# Results

April 5, 2021

# Tables of Friedman, Bonferroni-Dunn, Holm, Hochberg and Hommel Tests

Table 1: Average Rankings of the algorithms

| $\operatorname{Ranking}$ | 7.43181818181825 | 6.0227272727272 | 4.159090909090907 | 5.7954545454546 | 3.74999999999999999999999999999999999999 | 2.499999999999999 | 6.681818181818181 | 5.159090909090908 | 3.49999999999999 |
|--------------------------|------------------|-----------------|-------------------|-----------------|--|-------------------|-------------------|-------------------|------------------|
| Algorithm                | kmeans           | kmeans des      | kmeans desthr     | parzen          | parzen des                               | parzen desthr     | ppvs              | sydd des          | sydd desthr      |

Friedman statistic considering reduction performance (distributed according to chi-square with 8 degrees of freedom: 62.23333333333.

P-value computed by Friedman Test: 2.1730239829764741E-10.

Iman and Davenport statistic considering reduction performance (distributed according to F-distribution with 8 and 168 degrees of freedom: 11.487547612071491.

P-value computed by Iman and Daveport Test: 6.043801192459986E-13.

Table 2: Holm / Hochberg Table for  $\alpha = 0.05$ 

|   | Holm/Hochberg/Hommel | 0.00625             | 0.0071428571428571435 | 0.00833333333333333   | 0.01                 | 0.0125                | 0.01666666666666666  | 0.025               | 0.05                |
|---|----------------------|---------------------|-----------------------|-----------------------|----------------------|-----------------------|----------------------|---------------------|---------------------|
| כ | d                    | 2.33318119879579E-9 | 4.0961697785329716E-7 | 1.9879974243027805E-5 | 6.579704265067839E-5 | 0.0012804801761014165 | 0.044509687216435126 | 0.13007018523428457 | 0.22587235588913843 |
|   | $z = (R_0 - R_i)/SE$ | 5.972728425810508   | 5.0644333195812585    | 4.266234589864647     | 3.990993648583059    | 3.220319012994605     | 2.009258871355607    | 1.5138251770487463  | 1.2110601416389966  |
|   | algorithm            | kmeans              | ppvs                  | kmeans des            | parzen               | sydd des              | kmeans desthr        | parzen des          | svdd desthr         |
|   | .2                   | œ                   | -1                    | 9                     | ю                    | 4                     | က                    | 7                   | -                   |

Table 3: Holm / Hochberg Table for  $\alpha = 0.10$ 

|              | Lal           | ле э: пошт / пс      | Table 3. Hollii / Hochberg Table for $\alpha = 0.10$ | = 0.10               |
|--------------|---------------|----------------------|--|----------------------|
| .2           | algorithm     | $z = (R_0 - R_i)/SE$ | d  | Holm/Hochberg/Hommel |
| <sub>∞</sub> | kmeans        | 5.972728425810508    | 2.33318119879579E-9                                  | 0.0125               |
| -1           | ppvs          | 5.0644333195812585   | 4.0961697785329716E-7                                | 0.014285714285714287 |
| 9            | kmeans des    | 4.266234589864647    | 1.9879974243027805E-5                                | 0.01666666666666666  |
| n            | parzen        | 3.990993648583059    | 6.579704265067839E-5                                 | 0.02                 |
| 4            | sydd des      | 3.220319012994605    | 0.0012804801761014165                                | 0.025                |
| က            | kmeans desthr | 2.009258871355607    | 0.044509687216435126                                 | 0.03333333333333333  |
| 21           | parzen des    | 1.5138251770487463   | 0.13007018523428457                                  | 0.05                 |
| _            | sydd desthr   | 1.2110601416389966   | 0.22587235588913843                                  | 0.1                  |

Bonferroni-Dunn's procedure rejects those hypotheses that have a p-value  $\leq 0.0125$ . Holm's procedure rejects those hypotheses that have a p-value  $\leq 0.033333333333333$ .

Hochberg's procedure rejects those hypotheses that have a p-value  $\leq 0.025$ . Hommel's procedure rejects those hypotheses that have a p-value  $\leq 0.01666666666666666$ .

