

# **Yolah Board Game**

Building a Two-Player Perfect-Information Game with  
Artificial Players

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MrCoder

C'est en forgeant qu'on devient  
forgeron

*À Sarah, Hugo et Célya* ❤️

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# Chapter 1

## Introduction

### 1.1 The Yolah Game

We designed Yolah, a variant of the Pingouins game, to illustrate effective techniques for implementing board games and artificial players for our students. You can see the Pingouins game box in Figure 1.1.

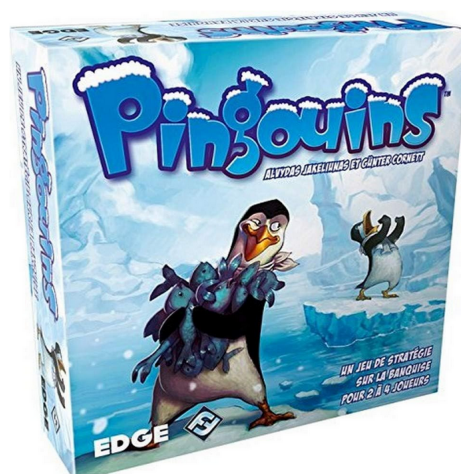


Figure 1.1: The Pingouins game box

#### Important

We have done our best with our current knowledge (*ars longa, vita brevis*) to implement our game and the associated AIs. But like any good scientist, you should look at our work with a critical eye. We wrote the book in French (easier for us) and asked an AI assistant (Claude [1]) to translate it for us.

We will now describe the rules of the game, then we will explain why we chose these rules, we will give an example of a game between two AIs and then we will present the rest of the book.

#### 1.1.1 Game Rules

The Yolah game board is shown in Figure 1.2. You can see four black pieces and four white pieces placed symmetrically. Black starts by choosing one of their four

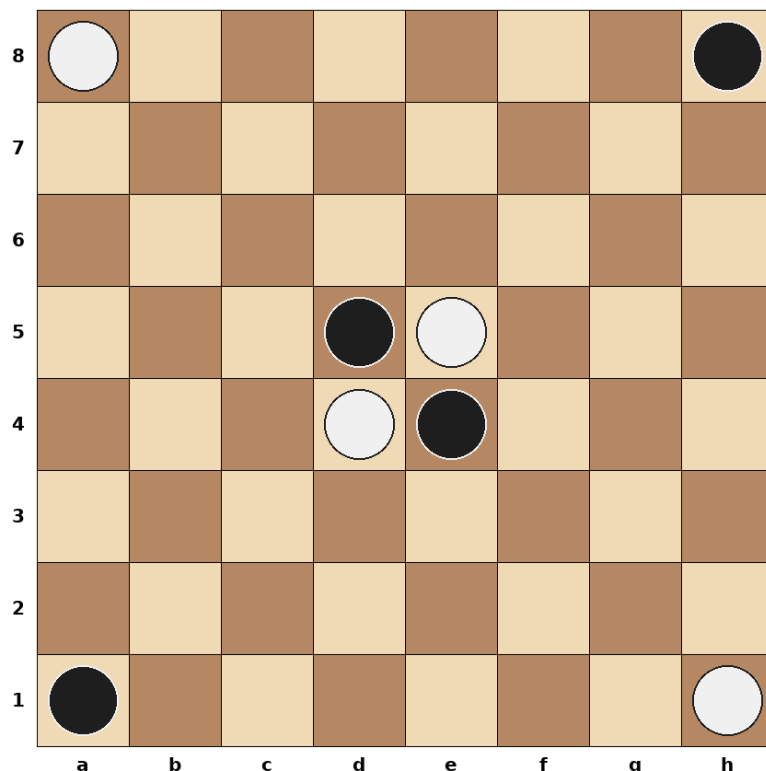


Figure 1.2: The initial configuration of the Yolah game

pieces. A piece can never disappear from the board because Yolah is a game without captures. A piece moves in all eight directions as far as it wishes as long as it is not blocked by another piece or a hole (a concept we will soon discuss). For example, if black chooses to move their piece located at **d5**, the squares where it can land are indicated by black crosses in Figure 1.3.

Now, if the black piece at **d5** moves to **b7**, which we will denote as **d5:b7**, we get the configuration shown in Figure 1.4. Notice that the starting square of the black piece disappears and becomes a hole! This square (this hole) becomes inaccessible and impassable for the rest of the game! This will create opportunities to block the opponent and try to create areas where the opponent cannot go.

A move earns one point for the player who just moved. For example, in the configuration of Figure 1.4, the black player has one point and the white player who has not yet moved has zero points. The goal of the game is quite simple to summarize: you must move longer than your opponent!

Now it is white's turn to play. They must decide which white piece they will move. Suppose it is the piece at **e5**. The possible moves for this white piece are shown in Figure 1.5. If white decides to make the move **e5:f5**, we end up in the configuration of Figure 1.6 and the score is one point each (each player has played one move).

To summarize, the rules of Yolah are as follows:

- The game is a two-player game (black and white) played in turns.

