- **Filtering in the Frequency Domain**

- Basic idea: Modify *importance* of cosines of certain frequencies by multiplying $F(u, v)$ by a filter function $H(u, v)$ and take inverse DFT (IDFT)!

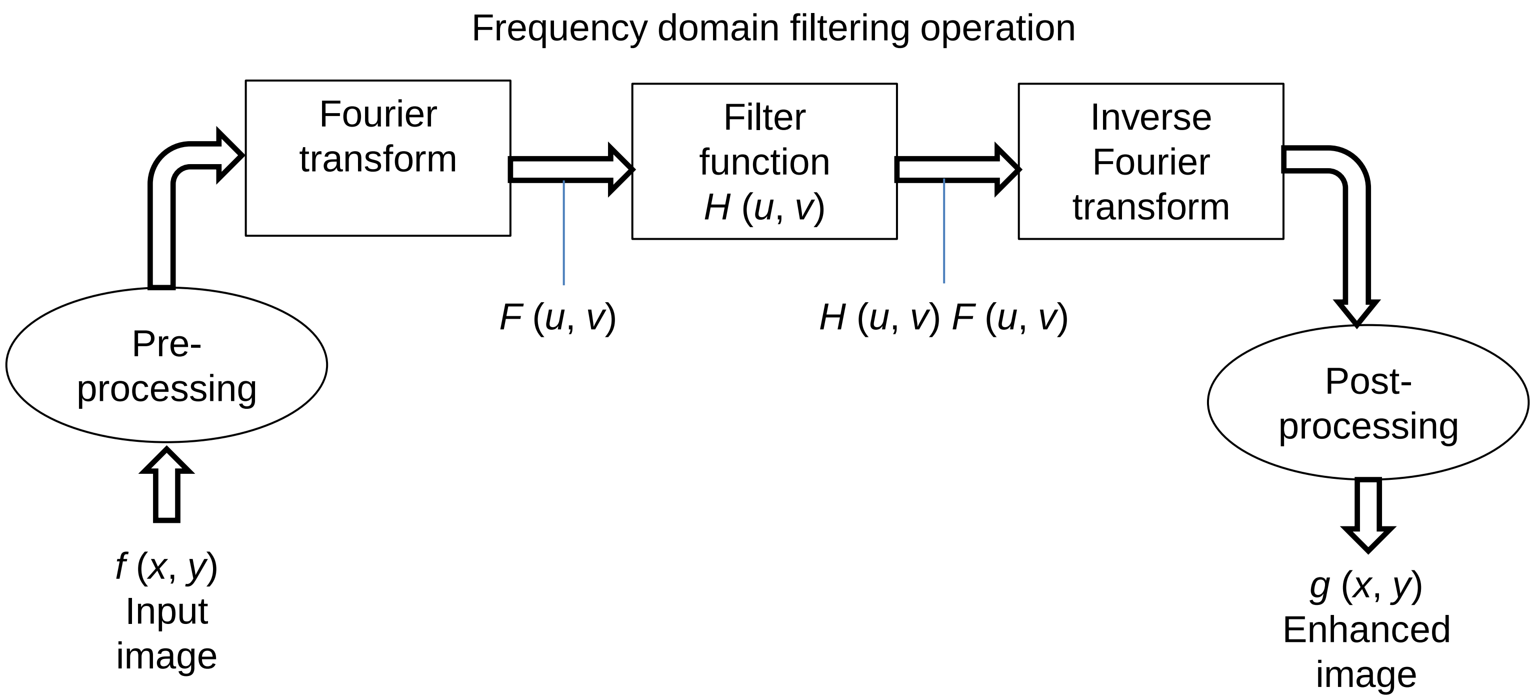


Figure 4.5 Basic steps for filtering in the frequency domain.

- Filtering in the Frequency Domain

- Convolution theorem:

  - $g(x, y) = \text{IDFT}\{H(u, v)F(u, v)\}$

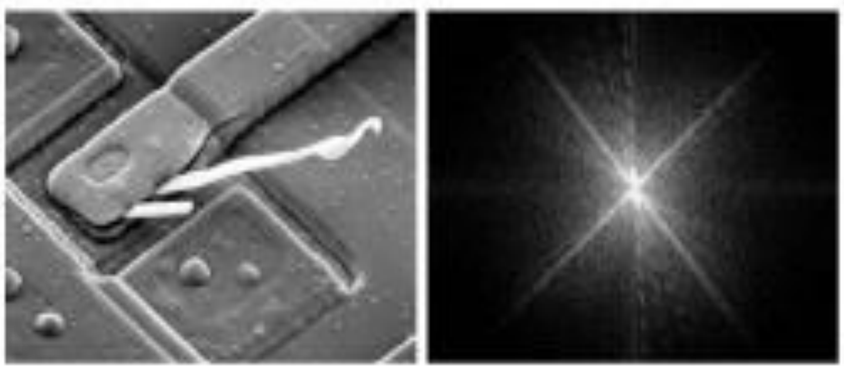
    $= \text{IDFT}\ \{H(u, v)\} * \text{IDFT}\{F(u, v)\}$