Table 4: Adjusted p-values

|    |               |                       | ,                     |                       |                       |                       |
|----|---------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
|    | algorithm     | unadjusted $p$        | $p_{Bonf}$            | $p_{Holm}$            | $^{pHoch}$            | $^{pHomm}$            |
| -  | kmeans        | 2.33318119879579E-9   | 1.866544959036632E-8  | 1.866544959036632E-8  | 1.866544959036632E-8  | 1.866544959036632E-8  |
| 2  | ppvs          | 4.0961697785329716E-7 | 3.2769358228263773E-6 | 2.86731884497308E-6   | 2.86731884497308E-6   | 2.86731884497308E-6   |
| က  | kmeans des    | 1.9879974243027805E-5 | 1.5903979394422244E-4 | 1.1927984545816682E-4 | 1.1927984545816682E-4 | 1.1927984545816682E-4 |
| 4  | parzen        | 6.579704265067839E-5  | 5.263763412054271E-4  | 3.2898521325339195E-4 | 3.2898521325339195E-4 | 3.2898521325339195E-4 |
| n  | svdd des      | 0.0012804801761014165 | 0.010243841408811332  | 0.005121920704405666  | 0.005121920704405666  | 0.005121920704405666  |
| 9  | kmeans desthr | 0.044509687216435126  | 0.356077497731481     | 0.1335290616493054    | 0.1335290616493054    | 0.1335290616493054    |
| -1 | parzen des    | 0.13007018523428457   | 1.0405614818742766    | 0.26014037046856914   | 0.22587235588913843   | 0.22587235588913843   |
| œ  | svdd desthr   | 0.22587235588913843   | 1.8069788471131074    | 0.26014037046856914   | 0.22587235588913843   | 0.22587235588913843   |

