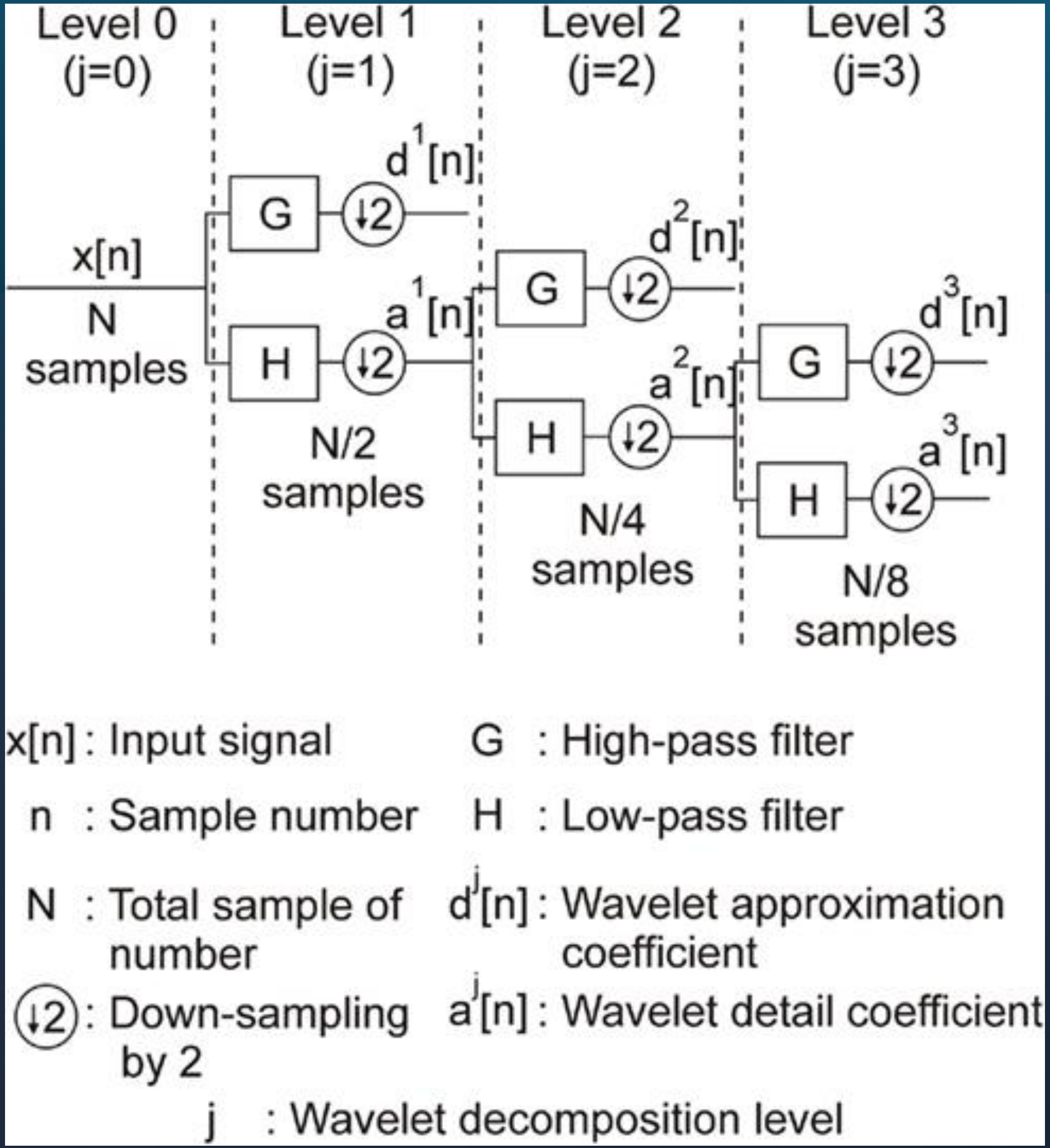


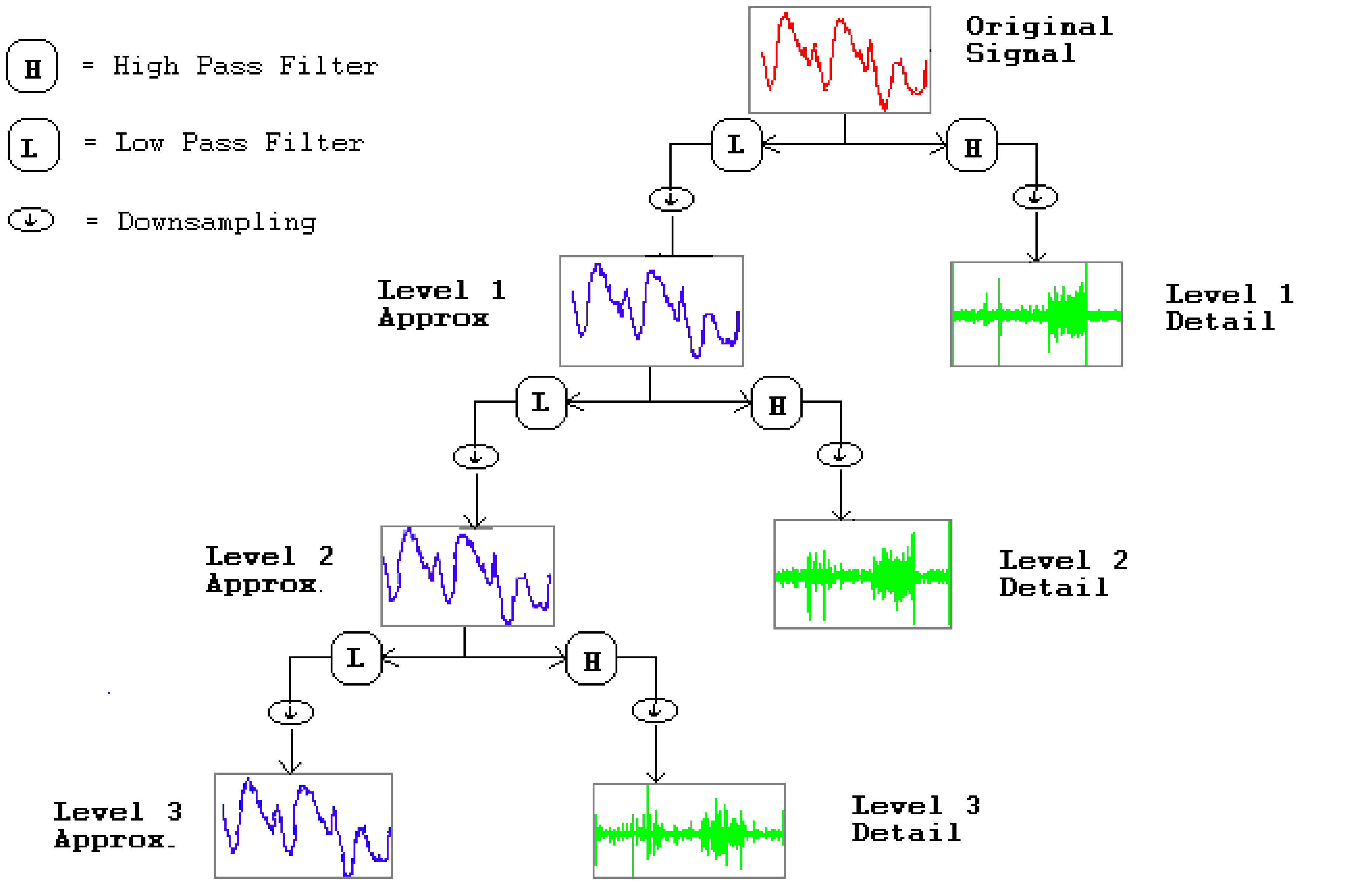
