• For the third problem ($x^2 + y^2 = z^n$) you can consider z odd and z even:

$$x^{2}+y^{2}=z^{2k+1}$$
 $x^{2}+y^{2}=z^{2k}$

For z even we have

$$x^2 + y^2 = z^{2k} = (z^k)^2$$

This look like the case where n = 2.