## Table 5: Holm / Shaffer Table for $\alpha=0.05$

| i  | algorithms                      | $z = (R_0 - R_i)/SE'$ | p                     | Holm                    | Shaffer               |
|----|---------------------------------|-----------------------|-----------------------|-------------------------|-----------------------|
| 36 | kmeans vs. parzen desthr        | 5.972728425810508     | 2.33318119879579E-9   | 0.00138888888888889     | 0.0013888888888888    |
| 35 | parzen desthr vs. svdd          | 5.0644333195812585    | 4.0961697785329716E-7 | 0.0014285714285714286   | 0.0017857142857142859 |
| 34 | kmeans vs. svdd desthr          | 4.7616682841715114    | 1.9199909495562918E-6 | 0.0014705882352941176   | 0.0017857142857142859 |
| 33 | kmeans vs. parzen des           | 4.458903248761762     | 8.23800935461124E-6   | 0.001515151515151515152 | 0.0017857142857142859 |
| 32 | kmeans des vs. parzen desthr    | 4.266234589864647     | 1.9879974243027805E-5 | 0.0015625               | 0.0017857142857142859 |
| 31 | parzen vs. parzen desthr        | 3.990993648583059     | 6.579704265067839E-5  | 0.0016129032258064516   | 0.0017857142857142859 |
| 30 | kmeans vs. kmeans desthr        | 3.963469554454901     | 7.386828793759703E-5  | 0.001666666666666668    | 0.0017857142857142859 |
| 29 | svdd vs. svdd desthr            | 3.8533731779422618    | 1.1650161951949738E-4 | 0.001724137931034483    | 0.0017857142857142859 |
| 28 | parzen des vs. svdd             | 3.550608142532512     | 3.8434219869509887E-4 | 0.0017857142857142859   | 0.0017857142857142859 |
| 27 | parzen desthr vs. svdd des      | 3.220319012994605     | 0.0012804801761014165 | 0.001851851851851852    | 0.002272727272727273  |
| 26 | kmeans desthr vs. svdd          | 3.0551744482256513    | 0.002249296490663528  | 0.0019230769230769232   | 0.002272727272727273  |
| 25 | kmeans des vs. svdd desthr      | 3.0551744482256504    | 0.0022492964906635346 | 0.002                   | 0.002272727272727273  |
| 24 | parzen vs. svdd desthr          | 2.779933506944062     | 0.005437003000700588  | 0.00208333333333333333  | 0.002272727272727273  |
| 23 | kmeans vs. svdd des             | 2.7524094128159033    | 0.005915851074237986  | 0.002173913043478261    | 0.002272727272727273  |
| 22 | kmeans des vs. parzen des       | 2.7524094128159007    | 0.005915851074238034  | 0.002272727272727273    | 0.002272727272727273  |
| 21 | parzen vs. parzen des           | 2.4771684715343123    | 0.013242937567462025  | 0.002380952380952381    | 0.002380952380952381  |
| 20 | kmeans des vs. kmeans desthr    | 2.25697571850904      | 0.02400959205819217   | 0.0025                  | 0.0025                |
| 19 | svdd des vs. svdd desthr        | 2.009258871355608     | 0.04450968721643503   | 0.002631578947368421    | 0.002631578947368421  |
| 18 | kmeans desthr vs. parzen desthr | 2.009258871355607     | 0.044509687216435126  | 0.00277777777777778     | 0.00277777777777778   |
| 17 | kmeans desthr vs. parzen        | 1.9817347772274516    | 0.047508932706608505  | 0.0029411764705882353   | 0.0029411764705882353 |
| 16 | kmeans vs. parzen               | 1.9817347772274494    | 0.047508932706608754  | 0.003125                | 0.003125              |
| 15 | svdd vs. svdd des               | 1.8441143065866537    | 0.06516648336431852   | 0.003333333333333333    | 0.003333333333333333  |
| 14 | kmeans vs. kmeans des           | 1.7064938359458612    | 0.08791617299952849   | 0.0035714285714285718   | 0.0035714285714285718 |
| 13 | parzen des vs. svdd des         | 1.7064938359458586    | 0.08791617299952897   | 0.0038461538461538464   | 0.0038461538461538464 |
| 12 | parzen des vs. parzen desthr    | 1.5138251770487463    | 0.13007018523428457   | 0.004166666666666667    | 0.004166666666666667  |
| 11 | kmeans desthr vs. svdd des      | 1.2110601416389977    | 0.22587235588913804   | 0.004545454545454546    | 0.004545454545454546  |
| 10 | parzen desthr vs. svdd desthr   | 1.2110601416389966    | 0.22587235588913843   | 0.005                   | 0.005                 |
| 9  | parzen vs. svdd                 | 1.0734396709981997    | 0.2830738931528096    | 0.00555555555555556     | 0.00555555555555556   |
| 8  | kmeans des vs. svdd des         | 1.045915576870042     | 0.29560001190371177   | 0.00625                 | 0.00625               |
| 7  | kmeans vs. svdd                 | 0.9082951062292496    | 0.36372232738071036   | 0.0071428571428571435   | 0.0071428571428571435 |
| 6  | kmeans des vs. svdd             | 0.7981987297166115    | 0.4247551744745131    | 0.008333333333333333    | 0.008333333333333333  |
| 5  | kmeans desthr vs. svdd desthr   | 0.7981987297166104    | 0.4247551744745137    | 0.01                    | 0.01                  |
| 4  | parzen vs. svdd des             | 0.7706746355884538    | 0.44089980988589994   | 0.0125                  | 0.0125                |
| 3  | kmeans desthr vs. parzen des    | 0.49543369430686074   | 0.620294015349162     | 0.01666666666666666     | 0.01666666666666666   |
| 2  | parzen des vs. svdd desthr      | 0.3027650354097497    | 0.7620689311980031    | 0.025                   | 0.025                 |
| 1  | kmeans des vs. parzen           | 0.27524094128158827   | 0.7831311345963253    | 0.05                    | 0.05                  |

