

Problem Set — Perfect Numbers

Difficulty: Beginner → Intermediate **Topic:** Loops, Number Theory

Problem 1 — Perfect Number Checker

A **Perfect Number** is a positive integer equal to the sum of its **proper divisors** (excluding itself).

For example:

- $6 \rightarrow \text{divisors: } 1, 2, 3 \rightarrow \text{sum} = 6 \rightarrow \text{Perfect}$
- 28 → divisors: 1, 2, 4, 7, 14 → sum = 28 → **Perfect**
- 12 \rightarrow divisors: 1, 2, 3, 4, 6 \rightarrow sum = 16 \neq 12 \rightarrow **Not Perfect**

Your task is to:

- Read an integer n.
- Print:
 - 1 if n is a perfect number.
 - 0 if it is not.

Input Format

• One integer (n) $(1 \le n \le 10^9)$.

Output Format

• Print 1 if n is perfect, otherwise 0.

Sample Input 1

6

Sample Output 1

1

Sample Input 2

12

Sample Output 2

0

Explanation

- For (n = 6): divisors are $\{1, 2, 3\}$, sum = $6 \rightarrow$ perfect \rightarrow output 1.
- For (n = 12): divisors are $\{1, 2, 3, 4, 6\}$, sum = $16 \rightarrow$ not perfect \rightarrow output 0.

Constraints & Notes

• Perfect numbers are rare. The first few are:

6, 28, 496, 8128, 33550336

Problem 2 — List All Perfect Numbers Less Than n

Now let's take it further: instead of checking one number, print all perfect numbers strictly less than a given (n).

Input Format

• One integer n $(1 \le n \le 10^9)$.

Output Format

- Print all perfect numbers less than (n) in ascending order, separated by spaces.
- If no perfect number exists below (n), print nothing.

