0.1 The equal sign, amounts and number lines

The equal sign

As the name implies, the *equal sign* $=$ refers to things that are the same. In what sense some things are the same is a philosophical question and initially we are bound to this: What equality $=$ points to must be understood by the context in which the sign is used. With this understanding of $=$ we can study some basic properties of our numbers and then later return to more precise meanings of the sign.

> **The language box**
>
> Common ways of expressing $=$ is
>
> - ”equals”
> - ”is the same as”

Amounts and number lines

There are so many things numbers can represent, however, in this book we shall stick to two ways of interpreting a number; a number as an *amount* and a number as a *placement on a line*. All representations of numbers relies on the understanding of 0 and 1.

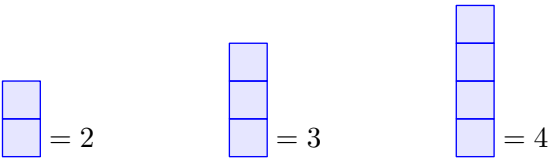
Numbers as amounts

Talking about an amount, the number 0 is[1] connected to ”nothing”. A figure showing nothing will therefore equal 0:

$$= 0$$

1 we’ll draw like a box:

In this way, other numbers are defined by how many one-boxes (ones) we have:



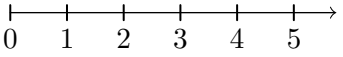
[1] In *Chapter* **??** we’ll se that there are also other interpretations of 0.

Numbers as placements on a line

When placing numbers on a line, 0 is our starting point:

Now we place 1 a certain length to the right of 0:

Other numbers are now defined by how many one-lengths (ones) we are away from 0:



Positive integers

We'll soon see that numbers do not necessarily have to be a *whole* amount of ones, but those who *are* have their own name:

0.1 Positive integers

Numbers which are a whole amount of ones are called *positive*[1] *integers*. The positive integers are

$$1, 2, 3, 4, 5 \text{ and so on.}$$

Positive integers are also called *natural numbers*.

What about 0?

Some authors also include 0 in the definition of positive integers/natural numbers. This is in some cases beneficial, in others not.

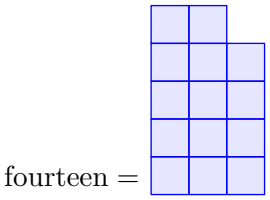
[1] We'll see what the the word *positive* refers to in chapter *chapter* **??**.

0.2 Numbers, digits and value

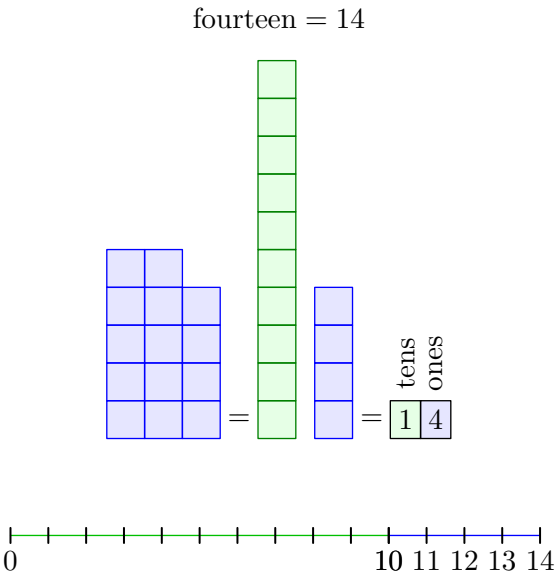
Our numbers consists of the *digits* $0, 1, 2, 3, 4, 5, 6, 7, 8$ and 9 and their *position*. The digits and their positions defines[1] the *value* of numbers.

Integers larger then 10

Let's, as an example, write the number *fourteen* by our digits.



