OUTILS MATHÉMATIQUES 1
Trigonométrie

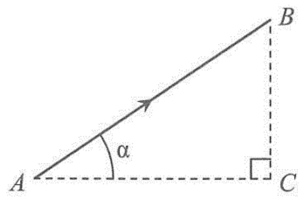
1 Relations entre les distances et les angles

1.1 Triangle rectangle

➢ Distances et angles comptés positivement

$\boxed{\cos(\alpha) = \dfrac{AC}{AB} = \dfrac{\text{côté adjacent}}{\text{hypoténuse}}}$ $\boxed{\sin(\alpha) = \dfrac{BC}{AB} = \dfrac{\text{côté opposé}}{\text{hypoténuse}}}$

$$\boxed{\tan(\alpha) = \frac{BC}{AC} = \frac{\text{côté opposé}}{\text{côté adjacent}}}$$

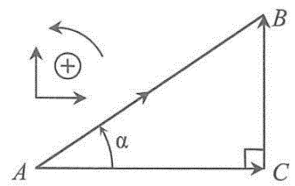


➢ Distances et angles algébriques

- Signe des distances algébriques :

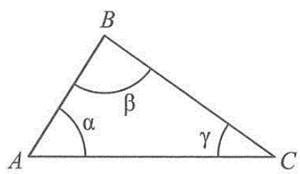
$$\begin{cases} \overline{AC} > 0 \text{ et } \overline{CB} > 0 \\ \overline{CA} = -\overline{AC} < 0 \text{ et } \overline{BC} = -\overline{CB} < 0 \end{cases}$$

- Signe des angles : $\alpha = CAB > 0$ et $BAC = -\alpha < 0$
- Relation : $\boxed{\tan(\alpha) = \dfrac{\overline{CB}}{\overline{AC}} > 0}$



1.2 Triangle quelconque

$$\frac{BC}{\sin(\alpha)} = \frac{AC}{\sin(\beta)} = \frac{AB}{\sin(\gamma)}$$



2 Relations trigonométriques

➢ Les indispensables ! $\boxed{\cos^2(\alpha) + \sin^2(\alpha) = 1}$ et $\boxed{\tan(\alpha) = \dfrac{\sin(\alpha)}{\cos(\alpha)}}$

➢ Transformations remarquables

$\cos(-\alpha) = \cos(\alpha)$ $\sin(-\alpha) = -\sin(\alpha)$ $\tan(-\alpha) = -\tan(\alpha)$	$\cos(2\pi + \alpha) = \cos(\alpha)$ $\sin(2\pi + \alpha) = \sin(\alpha)$ $\tan(2\pi + \alpha) = \tan(\alpha)$	$\cos(\pi - \alpha) = -\cos(\alpha)$ $\sin(\pi - \alpha) = \sin(\alpha)$ $\tan(\pi - \alpha) = -\tan(\alpha)$	$\cos(\pi + \alpha) = -\cos(\alpha)$ $\sin(\pi + \alpha) = -\sin(\alpha)$ $\tan(\pi + \alpha) = \tan(\alpha)$
$\cos\left(\dfrac{\pi}{2} - \alpha\right) = \sin(\alpha)$	$\sin\left(\dfrac{\pi}{2} - \alpha\right) = \cos(\alpha)$	$\cos\left(\dfrac{\pi}{2} + \alpha\right) = -\sin(\alpha)$	$\sin\left(\dfrac{\pi}{2} + \alpha\right) = \cos(\alpha)$
$\tan\left(\dfrac{\pi}{2} - \alpha\right) = \dfrac{1}{\tan(\alpha)}$		$\tan\left(\dfrac{\pi}{2} + \alpha\right) = -\dfrac{1}{\tan(\alpha)}$	

- Formules d'addition

$$\cos(\alpha-\beta)=\cos(\alpha)\cos(\beta)+\sin(\alpha)\sin(\beta)$$
$$\cos(\alpha+\beta)=\cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta)$$
$$\sin(\alpha-\beta)=\sin(\alpha)\cos(\beta)-\sin(\beta)\cos(\alpha)$$
$$\sin(\alpha+\beta)=\sin(\alpha)\cos(\beta)+\sin(\beta)\cos(\alpha)$$

- Formules de duplication

$$\cos(2\alpha)=\cos^2(\alpha)-\sin^2(\alpha)$$
$$\sin(2\alpha)=2\sin(\alpha)\cos(\alpha)$$

- Formules de linéarisation

$$\boxed{\begin{aligned}\cos^2(\alpha)&=\frac{1}{2}\left(1+\cos(2\alpha)\right)\\ \sin^2(\alpha)&=\frac{1}{2}\left(1-\cos(2\alpha)\right)\end{aligned}}$$

- Formules de factorisation

$$\cos(\alpha)+\cos(\beta)=2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$
$$\cos(\alpha)-\cos(\beta)=-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$
$$\sin(\alpha)+\sin(\beta)=2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$
$$\sin(\alpha)-\sin(\beta)=2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

3 Angles remarquables

α	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
$\cos(\alpha)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1
$\sin(\alpha)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0
$\tan(\alpha)$	0	$\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$+\infty$	0