Sample Input 1

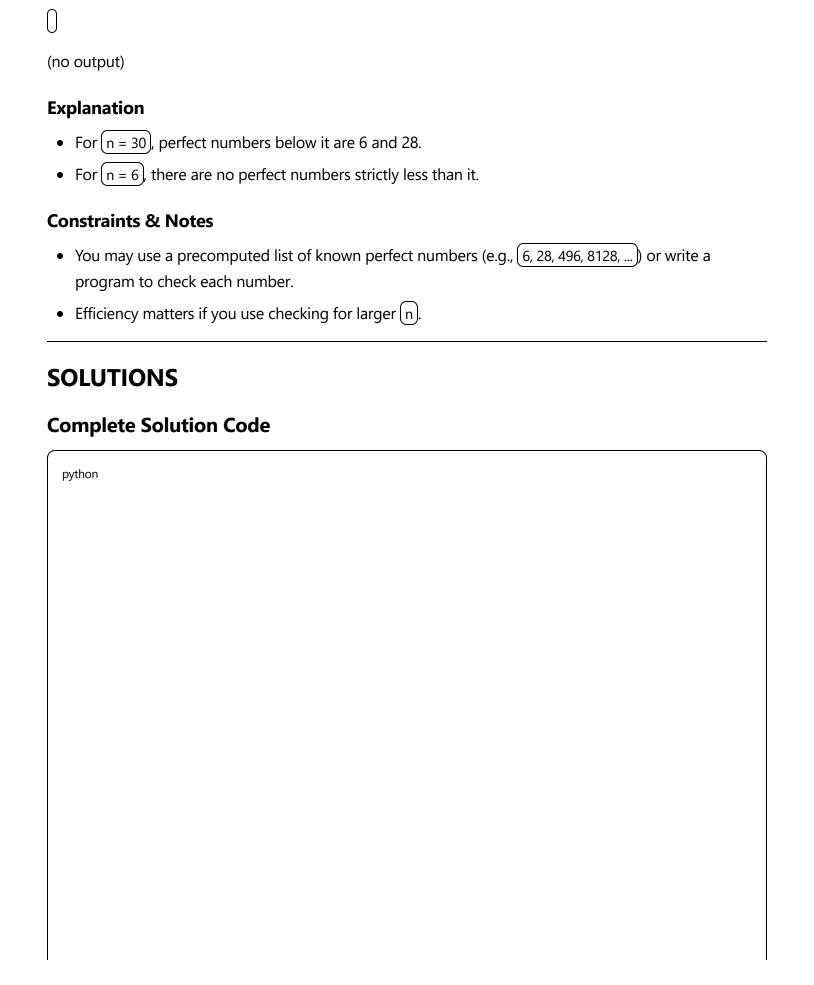
30

Sample Output 1

6 28

Sample Input 2

6



Sample Output 2

```
def is_perfect_number(n):
  Check if a number is a perfect number.
  A perfect number equals the sum of its proper divisors (excluding itself).
  if n <= 1:
    return False
  divisor\_sum = 1 # 1 is always a proper divisor for n > 1
  # Only check up to sqrt(n) for efficiency
  i = 2
  while i * i <= n:
    if n \% i == 0:
       divisor_sum += i
       # Add the corresponding divisor (n/i) if it's different from i
       if i * i != n:
         divisor_sum += n // i
    i += 1
  return divisor_sum == n
def solve_problem_1():
  Problem 1: Perfect Number Checker
  Read an integer n and print 1 if perfect, 0 otherwise.
  n = int(input().strip())
  result = 1 if is_perfect_number(n) else 0
  print(result)
def solve_problem_2():
  Problem 2: List All Perfect Numbers Less Than n
  Print all perfect numbers strictly less than n.
  n = int(input().strip())
  # Known perfect numbers up to reasonable limits
  # This is more efficient than checking each number for large n
  known_perfect = [6, 28, 496, 8128, 33550336, 8589869056, 137438691328]
```

```
perfect_below_n = [num for num in known_perfect if num < n]</pre>
  if perfect_below_n:
    print(' '.join(map(str, perfect_below_n)))
  # If no perfect numbers below n, print nothing (empty line)
def solve_problem_2_brute_force():
  Problem 2: Alternative solution using brute force checking
  This approach checks each number but is slower for large n.
  n = int(input().strip())
  perfect_numbers = []
  # For very large n, we can optimize by only checking up to a reasonable limit
  # since perfect numbers are extremely rare
  check_limit = min(n, 10000000) # Adjust based on time constraints
  for i in range(2, check_limit):
    if is_perfect_number(i):
       perfect_numbers.append(i)
  if perfect_numbers:
     print(' '.join(map(str, perfect_numbers)))
# Example usage and testing
if __name__ == "__main__":
  print("Testing Perfect Number functions:")
  # Test cases for Problem 1
  test_cases_1 = [6, 12, 28, 496, 100]
  print("\nProblem 1 test cases:")
  for num in test_cases_1:
    result = 1 if is_perfect_number(num) else 0
    print(f"n = {num}: {result}")
  # Test cases for Problem 2
  test_cases_2 = [30, 6, 500, 10000]
  print("\nProblem 2 test cases:")
  for num in test_cases_2:
```

```
known_perfect = [6, 28, 496, 8128, 33550336, 8589869056, 137438691328]
perfect_below = [n for n in known_perfect if n < num]
if perfect_below:
    print(f"n = {num}: {' '.join(map(str, perfect_below))}")
else:
    print(f"n = {num}: (no output)")

# Uncomment the line below to run the solution for Problem 1
# solve_problem_10

# Uncomment the line below to run the solution for Problem 2
# solve_problem_20</pre>
```

COMPETITIVE PROGRAMMING TEMPLATE - COPY THESE FOR SUBMISSION

Problem 1 Solution - Perfect Number Checker

Copy this code for Problem 1 submission:

```
python
# Problem 1 - Compact version for submission
def is_perfect(n):
  if n <= 1:
    return False
  divisor_sum = 1
  i = 2
  while i * i <= n:
    if n % i == 0:
       divisor_sum += i
       if i * i != n:
          divisor sum += n // i
    i += 1
  return divisor_sum == n
# Main code for Problem 1
n = int(input())
print(1 if is_perfect(n) else 0)
```

Copy this code for Problem 2 submission:

```
python

# Problem 2 - Optimized version using known perfect numbers

n = int(input())
known_perfect = [6, 28, 496, 8128, 33550336, 8589869056, 137438691328]
perfect_below_n = [num for num in known_perfect if num < n]
if perfect_below_n:
    print(' '.join(map(str, perfect_below_n)))</pre>
```

Alternative Problem 2 Solution - Brute Force

Copy this code if you need to check each number:

```
python
# Problem 2 - Brute force version
def is_perfect_bf(n):
  if n <= 1:
    return False
  divisor_sum = 1
  i = 2
  while i * i <= n:
    if n % i == 0:
       divisor_sum += i
       if i * i != n:
          divisor_sum += n // i
    i += 1
  return divisor_sum == n
n = int(input())
perfect_numbers = []
check_limit = min(n, 10000000) # Reasonable limit for time constraints
for i in range(2, check_limit):
  if is_perfect_bf(i):
    perfect_numbers.append(i)
if perfect_numbers:
  print(' '.join(map(str, perfect_numbers)))
```

Algorithm Notes

Time Complexity

- **is_perfect_number()**: O(√n) only checks divisors up to square root
- **Problem 1**: O(√n) per test case
- Problem 2 (optimized): O(1) uses precomputed list
- **Problem 2 (brute force)**: O(n√n) checks each number up to n

Space Complexity

- **Problem 1**: O(1)
- **Problem 2**: O(k) where k is the number of perfect numbers found

Key Optimizations

- 1. **Square root optimization**: Only check divisors up to \sqrt{n}
- 2. **Precomputed list**: Use known perfect numbers for faster lookup
- 3. **Early termination**: Stop checking when divisor sum exceeds n

The optimized versions will easily handle the constraint $n \le 10^9$ within typical competitive programming time limits.