

Results

April 5, 2021

1 Tables of Friedman, Bonferroni-Dunn, Holm, Hochberg and Hommel Tests

Table 1: Average Rankings of the algorithms

| Algorithm | Ranking |
|---------------|-------------------|
| kmeans | 5.31818181818175 |
| kmeans des | 4.840909090909 |
| kmeans desthr | 3.36363636363624 |
| parzen | 7.68181818181825 |
| parzen des | 5.4090909090909 |
| parzen desthr | 3.40909090909083 |
| svdd | 6.36363636363633 |
| svdd des | 5.27272727272725 |
| svdd desthr | 3.340909090909895 |

Friedman statistic considering reduction performance (distributed according to chi-square with 8 degrees of freedom: 50.984848484841).

P-value computed by Friedman Test: 2.6461138613598223E-8.

Iman and Davenport statistic considering reduction performance (distributed according to F-distribution with 8 and 168 degrees of freedom: 8.564416434371575.

P-value computed by Iman and Davenport Test: 9.352743981972688E-10.

Table 2: Holm / Hochberg Table for $\alpha = 0.05$

| i | algorithm | $z = (R_0 - R_i) / SE$ | p | Holm/Hochberg/Hommel |
|-----|---------------|------------------------|-----------------------|-----------------------|
| 8 | parzen | 5.257101978478375 | 1.4634317164520245E-7 | 0.00625 |
| 7 | svdd | 3.6607045190451504 | 2.5152269150530103E-4 | 0.0071428571428571435 |
| 6 | parzen_des | 2.5046925656624723 | 0.0122557870761371 | 0.008333333333333333 |
| 5 | kmeans | 2.3945961891498353 | 0.01663867854994565 | 0.01 |
| 4 | svdd_des | 2.3395480008935174 | 0.019307090374481563 | 0.0125 |
| 3 | kmeans_des | 1.8165902124584956 | 0.0692798832212019 | 0.016666666666666666 |
| 2 | parzen_desthr | 0.0825728238447778 | 0.9341916414436106 | 0.025 |
| 1 | kmeans_desthr | 0.02752409412815926 | 0.9780417227898052 | 0.05 |

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Bonferroni-Dunn's procedure rejects those hypotheses that have a p-value ≤ 0.00625 .
Holm's procedure rejects those hypotheses that have a p-value $\leq 0.008333333333333333$.
Hochberg's procedure rejects those hypotheses that have a p-value $\leq 0.0071428571428571435$.
Hommel's procedure rejects those hypotheses that have a p-value ≤ 0.0125 .

Table 3: Holm / Hochberg Table for $\alpha = 0.10$

| i | algorithm | $z = (R_0 - R_i) / SE$ | p | Holm/Hochberg/Hommel |
|-----|---------------|------------------------|-----------------------|----------------------|
| 8 | parzen | 5.257101978478375 | 1.4634317164520245E-7 | 0.0125 |
| 7 | svdd | 3.6607045190451504 | 2.5152269150530103E-4 | 0.014285714285714287 |
| 6 | parzen_des | 2.5046925656624723 | 0.0122557870761371 | 0.016666666666666666 |
| 5 | kmeans | 2.3945961891498353 | 0.01663867854994565 | 0.02 |
| 4 | svdd_des | 2.3395480008935174 | 0.019307090374481563 | 0.025 |
| 3 | kmeans_des | 1.8165902124584956 | 0.0692798832212019 | 0.03333333333333333 |
| 2 | parzen_desthr | 0.0825728238447778 | 0.9341916414436106 | 0.05 |
| 1 | kmeans_desthr | 0.02752409412815926 | 0.9780417227898052 | 0.1 |

Bonferroni-Dunn's procedure rejects those hypotheses that have a p-value ≤ 0.0125 .
Holm's procedure rejects those hypotheses that have a p-value ≤ 0.03333333333333333 .

Hochberg's procedure rejects those hypotheses that have a p-value ≤ 0.025 .
Hommel's procedure rejects those hypotheses that have a p-value $\leq 0.016666666666666666$.

