## 0.1 Order of operations

### Priority of the operations

Look at the following calculation:

$$2 + 3 \cdot 4$$

This *could* have been interpreted in two ways:

1. "2 plus 3 equals 5. 5 times 4 equals 20. The answer is 20."
2. "3 times 4 equals 12. 2 plus 12 equals 14. The answer is."

But the answers are not the same! This points out the need to have rules for what to calculate first. One of these rules is that multiplication and division is to be calculated *before* addition or subtraction, which means that

$$\begin{aligned} 2 + 3 \cdot 4 &= \text{"Calculate } 3 \cdot 4\text{, then add 2"} \\ &= 2 + 12 \\ &= 14 \end{aligned}$$

But what if we wanted to calculate $2 + 3$ first, then multiply the sum by 4? We use parentheses to tell that something is to be calculeted first:

$$\begin{aligned} (2 + 3) \cdot 4 &= \text{"Calculate } 2 + 3\text{, multiply by 4 afterwards"} \\ &= 5 \cdot 4 \\ &= 20 \end{aligned}$$

**0.1 Order of operations**

1. Expressions with parentheses
2. Multiplication or division
3. Addition or subtraction

**Example 1**

Calculate

$$23 - (3 + 9) + 4 \cdot 7$$

**Answer**

$$\begin{aligned} 23 - (3 + 9) + 4 \cdot 7 &= 23 - 12 + 4 \cdot 7 && \text{Parantheses} \\ &= 23 - 12 + 28 && \text{Multiplication} \\ &= 39 && \text{Addition and subtraction} \end{aligned}$$

**Example 2**

Calculate

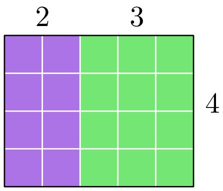
$$18 : (7 - 5) - 3$$

**Answer**

$$\begin{aligned} 18 : (7 - 5) &= 18 : 2 - 3 && \text{Parantheses} \\ &= 9 - 3 && \text{Division} \\ &= 6 && \text{Addition and subtraction} \end{aligned}$$

## Multiplication involving paranthesis

How many boxes are present in this figure?




Two correct interpretations include:

1. It is $2 \cdot 4 = 8$ purple boxes and $3 \cdot 4 = 12$ green boxes. In total there are $8 + 12 = 20$ boxes. This we can write as

$$2 \cdot 4 + 3 \cdot 4 = 20$$

2. It is $2 + 3 = 5$ boxes horizontally and 4 boxes vertically, so there are $5 \cdot 4 = 20$ boxes in total. This we can write as

$$(2 + 3) \cdot 4 = 20$$

From these two calculations it follows that

$$(2+3)\cdot 4 = 2\cdot 4 + 3\cdot 4$$

**0.2 Distributive law**

When an expression enclosed by a parenthesis is a factor, we can multiply the other factors with each term inside the parenthesis.

**Example 1**

$$(4+7)\cdot 8 = 4\cdot 8 + 7\cdot 8$$

**Example 2**

$$\begin{aligned}(10-7)\cdot 2 &= 10\cdot 2 - 7\cdot 2\\ &= 20-14\\ &= 6\end{aligned}$$

*Notice*: Obviously, it would be easier to calculate like this:

$$(10-7)\cdot 2 = 3\cdot 2 = 6$$

**Example 2**

Calculate $12\cdot 3$.

**Answer**

$$\begin{aligned}12\cdot 3 &= (10+2)\cdot 3\\ &= 10\cdot 3 + 2\cdot 3\\ &= 30+6\\ &= 36\end{aligned}$$

**Notice**

We introduced parenthesis as an indicator of what to calculate first, but *Rule 0.2* gives an alternative and equivalent interpretation of parenthesis . The rule is especially useful when working with algebra (see *Part* **??** ).

### Multiplying by 0

Earlier whe have seen that 0 can be expressed as the difference between two numbers, and this can help us calculate when multiplying by 0. Let's look at the calculation

$$(2-2)\cdot 3$$

By *Rule 0.2*, we get

$$\begin{aligned}(2-2)\cdot 3 &= 2\cdot 3 - 2\cdot 3\\ &= 6-6\\ &= 0\end{aligned}$$

Since $0 = 2-2$, this means that

$$0\cdot 3 = 0$$

**0.3 Multiplication by 0**

If 0 is a factor,the product equals 0.

**Example 1**

$$7\cdot 0 = 0$$

$$0\cdot 219 = 0$$

### Associative laws

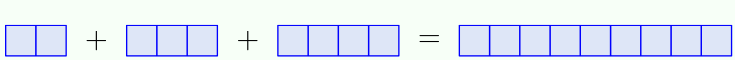
**0.4 Associative law for addition**

The placement of parentheses between terms have no impact on the sum.

**Example**

$$(2+3)+4 = 8$$

$$2+(3+4) = 8$$



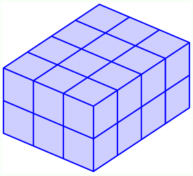
**0.5 Associative law for multiplication**

The placement of parentheses between factors have no impact on the product.

**Example**

$$(2 \cdot 3) \cdot 4 = 6 \cdot 4 = 24$$

$$2 \cdot (3 \cdot 4) = 2 \cdot 12 = 24$$



Opposite to addition and multiplication, neither subtraction nor divison is associative:

$$(12 - 5) - 4 = 7 - 4 = 3$$
$$12 - (5 - 4) = 12 - 1 = 11$$

$$(80 : 10) : 2 = 8 : 2 = 4$$
$$80 : (10 : 2) = 80 : 5 = 16$$

We have seen how parentheses hjelps indicating the *priority* of operations, but the fact that subtraction and divison is non-associative brings the need of having a rule of in which *direction* to calculate.

**0.6 Direction of calculations**

Operations which by *Rule 0.1* have equal priority, are to be calculated from left to right.

**Example 1**

$$\begin{aligned} 12 - 5 - 4 &= (12 - 5) - 4 \\ &= 7 - 4 \\ &= 3 \end{aligned}$$

**Example 2**

$$\begin{aligned} 80 : 10 : 2 &= (80 : 10) : 2 \\ &= 8 : 2 \\ &= 4 \end{aligned}$$

**Example 3**

$$\begin{aligned} 6 : 3 \cdot 4 &= (6 : 3) \cdot 4 \\ &= 2 \cdot 4 \\ &= 8 \end{aligned}$$