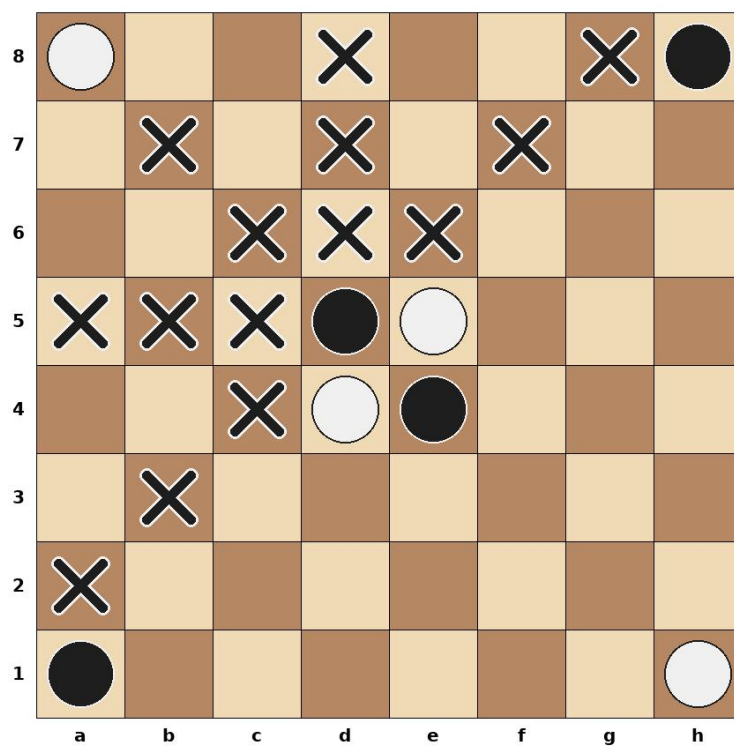


Figure 1.3: Possible moves (black crosses) for the black piece located on square d5

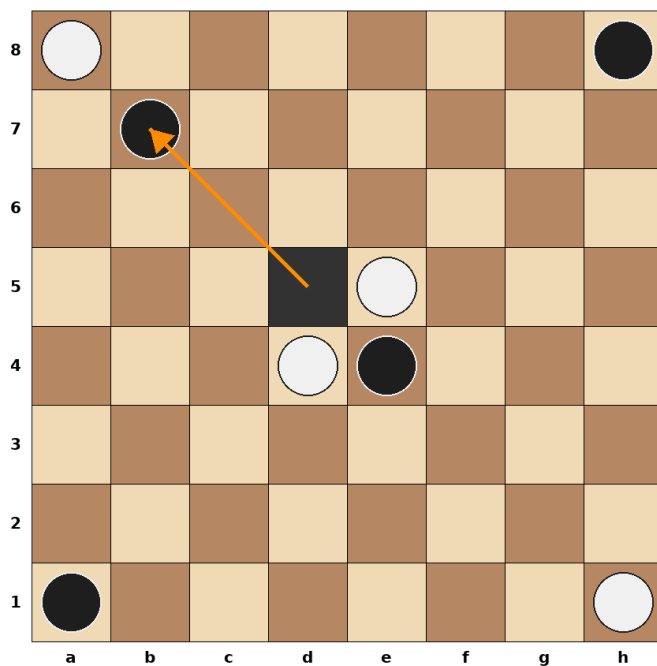


Figure 1.4: Black just moved from d5 to b7. The starting square d5 becomes inaccessible and impassable for the rest of the game

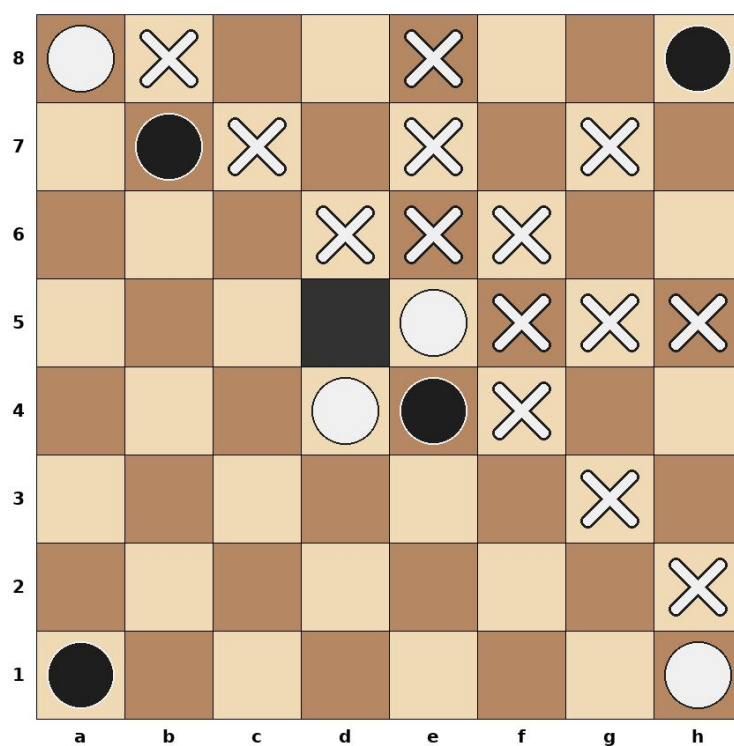


Figure 1.5: Possible moves (white crosses) for the white piece located on square **e5**