Table 4: Adjusted p-values

| i | algorithm | unadjusted p | p_{Bonf} | p_{Holm} | p_{Hoch} | p_{Hommel} |
|---|--------------|-----------------------|-----------------------|------------------------|------------------------|------------------------|
| 1 | parzen | 1.4634317164520245E-7 | 1.1707453731616196E-6 | 1.1707453731616196E-6 | 1.1707453731616196E-6 | 1.1707453731616196E-6 |
| 2 | svdd | 2.5152269150530105E-4 | 0.0020121815320424084 | 0.00176066588405371074 | 0.00176066588405371074 | 0.00176066588405371074 |
| 3 | parzen des | 0.0122357570261371 | 0.0950462962090968 | 0.07353472215682261 | 0.07353472215682261 | 0.0490231481045484 |
| 4 | kmeans | 0.01663867854994665 | 0.1331094283995652 | 0.08319339274972824 | 0.07722836149792625 | 0.0665547141197826 |
| 5 | svdd des | 0.019307090374481563 | 0.1344567229958525 | 0.08319339274972824 | 0.07722836149792625 | 0.07722836149792625 |
| 6 | kmeans des | 0.0692798832212019 | 0.5342390657696152 | 0.2078396496636057 | 0.2078396496636057 | 0.2078396496636057 |
| 7 | parzen destr | 0.9341916414436106 | 7.4735333131548884 | 1.868383282887221 | 0.9780417227898052 | 0.9780417227898052 |
| 8 | kmeans destr | 0.9780417227898052 | 7.824333782318441 | 1.868383282887221 | 0.9780417227898052 | 0.9780417227898052 |

Nemenyi's procedure rejects those hypotheses that have a p-value $\leq 0.001388888888888889$.
Holm's procedure rejects those hypotheses that have a p-value $\leq 0.001724137931034483$.
Shaffer's procedure rejects those hypotheses that have a p-value $\leq 0.001388888888888889$.
Nemenyi's procedure rejects those hypotheses that have a p-value $\leq 0.002777777777777778$.
Holm's procedure rejects those hypotheses that have a p-value $\leq 0.003448275862068966$.
Shaffer's procedure rejects those hypotheses that have a p-value $\leq 0.002777777777777778$.