wavelets-pdf

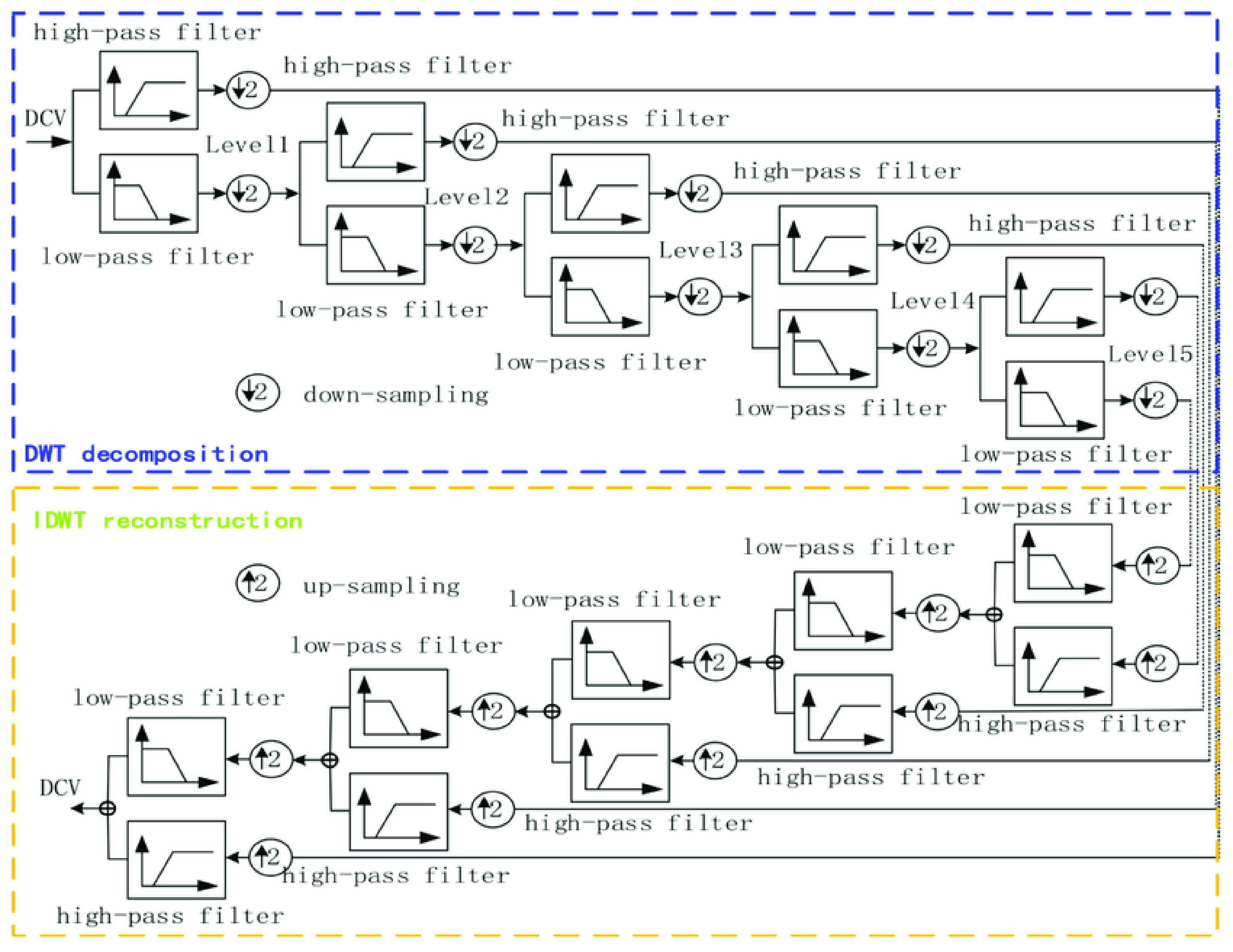
Random noise spreads everywhere

