A Compendium of Geothermal Reservoir Simulator Benchmark Problems

International Partnership for Geothermal Technology

April 8, 2014

1 Theis Problem Benchmark

1.1 Problem description

This benchmark problem provides the solution describing the pressure changes that occur when fluid is produced from a symmetric, infinite, homogeneous, isotropic aquifer of uniform thickness. The problem tests the ability of a code to conserve mass and model a compressible fluid and tests mass conservation equation and source boundary conditions. The governing equation considered is

$$\frac{\partial p}{\partial t} = \frac{T}{S}\left(\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r}\right) \tag{1}$$

The problem is defined for a disk-shaped domain with Neumann condition representing pumping on the inside of the domain at well radius, r_{well} with outer radial dimension sufficiently distant to effect a semi-infinite domain (Figure 1). The Fluid properties are prescribed for 50 °C. Output data for x-axis pressures are calculated for times of 2.0 s, 20 s and 200 s.

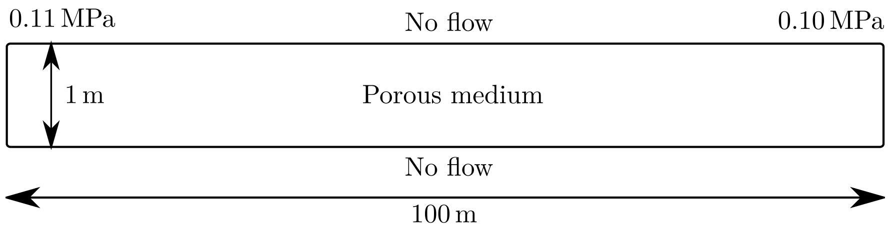


Figure 1: Model domain

The analytical solution of this problem introduces the similarity variable

$$u = \frac{r^2 S}{4Tt} \tag{2}$$

which is then used to determine the drawdown

$$s = \frac{Q_v}{4\pi T} f(u) \tag{3}$$

where $f(u)$ is the exponential integral, defined as

$$f(u) = \int_u^\infty \frac{e^{-t}}{t} dt. \tag{4}$$

The analytical solution is shown on Figure 2 using parameters from Tables 1 and 2.

The numerical solution was computed with the FALCON simulator from INL. Specified properties for the homogeneous solid and fluid are given in Tables 1 and 2. FALCON computes fluid properties (viscosity, density) from lookup tables based on the IAWPS-97 equation of state. The FALCON-calculated values match those used for calculation of the analytical solution.

Table 1: Reservoir properties

Radius of injection well	r_{well}	$:=$	1.000	m
Radius of outer limit	r_{outer}	$:=$	$1.000{\times}10^{2}$	m
Thickness	b	$:=$	1.000	m
Permeability	k	$:=$	$9.869{\times}10^{-13}$	m^2
Porosity of matrix	θ_{unit}	$:=$	$5.000{\times}10^{-1}$	$-$

Table 2: Fluid properties

Ambient fluid pressure	p_{amb}	$:=$	$1.000{\times}10^{5}$	Pa
Ambient fluid temperature	T_{amb}	$:=$	$3.232{\times}10^{2}$	K
Volumetric flow rate	Q_v	$:=$	$1.667{\times}10^{-5}$	$\mathrm{m}^3\,\mathrm{s}^{-1}$
Dynamic viscosity, water	μ	$:=$	$5.465{\times}10^{-4}$	Pa s
Fluid compressibility	β	$:=$	$4.417{\times}10^{-10}$	Pa^{-1}

1.2 Files

- Analytical results are stored in `TheisProblemBM-analytical.xlsx`
- Numerical results are stored in `TheisProblemBM-FALCON.xlsx`
- The numerical solution input file is `TheisProblem.i`
- The numerical solution exodus output file is `TheisProblem_out.e`

