0.1 Potensar

$$\text{grunntal} \longrightarrow 2^{3 \longleftarrow \text{eksponent}}$$

A power is composed by a *base* and an *exponent*. For example, 2^3 is a power with base 2 and exponent 3. An exponent which is a positive integer indicates the amount of instances of the base to be multiplied together, that is

$$2^3 = 2 \cdot 2 \cdot 2$$

0.1 Potenstall

a^n is a power with base a and exponent n.

If n is a natural number, a^n corresponds to n instances of a multiplied together.

Notice: $a^1 = a$

Example 1

$$\begin{aligned} 5^3 &= 5 \cdot 5 \cdot 5 \\ &= 125 \end{aligned}$$

Example 2

$$c^4 = c \cdot c \cdot c \cdot c$$

Example 3

$$\begin{aligned} (-7)^2 &= (-7) \cdot (-7) \\ &= 49 \end{aligned}$$

The language box

Common ways of saying 2^3 are[1]

- ”2 to the power of 3”
- ”2 to the third power”

In programming languages one usually writes the symbol `^` or the symbols `**` between the base and the exponent.

[1] Attention! The examples illustrates a small paradox of the English language. Thing is, *power* is also a (and in the spoken language the preferred) synonym for *exponent*.

Notice

The next pages declares rules concerning powers with corresponding explanations. Even though one wish to have these explanations as general as possible, we choose to use exponents which are not variables. Using variables as exponents would lead to less reader-friendly expressions and it is our claim that the general cases are well illustrated by the specific cases.

0.2 Multiplikasjon av potensar

$$a^m \cdot a^n = a^{m+n}$$

Example 1

$$\begin{aligned} 3^5 \cdot 3^2 &= 3^{5+2} \\ &= 3^7 \end{aligned}$$

Example 2

$$\begin{aligned} b^4 \cdot b^{11} &= b^{3+11} \\ &= b^{14} \end{aligned}$$

Example 3

$$\begin{aligned} a^5 \cdot a^{-7} &= a^{5-7} \\ &= a^{-2} \end{aligned}$$

(See *Rule 0.5* regarding how powers with negative exponents can be interpreted.)

0.2 Multiplikasjon av potensar (forklaring)

Let's study the case

$$a^2 \cdot a^3$$

We have

$$a^2 = 2 \cdot 2$$

$$a^3 = 2 \cdot 2 \cdot 2$$

Hence we can write

$$\begin{aligned} a^2 \cdot a^3 &= \overbrace{a \cdot a}^{a^2} \cdot \overbrace{a \cdot a \cdot a}^{a^3} \\ &= a^5 \end{aligned}$$

0.3 Divisjon med potensar

$$\frac{a^m}{a^n} = a^{m-n}$$

Example 1

$$\frac{3^5}{3^2} = 3^{5-2} = 3^3$$

Example 2

$$\begin{aligned} \frac{2^4 \cdot a^3}{a^2 \cdot 2^2} &= 2^{4-2} \cdot a^{3-2} \\ &= 2^2 a \\ &= 4a \end{aligned}$$

0.3 Divisjon med potensar (forklaring)

Let's examine the fraction

$$\frac{a^5}{a^2}$$

Expanding the powers, we get

$$\begin{aligned}
\frac{a^5}{a^2} &= \frac{a \cdot a \cdot a \cdot a \cdot a}{a \cdot a} \\
&= \frac{\cancel{a} \cdot \cancel{a} \cdot a \cdot a \cdot a}{\cancel{a} \cdot \cancel{a}} \\
&= a \cdot a \cdot a \\
&= a^3
\end{aligned}$$