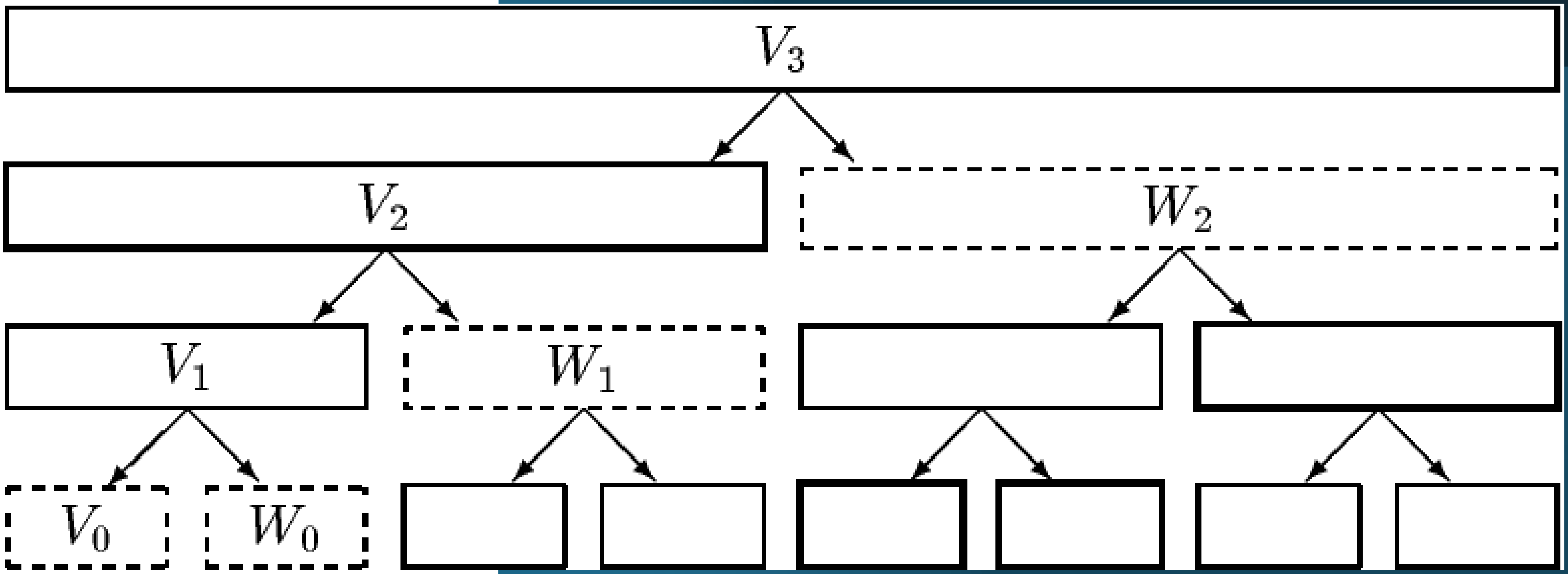
- Approximation: smoothed smaller image, smoothing remove (Gaussian) noise
- Scales: noise removed by thresholding (zero if below some small value)