Table 3: Derived parameters

Hydraulic diffusivity	D_{eff}	$:=$	$\dfrac{k}{\theta_{\mathrm{unit}}\beta\mu}$	$=$	8.176	$\mathrm{m^2\,s^{-1}}$
Transmissivity of unit	T_{unit}	$:=$	kb	$=$	9.869×10^{-13}	$\mathrm{m^3}$
Storativity of unit	S_{unit}	$:=$	$b\theta_{\mathrm{unit}}\beta$	$=$	2.209×10^{-10}	$\mathrm{m\,Pa^{-1}}$
Implied hydraulic K	K_g	$:=$	$\dfrac{k\rho g}{\mu}$	$=$	1.750×10^{-5}	$\mathrm{m\,d^{-1}}$
Injection velocity	v_{inj}	$:=$	$\dfrac{Q_v}{2\pi r_{\mathrm{well}} b}$	$=$	2.653×10^{-6}	$\mathrm{m\,s^{-1}}$
Reynolds number at well	Re	$:=$	$\dfrac{2r_{\mathrm{well}}\rho v_{\mathrm{inj}}}{\mu}$	$=$	9.591	
Transmissivity	T	$:=$	$\dfrac{1}{\mu}T_{\mathrm{unit}}\rho g$	$=$	1.774×10^{-5}	$\mathrm{m^2\,s^{-1}}$
Storativity	S	$:=$	$S_{\mathrm{unit}}\rho g$	$=$	2.141×10^{-6}	–

1.3 Results

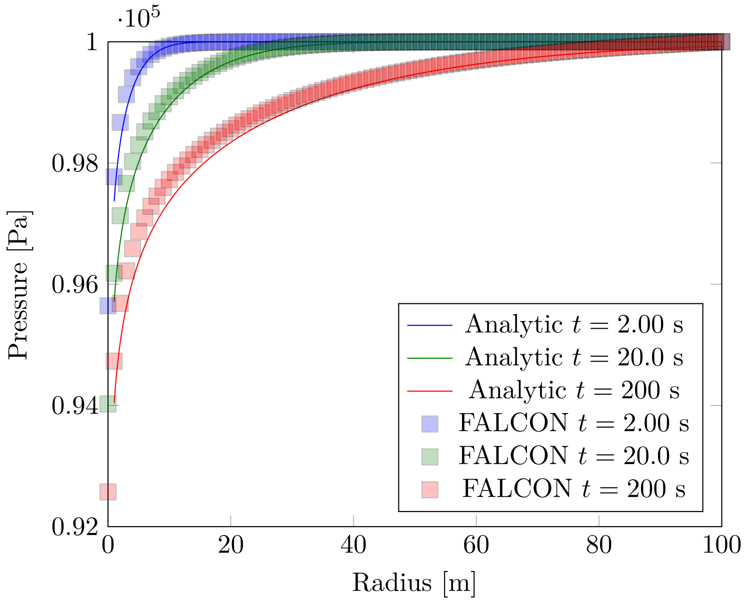


Figure 2: The solution to the Theis problem

Table 4: Problem dimensions

Length of beam	L_x	$:=$	1.000	m
Thickness of beam	h	$:=$	1.000×10^{-2}	m
Loading	q	$:=$	1.000×10^{3}	Pa

Table 5: Material properties

Density of steel	ρ_r	$:=$	7.850×10^{3}	$\mathrm{kg\,m^{-3}}$
Youngs modulus of steel	E_r	$:=$	2.000×10^{11}	Pa
Shear modulus of steel	G_r	$:=$	3.000×10^{10}	Pa
Poisson's ratio	ν_r	$:=$	2.250×10^{-1}	

2 Cantilevered Steel Beam Benchmark

2.1 Problem description

This example tests the ability of a code to describe a common mechanical behaviour, the bending of a cantilevered beam with a uniform load applied on its upper face. The problem is defined for a steel 1.0 m long, 0.10 m thick, steel beam. The initial condition is zero displacement. Output data for y-axis displacements are shown for steady state.

Table 6: Derived parameters

Total force applied	F	$:=$	qL_x	$= 1.000\times10^{3}$	$\mathrm{N\,m^{-1}}$
Flexural rigidity of plate	D	$:=$	$\dfrac{E_r h^3}{12(1-\nu_2^2)}$	$= 1.756\times10^{4}$	J
Moment of inertia	I	$:=$	$\dfrac{D}{E_r}$	$= 8.778\times10^{-8}$	$\mathrm{m^4}$

A reference for the analytical solution is Turcotte and Schubert, page 117. The solution was computed using Mathcad, Version 15, using parameter values and equations shown below. The numerical solution was computed with the FALCON simulator from INL. Specified dimensions and properties for the material (steel) are highlighted above.

