The Implementation of a 4D Helical Track Fitter in REDTOP Simulations

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The REDTOP (Rare Eta Decays with a TPC for Optical Photons) experiment is proposed project at Fermi National Accelerator Laboratory planning to have a continuous proton beam impinging on a low-Z target for the purpose of collecting 10^{13} η/η' mesons after one year of running. The decay products of these η/η' mesons will be observed to find discrepancies in the standard model or find new physics. The following paper outlines the present simulation framework, including a new algorithm that incorporates the time information into the reconstruction of the particles' helical paths.

I. INTRODUCTION

The REDTOP experiment is proposing to have a 1.9 GeV continuous proton beam imping on 10 sheets of lithium foil to produce about 10^{13} η/η' mesons after one year of running. The REDTOP Detector Complex, consisting of a silicon based LGAD pixel detector, fiber tracker, RICH detector, and ADRIANO-2 Dual-Readout Calorimeter, will attempt to capture the decay products of the η mesons. The REDTOP Collaboration is especially interested in the events that are either not expected or are suppressed at the 10^{-11} level. The following framework describes the simulation of the production and reconstructed tracks of the decay products, including the implementation of a new fitter that includes the time information into the reconstruction, allowing for a more precise fit.

II. PHYSICAL MOTIVATION

Decay products in the REDTOP experiment travel along a helical path through the detector complex due to a solenoidal magnetic field. The reconstruction of the helical path currently is described in 3D by five perigee parameters at the distance of closest approach to the origin:

C: The curvature of the track. The curvature is positive if the particle has a positive charge ad negative if the charge is negative.

φ_0: The azimuthal angle of the momentum at the position of closest approach to the reference point which is located at (0,0,0).

d_0: The distance of closest approach to the reference point.

$\tan(\lambda)$: The slope in the SZ plane, dz/ds.

z_0: The distance along the z-axis when the particle is at the distance of closest approach in the XY plane.

The accuracy of the reconstruction can be improved by including the time information (t_0) in the fit, so that the parametrization becomes 6-dimensional. This will further prevent the reconstruction from incorrectly fitting hits in the detector that belong to different decay products.

III. THE REDTOP SIMULATION FRAMEWORK

The following information was first presented in "The Implementation of a Secondary Vertex Search Algorithm in REDTOP Simulations" by Karime Maamari and Corrado Gatto [1].

A. The event generator: *GenieHad*

The REDTOP Collaboration has developed an event generator package named *GenieHad* [2] to overcome the limitations of the Geant4 simulation package (such as rescattering not being considered and nucleons in the final state being considered as free particles). *GenieHad* is an extension of the Genie framework that is widely utilized in the neutrino community. *GenieHad* consists of a collection of interfaces to existing event generator packages that are being/have been developed and maintained in the nuclear physics community. The initial nuclear interaction between the pron beam and nuclear matter is followed by a set of computer-based models of clusterization, de-excitation, and evaporation processes. The underlying architecture of the package takes care of the geometry of the apparatus, materials, beam profile and composition, and other environmental considerations, while the hadron interaction engine is called at run-time in order to simulate scattering at the nuclear level. This approach allows for effective testing of the strength and uncertainty of each event by comparing predictions from multiple nuclear interaction models.

For signal events, the beam interacts with the target – if an η or η' meson is produced (often along with other mesons and baryons coming as a result of nuclear fragmentation of the target), it subsequently decays according to the nuclear model of interest.

In the case of background events, the final state, as obtained at the end of the interaction-clusterization--evaporation-deexcitation chain, is fed into the following stage of the simulation (Geant4). Several nuclear models have been tested for the nuclear interaction and the results are reasonably consistent and within the approximation of the formulae used for the branching ratio [9].

B. The particle transport software: *slic*

Slic is an interface to Geant4, developed by the SID Collaboration within the ILC framework, and currently used for the simulation of the events of the HPS experiment at JLAB. *Slic's* framework allows for the description of both the geometry and the segmentation of the sensitive detectors as well as for the generation of the collections of hits. REDTOP employs an improved version of this framework, in which optical surfaces can also be described and the detection of the optical photons of metal-dielectric surfaces is fully implemented. Although the details of the geometry do not go as far as the individual fixtures, most of the passive material is taken into account.