- $H\ (u, v)$ will often pass certain frequencies (attenuate rest)

  - Lowpass - Highpass - Selective (also Ch. 5)

DC component removed
(average intensity)

- Filtering in the Frequency Domain



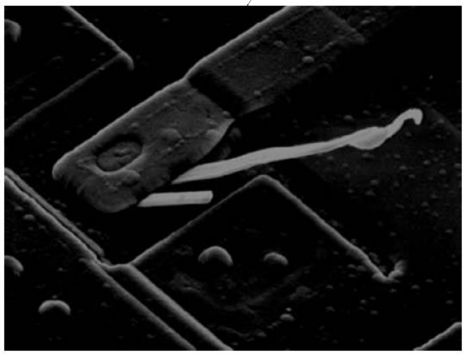
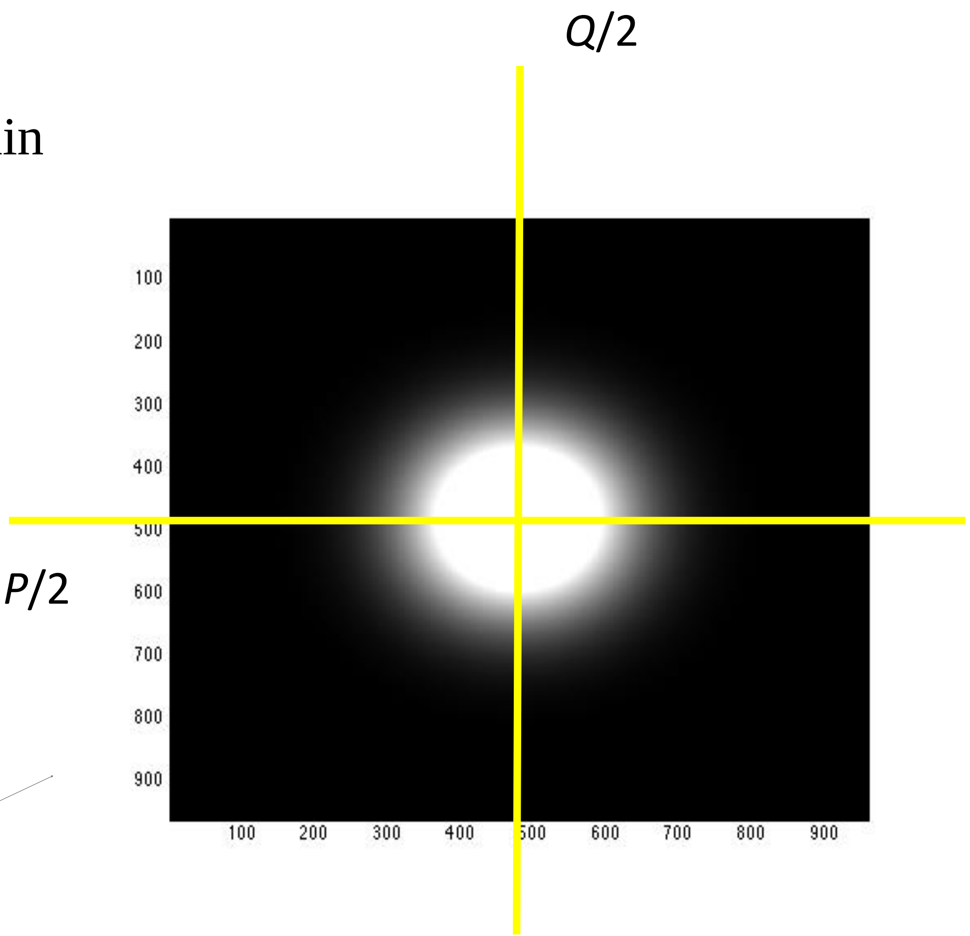


FIGURE 4.30 Result of filtering the image in Fig. 4.29(a) by setting to 0 the term $F(M/2, N/2)$ in the Fourier transform.

- Filtering in the Frequency Domain
  $F(u, v) = \mathrm{DFT}\{ f_p(x, y)\}$
  is (P x Q)

- Design filter $H(u, v)$ of size
  (P x Q) centered at
  $P/2$, $Q/2$!