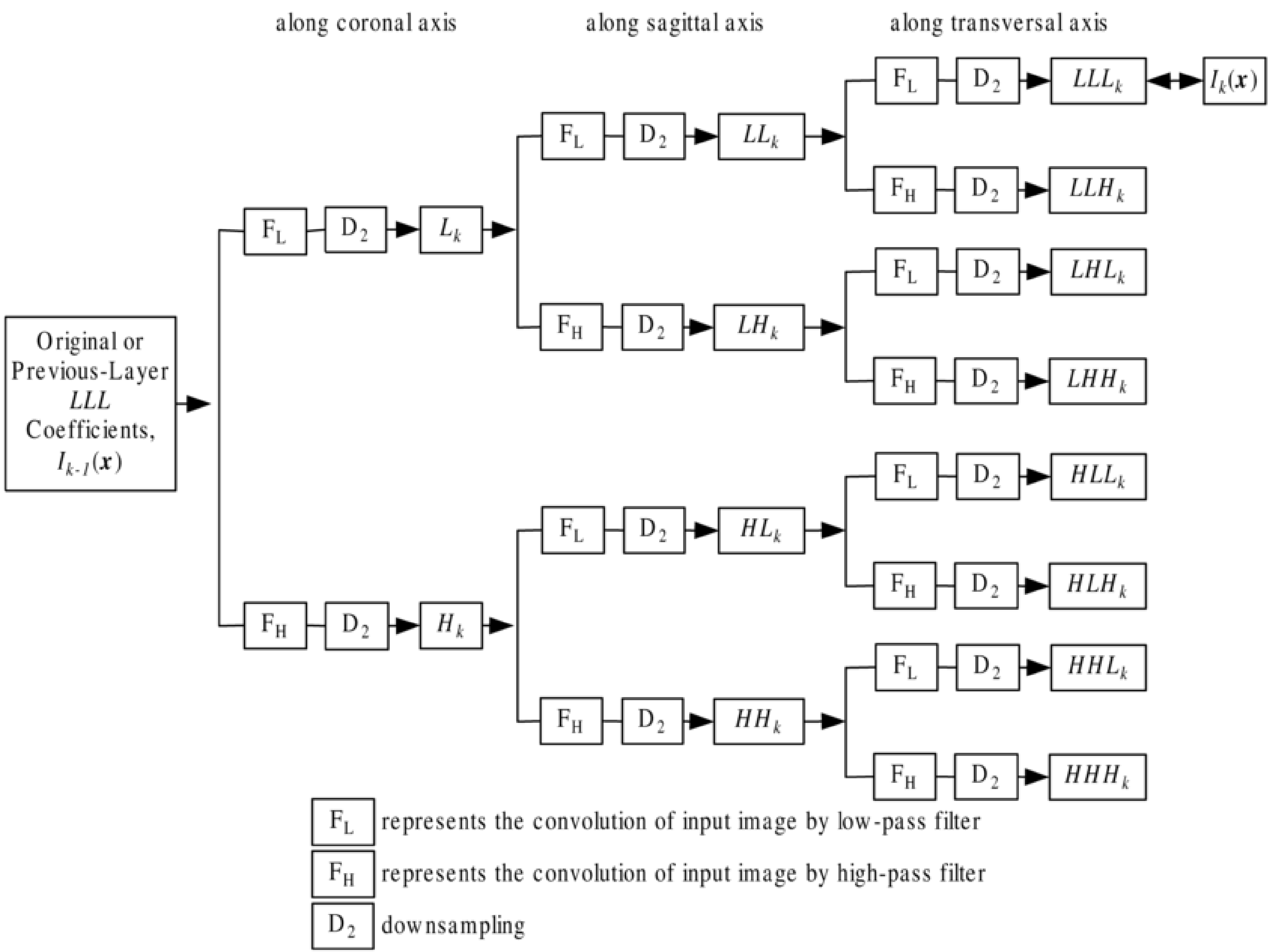

Level 0 (j=0)
Level 1 (j=1)
Level 2 (j=2)
Level 3 (j=3)
x[n]
N samples
G
H
↓2
d¹[n]
a¹[n]
N/2 samples
d²[n]
a²[n]
N/4 samples
d³[n]
a³[n]
N/8 samples
x[n] : Input signal
n : Sample number
N : Total sample of number
↓2 : Down-sampling by 2
G : High-pass filter
H : Low-pass filter
dʲ[n] : Wavelet approximation coefficient
aʲ[n] : Wavelet detail coefficient
j : Wavelet decomposition level


H = High Pass Filter
L = Low Pass Filter
= Downsampling
Original Signal
L
H
Level 1 Approx
Level 1 Detail
L
H
Level 2 Approx.
Level 2 Detail
L
H
Level 3 Approx.
Level 3 Detail

