

## Exercise Sheet 3

Complete before class on Friday, September 26th

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**Exercise 1.** In the Point Line Cover problem, we are given a set of  $n$  points in the plane and an integer  $k$ , and the goal is to check if there exists a set of  $k$  lines on the plane that contain all the input points. Give a kernel for this problem with  $O(k^2)$  points.

*Hint: What can you say if more than  $k$  points lie on one line?*

**Exercise 2.** Using the preprocessing rules from the lecture, simplify the following integer linear program as much as possible.

$$\begin{aligned} \min & x_1 + 3x_2 - x_3 + 3x_4 \\ & x_1 - x_3 + 3x_4 \leq 6 \\ & x_3 + x_4 \leq 2 \\ & 2x_2 + 3x_4 \geq 4 \\ & x_2, x_4 \in \{0, 1\} \\ & x_1, x_3 \in \mathbb{Z}_{\geq 0} \end{aligned}$$

**Exercise 3.** In the  $d$ -Set Packing problem we are given a family of sets  $\mathcal{A}$  over universe  $U$ , each of cardinality at most  $d$ , and a number  $k \in \mathbb{N}$ . Our goal is to find  $k$  disjoint sets  $S_1, \dots, S_k$ . Use the sunflower lemma to derive a kernel for this problem with  $f(d)k^d$  sets, where  $f$  is an arbitrary function.

*Hint: Consider a  $(kd + 2)$ -sunflower. You can remove any of the sets in the sunflower without changing the answer to the problem. Why?*