C. The reconstruction and analysis software: *lcsim*

Lcsim is a java-based software package, also initially developed by the SID Collaboration and used for the reconstruction of the events by the HPS experiment at JLAB. The reconstruction in *lcsim* proceeds according to the following steps:

1. Digitization of the hits produced by Geant4
2. Level-0 trigger for rejection of background
3. Pattern recognition
4. Track reconstruction
5. Shower reconstruction
6. Final analysis

The following section further details some of these steps, namely the digitization, pattern recognition, and track reconstruction.

IV. THE TRACK RECONSTRUCTION STRATEGY

After the generation of signal and background events with *GenieHad*, the events are transported throughout the detector using the *slic/Geant4* software package and then the produced hits are passed to *lcsim* for the digitization and reconstruction. Details of the implementation are presented below:

A. Digitization

Initially, the digitization of the hits in the three subdetectors is performed, taking into account the energy response, gain discrimination, and efficiency of the fiber tracker and LGAD pixel detector. Then, overall light yield and efficiency of ADRIANO-2 are calibrated to the results of several test beams of a 1.1 meters long prototype built by T1015 Collaboration [3, 4, 5]. New prototypes of the ADRIANO-2 Dual-Readout Calorimeter are presently in development and consequently calibration is soon to be updated.

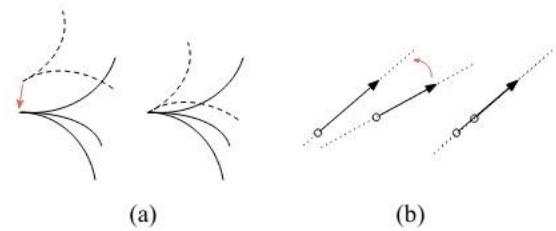

FIG. 1. The two constraint strategies used. (a) demonstrates the constraining of a constructed vertex to the target. (b) highlights the constraint of the direction of the momentum of the newly constructed vertex to that of the decayed eta.

B. Pattern recognition and PID (cheat)

Thus far, REDTOP's pattern recognition has only been implemented for the fiber tracking and the vertexing processes. Outside of these processes, a cheating strategy is implemented where the digi are assigned to a particle using the Montecarlo truth. The same is true for the assigned identity of each reconstructed particle through the use of a LUT, where misidentification probabilities and efficiencies are evaluated based on the performance of each sub-detector.

C. Helix fitting from the Fiber Tracker

In the fiber tracker, a full helix fit of the tracks propagating through the solenoidal magnetic field is performed. Since the paths follow a helix along the z-axis, the fit can be broken up into two uncorrelated fitters: a circular fitter that fits the (d_0,C,φ_0) parameters on the XY plane, and a line fitter that fits the (tan(λ),z_0) parameters on the SZ plane. The associated covariance matrix accounts for all materials traversed by the tracks. Only those tracks successfully fitted, with a satisfactory χ^2, are further passed to the next step of the reconstruction.

D. Simulation of the LGAD pixel detector

The tracks successfully reconstructed in the fiber tracker are associated with the digi in the LGAD pixel detector. Consequently, a parametric smearing is performed to the momentum of the tracks. The parameters are chosen based on geometrical uncertainties related to reconstruction of the Cerenkov cone. Multiple scattering in the aerogel and in the gas is also taken into account, as well as the dimensions of the SiPM's.

V. STAGE 1: Preliminary Time Fitter

The process for implementing a time fitter was broken up into two separate stages. Stage 1 is a preliminary fitter to act as a proof of concept, while stage 2 would implement a full chi-square fit.

To verify the idea of incorporating the time information into the reconstruction, a linear chi-square was used to fit the preliminary t_0, where all error is projected onto the dependent variable.

$$\chi^2 = \sum \frac{(y_i - bx_i - a)^2}{\sigma_{y_i}^2}$$

Parameter Definitions:

χ^2 = The chi-square

y_i = The time of the i-th hits

b = The inverse of velocity

x_i= The path length of the i-th hits

a = t_0

$\sigma_{y_i}^2$ = The variance of the time of the i-th hits

Stage 1 was completed in 4 steps:

1. Calculation of the decay product's velocity
2. Calculation of the measured time's variance
3. Fitting t_0 and velocity
4. Calculation of the chi-square

A. Calculation of the Decay Product's velocity

The momentum of the decay products can be directly calculated from the five helical parameters previously fitted by the circle and line fitters. The simulation already accounts for all loss of energy during the individual hits. Using the transversal momentum, p_t, and assuming the decay products have the mass of a pion, the velocity can be calculated relativistically.

$$v = c\sqrt{\frac{\left(\frac{p_t}{m}\right)^2}{1+\left(\frac{p_t}{m}\right)^2}}$$

Note that the convention in the REDTOP framework uses GeV as the unit for all mass, momentum, and energy by evaluating c, the speed of light, as one. However, in the above equation it is reconverted from a fraction of c to an actual value in nm/s.