Table 5: Holm / Shaffer Table for $\alpha = 0.05$

| i | algorithms | $z = (R_0 - R_i)/SE$ | p | Holm | Shaffer |
|-----|---------------------------------|----------------------|-----------------------|-----------------------|-----------------------|
| 36 | parzen vs. svdd desthr | 5.257101978478375 | 1.4634317164520245E-7 | 0.00138888888888889 | 0.00138888888888889 |
| 35 | kmeans desthr vs. parzen | 5.229577884350215 | 1.698975151974438E-7 | 0.0014285714285714286 | 0.0017857142857142859 |
| 34 | parzen vs. parzen desthr | 5.174529696093897 | 2.2848576992066736E-7 | 0.0014705882352941176 | 0.0017857142857142859 |
| 33 | svdd vs. svdd desthr | 3.6607045190451504 | 2.5152269150530105E-4 | 0.0015151515151515152 | 0.0017857142857142859 |
| 32 | kmeans desthr vs. svdd | 3.6331804249169912 | 2.7994911788004187E-4 | 0.0015625 | 0.0017857142857142859 |
| 31 | parzen desthr vs. svdd | 3.5781322366606725 | 3.4605836543277237E-4 | 0.0016129032258064516 | 0.0017857142857142859 |
| 30 | kmeans des vs. parzen | 3.440511766019879 | 5.806151916939471E-4 | 0.0016666666666666668 | 0.0017857142857142859 |
| 29 | parzen vs. svdd des | 2.917553977584857 | 0.0035278858210090543 | 0.001724137931034483 | 0.0017857142857142859 |
| 28 | kmeans vs. parzen | 2.8625057893285395 | 0.004203055819850793 | 0.0017857142857142859 | 0.0017857142857142859 |
| 27 | parzen vs. parzen des | 2.7524094128159025 | 0.005915851074238003 | 0.001851851851851852 | 0.001851851851851852 |
| 26 | parzen des vs. svdd desthr | 2.5046925656624723 | 0.0122557870261371 | 0.0019230769230769232 | 0.0019230769230769232 |
| 25 | kmeans desthr vs. parzen des | 2.4771684715343127 | 0.013242937567462013 | 0.002 | 0.002 |
| 24 | parzen des vs. parzen desthr | 2.4221202832779944 | 0.01543024222338637 | 0.0020833333333333333 | 0.0020833333333333333 |
| 23 | kmeans vs. svdd desthr | 2.3945961891498353 | 0.01663867854994565 | 0.002173913043478261 | 0.002173913043478261 |
| 22 | kmeans vs. kmeans desthr | 2.3670720950216757 | 0.017929439801965402 | 0.002272727272727273 | 0.002272727272727273 |
| 21 | svdd des vs. svdd desthr | 2.3395480008935174 | 0.019307090374481563 | 0.002380952380952381 | 0.002380952380952381 |
| 20 | kmeans desthr vs. svdd des | 2.3120239067653583 | 0.020776365899404393 | 0.0025 | 0.0025 |
| 19 | kmeans vs. parzen desthr | 2.3120239067653574 | 0.020776365899404445 | 0.002631578947368421 | 0.002631578947368421 |
| 18 | parzen desthr vs. svdd des | 2.25697571850904 | 0.02400959205819217 | 0.0027777777777777778 | 0.0027777777777777778 |
| 17 | kmeans des vs. svdd | 1.8441143065866548 | 0.06516648336431836 | 0.0029411764705882353 | 0.0029411764705882353 |
| 16 | kmeans des vs. svdd desthr | 1.8165902124584956 | 0.0692798832212019 | 0.003125 | 0.003125 |
| 15 | kmeans des vs. kmeans desthr | 1.7890661183303362 | 0.07360416794958406 | 0.0033333333333333335 | 0.0033333333333333335 |
| 14 | kmeans des vs. parzen desthr | 1.7340179300740177 | 0.08291490185366591 | 0.0035714285714285718 | 0.0035714285714285718 |
| 13 | parzen vs. svdd | 1.5963974594332242 | 0.11040008300997517 | 0.0038461538461538464 | 0.0038461538461538464 |
| 12 | svdd vs. svdd des | 1.3211565181516327 | 0.18644918121998139 | 0.004166666666666667 | 0.004166666666666667 |
| 11 | kmeans vs. svdd | 1.266108329895315 | 0.20547430075806158 | 0.004545454545454546 | 0.004545454545454546 |
| 10 | parzen des vs. svdd | 1.156011953382678 | 0.2476762676918937 | 0.005 | 0.005 |
| 9 | kmeans des vs. parzen des | 0.6881023532039766 | 0.49138832870478993 | 0.005555555555555556 | 0.005555555555555556 |
| 8 | kmeans vs. kmeans des | 0.5780059766913396 | 0.5632600841286354 | 0.00625 | 0.00625 |
| 7 | kmeans des vs. svdd des | 0.5229577884350222 | 0.6010036284073753 | 0.0071428571428571435 | 0.0071428571428571435 |
| 6 | parzen des vs. svdd des | 0.16514456476895448 | 0.8688301969253134 | 0.008333333333333333 | 0.008333333333333333 |
| 5 | kmeans vs. parzen des | 0.11009637651263704 | 0.9123329418135497 | 0.01 | 0.01 |
| 4 | parzen desthr vs. svdd desthr | 0.08257228238447778 | 0.9341916414436106 | 0.0125 | 0.0125 |
| 3 | kmeans desthr vs. parzen desthr | 0.05504818825631852 | 0.9561000732891639 | 0.016666666666666666 | 0.016666666666666666 |
| 2 | kmeans vs. svdd des | 0.055048188256317444 | 0.9561000732891648 | 0.025 | 0.025 |
| 1 | kmeans desthr vs. svdd desthr | 0.02752409412815926 | 0.9780417227898052 | 0.05 | 0.05 |

