0.1 The equal sign, amounts and number lines

The equal sign

As the name implies, the *equal sign* $=$ refers to things that are the same. In what sense some things are the same is a philosophical question and initially we are bound to this: What equality $=$ points to must be understood by the context in which the sign is used. With this understanding of $=$ we can study some basic properties of our numbers and then later return to more precise meanings of the sign.

> **The language box**
>
> Common ways of expressing $=$ is
>
> - "equals"
>
> - "is the same as"

Amounts and number lines

There are so many things numbers can represent, however, in this book we shall stick to two ways of interpreting a number; a number as an *amount* and a number as a *placement on a line*. All representations of numbers relies on the understanding of 0 and 1.

Numbers as amounts

Talking about an amount, the number 0 is[1] connected to "nothing". A figure showing nothing will therefore equal 0:

$$= 0$$

1 we'll draw like a box:

$$\square = 1$$

In this way, other numbers are defined by how many one-boxes (ones) we have:

$$\boxed{} = 2 \qquad \boxed{} = 3 \qquad \boxed{} = 4$$
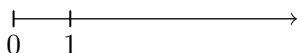
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[1]In *Chapter* **??** we'll se that there are also other interpretations of 0.

Numbers as placements on a line

When placing numbers on a line, 0 is our starting point:

$$\vdash\!\!\!\longrightarrow$$
$$0$$

Now we place 1 a certain length to the right of 0:

$$\vdash\!\!\!\!\!\dashv\!\!\!\longrightarrow$$
$$0 \quad 1$$

Other numbers are now defined by how many one-lengths (ones) we are away from 0:

$$\vdash\!\!\dashv\!\!\dashv\!\!\dashv\!\!\dashv\!\!\dashv\!\!\longrightarrow$$
$$0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$$
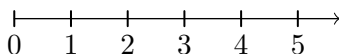
Positive integers

We'll soon see that numbers do not necessarily have to be a *whole* amount of ones, but those who *are* have their own name:

0.1 Positive integers

Numbers which are a whole amount of ones are called *positive*[1] *integers*. The positive integers are

$$1, 2, 3, 4, 5 \text{ and so on.}$$

Positive integers are also called *natural numbers*.

What about 0?

Some authors also include 0 in the definition of positive integers/natural numbers. This is in some cases beneficial, in others not.
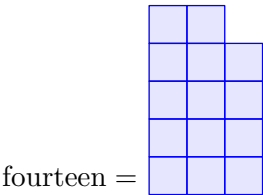
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[1]We'll see what the the word *positive* refers to in chapter chapter **??**.

0.2 Numbers, digits and value

Our numbers consists of the *digits* 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 and their *position*. The digits and their positions defines[1] the *value* of numbers.
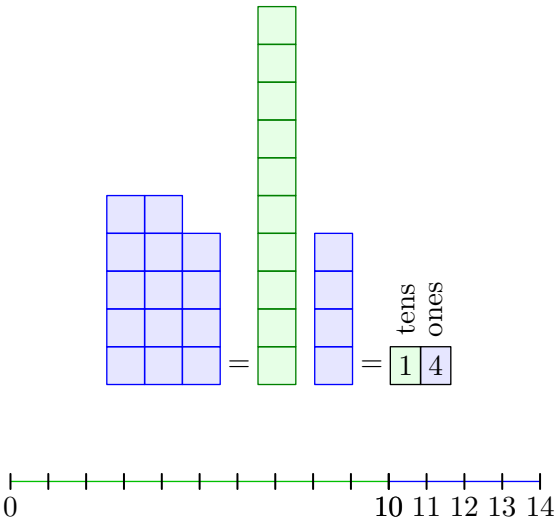
Integers larger then 10

Let's, as an example, write the number *fourteen* by our digits.