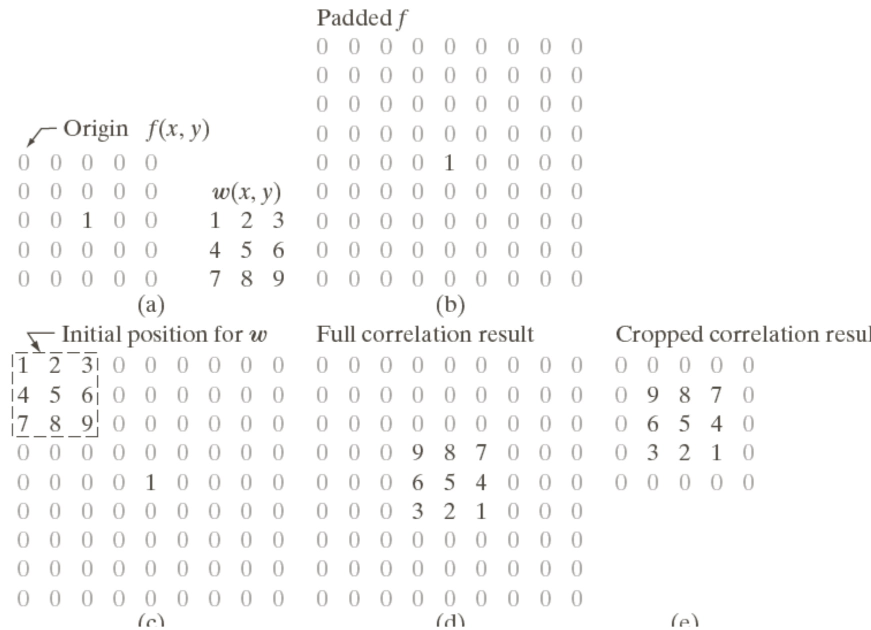


Even symmetry

- Filtering in the Frequency Domain

1. Given $f(x, y)$ of size (M x N) find padding parameters $P = 2M$, $Q=2N$
2. Form $f_p(x, y)$ of size (P x Q) by zero-padding
3. Compute $F(u, v) = \text{DFT}\{ f_p(x, y) \}$
4. Center at (P/2, Q/2) with fftshift
5. Design $H(u, v)$ of size (P x Q) centered at (P/2, Q/2)
6. Filter by $G(u, v) = H(u, v)F(u, v)$
7. «Center» back at (0, 0) with fftshift
8. Obtain $g_p(x, y) = \text{real}[\text{IDFT}\{G(u, v)\}]$
9. Obtain $g(x, y)$ by cropping out the (M x N) top left part of $g_p(x, y)$

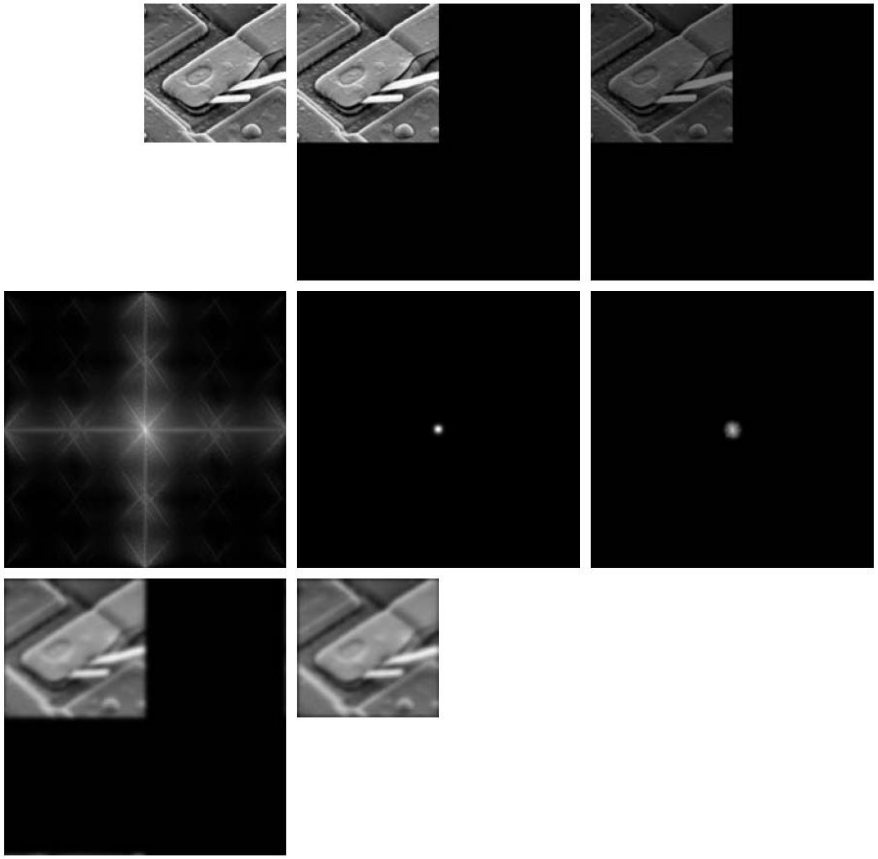
- *Why padding to 2M and 2N?*
- Spatial padding is only +2...