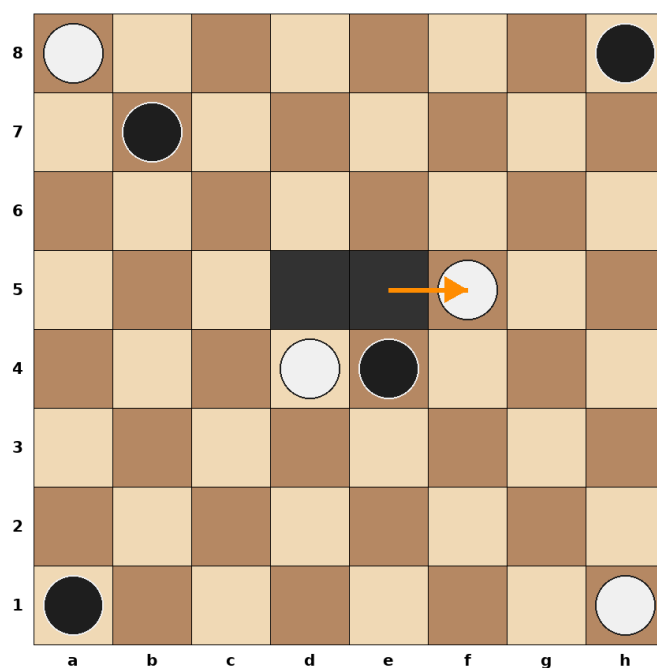


Figure 1.6: White just moved from **e5** to **f5**. The starting square **e5** becomes inaccessible and impassable for the rest of the game. The score is one point each (each player has moved once)

- Each player has four pieces.
- On their turn, the player chooses one of the pieces that can still move; if no piece can move, they pass their turn (we will denote by the move **a1:a1** to skip one's turn).
- They must move the chosen piece in one of the eight directions, as many squares as desired, but must not land on or be blocked by a piece or a hole.
- After moving the chosen piece, the starting square of the move becomes a hole and can no longer be crossed or landed on.
- After each move, the player earns one point.
- The game ends when both players can no longer move.
- The player with the most points wins the game.
- If both players have the same number of points, the game is declared a draw.

### 1.1.2 Interesting Characteristics of Yolah for Developing AIs

We chose to modify the rules of the Pingouins game to create our Yolah variant, allowing us to reuse concepts from chess implementation. This is why we use a board identical to that of chess, rather than the hexagonal tiles of the Pingouins game. Piece movement thus becomes identical to that of a Queen in chess. As in the Pingouins game, there are no cycles in the game, so there is no need for special rules to ensure a game terminates. The number of moves available to each player is quite large, but reasonable, at the start (larger than for the Pingouins game) <sup>1</sup>, but it gradually decreases as holes appear <sup>2</sup>. This allows artificial players to look ahead a fairly large number of moves. Compared to the Pingouins game, we removed the varying point values of tiles and defined a fixed starting configuration to simplify the game and make it symmetric.

### 1.1.3 Game Example

To get an idea of how a Yolah game unfolds, we will have two artificial intelligences play against each other. The first AI will be based on Monte Carlo Tree Search and the second will be based on Minimax with a neural network. We will study both of these AIs later in the book. The second AI is stronger and you will see its zone isolation strategy in action!

The progression of the game is described in Figures 1.7, 1.8 and 1.9.

The white AI estimates that it is winning starting from move 10 (see Figure 1.7k). We can see at move 30 (see Figure 1.8k) that it has successfully isolated a zone where black can no longer access. At move 32 (see Figure 1.8m) it moves one of its pieces out

---

<sup>1</sup>56 possible moves for the black player at the start of the game.

<sup>2</sup>The number of possible moves does not necessarily decrease after each move; there are configurations where a player has more than 56 moves available.

of the isolated zone because the other piece will be able to collect all the points from that zone. It is more useful to use the other piece to gather points elsewhere. Note that starting from move 47 (see Figure 1.9h onward), black has no more available moves and must therefore pass their turn.

The game is won by the white player 32 to 24, which is a very good score because the Yolah game seems to favor black.

## 1.2 What's Next

In the next chapter, we will study the implementation of the Yolah game in C++ [2]. This implementation is designed to be efficient because it will be important for the AIs to be able to play many games per second; their level of play will depend on it.

Chapter 3 describes the common interface for our different AIs. Chapter 4 presents a very simple AI based on Monte Carlo search. The next AI, described in Chapter ??, is an evolution of the previous one and will allow, unlike the Monte Carlo AI, the development of a game tree. Note that these two AIs will not require heuristics provided by humans and only need the rules of the game. Chapter 6 presents an AI based on minimax tree search with heuristics provided by humans. The heuristic used by the AI is a linear combination of the heuristics provided by humans; the weights of each heuristic in this linear combination are learned using a genetic algorithm. Our last and strongest AI is presented in Chapter 7. A neural network is used instead of heuristics. This neural network is trained on a set of games played by the previous AI.

Chapter 8 then evaluates all these AIs by having them compete in a tournament. We will then conclude and propose different directions for creating other artificial players.

Happy reading!