2.2 Files

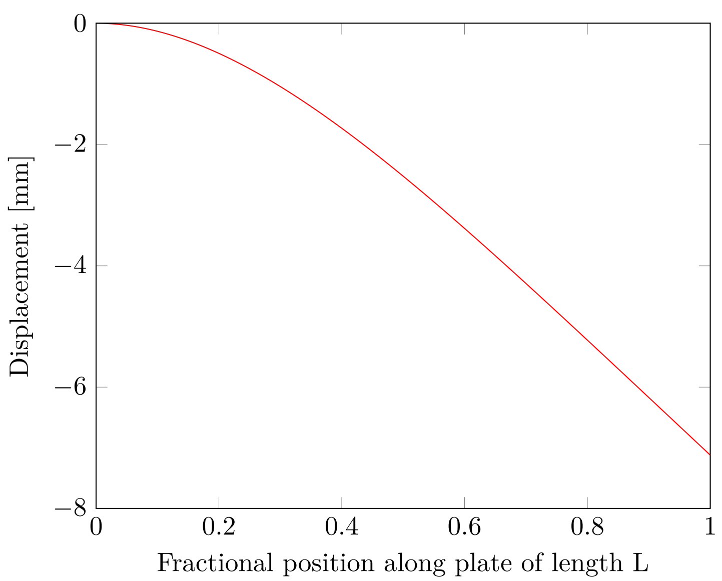
- Analytical results are stored in `CantileveredSteelBeamBM-analytical.xlsx`
- Numerical results are stored in `CantileveredSteelBeamBM-FALCON.xlsx`

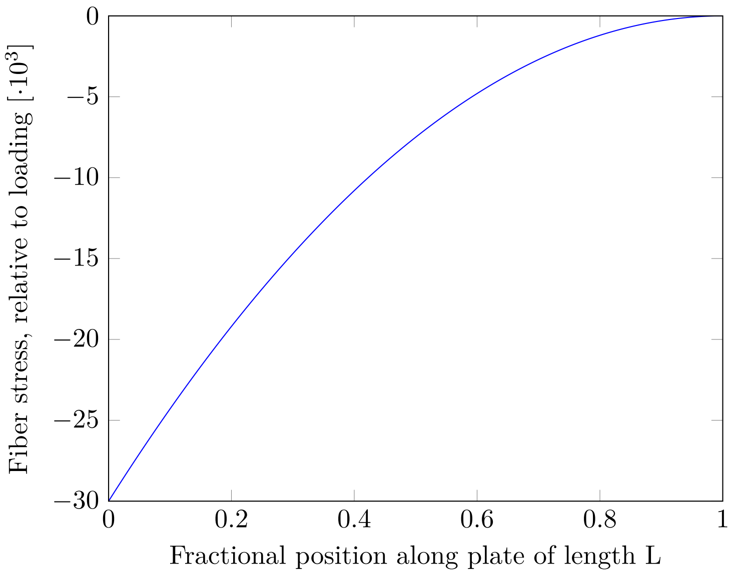
- The numerical solution input file is `CantileveredSteelBeamBM.i`
- The numerical solution exodus output file is `CantileveredSteelBeamBM_out.e`

2.3 Results

Table 7: Analytical solution of the beam problem

Bending moment	$M(x)$	$\coloneqq$	$\frac{1}{2}q(L_x - x)^2$
Displacement solution	$w(x)$	$\coloneqq$	$\frac{qx^2}{D}\left(\frac{x^2}{24} - \frac{L_x x}{6} + \frac{L_x^2}{4}\right)$
Maximum bending stress	$\sigma_{xx,\max}$	$\coloneqq$	$-\frac{6M(x)}{h^2}$





Fiber stress, relative to loading [$\cdot 10^3$]
0
−5
−10
−15
−20
−25
−30
0
0.2
0.4
0.6
0.8
1
Fractional position along plate of length L

3 Heat Diffusion Bencmark

3.1 Problem description

This example tests the ability of a code to represent heat dissipation by diffusion alone. The problem is defined for a solid with Dirichlet conditions on the left and right sides of the 100 m long by 10 m thick domain (Figure 3<++>). The initial initial condition is uniform temperature of 0. °C. Output data for x-axis pressures are calculated for times of $1.0{\times}10^{6}$ s and $1.0{\times}10^{7}$ s.