- In Fourier:
- The filter is not 3x3!
- Fourier filter can have the size of the entire image

Note:

Multiplying by $(-1)^{x+y}$ in image equivalent to fftshift in Fourier

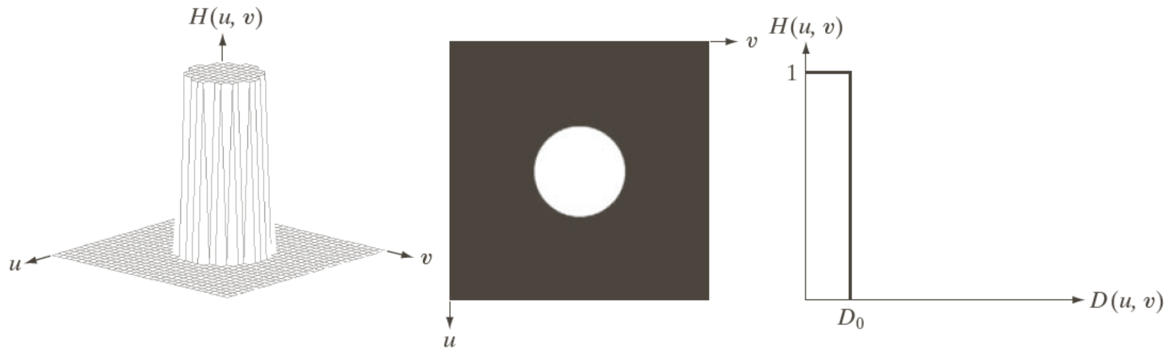


a b c
d e f
g h

FIGURE 4.36 (a) An $M \times N$ image, f. (b) Padded image, f_p of size $P \times Q$. (c) Result of multiplying f_p by $(-1)^{x+y}$. (d) Spectrum of F_p. (e) Centered Gaussian lowpass filter, H, of size $P \times Q$. (f) Spectrum of the product HF_p. (g) g_p, the product of $(-1)^{x+y}$ and the real part of the IDFT of HF_p. (h) Final result, g, obtained by cropping the first M rows and N columns of g_p.

- Filtering in the Frequency Domain

- Ideal lowpass filter (ILPF)



a b c

FIGURE 4.40 (a) Perspective plot of an ideal lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

- Frequencies higher than D_0 are «cut off»

$$H(u,v) = \begin{cases} 1, tD(u,v) \leq D_0 \\ 0, tD(u,v) > D_0 \end{cases}$$

- $D(u, v)$ a distance from the center:

$$D(u,v) = \sqrt{(u - P/2)^2 + (v - Q/2)^2}$$

- Filtering in the Frequency Domain, choosing the appropriate D_0

- Specify D_0 to encompass a percentage of the *power* of $f_p(x, y)$
- Power spectrum: $P(u, v) = |F(u, v)|^2$

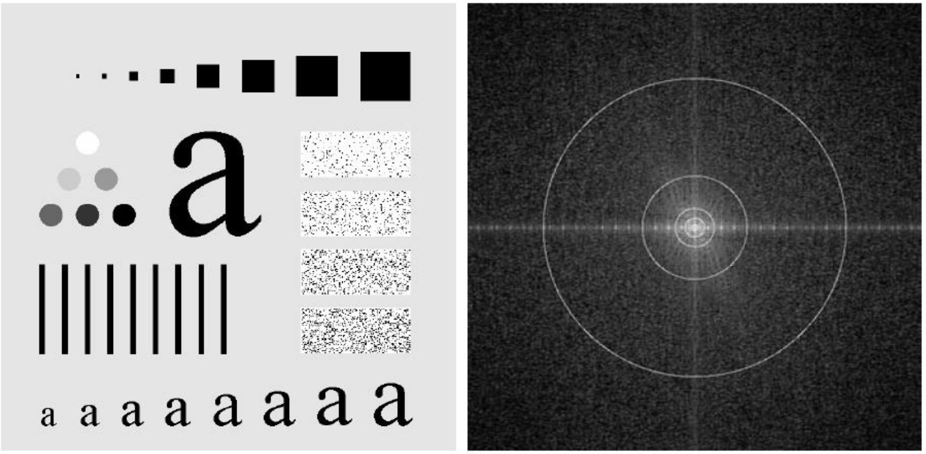


FIGURE 4.41 (a) Test pattern of size 688 × 688 pixels, and (b) its Fourier spectrum. The spectrum is double the image size due to padding but is shown in half size so that it fits in the page. The superimposed circles have radii equal to 10, 30, 60, 160, and 460 with respect to the full-size spectrum image. These radii enclose 87.0, 93.1, 95.7, 97.8, and 99.2% of the padded image power, respectively.