Table 6: Holm / Shaffer Table for $\alpha = 0.10$

| i | algorithms | $z = (R_0 - R_i)/SE$ | p | Holm | Shaffer |
|-----|---------------------------------|----------------------|-----------------------|-----------------------|-----------------------|
| 36 | parzen vs. svdd desthr | 5.257101978478375 | 1.4634317164520245E-7 | 0.002777777777777778 | 0.002777777777777778 |
| 35 | kmeans desthr vs. parzen | 5.229577884350215 | 1.698975151974438E-7 | 0.002857142857142857 | 0.003571428571428571 |
| 34 | parzen vs. parzen desthr | 5.174529696093897 | 2.2848576992066736E-7 | 0.0029411764705882353 | 0.003571428571428571 |
| 33 | svdd vs. svdd desthr | 3.6607045190451504 | 2.5152269150530105E-4 | 0.0030303030303030303 | 0.003571428571428571 |
| 32 | kmeans desthr vs. svdd | 3.6331804249169912 | 2.7994911788004187E-4 | 0.003125 | 0.003571428571428571 |
| 31 | parzen desthr vs. svdd | 3.5781322366606725 | 3.4605836543277237E-4 | 0.0032258064516129032 | 0.003571428571428571 |
| 30 | kmeans des vs. parzen | 3.440511766019879 | 5.806151916939471E-4 | 0.0033333333333333335 | 0.003571428571428571 |
| 29 | parzen vs. svdd des | 2.917553977584857 | 0.0035278858210090543 | 0.003448275862068966 | 0.003571428571428571 |
| 28 | kmeans vs. parzen | 2.8625057893285395 | 0.004203055819850793 | 0.0035714285714285718 | 0.003571428571428571 |
| 27 | parzen vs. parzen des | 2.7524094128159025 | 0.005915851074238003 | 0.003703703703703704 | 0.003703703703703704 |
| 26 | parzen des vs. svdd desthr | 2.5046925656624723 | 0.0122557870261371 | 0.0038461538461538464 | 0.0038461538461538464 |
| 25 | kmeans desthr vs. parzen des | 2.4771684715343127 | 0.013242937567462013 | 0.004 | 0.004 |
| 24 | parzen des vs. parzen desthr | 2.4221202832779944 | 0.01543024222338637 | 0.004166666666666667 | 0.004166666666666667 |
| 23 | kmeans vs. svdd desthr | 2.3945961891498353 | 0.01663867854994565 | 0.004347826086956522 | 0.004347826086956522 |
| 22 | kmeans vs. kmeans desthr | 2.3670720950216757 | 0.017929439801965402 | 0.004545454545454546 | 0.004545454545454546 |
| 21 | svdd des vs. svdd desthr | 2.3395480008935174 | 0.019307090374481563 | 0.004761904761904762 | 0.004761904761904762 |
| 20 | kmeans desthr vs. svdd des | 2.3120239067653583 | 0.020776365899404393 | 0.005 | 0.005 |
| 19 | kmeans vs. parzen desthr | 2.3120239067653574 | 0.020776365899404445 | 0.005263157894736842 | 0.005263157894736842 |
| 18 | parzen desthr vs. svdd des | 2.25697571850904 | 0.02400959205819217 | 0.005555555555555556 | 0.005555555555555556 |
| 17 | kmeans des vs. svdd | 1.8441143065866548 | 0.06516648336431836 | 0.0058823529411764705 | 0.0058823529411764705 |
| 16 | kmeans des vs. svdd desthr | 1.8165902124584956 | 0.0692798832212019 | 0.00625 | 0.00625 |
| 15 | kmeans des vs. kmeans desthr | 1.7890661183303362 | 0.07360416794958406 | 0.006666666666666667 | 0.006666666666666667 |
| 14 | kmeans des vs. parzen desthr | 1.7340179300740177 | 0.08291490185366591 | 0.0071428571428571435 | 0.0071428571428571435 |
| 13 | parzen vs. svdd | 1.5963974594332242 | 0.11040008300997517 | 0.007692307692307693 | 0.007692307692307693 |
| 12 | svdd vs. svdd des | 1.3211565181516327 | 0.18644918121998139 | 0.008333333333333333 | 0.008333333333333333 |
| 11 | kmeans vs. svdd | 1.266108329895315 | 0.20547430075806158 | 0.009090909090909092 | 0.009090909090909092 |
| 10 | parzen des vs. svdd | 1.156011953382678 | 0.2476762676918937 | 0.01 | 0.01 |
| 9 | kmeans des vs. parzen des | 0.6881023532039766 | 0.49138832870478993 | 0.011111111111111112 | 0.011111111111111112 |
| 8 | kmeans vs. kmeans des | 0.5780059766913396 | 0.5632600841286354 | 0.0125 | 0.0125 |
| 7 | kmeans des vs. svdd des | 0.5229577884350222 | 0.6010036284073753 | 0.014285714285714287 | 0.014285714285714287 |
| 6 | parzen des vs. svdd des | 0.16514456476895448 | 0.8688301969253134 | 0.016666666666666666 | 0.016666666666666666 |
| 5 | kmeans vs. parzen des | 0.11009637651263704 | 0.9123329418135497 | 0.02 | 0.02 |
| 4 | parzen desthr vs. svdd desthr | 0.08257228238447778 | 0.9341916414436106 | 0.025 | 0.025 |
| 3 | kmeans desthr vs. parzen desthr | 0.05504818825631852 | 0.9561000732891639 | 0.033333333333333333 | 0.033333333333333333 |
| 2 | kmeans vs. svdd des | 0.055048188256317444 | 0.9561000732891648 | 0.05 | 0.05 |
| 1 | kmeans desthr vs. svdd desthr | 0.02752409412815926 | 0.9780417227898052 | 0.1 | 0.1 |