$$\text{fourteen} =$$

We can now make a group of 10 ones, then we also have 4 ones. By this, we write fourteen as

$$\text{fourteen} = 14$$

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[1]Later on, we'll also se that *signs* have an impact on a numbers value (see *Chapter* ??).
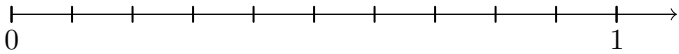
Decimal numbers

Sometimes we don't have a whole amount of ones, and this brings the need of dividing 1 into smaller pieces. Let's start off by drawing a one:
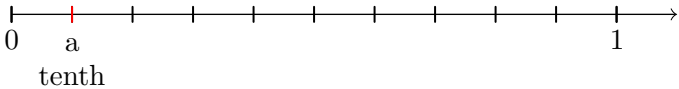
$$\square = 1$$

$$0 \qquad\qquad\qquad\qquad\qquad\qquad 1$$

Now we divide our one into 10 smaller pieces:

$$\boxplus = 1$$

$$0 \qquad\qquad\qquad\qquad\qquad\qquad 1$$

Since we have divided 1 into 10 pieces, we name one such piece *a tenth*:

$$\square = \text{a tenth}$$

$$0 \quad \begin{matrix}\text{a}\\\text{tenth}\end{matrix} \qquad\qquad\qquad\qquad\qquad 1$$

We indicate tenths by using the *decimal mark* $\boxed{.}$:

$$\square = 0.1$$

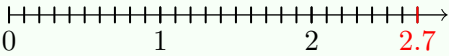$$0 \quad 0.1 \;\; 0.2 \;\; 0.3 \;\; 0.4 \;\; 0.5 \;\; 0.6 \;\; 0.7 \;\; 0.8 \;\; 0.9 \;\; 1$$

Example

$$\square\;\boxplus = 2.7$$
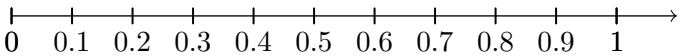
$$0 \qquad\quad 1 \qquad\quad 2 \quad\; 2.7$$

The language box

In a lot of countires, comma $\boxed{,}$ is used as decimal mark instead of dot.

$$3{,}5 \quad (\textit{other})$$
$$3.5 \quad (\textit{english})$$

4

Base-10 positional notation

So far we have seen how we can express the value of a number by placing digits according to the amount of tens, ones and tenths, and the pattern continues:

0.2 Base-10 positional notation

The value of a number is given by the digits $0, 1, 2, 3, 4, 5, 6, 7, 8$ and 9 and their position. In respect of the digit indicating ones,

- digits to the left (respectively) indicate amount of tens, hundeds, thousands etc.

- digits to the left (respectively) indicate amount of tenths, hundedths, thousandths etc.

Example 1

$$= 4\ 7.8$$
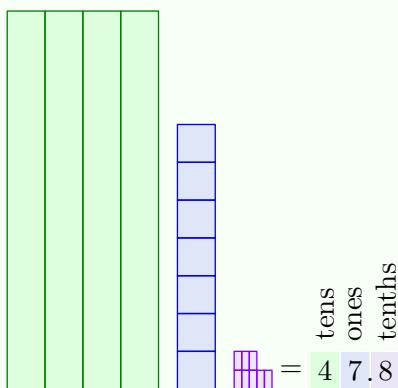
tens ones tenths

Example 2

thousands hundreds tens ones tenths hundredths

3805.72

0.3   Koordinatsystem

Two number lines can be put together to form a *coordinate system*. In that case we place one number line *horizontally* and one *vertically*. A position in a coordinate system is called a *point*.

In fact, there are a lot of types of coordinate systems but in this book we'll use the term about the *cartesian coordinate system*. It is named after the french mathematician and philosopher René Descartes.

A point is written as two numbers inside a bracket. We shall call these two numbers the *first coordinate* and the *second coordinate*.

- The first coordinate tells how many units to move along the horizontal axis.

- The second coordinate tells how many units to move along the vertical axis.

In the figure, the points (2,3), (5,1) and (0,0) are shown. The point where the axis intersect, that is (0,0), is called *origo*.