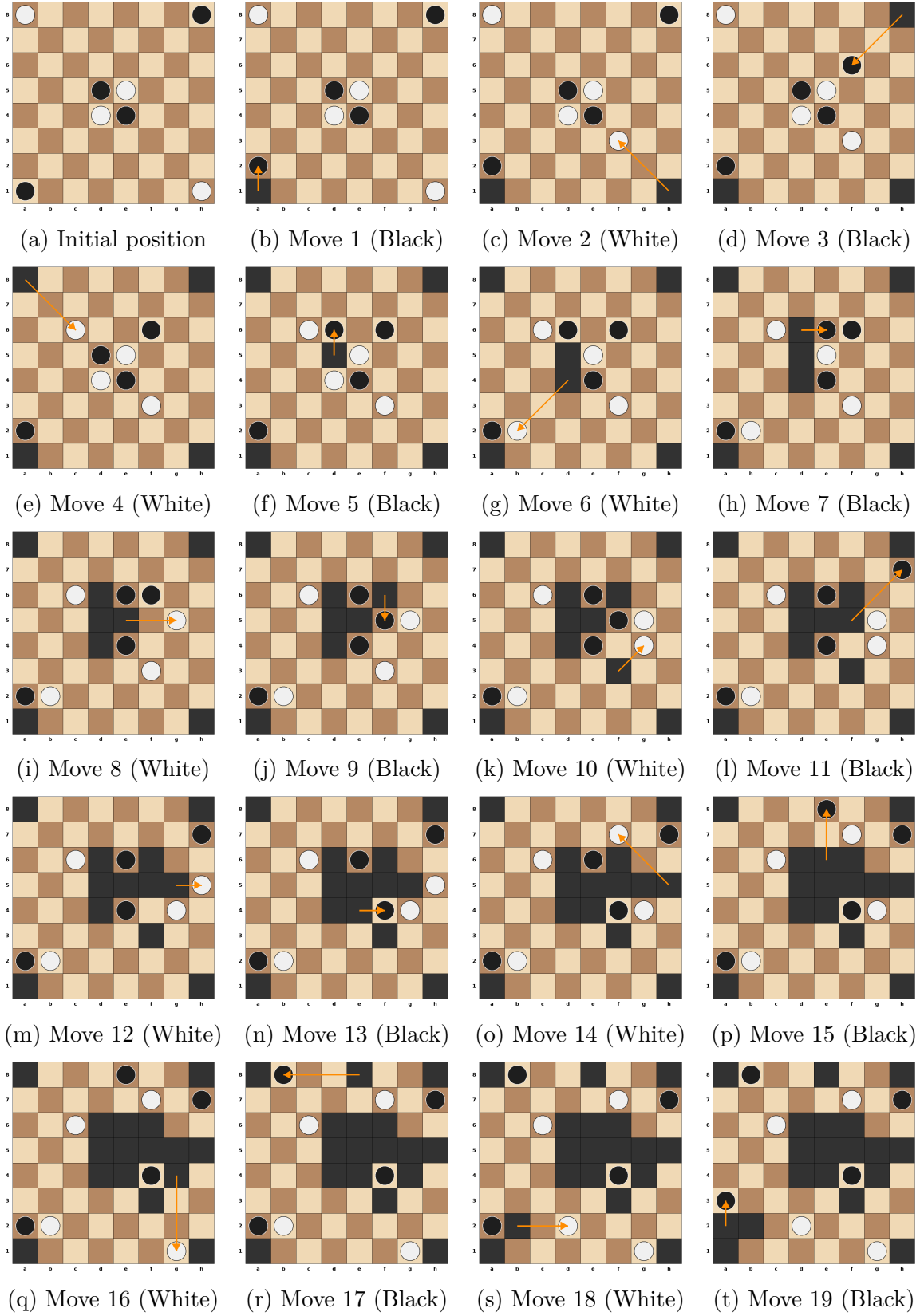


Figure 1.7: Game example between two AIs - moves 1 to 19

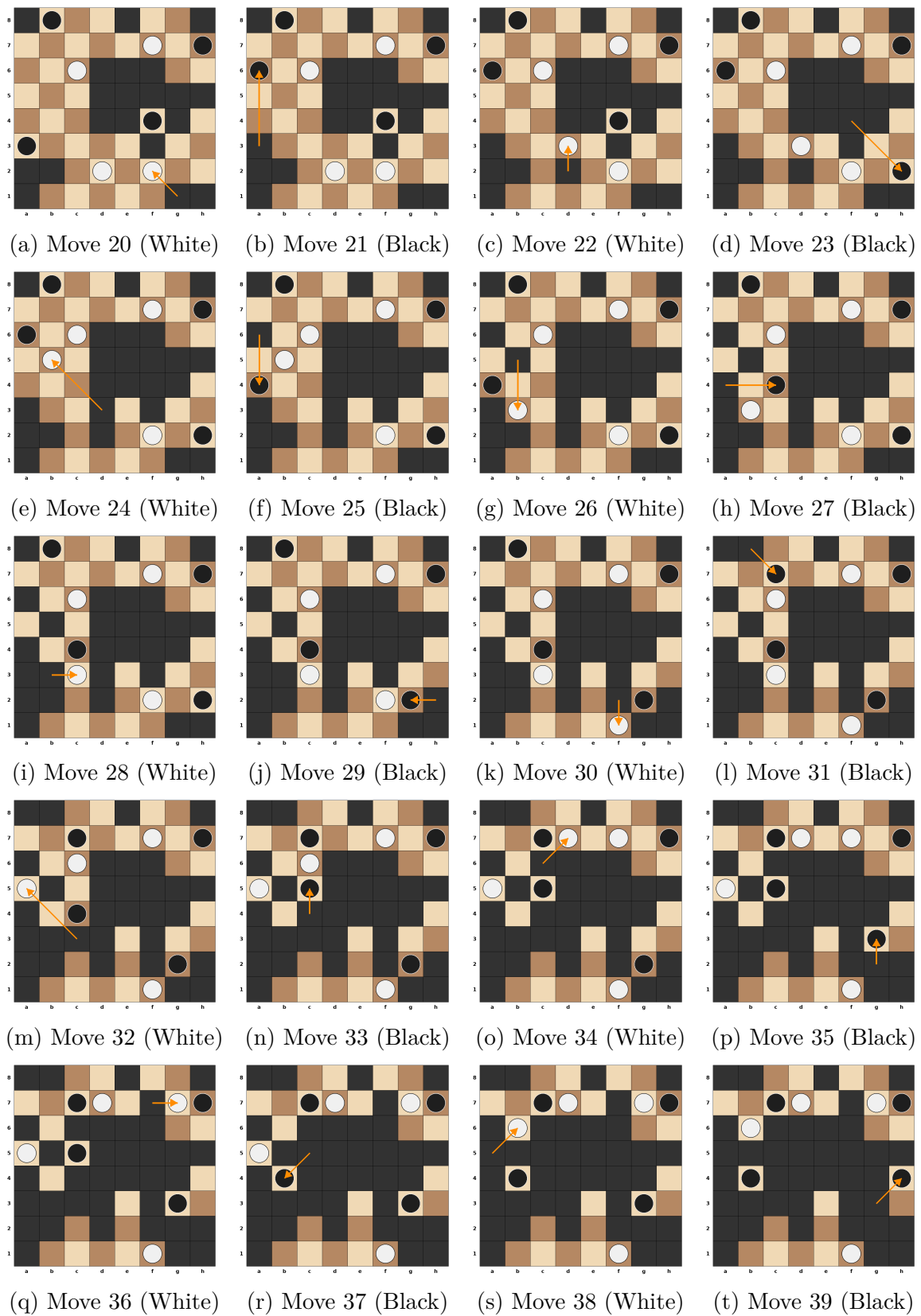


Figure 1.8: Game example between two AIs - moves 20 to 39

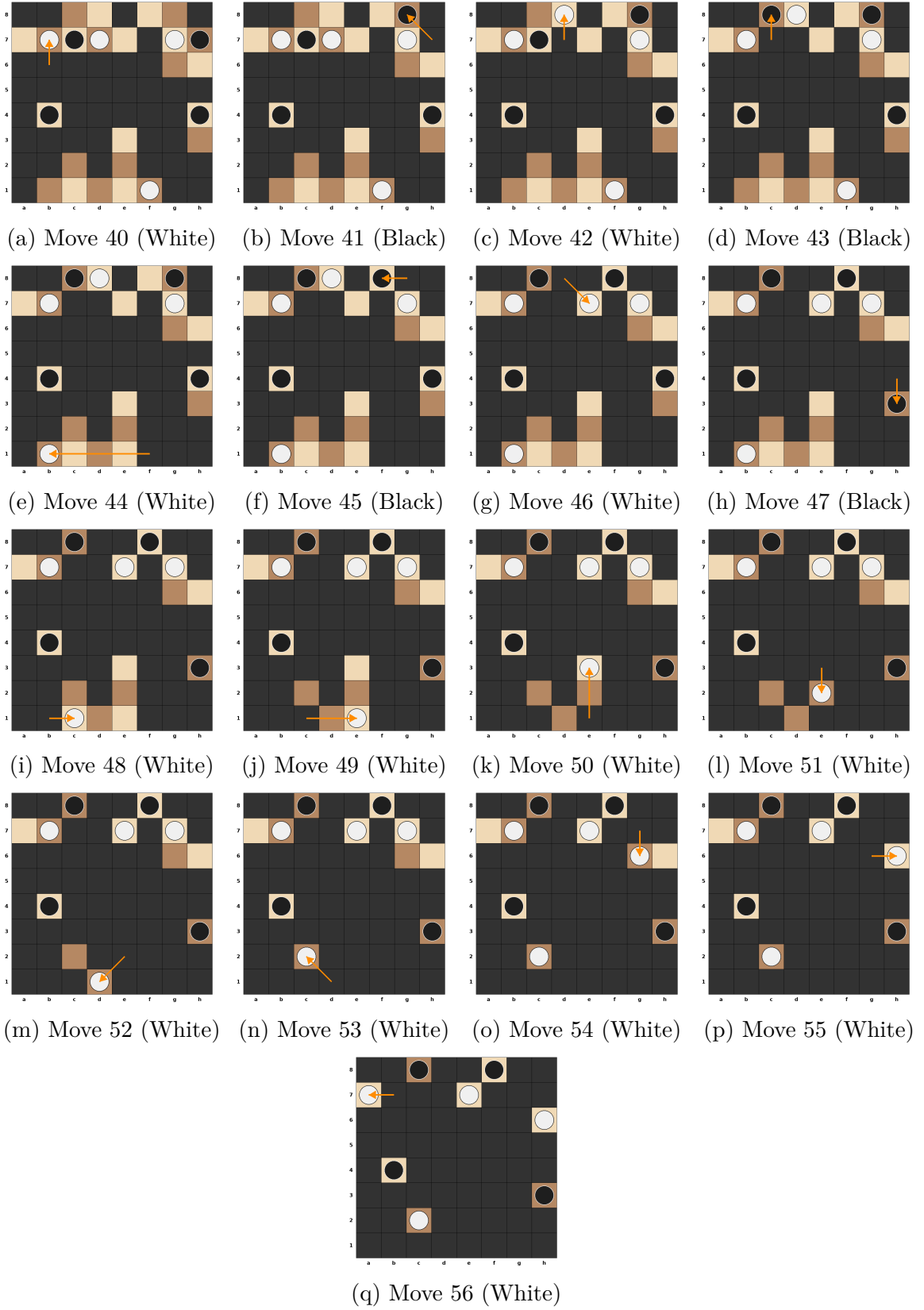


Figure 1.9: Game example between two AIs - moves 40 to 56. White wins 32 to 24



# Chapter 2

## Game Engine

We will implement game management in C++, striving to create an efficient implementation capable of running as many games per second as possible—this will be crucial for developing high-level AIs. We will test our implementation by generating as many random games as possible within a given time at the end of this chapter. We will draw inspiration from the excellent Stockfish chess engine [3] for Yolah’s data structures.

### 2.1 Data Structures

We will represent the positions of black pieces, white pieces, and destroyed squares using 64-bit unsigned integers: `uint64_t`. We call these integers bitboards. We use `uint64_t` because the Yolah board contains 64 squares, giving us one bit per square. Table 2.1 shows the position of each board square in the bitboard. This information is represented in code by the enumeration in Listing 1<sup>1</sup>.

Table 2.1: Position of each board square in the bitboard

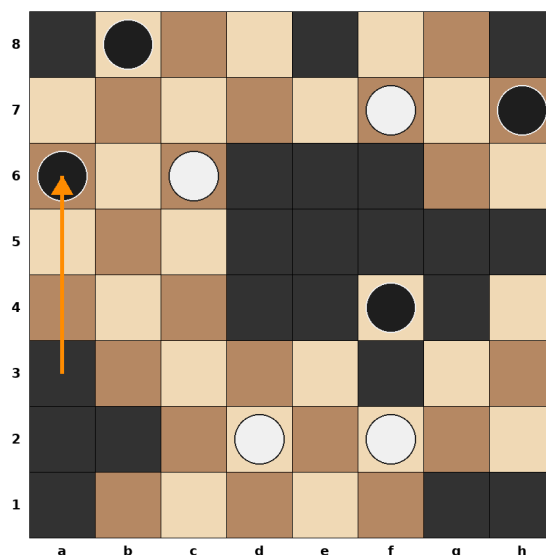
8	bit <sub>56</sub>	bit <sub>57</sub>	bit <sub>58</sub>	bit <sub>59</sub>	bit <sub>60</sub>	bit <sub>61</sub>	bit <sub>62</sub>	bit <sub>63</sub>
7	bit <sub>48</sub>	bit <sub>49</sub>	bit <sub>50</sub>	bit <sub>51</sub>	bit <sub>52</sub>	bit <sub>53</sub>	bit <sub>54</sub>	bit <sub>55</sub>
6	bit <sub>40</sub>	bit <sub>41</sub>	bit <sub>42</sub>	bit <sub>43</sub>	bit <sub>44</sub>	bit <sub>45</sub>	bit <sub>46</sub>	bit <sub>47</sub>
5	bit <sub>32</sub>	bit <sub>33</sub>	bit <sub>34</sub>	bit <sub>35</sub>	bit <sub>36</sub>	bit <sub>37</sub>	bit <sub>38</sub>	bit <sub>39</sub>