Table 7: Adjusted p -values

| i | hypothesis | unadjusted p | P_{Neme} | P_{Holm} | P_{Shaf} | P_{Ben} |
|-----|---------------------------------|-----------------------|----------------------|----------------------|-----------------------|--------------------|
| 1 | parzen vs .svdd desthr | 1.4634317164520245E-7 | 5.268354179227289E-6 | 5.268354179227289E-6 | 5.268354179227289E-6 | 0.0000000000000000 |
| 2 | kmeans desthr vs .parzen | 1.698975151974438E-7 | 6.116310547107977E-6 | 5.946413031910533E-6 | 5.268354179227289E-6 | 0.0000000000000000 |
| 3 | parzen vs .parzen desthr | 2.2848576992066736E-7 | 8.225487717144025E-6 | 7.76851617730269E-6 | 6.397601557778686E-6 | 0.0000000000000000 |
| 4 | svdd vs .svdd desthr | 2.5152269150530105E-4 | 0.009054816894190838 | 0.008300248819674935 | 0.0070426353621484295 | 0.0000000000000000 |
| 5 | kmeans desthr vs .svdd | 2.7994911788004187E-4 | 0.010078168243681507 | 0.00895837177216134 | 0.007838575300641172 | 0.0000000000000000 |
| 6 | parzen desthr vs .svdd | 3.4605836543277237E-4 | 0.012458101155579805 | 0.010727809328415943 | 0.009689634232117626 | 0.0000000000000000 |
| 7 | kmeans des vs .parzen | 5.806151916939471E-4 | 0.020902146900982094 | 0.017418455750818414 | 0.01625722536743052 | 0.0000000000000000 |
| 8 | parzen vs .svdd des | 0.0035278858210090543 | 0.12700388955632597 | 0.10230868880926257 | 0.09878080298825352 | 0.0000000000000000 |
| 9 | kmeans vs .parzen | 0.004203055819850793 | 0.15131000951462856 | 0.1176855629558222 | 0.1176855629558222 | 0.0000000000000000 |
| 10 | parzen vs .parzen des | 0.005915851074238003 | 0.2129706386725681 | 0.15972797900442606 | 0.13014872363323607 | 0.0000000000000000 |
| 11 | parzen des vs .svdd desthr | 0.0122557870261371 | 0.4412083329409356 | 0.3186504626795646 | 0.2696273145750162 | 0.0000000000000000 |
| 12 | kmeans desthr vs .parzen des | 0.013242937567462013 | 0.4767457524286325 | 0.3310734391865503 | 0.29134462648416426 | 0.0000000000000000 |
| 13 | parzen des vs .parzen desthr | 0.01543024222338637 | 0.5554887200419093 | 0.3703258133612729 | 0.33946532891450015 | 0.0000000000000000 |
| 14 | kmeans vs .svdd desthr | 0.01663867854994565 | 0.5989924277980434 | 0.3826896066487499 | 0.36605092809880424 | 0.0000000000000000 |
| 15 | kmeans vs .kmeans desthr | 0.017929439801965402 | 0.6454598328707545 | 0.39444767564323885 | 0.39444767564323885 | 0.0000000000000000 |
| 16 | svdd des vs .svdd desthr | 0.019307090374481563 | 0.6950552534813362 | 0.4054488978641128 | 0.4054488978641128 | 0.0000000000000000 |
| 17 | kmeans desthr vs .svdd des | 0.020776365899404393 | 0.7479491723785582 | 0.41552731798808784 | 0.4054488978641128 | 0.0000000000000000 |