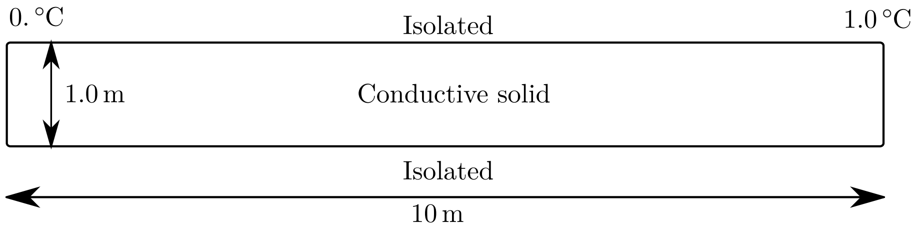


Figure 3: Model domain

The analytical solution applied is for diffusion in a plane sheet in a semi-infiite domain:

$$T(x,t) = \operatorname{erf}\left(\frac{x}{2\sqrt{D_{\text{eff}}t}}\right) \tag{5}$$

The solution was computed using Mathcad, Version 15, using parameter values given below.

Table 8: Material properties

Thermal conductivity of fluid	λ_f	:=	$6.800{\times}10^{-1}$	$\mathrm{W\,m^{-1}\,K^{-1}}$
Thermal conductivity of rock	λ_r	:=	2.955	$\mathrm{W\,m^{-1}\,K^{-1}}$
Specific heat of fluid	$c_{p,f}$	:=	$4.186{\times}10^{3}$	$\mathrm{J\,kg^{-1}\,K^{-1}}$
Specific heat of rock	$c_{p,r}$	:=	$9.200{\times}10^{2}$	$\mathrm{J\,kg^{-1}\,K}$
Density of fluid	ρ_f	:=	$1.000{\times}10^{3}$	$\mathrm{kg\,m^{-3}}$
Density of rock	ρ_r	:=	$2.500{\times}10^{3}$	$\mathrm{kg\,m^{-3}}$
Porosity	θ	:=	$2.000{\times}10^{-1}$	

3.2 Files

- Analytical results are stored in `HeatDiffusionBM-analytical.xlsx`
- Numerical results are stored in `HeatDiffusionBM-FALCON.xlsx`
- The numerical solution input file is `H1_heat_diffusion.i`
- The numerical solution on exodus output file is `H1_heat_diffusion_out.e`

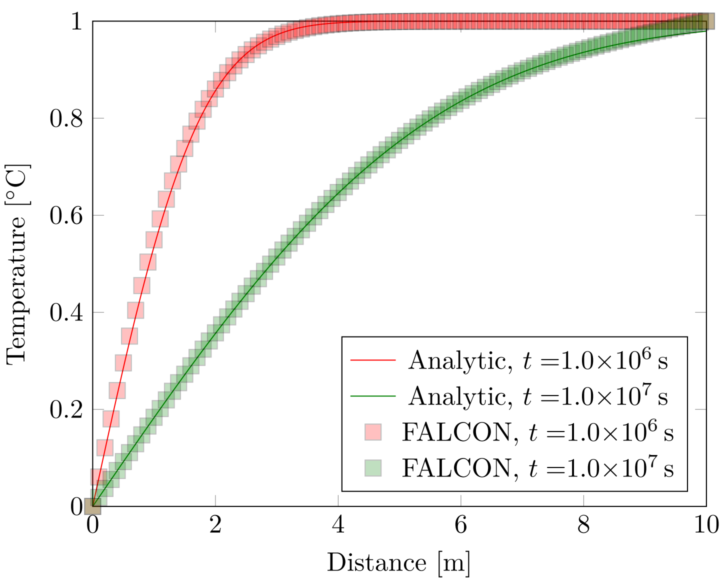
Table 9: Domain parameters

Initial temperature	T_0	$:=$	0.	°C
Injection temperature	T_{inj}	$:=$	1.000	°C
Flow length	X	$:=$	1.000×10^{1}	m
Simulation end time	t_1	$:=$	1.000×10^{5}	s
	t_2	$:=$	5.000×10^{5}	s