B. Calculation of the Measured Time's Variance

The error to be used in the final chi-square equation projects all error onto y_i , the dependent variable. The error propagation from momentum onto this time variable is computed as follows:

$$y_i = bx_i + a$$

Switching the variables from the conventional linear format to what they represent in the REDTOP simulation.

$$t_i = \frac{x_i}{v} + t_0$$

Taking the standard deviation of each component, where the fitting value's error (σ_{t_0}) is zero.

$$\sigma_{t_i} = \sigma_{\frac{x_i}{v}} + \sigma_{t_0}$$

$$\sigma_{\frac{x_i}{v}} = x_i \frac{\partial}{\partial \frac{p}{m}}\left(\frac{1}{v}\right)\sigma_{\frac{p}{m}} + \left(\frac{1}{v}\right)\sigma_{x_i}$$

The final step of squaring the equation to get time variance yields the final equation.

$$\sigma_{t_i}^2 = \left(\frac{v x_i m^4}{p^4}\right)^2 \begin{bmatrix} p_x & p_y & p_z \end{bmatrix} \begin{bmatrix} \sigma_{p_x}^2 & \sigma_{p_x}\sigma_{p_y} & \sigma_{p_x}\sigma_{p_z} \\ \sigma_{p_y}\sigma_{p_x} & \sigma_{p_y}^2 & \sigma_{p_y}\sigma_{p_z} \\ \sigma_{p_z}\sigma_{p_x} & \sigma_{p_z}\sigma_{p_y} & \sigma_{p_z}^2 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}$$

The square of the inherent error of the detectors is omitted from this equation because it is dependent on which detector measured the particle. The resolution of the LGAD pixel detector and fiber tracker are 30ps and 300ps, respectively.

C. Fitting t_0 and Velocity

Using the original chi-square equation, the sigma can be distributed to all iterative variables.

$$\chi^2 = \frac{\left(\sum t_i - \left(\frac{1}{v}\right)\sum x_i - t_0\right)^2}{\sum \sigma_{t_i}^2}$$

Taking the partial derivative of the chi-square with respect to velocity, and the partial derivative with respect to t_0, and then solving for each parameter in a linear system leads to the following solutions for velocity and t_0

$$v = \frac{\sum y_i \sum x_i^2 - \sum x_i \sum x_i}{\sum \sigma_{t_i}^2 \sum x_i\, y_i - \sum x_i \sum y_i}$$

$$t_0 = \frac{\sum y_i \sum x_i^2 - \sum x_i\, y_i \sum x_i}{\sum y_i \sum x_i^2 - \sum x_i \sum x_i}$$

Using 1000 simulated events of 200 MeV π^+, the following histograms demonstrate the fitted values of velocity and t_0

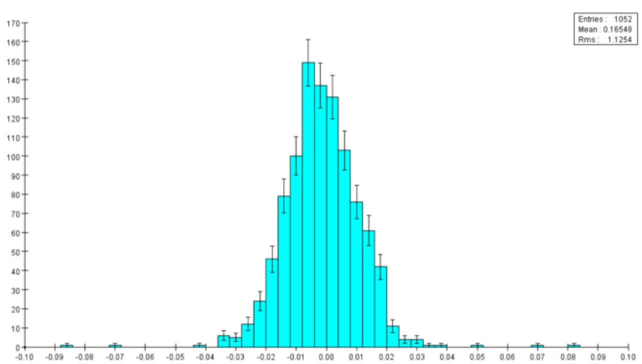

FIG. 2. Distribution of the fitted t_0 parameter (Events vs. t_0)

The evaluated mass of the particle was found by using the fitted velocity value. The residual of the mass is the evaluated value, minus the true mass of a pion (0.13957061 GeV/c²).