The above calculations are equivalent to writing

$$\begin{aligned}
\frac{a^5}{a^2} &= a^{5-2} \\
&= a^3
\end{aligned}$$

0.4 Spesialtilfellet a^0

$$a^0 = 1$$

Example 1

$$1000^0 = 1$$

Example 2

$$(-b)^0 = 1$$

0.4 Spesialtilfellet a^0 (forklaring)

A number divided by itself always equals 1, therefore

$$\frac{a^n}{a^n} = 1$$

From this, and *Rule 0.3*, it follows that

$$\begin{aligned}
1 &= \frac{a^n}{a^n} \\
&= a^{n-n} \\
&= a^0
\end{aligned}$$

0.5 Potens med negativ eksponent

$$a^{-n} = \frac{1}{a^n}$$

Example 1

$$a^{-8} = \frac{1}{a^8}$$

Example 2

$$(-4)^{-3} = \frac{1}{(-4)^3} = -\frac{1}{64}$$

0.5 Potens med negativ eksponent (forklaring)

By *Rule 0.4*, we have $a^0 = 1$. Thus

$$\frac{1}{a^n} = \frac{a^0}{a^n}$$

By *Rule 0.3*, we obtain

$$\begin{aligned} \frac{a^0}{a^n} &= a^{0-n} \\ &= a^{-n} \end{aligned}$$

0.6 Brøk som grunntal

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Example 1

$$\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$$

Example 2

$$\left(\frac{a}{7}\right)^3 = \frac{a^3}{7^3} = \frac{a^3}{343}$$

0.6 Brøk som grunntal (forklaring)

Let's study

$$\left(\frac{a}{b}\right)^3$$

We have

$$\begin{aligned}\left(\frac{a}{b}\right)^3 &= \frac{a \cdot a \cdot a}{b \cdot b \cdot b} \\ &= \frac{a^3}{b^3}\end{aligned}$$

0.7 Faktorar som grunntal

$$(ab)^m = a^m b^m$$

Example 1

$$\begin{aligned}(3a)^5 &= 3^5 a^5 \\ &= 243a^5\end{aligned}$$

Example 2

$$(ab)^4 = a^4 b^4$$

0.7 Faktorar som grunntal (forklaring)

Let's use $(a \cdot b)^3$ as an example. We have

$$\begin{aligned}(a \cdot b)^3 &= (a \cdot b) \cdot (a \cdot b) \cdot (a \cdot b) \\ &= a \cdot a \cdot a \cdot b \cdot b \cdot b \\ &= a^3 b^3\end{aligned}$$

0.8 Potens som grunntal

$$(a^m)^n = a^{m \cdot n}$$

Example 1

$$\begin{aligned}\left(c^4\right)^5 &= c^{4 \cdot 5} \\ &= c^{20}\end{aligned}$$

Example 2

$$\begin{aligned}\left(3^{\frac{5}{4}}\right)^8 &= 3^{\frac{5}{4} \cdot 8} \\ &= 3^{10}\end{aligned}$$

0.8 Potens som grunntal (forklaring)

Let's use $(a^3)^4$ as an expample. We have

$$\left(a^3\right)^4 = a^3 \cdot a^3 \cdot a^3 \cdot a^3$$

By *Rule 0.2*, we get

$$\begin{aligned}a^3 \cdot a^3 \cdot a^3 \cdot a^3 &= a^{3+3+3+3} \\ &= a^{3 \cdot 4} \\ &= a^{12}\end{aligned}$$

0.9 n-rot

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

The symbol $\sqrt{\ }$ is called the *radical sign*. In the case of an exponent equal to $\frac{1}{2}$ it is common to omit 2 from the radical:

$$a^{\frac{1}{2}} = \sqrt{a}$$

Example

By *Rule 0.8*, we have

$$\begin{aligned} \left(a^b\right)^{\frac{1}{b}} &= a^{b\cdot\frac{1}{b}} \\ &= a \end{aligned}$$

For example is

$$9^{\frac{1}{2}} = \sqrt{9} = 3, \text{ since } 3^2 = 9$$

$$125^{\frac{1}{3}} = \sqrt[3]{125} = 5, \text{ since } 5^3 = 125$$

$$16^{\frac{1}{4}} = \sqrt[4]{16} = 2, \text{ since } 2^4 = 16$$

The language box

$\sqrt{9}$ is called " the square root of 9"

$\sqrt[3]{8}$ is called "the cube root of 8"

$\sqrt[5]{9}$ is called "the 5th root of 9".

0.2 Irrasjonale tal

0.10 Irrational numbers

A number which is *not* a rational number, is an irrational number[1].

The value of an irrational number are decimal numbers with infinite digits in a non-repeating manner.

Example 1

$\sqrt{2}$ is an irrational number.

$$\sqrt{2} = 1.414213562373...$$

[1]Strictly speaking, irrational numbers are all *real* numbers which are not rational numbers. But to explain what *real* numbers are, we have to mention *imaginary* numbers, and this we choose not to do in this book.