4	bit <sub>24</sub>	bit <sub>25</sub>	bit <sub>26</sub>	bit <sub>27</sub>	bit <sub>28</sub>	bit <sub>29</sub>	bit <sub>30</sub>	bit <sub>31</sub>
3	bit <sub>16</sub>	bit <sub>17</sub>	bit <sub>18</sub>	bit <sub>19</sub>	bit <sub>20</sub>	bit <sub>21</sub>	bit <sub>22</sub>	bit <sub>23</sub>
2	bit <sub>8</sub>	bit <sub>9</sub>	bit <sub>10</sub>	bit <sub>11</sub>	bit <sub>12</sub>	bit <sub>13</sub>	bit <sub>14</sub>	bit <sub>15</sub>
1	bit <sub>0</sub>	bit <sub>1</sub>	bit <sub>2</sub>	bit <sub>3</sub>	bit <sub>4</sub>	bit <sub>5</sub>	bit <sub>6</sub>	bit <sub>7</sub>
	a	b	c	d	e	f	g	h

---

```
1 enum Square : int8_t {  
2     SQ_A1, SQ_B1, SQ_C1, SQ_D1, SQ_E1, SQ_F1, SQ_G1, SQ_H1,  
3     SQ_A2, SQ_B2, SQ_C2, SQ_D2, SQ_E2, SQ_F2, SQ_G2, SQ_H2,
```

---

<sup>1</sup>By default, the enumeration starts at value 0, so `SQ_A1` equals 0, `SQ_B1` equals 1, ...



