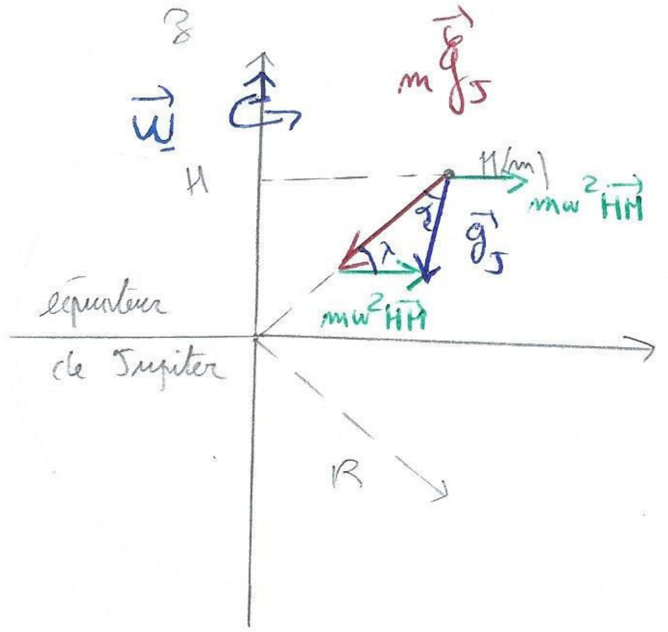
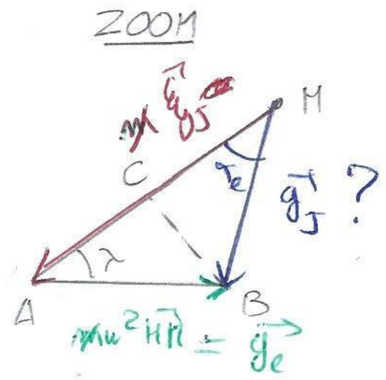
Exercice n°3 : Pesanteur apparente (Jupiter)





Géométriquement on tire 2 équations :

$$\begin{cases} g_J \cos\alpha_e + g_e \cos\lambda = \mathcal{G}_J & (1) \\ g_e \sin\lambda = g_J \sin\alpha_e & (2) \end{cases}$$

$$(2) \Rightarrow \boxed{g_J = g_e \frac{\sin\lambda}{\sin\alpha_e}}$$

$$(1) \Rightarrow g_e \frac{\sin\lambda\cos\alpha_e}{\sin\alpha_e} + g_e \cos\lambda = \mathcal{G}_J$$

$$\Rightarrow \cos\lambda + \sin\lambda \underbrace{\frac{\cos\alpha_e}{\sin\alpha_e}}_{\frac{1}{tg\,\alpha_e}} = \frac{\mathcal{G}_J}{g_e}$$

$$\Rightarrow tg\,\alpha_e = \frac{\sin\lambda}{\frac{\mathcal{G}_J}{g_e} - \cos\lambda}$$

↳ à expliciter

$$\frac{\mathcal{G}_J}{g_e} = \frac{GM_J}{R^2\omega^2 R\cos\lambda}$$

$$\text{d'où } tg\,\alpha_e = \frac{\sin\lambda}{\frac{GM_J}{R^2\omega^2R\cos\lambda} - \cos\lambda} = \frac{\sin\lambda\cos\lambda}{\underbrace{\frac{GM_J}{R^2\omega^2R}}_{A} - \cos^2\lambda}$$

Calcul de λ_{max} pour α_e maxi

$$\frac{d \tan \alpha_e}{d\lambda} = \frac{(\cos^2\lambda - \sin^2\lambda)(A-\cos^2\lambda) + \sin\lambda\cos\lambda \times 2\cos\lambda\sin\lambda}{D^2} = 0$$

$$\Rightarrow \cos(2\lambda)\,(A - \cos^2\lambda) + 2\sin^2\lambda\cos^2\lambda = 0 \quad (e)$$

A résoudre numériquement : $A = \frac{GM_J}{R^3\omega^2} = \frac{g_J}{R\left(\frac{2\pi}{T_J}\right)^2}$

$$= \frac{26{,}5}{7.10^7 \times \left(\frac{2\pi}{(9\times3600+52\times60)}\right)^2}$$

$$= 12{,}05$$

$(e) \Rightarrow \lambda_{max} = 43{,}76°$

d'où $\tan\alpha_e = \frac{\sin\lambda\cos\lambda}{A - \cos^2\lambda} = 0{,}043.$

soit $\alpha_e = 2{,}479 \simeq 2{,}48°$