• For the third problem ($x^2 + y^2 = z^n$) you can consider z odd and z even:

$$x^{2}+y^{2}=z^{2k+1}$$
 $x^{2}+y^{2}=z^{2k}$