- Effect of Ideal LPF

- Passing only low frequencies blurres out edges, which correspond to high frequencies
- Slowly varying areas are preserved

This kind of effect
is called «ringing»

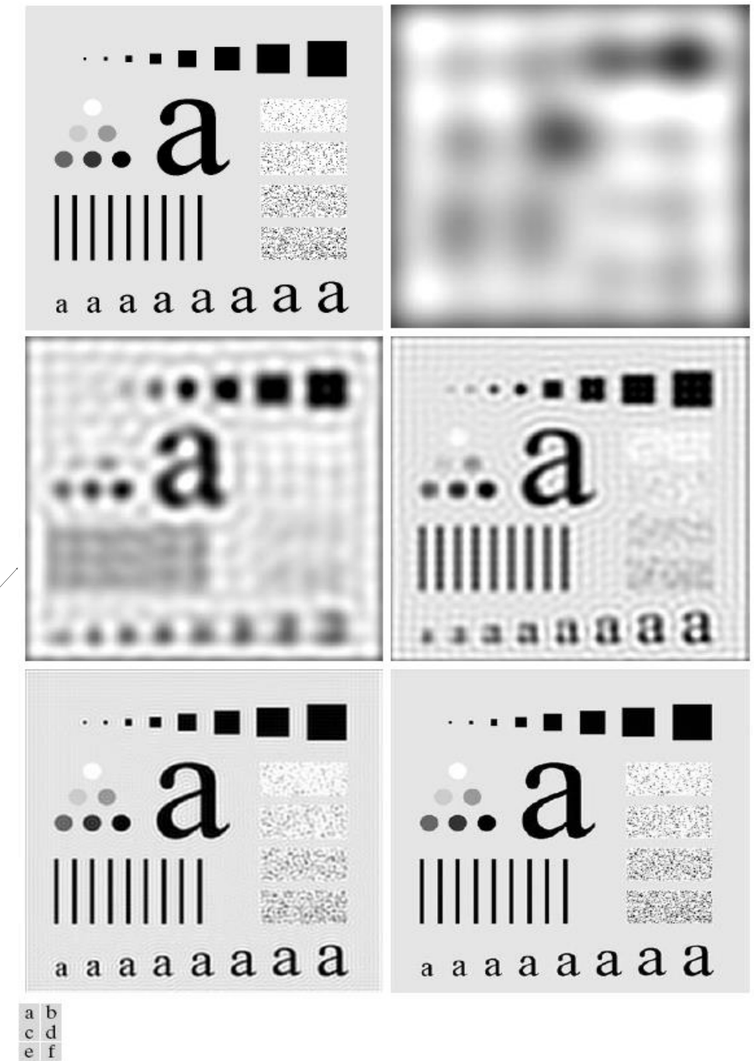
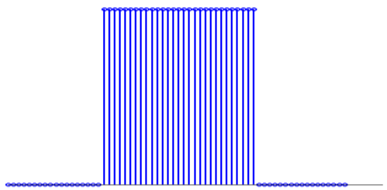


FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.

- The ringing effect

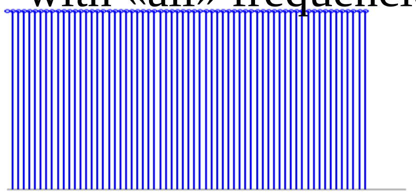
- «Ideal» filters will always cause ringing!

Ideal Low-pass $H(u, v)$



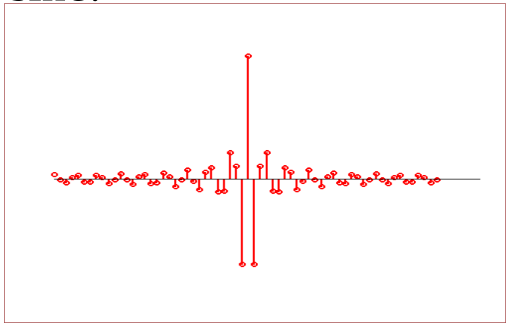
x

$F(u, v)$ of «image» with «all» frequencies

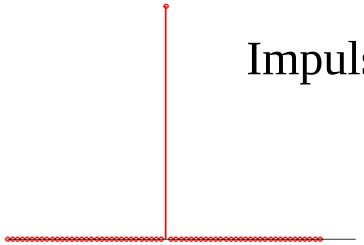


IDFT

IDFT of $H(u, v)$ is sinc!