## Table 6: Holm / Shaffer Table for $\alpha=0.10$

| i  | algorithms                      | $z = (R_0 - R_i)/SE$ | p                     | Holm                                   | Shaffer                                |
|----|---------------------------------|----------------------|-----------------------|--|--|
| 36 | kmeans vs. parzen desthr        | 5.972728425810508    | 2.33318119879579E-9   | 0.00277777777777778                    | 0.00277777777777778                    |
| 35 | parzen desthr vs. svdd          | 5.0644333195812585   | 4.0961697785329716E-7 | 0.002857142857142857                   | 0.0035714285714285718                  |
| 34 | kmeans vs. svdd desthr          | 4.7616682841715114   | 1.9199909495562918E-6 | 0.0029411764705882353                  | 0.0035714285714285718                  |
| 33 | kmeans vs. parzen des           | 4.458903248761762    | 8.23800935461124E-6   | 0.0030303030303030303                  | 0.0035714285714285718                  |
| 32 | kmeans des vs. parzen desthr    | 4.266234589864647    | 1.9879974243027805E-5 | 0.003125                               | 0.0035714285714285718                  |
| 31 | parzen vs. parzen desthr        | 3.990993648583059    | 6.579704265067839E-5  | 0.0032258064516129032                  | 0.0035714285714285718                  |
| 30 | kmeans vs. kmeans desthr        | 3.963469554454901    | 7.386828793759703E-5  | 0.003333333333333333                   | 0.0035714285714285718                  |
| 29 | svdd vs. svdd desthr            | 3.8533731779422618   | 1.1650161951949738E-4 | 0.003448275862068966                   | 0.0035714285714285718                  |
| 28 | parzen des vs. svdd             | 3.550608142532512    | 3.8434219869509887E-4 | 0.0035714285714285718                  | 0.0035714285714285718                  |
| 27 | parzen desthr vs. svdd des      | 3.220319012994605    | 0.0012804801761014165 | 0.003703703703703704                   | 0.004545454545454546                   |
| 26 | kmeans desthr vs. svdd          | 3.0551744482256513   | 0.002249296490663528  | 0.0038461538461538464                  | 0.004545454545454546                   |
| 25 | kmeans des vs. svdd desthr      | 3.0551744482256504   | 0.0022492964906635346 | 0.004                                  | 0.004545454545454546                   |
| 24 | parzen vs. svdd desthr          | 2.779933506944062    | 0.005437003000700588  | 0.004166666666666667                   | 0.004545454545454546                   |
| 23 | kmeans vs. svdd des             | 2.7524094128159033   | 0.005915851074237986  | 0.004347826086956522                   | 0.004545454545454546                   |
| 22 | kmeans des vs. parzen des       | 2.7524094128159007   | 0.005915851074238034  | 0.004545454545454546                   | 0.004545454545454546                   |
| 21 | parzen vs. parzen des           | 2.4771684715343123   | 0.013242937567462025  | 0.004761904761904762                   | 0.004761904761904762                   |
| 20 | kmeans des vs. kmeans desthr    | 2.25697571850904     | 0.02400959205819217   | 0.005                                  | 0.005                                  |
| 19 | svdd des vs. svdd desthr        | 2.009258871355608    | 0.04450968721643503   | 0.005263157894736842                   | 0.005263157894736842                   |
| 18 | kmeans desthr vs. parzen desthr | 2.009258871355607    | 0.044509687216435126  | 0.00555555555555556                    | 0.0055555555555556                     |
| 17 | kmeans desthr vs. parzen        | 1.9817347772274516   | 0.047508932706608505  | 0.0058823529411764705                  | 0.0058823529411764705                  |
| 16 | kmeans vs. parzen               | 1.9817347772274494   | 0.047508932706608754  | 0.00625                                | 0.00625                                |
| 15 | svdd vs. svdd des               | 1.8441143065866537   | 0.06516648336431852   | 0.0066666666666666                     | 0.00666666666666667                    |
| 14 | kmeans vs. kmeans des           | 1.7064938359458612   | 0.08791617299952849   | 0.0071428571428571435                  | 0.0071428571428571435                  |
| 13 | parzen des vs. svdd des         | 1.7064938359458586   | 0.08791617299952897   | 0.007692307692307693                   | 0.007692307692307693                   |
| 12 | parzen des vs. parzen desthr    | 1.5138251770487463   | 0.13007018523428457   | 0.008333333333333333                   | 0.008333333333333333                   |
| 11 | kmeans desthr vs. svdd des      | 1.2110601416389977   | 0.22587235588913804   | 0.009090909090909092                   | 0.009090909090909092                   |
| 10 | parzen desthr vs. svdd desthr   | 1.2110601416389966   | 0.22587235588913843   | 0.01                                   | 0.01                                   |
| 9  | parzen vs. svdd                 | 1.0734396709981997   | 0.2830738931528096    | 0.011111111111111111111111111111111111 | 0.011111111111111111111111111111111111 |
| 8  | kmeans des vs. svdd des         | 1.045915576870042    | 0.29560001190371177   | 0.0125                                 | 0.0125                                 |
| 7  | kmeans vs. svdd                 | 0.9082951062292496   | 0.36372232738071036   | 0.014285714285714287                   | 0.014285714285714287                   |
| 6  | kmeans des vs. svdd             | 0.7981987297166115   | 0.4247551744745131    | 0.016666666666666666                   | 0.01666666666666666                    |
| 5  | kmeans desthr vs. svdd desthr   | 0.7981987297166104   | 0.4247551744745137    | 0.02                                   | 0.02                                   |
| 4  | parzen vs. svdd des             | 0.7706746355884538   | 0.44089980988589994   | 0.025                                  | 0.025                                  |
| 3  | kmeans desthr vs. parzen des    | 0.49543369430686074  | 0.620294015349162     | 0.03333333333333333                    | 0.03333333333333333                    |
| 2  | parzen des vs. svdd desthr      | 0.3027650354097497   | 0.7620689311980031    | 0.05                                   | 0.05                                   |
| 1  | kmeans des vs. parzen           | 0.27524094128158827  | 0.7831311345963253    | 0.1                                    | 0.1                                    |

