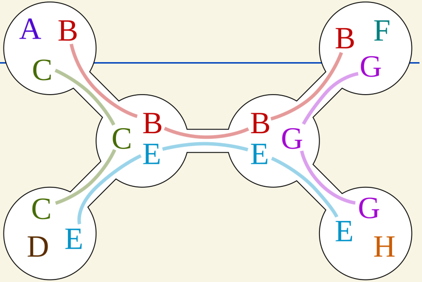
Dynamic Programming for Vertex Coloring



DM898: Parameterized Algorithms

Lars Rohwedder

Simple branching algorithm

Algorithm color(G, c)

Input: Graph $G = (V, E)$, number of colors c

Output: yes if G has proper coloring with c colors, no otherwise

- if $c = 0$ and $V \neq \emptyset$ return no
- if $c = 0$ and $V = \emptyset$ return yes
- result $\leftarrow$ no
- for each $U \subseteq V$ // Guess vertices of color c
    - if $(u, v) \in E$ for some $u, v \in U$ // U cannot be colored with one color
        - continue
    - else if color($G \setminus U, c - 1$) = yes
        - result $\leftarrow$ yes
- return result

Dynamic programming

We iteratively fill a table $D[V', c']$ containing yes if $G[V']$ has a proper c'-coloring and no otherwise.

Branching

- if $c = 0$ and $V \neq \emptyset$ return no
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- result $\leftarrow$ no
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  - if $(u, v) \in E$ for some $u, v \in U$
    - continue
  - else if color$(G \setminus U, c - 1) =$ yes
    - result $\leftarrow$ yes
- return result

Dynamic programming

$D[V', 0] = \text{no for all } V' \neq \emptyset$

$D[\emptyset, 0] = \text{yes}$

for each $V' \subseteq V$, $c' \in \{1, 2, \ldots, c\}$ in increasing order

- result $\leftarrow$ no
- for each $U \subseteq V'$
  - if $(u, v) \in E$ for some $u, v \in U$
    - continue
  - else if $D[V' \setminus U, c' - 1] =$ yes
    - result $\leftarrow$ yes
- $D[V', c'] \leftarrow$ result

return $D[V, c]$

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    - result $\leftarrow$ yes
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return $D[V, c]$

Running time

$n^{O(1)} \cdot c \cdot \sum_{V' \subseteq V} 2^{|V'|} = n^{O(1)} \cdot c \cdot 3^n$ by binomial theorem.