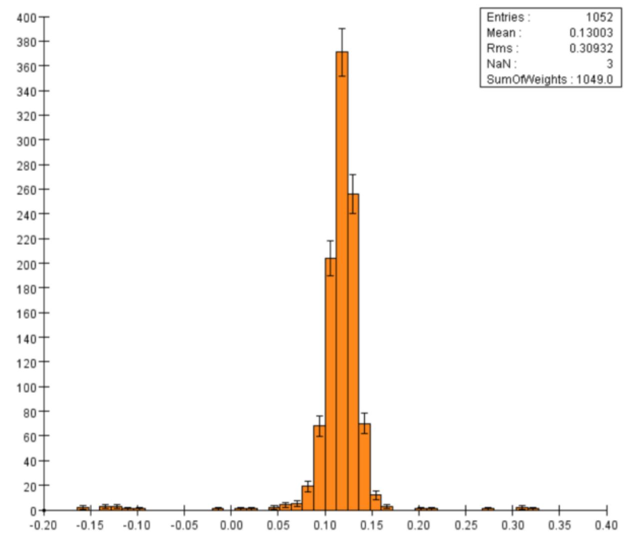

FIG. 3. Distribution of the fitted mass values (events vs. mass in GeV/c^2)

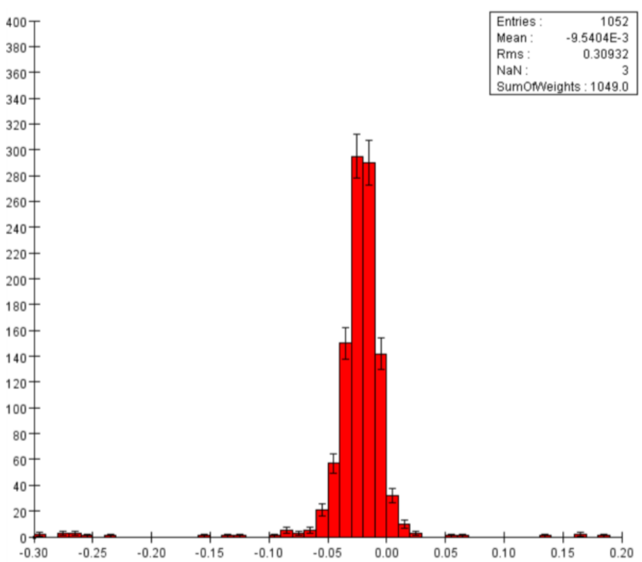

FIG. 4. Distribution of the residual of the mass (Events vs Residual)

D. Calculation of the Chi-Square

With velocity and t_0 values determined, they can be used to evaluate the chi-square.

$$\chi^2 = \frac{\left(\sum t_i - \left(\frac{1}{v}\right)\sum x_i - t_0\right)^2}{\sum \sigma_{t_i}^2}$$

The Normalized chi-square distributions are represented in the following histograms.

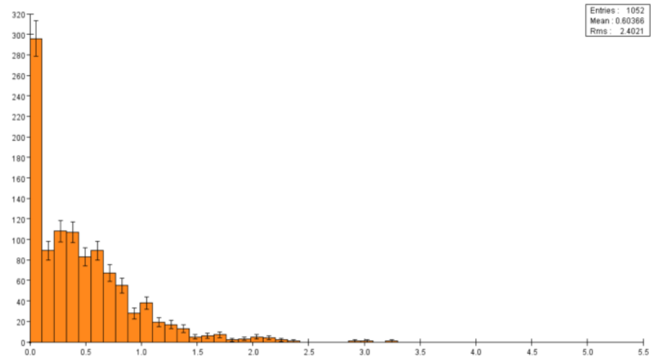

FIG. 5. Normalized distribution of the chi-square of the t_0 fit (events vs chi-square)

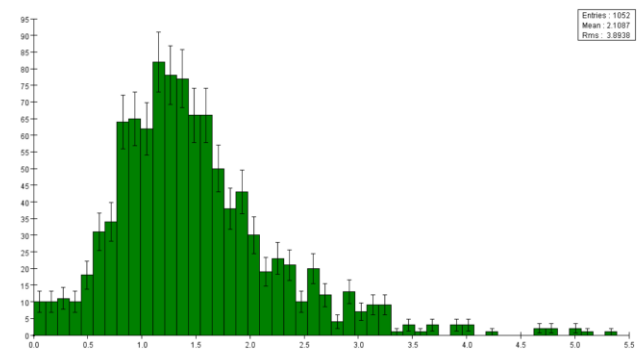

FIG. 6. Normalized distribution of the chi-square of all three fitters (Events vs. Chi-Square)