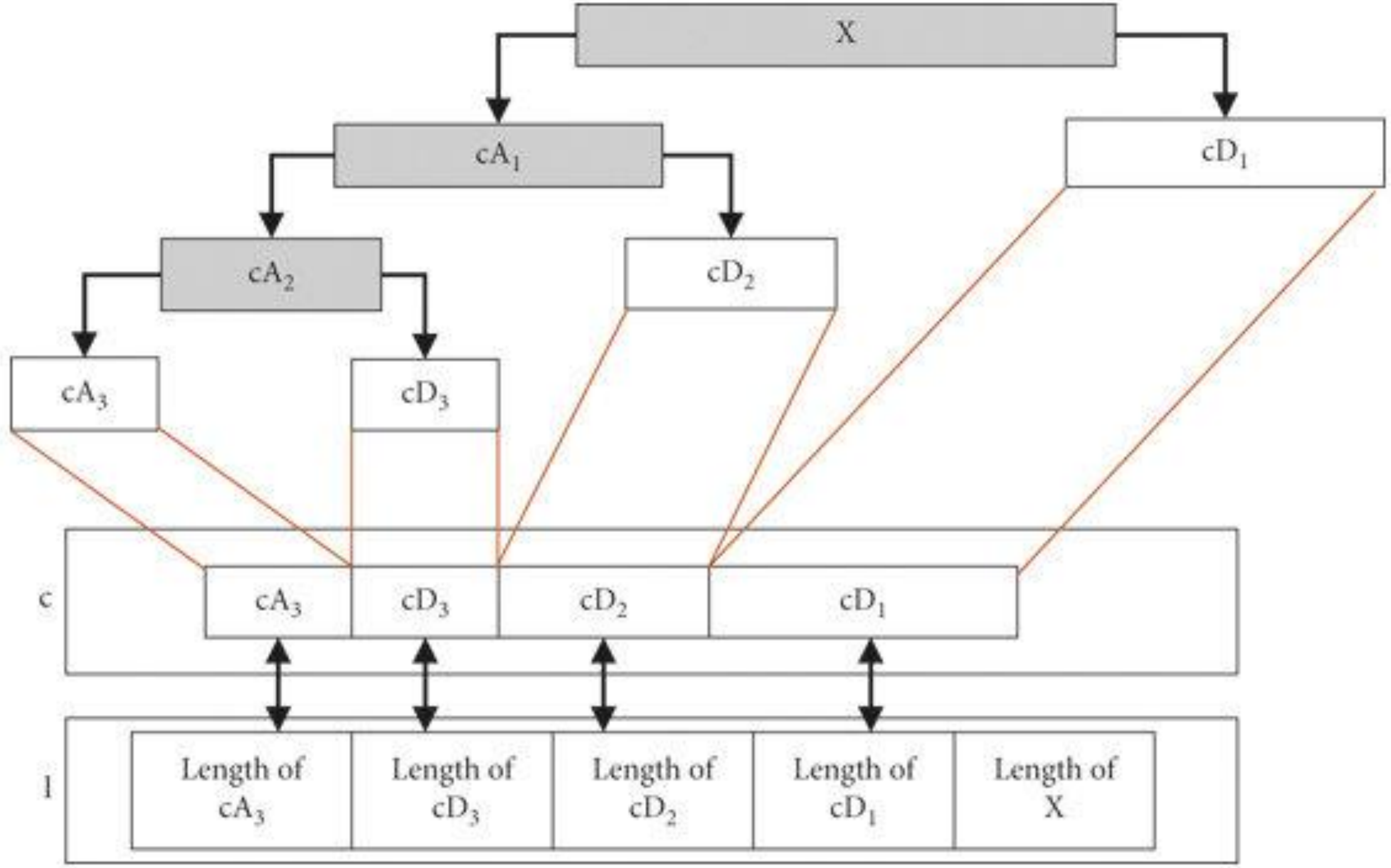

high-pass filter
high-pass filter
DCV
high-pass filter
Level1
high-pass filter
Level2
high-pass filter
low-pass filter
Level3
low-pass filter
Level4
low-pass filter
Level5
down-sampling
low-pass filter
low-pass filter
DWT decomposition
low-pass filter
IDWT reconstruction
low-pass filter
up-sampling
low-pass filter
low-pass filter
low-pass filter
high-pass filter
DCV
high-pass filter
high-pass filter
high-pass filter
high-pass filter



V_3
V_2
W_2
V_1
W_1
V_0
W_0



along coronal axis
along sagittal axis
along transversal axis
Original or Previous-Layer *LLL* Coefficients, $I_{k-1}(\boldsymbol{x})$
F_L
F_H
D_2
L_k
H_k
LL_k
LH_k
HL_k
HH_k
LLL_k
$I_k(\boldsymbol{x})$
LLH_k
LHL_k
LHH_k
HLL_k
HLH_k
HHL_k
HHH_k
F_L represents the convolution of input image by low-pass filter
F_H represents the convolution of input image by high-pass filter
D_2 downsampling