- The positions of black pieces on the board (Table 2.2) are represented by the bit-board: `0b000000010100000000000000100000000010000000000000000000000000`.
- The positions of white pieces on the board (Table 2.3) are represented by the bit-board: `0b00000000000100000000001000000000000000000000000000000010100000000000`.
- The positions of holes (Table 2.4) are represented by the bitboard:  
`0b10010001000000000001110001111100001011000001000010000001111000001`.

Table 2.2: Black piece positions

8	.	•	.	.	.	.	.	.
7	.	.	.	.	.	.	.	•
6	•	.	.	.	.	.	.	.
5	.	.	.	.	.	.	.	.
4	.	.	.	.	.	•	.	.
3	.	.	.	.	.	.	.	.
2	.	.	.	.	.	.	.	.
1	.	.	.	.	.	.	.	.
	a	b	c	d	e	f	g	h

Table 2.3: White piece positions

8	.	.	.	.	.	.	.	.
7	.	.	.	.	.	○	.	.
6	.	.	○	.	.	.	.	.
5	.	.	.	.	.	.	.	.
4	.	.	.	.	.	.	.	.
3	.	.	.	.	.	.	.	.
2	.	.	.	○	.	○	.	.
1	.	.	.	.	.	.	.	.
	a	b	c	d	e	f	g	h

Table 2.4: Destroyed square positions (holes)

8	■	.	.	.	■	.	.	■
7	.	.	.	.	.	.	.	.
6	.	.	.	■	■	■	.	.
5	.	.	.	■	■	■	■	■
4	.	.	.	■	■	.	■	.
3	■	.	.	.	.	■	.	.
2	■	■	.	.	.	.	.	.
1	■	.	.	.	.	.	■	■
	a	b	c	d	e	f	g	h

Bitboards allow us to efficiently manipulate the board using bitwise operations while requiring minimal memory to represent it.

We represent the game board with the `Yolah` class, an excerpt of which is given in Listing 2.

---

```
1 constexpr uint64_t BLACK_INITIAL_POSITION =
2 0b100000000'000000000'000000000'00001000'00010000'000000000'000000000'000000001;
3 constexpr uint64_t WHITE_INITIAL_POSITION =
4 0b000000001'000000000'000000000'00010000'00001000'000000000'000000000'100000000;
5 class Yolah {
6     uint64_t black = BLACK_INITIAL_POSITION;
7     uint64_t white = WHITE_INITIAL_POSITION;
8     uint64_t holes = 0;
9     uint8_t black_score = 0;
10    uint8_t white_score = 0;
11    uint8_t ply = 0;
12 public:
13     // ...
14 };
```

---

Listing 2: Attributes of the `Yolah` class representing the game board

The attributes of the `Yolah` class are:

- Line 6, `black`: the bitboard for black pieces.
- Line 7, `white`: the bitboard for white pieces.
- Line 8, `holes`: the bitboard for holes (destroyed squares).
- Line 9, `black_score`: the score, or number of moves, of the black player.
- Line 10, `white_score`: the score, or number of moves, of the white player.
- Line 11, `ply`: the number of moves played by both players since the start of the game.

A `Yolah` object will occupy `sizeof(Yolah) == 32` bytes in memory. Note that due to padding<sup>2</sup>, it would not have been wise to write, for example,

---

```
1 class Yolah {
2     uint64_t black = BLACK_INITIAL_POSITION;
3     uint8_t black_score = 0;
```

---

<sup>2</sup>[https://en.wikipedia.org/wiki/Data\\_structure\\_alignment](https://en.wikipedia.org/wiki/Data_structure_alignment).



---

```

4     uint64_t white = WHITE_INITIAL_POSITION;
5     uint8_t white_score = 0;
6     uint64_t holes = 0;
7     uint8_t ply = 0;
8 public:
9     // ...
10 };

```

---

because with this implementation, we would have `sizeof(Yolah) == 48` bytes.

The bitboard representation allows for efficiently obtaining game information. For example, to see the squares occupied by black and white pieces, simply perform: `black | white`. For the positions in Tables 2.2 and 2.3, we would obtain in a single highly efficient operation<sup>3</sup> the black and white piece positions shown in Table 2.5. In binary notation, we get:

```

black | white
== 00000010100000000000000010000000000100000000000000000000000000 |
   00000000001000000000010000000000000000000000000000001010000000000
== 000000101010000000000101000000000010000000000000010100000000000

```

Table 2.5: Black and white piece positions obtained by computing: `black | white`

8	.	•	.	.	.	.	.	.
7	.	.	.	.	.	○	.	•
6	•	.	○	.	.	.	.	.
5	.	.	.	.	.	.	.	.
4	.	.	.	.	.	•	.	.
3	.	.	.	.	.	.	.	.
2	.	.	.	○	.	○	.	.
1	.	.	.	.	.	.	.	.
	a	b	c	d	e	f	g	h

We will continue using bitwise operations throughout this chapter, particularly for testing game over conditions and generating possible moves.

## 2.2 Game Over Test

To test for game over, we will need some enumerations and constants (Listings 3 and 5) and the `shift` function (Listing 4). The `NORTH` constant in the `Direction` enumeration (Listing 3) equals 8. Why this value? Let us revisit below, see Table 2.6, the table from the previous chapter showing the position of each bit in a bitboard on

---

<sup>3</sup>You can find information on instruction latency and throughput for various processors at: [https://agner.org/optimize/instruction\\_tables.pdf](https://agner.org/optimize/instruction_tables.pdf)

the board. We can see that adding 8 to the bit number in any square gives us the bit number of the square to the north (unless we exit the board). The same applies to the other constant values in the **Direction** enumeration (Listing 3).