| 18 | kmeans vs .parzen desthr | 0.020776365899404445 | 0.7479491723785601 | 0.41552731798808784 | 0.4054488978641128 | 0.0000000000000000 |
| 19 | parzen desthr vs .svdd des | 0.02400959205819217 | 0.8643453140949181 | 0.43217265704745905 | 0.43217265704745905 | 0.0000000000000000 |
| 20 | kmeans des vs .svdd | 0.06516648336431836 | 2.3459934011154613 | 1.1078302171934122 | 1.0426637338290938 | 0.0000000000000000 |
| 21 | kmeans des vs .svdd desthr | 0.0692798832212019 | 2.4940757959632687 | 1.1084781315392305 | 1.1084781315392305 | 0.0000000000000000 |
| 22 | kmeans des vs .kmeans desthr | 0.07360416794958406 | 2.649750046185026 | 1.1084781315392305 | 1.1084781315392305 | 0.0000000000000000 |
| 23 | kmeans des vs .parzen desthr | 0.08291490185366591 | 2.984936466731973 | 1.1608086259513228 | 1.1084781315392305 | 0.0000000000000000 |
| 24 | parzen vs .svdd | 0.11040008300997517 | 3.974402988359106 | 1.4352010791296772 | 1.4352010791296772 | 0.0000000000000000 |
| 25 | svdd vs .svdd des | 0.18644918121998139 | 6.71217052391933 | 2.237390174639777 | 2.237390174639777 | 0.0000000000000000 |
| 26 | kmeans vs .svdd | 0.20547430075806158 | 7.397074827290217 | 2.2602173083386776 | 2.2602173083386776 | 0.0000000000000000 |
| 27 | parzen des vs .svdd | 0.2476762676918937 | 8.916345636908172 | 2.476762676918937 | 2.476762676918937 | 0.0000000000000000 |
| 28 | kmeans des vs .parzen des | 0.49138832870478993 | 17.689979833372437 | 4.422494958343109 | 4.422494958343109 | 0.0000000000000000 |
| 29 | kmeans vs .kmeans des | 0.5632600841286354 | 20.277363028630873 | 4.506080673029083 | 4.506080673029083 | 0.0000000000000000 |
| 30 | kmeans des vs .svdd des | 0.6010036284073753 | 21.63613062266551 | 4.506080673029083 | 4.506080673029083 | 0.0000000000000000 |
| 31 | parzen des vs .svdd des | 0.8688301969253134 | 31.27788708931128 | 5.21298118155188 | 5.21298118155188 | 0.0000000000000000 |
| 32 | kmeans vs .parzen des | 0.9123329418135497 | 32.84398590528779 | 5.21298118155188 | 5.21298118155188 | 0.0000000000000000 |
| 33 | parzen desthr vs .svdd desthr | 0.9341916414436106 | 33.63089909196998 | 5.21298118155188 | 5.21298118155188 | 0.0000000000000000 |
| 34 | kmeans desthr vs .parzen desthr | 0.9561000732891639 | 34.4196026384099 | 5.21298118155188 | 5.21298118155188 | 0.0000000000000000 |
| 35 | kmeans vs .svdd des | 0.9561000732891648 | 34.419602638409934 | 5.21298118155188 | 5.21298118155188 | 0.0000000000000000 |
| 36 | kmeans desthr vs .svdd desthr | 0.9780417227898052 | 35.209502020432986 | 5.21298118155188 | 5.21298118155188 | 0.0000000000000000 |