X
cA1
cD1
cA2
cD2
cA3
cD3
c
cA3
cD3
cD2
cD1
l
Length of cA3
Length of cD3
Length of cD2
Length of cD1
Length of X

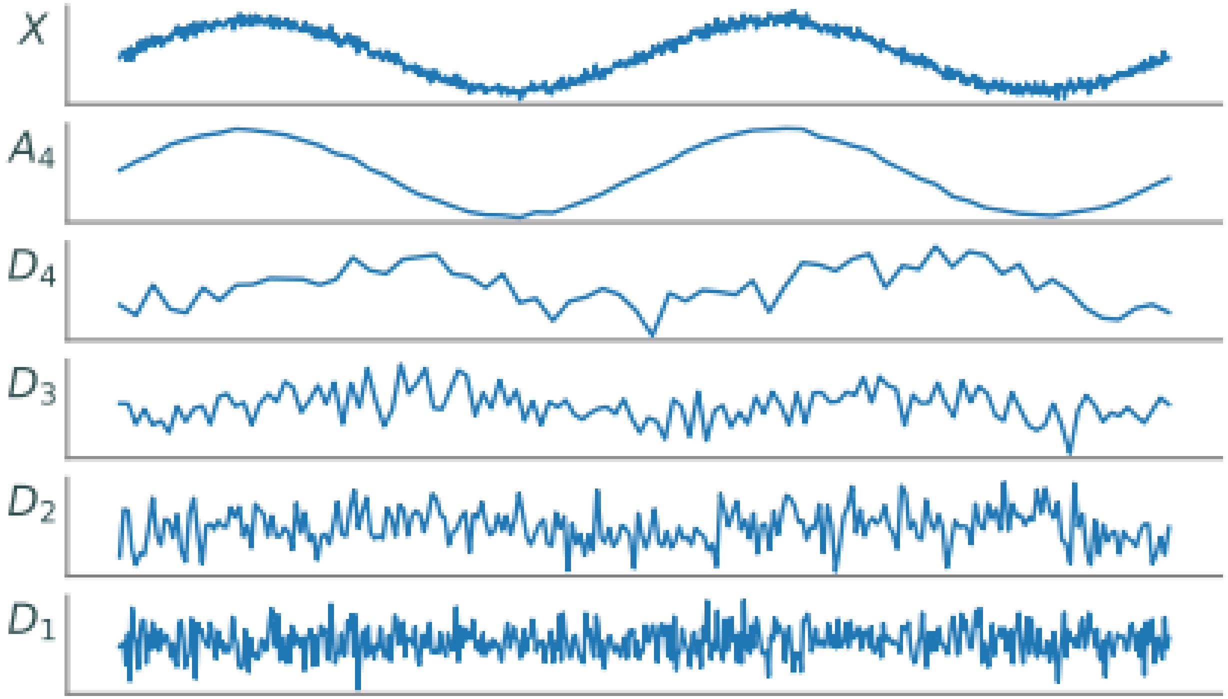


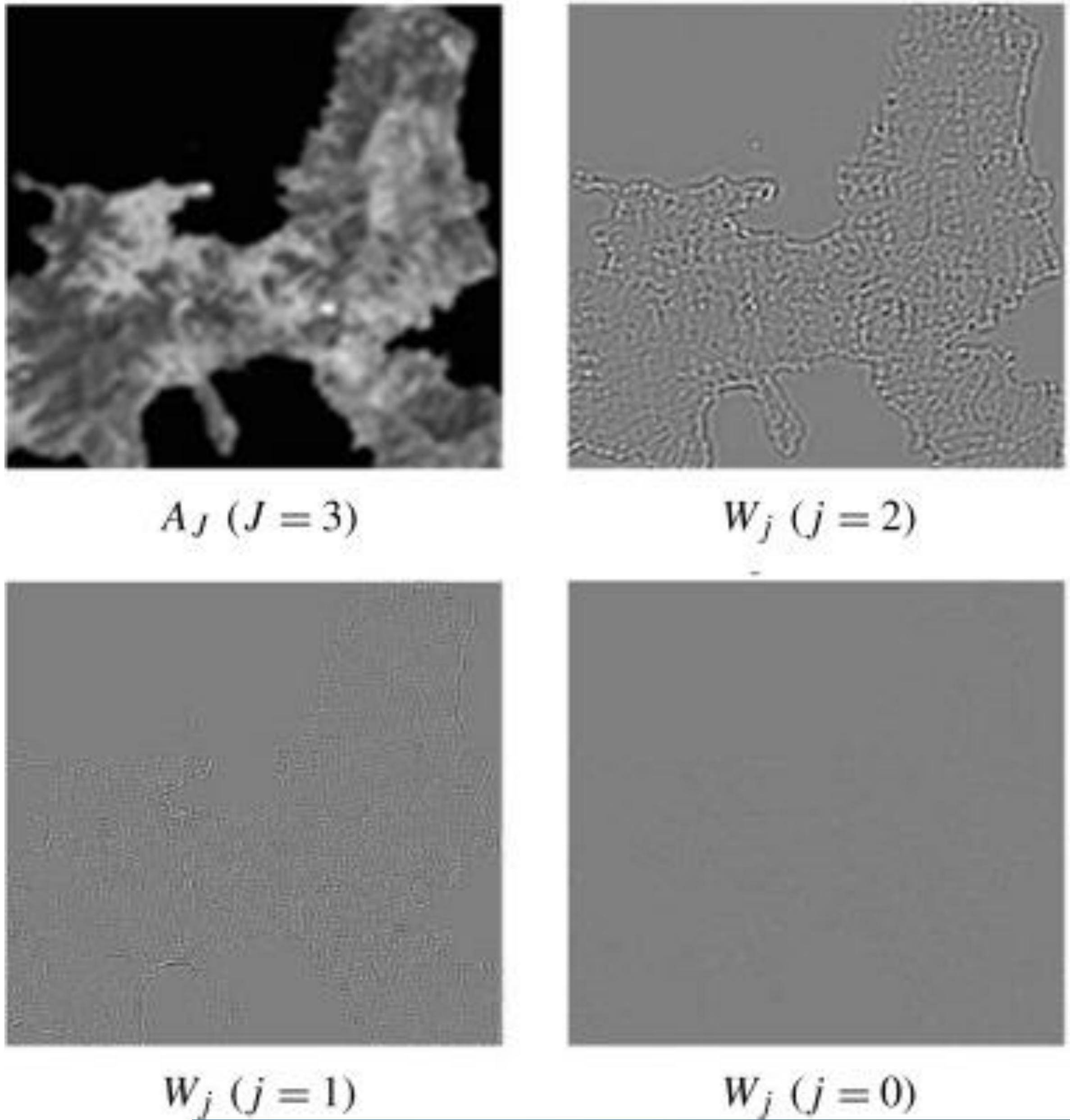
Figure: A level four wavelet decomposition of a signal. The top panel is the original signal, the next panel down is the approximation, and the remaining panels are the detail coefficients. Notice how the approximation resembles a smoothed version of the original signal, while the details capture the high-frequency oscillations and noise.

Image Compression

(a) Uncompressed (b) 12:1 compressed (c) 26:1 compressed

Fingerprint scan at different levels of compression. Original image source: http://www.nist.gov/itl/iad/ig/wsq.cfm.

wavelets-pdf



https://www.google.com/url?sa=i&url=https%3A%2F%2Fwww.sciencedirect.com%2Ftopics%2Fcomputer-science%2Fmultiresolution-analysis&psig=AOvVaw37HiNPj9VgozXrITM-vog0&ust=1740819657544000&source=images&cd=vfe&opi=89978449&ved=0CBcQjhxqGAoTCIiM596A5osDFQAAAAAdAAAAABDcCw