VI. STAGE 2: Full Time Fitter

As described in section IV subsection C, the circle fitting procedure will calculate a chi-square for the φ_0, C, and d_0 parameters by using the hit coordinates on the XY plane and the weights of the hits; however, an XY coordinate pair can also be found by rederiving it from the measured time of the hit, and it can be passed to the circle fitter again as a hit object. By using this method, the chi-square of the time information can be processed in the circle fitter without the need for a separate fitter.

Similarly, the line fitter on the SZ plane can directly process the time information by rederiving the SZ coordinate of each hit based on the measured time.

A. Including Time Information in the Circle Fitter

In order to derive the XY coordinate pair from the time information, the following equations can be used. S is the arc length, x_c is the X coordinate of the circle's center, and y_c is the Y coordinate of the circle's center.

$$x_i = x_c - \frac{sin(\phi_0 - SC)}{C}$$

$$y_i = y_c + \frac{cos(\phi_0 - SC)}{C}$$

The weight of the coordinate pair is found by calculating the inverse of the time variance. This can be calculated by the following error propagation function, where $t=S/v_t$. S is the arc length and V_t is the transversal velocity. r represents the evaluated radius of curvature.

$$\frac{1}{\sigma_{t_i}^2} = \begin{bmatrix} \frac{\partial t}{\partial d_0} & \frac{\partial t}{\partial \phi_0} & \frac{\partial t}{\partial r} \end{bmatrix} \begin{bmatrix} \sigma_{d_0}^2 & \sigma_{d_0}\sigma_{\phi_0} & \sigma_{d_0}\sigma_r \\ \sigma_{d_0}\sigma_{\phi_0} & \sigma_{\phi_0}^2 & \sigma_{\phi_0}\sigma_r \\ \sigma_{d_0}\sigma_r & \sigma_r\sigma_{\phi_0} & \sigma_r^2 \end{bmatrix} \begin{bmatrix} \frac{\partial t}{\partial d_0} \\ \frac{\partial t}{\partial \phi_0} \\ \frac{\partial t}{\partial r} \end{bmatrix}$$

B. Including Time Information in the Line Fitter

Rederiving the SZ coordinate is found by two simple equations. S is the path length and Z is the coordinate on the z-axis.

$$\mathrm{S_i} = t_i \mathrm{c} \sqrt{\frac{\left(\frac{p_z}{m}\right)^2}{1 + \left(\frac{p_z}{m}\right)^2}}$$

$$\mathrm{z} = \mathrm{z_0} + \mathrm{S_i} * \tan(\lambda)$$

The line fitter calculates the chi-square of the hits on the SZ plane in a similar way to stage 1 of the time fitter. All error is projected onto the dependent variable in a linear chi-square, so only the standard deviation of the Z coordinate needs to be calculated.

The standard deviation of the Z coordinate is dependent on the standard deviation of time.

$$\sigma_{t_i} = \sqrt{\frac{\sigma_S^2}{v_z^2} + \frac{\sigma_{v_z}^2 S^2}{v_z^4}}$$

Using this information, the final error in the Z coordinate can be found.

$$\sigma_z = \left(\frac{p_z}{m}\right) \tan(\lambda) \sigma_{t_i}$$

VII. RESULTS

While stage 1 of the time fitter successfully calculates a t_0 parameter as shown in figure 2, the chi-square, shown in figure 5, does not possess a distribution maximizing at one, which is what would demonstrate a good fit

Stage 2 is still a work in progress, but once fully implemented, the REDTOP simulation will have a powerful fitter that will create a 4-dimensional helix reconstruction.

[1] K. Maamari, C. Gatto, The Implementation of a Secondary Vertex Search Algorithm in REDTOP Simulations (2019).

[2] C.Gatto,https://redtop.fnal.gov/the-geniehadeventgeneration-framework (2019). et a.

[3] C. Gatto, Talk at LCWS15, Whistler, Canada (2015).

[4] C. Gatto et al., J. Phys. Conf. Ser. Vol. 404 - 012030 (2015).

[5] C. Gatto et al., J. Phys. Conf. Ser. Vol. 587 - 012060 (2015).