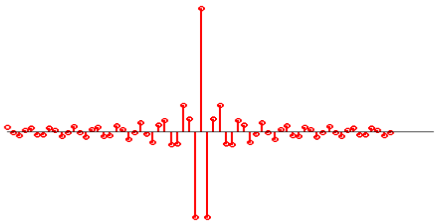


*



Impulse

Convolution of sinc with impulse is sinc



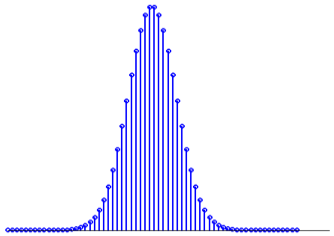
- ILPF are equivalent to convolving an image with a sinc-function
- The alternating ± behavior of the sinc causes the ringing
- When D_0 gets larger, the ringing decreases

•Because of ringing, ILPFs are basically not used in practice!

Q: Ways to reduce ringing?

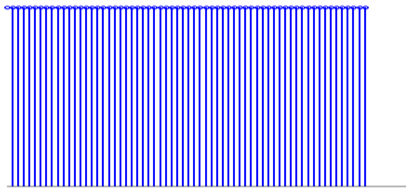
- Gaussian lowpass filters (GLPFs)

lowpass: $H(u, v)$



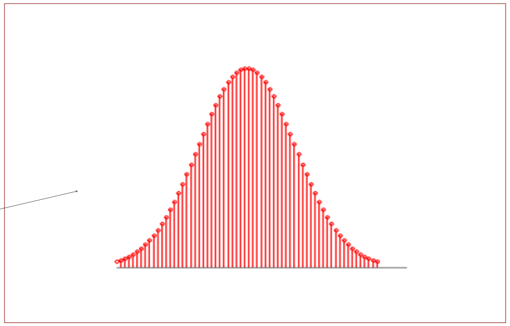
x

$F(u, v)$ of «image» with «all» frequencies

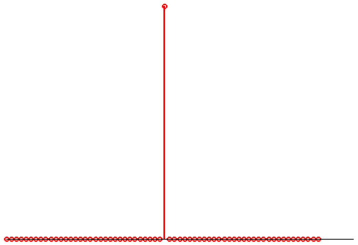


IDFT

IDFT of $H(u, v)$ is also Gaussian!
---> smooth positive



*



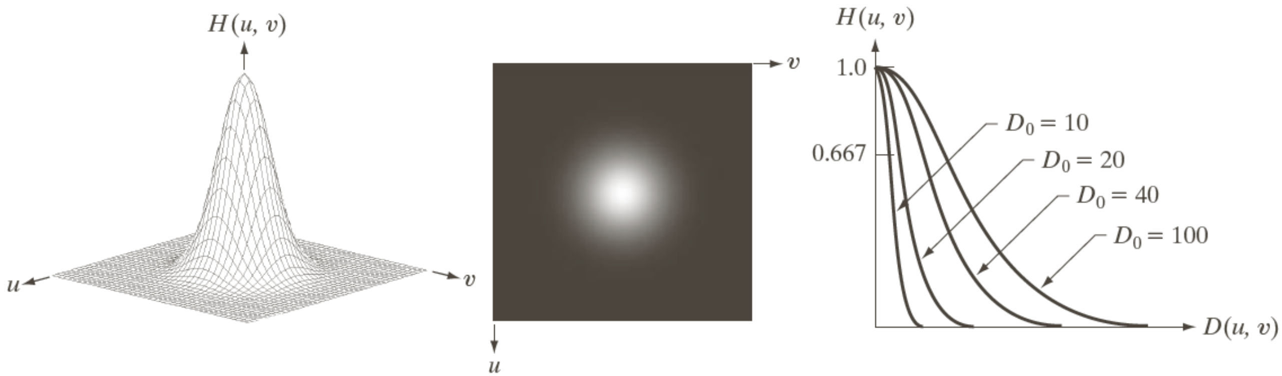
Smoothing/blurring but no ringing!

- Gaussian in 2D:

$$H(u,v) = e^{-\frac{D^2(u,v)}{2D_0^2}}$$

with $D^2(u,v) = (u - P/2)^2 + (v - Q/2)^2$

Typically want smooth filters like the Gaussian



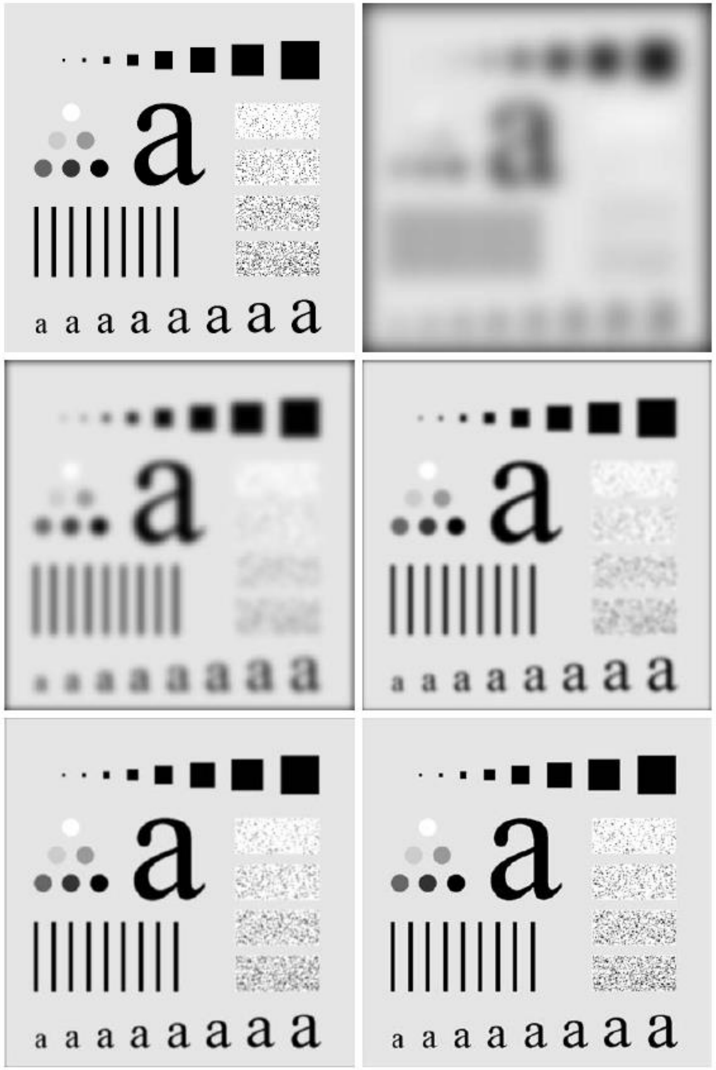
a b c

FIGURE 4.47 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

- Effect of Gaussian LPF

GLPF visually quite different from ILPF

Blurring but no ringing



a b
c d
e f

FIGURE 4.48 (a) Original image. (b)–(f) Results of filtering using GLPFs with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Figs. 4.42 and 4.45.

- Other filters coming from signal processing
- Butterworth lowpass filters (BLPFs)
  - Compromise between ILPF and GLPF
  - May have ringing depending on the order *n* of the filter:

$$H(u,v)=\frac{1}{1+\left[\frac{D(u,v)}{D_0}\right]^{2n}}$$

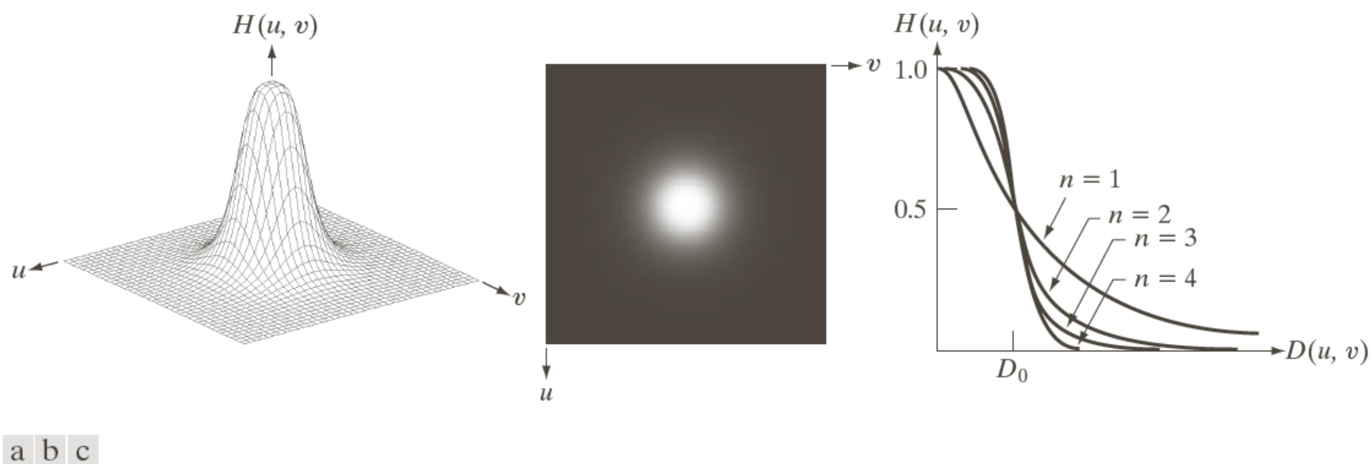
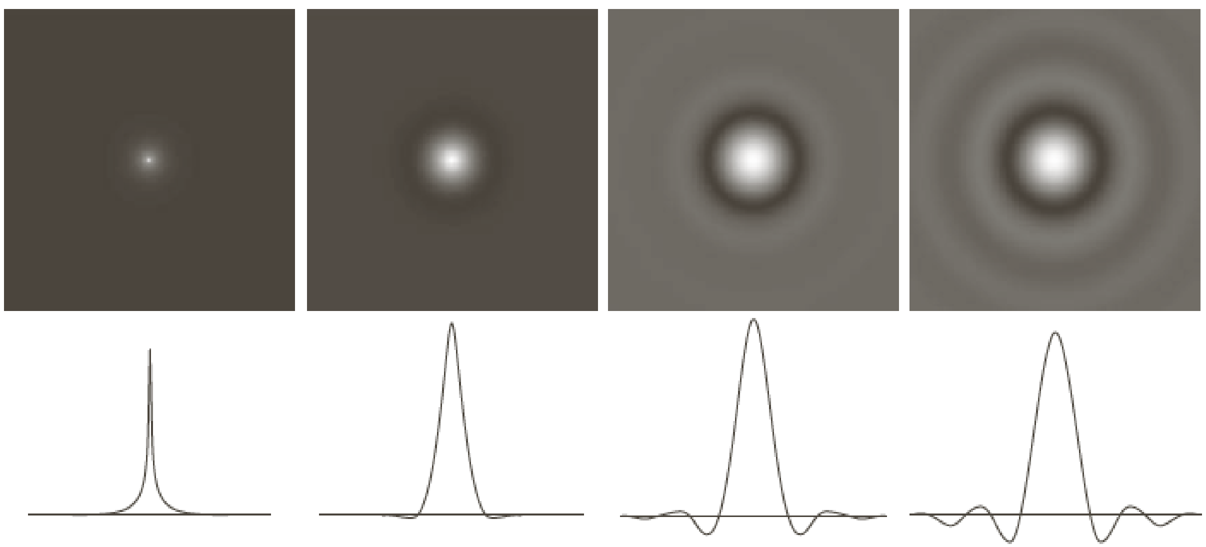