### Table 7: Adjusted p-values

|    |                                 | 10010                 | 1. Hajabada p van     | acb                   |                       |       |
|----|---------------------------------|-----------------------|-----------------------|-----------------------|-----------------------|-------|
| i  | hypothesis                      | unadjusted $p$        | $p_{Neme}$            | $p_{Holm}$            | $p_{Shaf}$            | $p_B$ |
| 1  | kmeans vs .parzen desthr        | 2.33318119879579E-9   | 8.399452315664844E-8  | 8.399452315664844E-8  | 8.399452315664844E-8  | 0     |
| 2  | parzen desthr vs .svdd          | 4.0961697785329716E-7 | 1.4746211202718697E-5 | 1.43365942248654E-5   | 1.146927537989232E-5  | 0     |
| 3  | kmeans vs .svdd desthr          | 1.9199909495562918E-6 | 6.91196741840265E-5   | 6.527969228491392E-5  | 5.375974658757617E-5  | 0     |
| 4  | kmeans vs .parzen des           | 8.23800935461124E-6   | 2.9656833676600466E-4 | 2.7185430870217095E-4 | 2.3066426192911474E-4 | 0     |
| 5  | kmeans des vs .parzen desthr    | 1.9879974243027805E-5 | 7.156790727490009E-4  | 6.361591757768898E-4  | 5.566392788047786E-4  | 0     |
| 6  | parzen vs .parzen desthr        | 6.579704265067839E-5  | 0.002368693535424422  | 0.00203970832217103   | 0.001842317194218995  | 0     |
| 7  | kmeans vs .kmeans desthr        | 7.386828793759703E-5  | 0.002659258365753493  | 0.0022160486381279107 | 0.002068312062252717  | 0     |
| 8  | svdd vs .svdd desthr            | 1.1650161951949738E-4 | 0.004194058302701906  | 0.003378546966065424  | 0.0032620453465459264 | 0     |
| 9  | parzen des vs .svdd             | 3.8434219869509887E-4 | 0.013836319153023559  | 0.010761581563462769  | 0.010761581563462769  | 0     |
| 10 | parzen desthr vs .svdd des      | 0.0012804801761014165 | 0.046097286339650996  | 0.03457296475473825   | 0.028170563874231162  | 0     |
| 11 | kmeans desthr vs .svdd          | 0.002249296490663528  | 0.08097467366388701   | 0.05848170875725173   | 0.04948452279459762   | 0     |
| 12 | kmeans des vs .svdd desthr      | 0.0022492964906635346 | 0.08097467366388725   | 0.05848170875725173   | 0.04948452279459776   | 0     |
| 13 | parzen vs .svdd desthr          | 0.005437003000700588  | 0.19573210802522117   | 0.13048807201681412   | 0.11961406601541294   | 0     |
| 14 | kmeans vs .svdd des             | 0.005915851074237986  | 0.2129706386725675    | 0.13606457470747368   | 0.1301487236332357    | 0     |
| 15 | kmeans des vs .parzen des       | 0.005915851074238034  | 0.2129706386725692    | 0.13606457470747368   | 0.13014872363323673   | 0     |
| 16 | parzen vs .parzen des           | 0.013242937567462025  | 0.47674575242863293   | 0.2781016889167025    | 0.2781016889167025    | 0     |
