

0.1 Multiplication

Multiplication by integers: initial definition

When adding equal numbers, we can use the multiplication symbol \cdot to write our calculations more compact:

Example

$$4 + 4 + 4 = 4 \cdot 3$$

$$8 + 8 = 8 \cdot 2$$

$$1 + 1 + 1 + 1 + 1 = 1 \cdot 5$$

The language box

A calculation involving multiplication includes several *factors* and one *product*. In the calculation

$$4 \cdot 3 = 12$$

both 4 and 3 are factors, while 12 is the product.

Common ways of saying $4 \cdot 3$ are

- "4 times 3"
- "4 multiplied by 3"
- "4 and 3 multiplied together"

A lot of texts use \times instead of \cdot . In computer programming, $*$ is the most common symbol for multiplication.

Multiplication involving amounts

Let us illustrate $2 \cdot 3$:

$$2 \cdot 3 = \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} + \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} + \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \square & \square & \square \\ \hline \end{array}$$

Now notice the product of $3 \cdot 2$:

$$3 \cdot 2 = \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \square \\ \hline \end{array} + \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \square \\ \hline \end{array} = \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \square & \square \\ \hline \end{array}$$

0.1 Multiplication is commutative

The order of the factors have no impact on the product.

Example

$$3 \cdot 4 = 12 = 4 \cdot 3$$

$$6 \cdot 7 = 42 = 7 \cdot 6$$

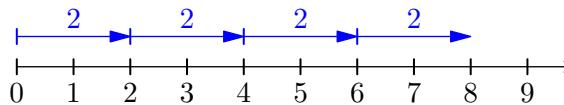
$$8 \cdot 9 = 72 = 9 \cdot 8$$

Multiplication on the number line

We can also use the number line to calculate multiplications. In the case of $2 \cdot 4$ we can think like this:

" $2 \cdot 4$ means moving 2 places to the right, 4 times."

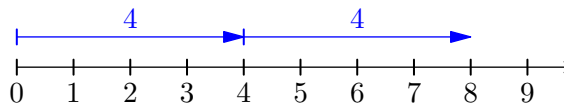
$$2 \cdot 4 = 8$$



We can also use the number line to convince ourselves that multiplication is commutative:

" $4 \cdot 2$ means moving 4 places to the right, 2 times."

$$4 \cdot 2 = 8$$



Final definition of multiplication by positive integers

It may be the most intuitive to interpret " 2 times 3 " as " 3 , 2 times". Then

$$2 \text{ times } 3 = 3 + 3$$

In this section we introduced $2 \cdot 3$, that is " 2 times 3 ", as $2 + 2 + 2$. With this interpretation, $3 + 3$ corresponds to $3 \cdot 2$, but the fact that multiplication is a commutative operation ([Rule 0.1](#)) ensures that the one interpretation does not exclude the other; $2 \cdot 3 = 2 + 2 + 2$ and $2 \cdot 3 = 3 + 3$ are two expressions of same value.

0.2 Multiplication as repeated addition

Multiplication involving a positive integer can be expressed as repeated addition.

Example 1

$$4 + 4 + 4 = 4 \cdot 3 = 3 + 3 + 3 + 3$$

$$8 + 8 = 8 \cdot 2 = 2 + 2 + 2 + 2 + 2 + 2 + 2$$

$$1 + 1 + 1 + 1 + 1 = 1 \cdot 5 = 5$$

Notice

The fact that multiplication with positive integers can be expressed as repeated addition does not exclude other expressions. It's nothing wrong with writing $2 \cdot 3 = 1 + 5$.