FIGURE 4.44 (a) Perspective plot of a Butterworth lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

- Butterworth in the spatial domain

- Ringing starts for $n > 1$

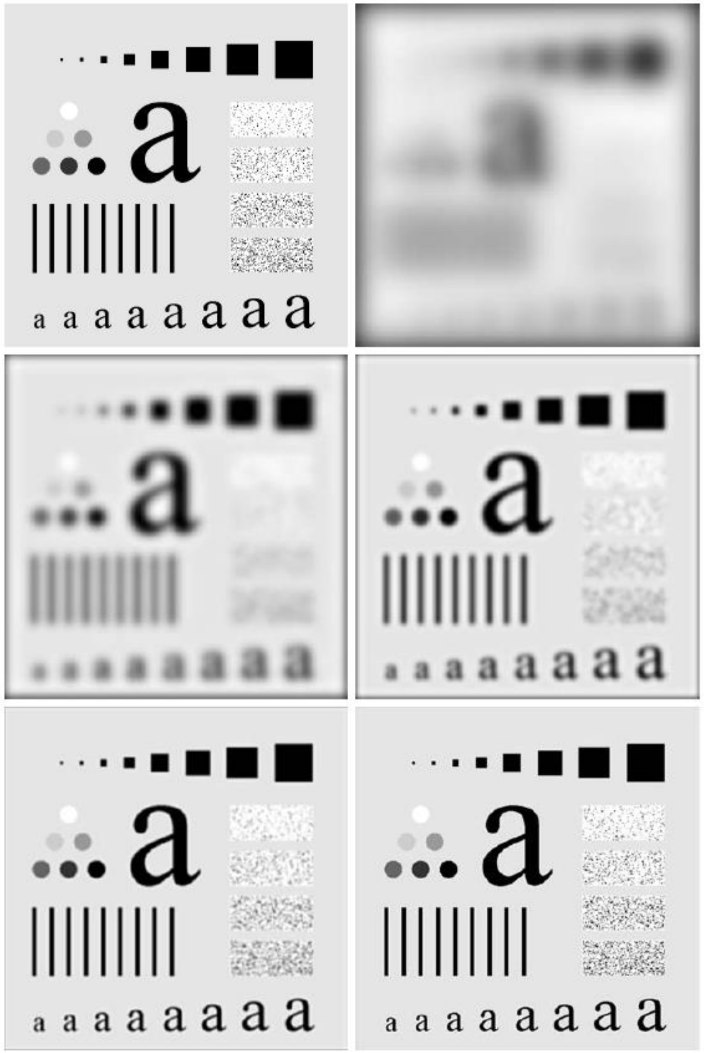


a b c d

FIGURE 4.46 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding intensity profiles through the center of the filters (the size in all cases is 1000×1000 and the cutoff frequency is 5). Observe how ringing increases as a function of filter order.

Effect of Butterworth LPF

BLPF of order 2 obtains very similar results to a GLPF



a b
c d
e f

FIGURE 4.45 (a) Original image. (b)–(f) Results of filtering using BLPFs of order 2, with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Fig. 4.42.