| 17 | kmeans des vs .kmeans desthr    | 0.02400959205819217   | 0.8643453140949181    | 0.4801918411638434    | 0.43217265704745905   | 0     |
| 18 | svdd des vs .svdd desthr        | 0.04450968721643503   | 1.602348739791661     | 0.8456840571122656    | 0.8011743698958305    | 0     |
| 19 | kmeans desthr vs .parzen desthr | 0.044509687216435126  | 1.6023487397916645    | 0.8456840571122656    | 0.8011743698958322    | 0     |
| 20 | kmeans desthr vs .parzen        | 0.047508932706608505  | 1.7103215774379061    | 0.8456840571122656    | 0.8011743698958322    | 0     |
| 21 | kmeans vs .parzen               | 0.047508932706608754  | 1.7103215774379152    | 0.8456840571122656    | 0.8011743698958322    | 0     |
| 22 | svdd vs .svdd des               | 0.06516648336431852   | 2.3459934011154666    | 0.9774972504647778    | 0.9774972504647778    | 0     |
| 23 | kmeans vs .kmeans des           | 0.08791617299952849   | 3.1649822279830255    | 1.2308264219933989    | 1.1429102489938703    | 0     |
| 24 | parzen des vs .svdd des         | 0.08791617299952897   | 3.1649822279830433    | 1.2308264219933989    | 1.1429102489938767    | 0     |
| 25 | parzen des vs .parzen desthr    | 0.13007018523428457   | 4.682526668434244     | 1.5608422228114147    | 1.5608422228114147    | 0     |
| 26 | kmeans desthr vs .svdd des      | 0.22587235588913804   | 8.13140481200897      | 2.4845959147805186    | 2.4845959147805186    | 0     |
| 27 | parzen desthr vs .svdd desthr   | 0.22587235588913843   | 8.131404812008983     | 2.4845959147805186    | 2.4845959147805186    | 0     |
| 28 | parzen vs .svdd                 | 0.2830738931528096    | 10.190660153501145    | 2.5476650383752864    | 2.5476650383752864    | 0     |
| 29 | kmeans des vs .svdd des         | 0.29560001190371177   | 10.641600428533623    | 2.5476650383752864    | 2.5476650383752864    | 0     |
| 30 | kmeans vs .svdd                 | 0.36372232738071036   | 13.094003785705572    | 2.5476650383752864    | 2.5476650383752864    | 0     |
| 31 | kmeans des vs .svdd             | 0.4247551744745131    | 15.291186281082473    | 2.5485310468470788    | 2.5485310468470788    | 0     |
| 32 | kmeans desthr vs .svdd desthr   | 0.4247551744745137    | 15.291186281082494    | 2.5485310468470788    | 2.5485310468470788    | 0     |
| 33 | parzen vs .svdd des             | 0.44089980988589994   | 15.872393155892398    | 2.5485310468470788    | 2.5485310468470788    | 0     |
| 34 | kmeans desthr vs .parzen des    | 0.620294015349162     | 22.33058455256983     | 2.5485310468470788    | 2.5485310468470788    | 0     |
| 35 | parzen des vs .svdd desthr      | 0.7620689311980031    | 27.434481523128113    | 2.5485310468470788    | 2.5485310468470788    | 0     |
| 36 | kmeans des vs .parzen           | 0.7831311345963253    | 28.19272084546771     | 2.5485310468470788    | 2.5485310468470788    | 0     |