Table 10: Derived parameters

Eff. thermal cond.	λ_{eff}	$:=$	$\lambda_f\theta + \lambda_r(1-\theta)$	$=$	2.500	$\mathrm{W\,m^{-1}\,K^{-1}}$
Eff. heat capacity	$c_{p,\mathrm{eff}}$	$:=$	$\rho_f c_{p,f}\theta + \rho_r c_{p,r}(1-\theta)$	$=$	2.677×10^{6}	$\mathrm{J\,m^{-3}\,K^{-1}}$
Thermal diffusivity	D_{eff}	$:=$	$\dfrac{\lambda_{\mathrm{eff}}}{\rho_f c_{p,f}}$	$=$	5.972×10^{-7}	$\mathrm{m^2\,s^{-1}}$
Thermal retardation	R	$:=$	$\dfrac{c_{p,\mathrm{eff}}}{\theta\rho_f c_{p,f}}$	$=$	3.198	

3.3 Results



4 Pressure Diffusion Benchmark

4.1 Problem description

This example Tests the ability of a code to represent pressure dissipation with a slightly compressible fluid. The problem is defined assuming Darcian flow in a porous medium with Dirichlet conditions on the left and right sides of the 100 m long by 10 m thick domain (Figure 4). The initial initial condition is uniform pressure (0.10 MPa). The temperature in the simulation is fixed at 50 °C. Output data for x-axis pressures are calculated for times of 2.0 s, 20 s and 200 s. The rock matrix is assumed to be incompressible.

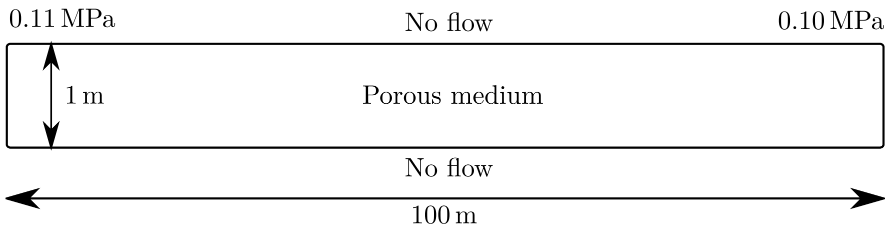


Figure 4: Model domain

The analytical solution applied is eq. 4.2.2 of Crank (1975) for that of Section 4.3.3 Uniform initial distribution. Surface concentrations different:

$$
\begin{aligned}
f(x,t) = & P_1 + (P_2 - P_1)\frac{x}{X} + \frac{2}{\pi}\sum_{n=1}^{\infty}\left[\frac{1}{n}(P_2\cos(n\pi) - P_1)\sin\left(\frac{n\pi x}{X}\right)\exp\left(-\frac{D_{\text{eff}} n^2\pi^2 t}{X^2}\right)\right] \\
& + \frac{4P_0}{\pi}\sum_{m=0}^{\infty}\left[\frac{1}{2m+1}\sin\left(\frac{(2m+1)\pi x}{X}\right)\exp\left(-\frac{D_{\text{eff}}(2m+1)^2\pi^2 t}{X^2}\right)\right]
\end{aligned}
\tag{6}
$$

The solution was computed using Mathcad, Version 15, using parameter values given below.

Table 11: Dimensions and material properties

System length	X	$:=$	1.000×10^{2}	m
Thickness	b	$:=$	1.000×10^{1}	m
Permeability	k	$:=$	1.000×10^{-12}	m^2
Porosity of matrix	θ_{unit}	$:=$	5.000×10^{-1}	
Dynamic viscosity, water	μ	$:=$	5.465×10^{-4}	Pa s
Fluid compressibility	β	$:=$	4.417×10^{-10}	Pa^{-1}

Table 12: Boundary and initial conditions

Initial pressure	P_0	$:=$	1.000×10^5	Pa
Pressure, left side	P_1	$:=$	1.000×10^5	Pa
Pressure, right side	P_2	$:=$	1.100×10^5	Pa

Table 13: Derived parameters

Hydraulic diffusivity	D_{eff}	$:=$	$\dfrac{k}{\theta_{\text{unit}}\beta\mu}$	$= 8.285$	$\mathrm{m^2\,s^{-1}}$

4.2 Files

- Analytical results are stored in `FluidDiffusionBM-analytical.xlsx`
- Numerical results are stored in `FluidDiffusionBM-FALCON.xlsx`
- The numerical solution input file is `H1_pressure_diffusion.i`
- The numerical solution exodus output file is `H1_pressure_diffusion_outt.e`

4.3 Results