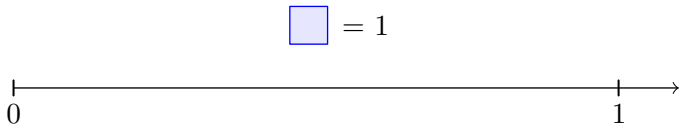
We can now make a group of 10 ones, then we also have 4 ones. By this, we write fourteen as



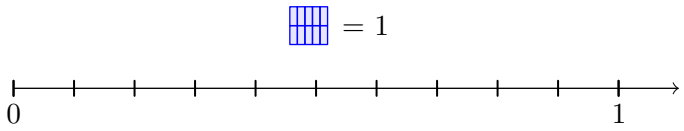
[1] Later on, we'll also see that *signs* have an impact on a numbers value (see *Chapter ??*).

Decimal numbers

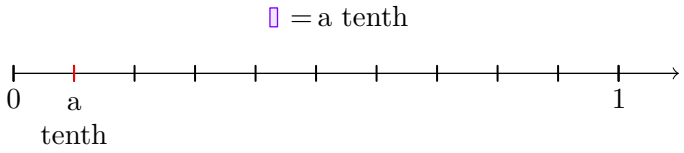
Sometimes we don't have a whole amount of ones, and this brings the need of dividing 1 into smaller pieces. Let's start off by drawing a one:



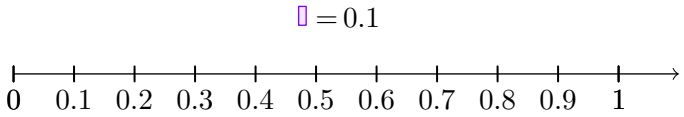
Now we divide our one into 10 smaller pieces:



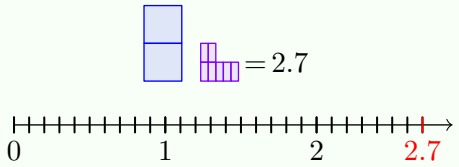
Since we have divided 1 into 10 pieces, we name one such piece *a tenth*:



We indicate tenths by using the *decimal mark* . :



Example



The language box

In a lot of countires, comma , is used as decimal mark instead of dot.

3,5 (*other*)

3.5 (*english*)

Base-10 positional notation

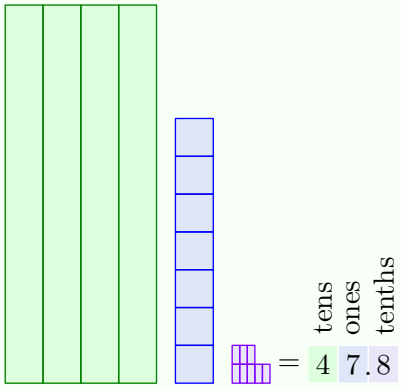
So far we have seen how we can express the value of a number by placing digits according to the amount of tens, ones and tenths, and the pattern continues:

0.2 Base-10 positional notation

The value of a number is given by the digits $0, 1, 2, 3, 4, 5, 6, 7, 8$ and 9 and their position. In respect of the digit indicating ones,

- digits to the left (respectively) indicate amount of tens, hundeds, thousands etc.
- digits to the left (respectively) indicate amount of tenths, hundedths, thousandths etc.

Example 1



Example 2

0.3 Coordinate systems

Two number lines can be put together to form a *coordinate system*. In that case we place one number line *horizontally* and one *vertically*. A position in a coordinate system is called a *point*.

In fact, there are a lot of types of coordinate systems but in this book we'll use the term about the *cartesian coordinate system*. It is named after the french mathematician and philosopher René Descartes.

A point is written as two numbers inside a bracket. We shall call these two numbers the *first coordinate* and the *second coordinate*.

- The first coordinate tells how many units to move along the horizontal axis.
- The second coordinate tells how many units to move along the vertical axis.

In the figure, the points (2,3), (5,1) and $(0, 0)$ are shown. The point where the axis intersect, that is $(0, 0)$, is called *origo*.