Table 2.6: Position of each board square in the bitboard. Note that for a square not in rank 8,  $\text{bit}_{i+8}$  corresponds to the square north of  $\text{bit}_i$

8	$\text{bit}_{56}$	$\text{bit}_{57}$	$\text{bit}_{58}$	$\text{bit}_{59}$	$\text{bit}_{60}$	$\text{bit}_{61}$	$\text{bit}_{62}$	$\text{bit}_{63}$
7	$\text{bit}_{48}$	$\text{bit}_{49}$	$\text{bit}_{50}$	$\text{bit}_{51}$	$\text{bit}_{52}$	$\text{bit}_{53}$	$\text{bit}_{54}$	$\text{bit}_{55}$
6	$\text{bit}_{40}$	$\text{bit}_{41}$	$\text{bit}_{42}$	$\text{bit}_{43}$	$\text{bit}_{44}$	$\text{bit}_{45}$	$\text{bit}_{46}$	$\text{bit}_{47}$
5	$\text{bit}_{32}$	$\text{bit}_{33}$	$\text{bit}_{34}$	$\text{bit}_{35}$	$\text{bit}_{36}$	$\text{bit}_{37}$	$\text{bit}_{38}$	$\text{bit}_{39}$
4	$\text{bit}_{24}$	$\text{bit}_{25}$	$\text{bit}_{26}$	$\text{bit}_{27}$	$\text{bit}_{28}$	$\text{bit}_{29}$	$\text{bit}_{30}$	$\text{bit}_{31}$
3	$\text{bit}_{16}$	$\text{bit}_{17}$	$\text{bit}_{18}$	$\text{bit}_{19}$	$\text{bit}_{20}$	$\text{bit}_{21}$	$\text{bit}_{22}$	$\text{bit}_{23}$
2	$\text{bit}_8$	$\text{bit}_9$	$\text{bit}_{10}$	$\text{bit}_{11}$	$\text{bit}_{12}$	$\text{bit}_{13}$	$\text{bit}_{14}$	$\text{bit}_{15}$
1	$\text{bit}_0$	$\text{bit}_1$	$\text{bit}_2$	$\text{bit}_3$	$\text{bit}_4$	$\text{bit}_5$	$\text{bit}_6$	$\text{bit}_7$
	a	b	c	d	e	f	g	h

---

```

1  enum Direction : int8_t {
2      NORTH = 8,
3      EAST  = 1,
4      SOUTH = -NORTH,
5      WEST  = -EAST,
6      NORTH_EAST = NORTH + EAST,
7      SOUTH_EAST = SOUTH + EAST,
8      SOUTH_WEST = SOUTH + WEST,
9      NORTH_WEST = NORTH + WEST
10 };

```

---

Listing 3: Directions

---

```

1  template<Direction D>
2  constexpr uint64_t shift(uint64_t b) {
3      if constexpr (D == NORTH)
4          return b << NORTH;
5      else if constexpr (D == SOUTH)
6          return b >> -SOUTH;
7      else if constexpr (D == EAST)
8          return (b & ~FileHBB) << EAST;
9      else if constexpr (D == WEST)
10         return (b & ~FileABB) >> -WEST;
11     else if constexpr (D == NORTH_EAST)

```

```

12         return (b & ~FileHBB) << NORTH_EAST;
13     else if constexpr (D == NORTH_WEST)
14         return (b & ~FileABB) << NORTH_WEST;
15     else if constexpr (D == SOUTH_EAST)
16         return (b & ~FileHBB) >> -SOUTH_EAST;
17     else if constexpr (D == SOUTH_WEST)
18         return (b & ~FileABB) >> -SOUTH_WEST;
19     else return 0;
20 }

```

Listing 4: Shift by direction

The `shift(uint64_t b)` function given in Listing 4 shifts all 1-bits in the bitboard parameter in direction `D`<sup>4</sup>. Given

[illegible][illegible]

Table 2.7: Game board before applying `shift<NORTH>`

8	.	.	.	.	.	.	.	●
7	.	.	.	.	.	.	.	.
6	.	.	.	.	.	.	.	.
5	.	.	.	●	.	.	.	.
4	.	.	.	.	.	●	.	.
3	.	.	.	.	.	.	.	.
2	.	.	.	.	.	.	.	.
1	●	.	.	.	.	.	.	.
	a	b	c	d	e	f	g	h

We can thus easily obtain all squares directly adjacent to the pieces on a given **board** by performing the following operation:

<sup>4</sup>Thanks to **if constexpr**, when we call `shift<NORTH>(b)` for example, the compiler will transform the code to: `return b << NORTH;`

For the board in Table 2.9, we get the positions represented by stars in Table 2.10.

Table 2.9: Game board

8	○	.	.	.	.	.	.	●
7	.	●	.	.	.	.	.	.
6	.	.	.	.	.	.	.	.
5	.	.	.	■	○	.	.	.
4	.	.	.	■	■	●	.	.
3	.	.	.	.	.	.	.	.
2	.	.	.	○	.	.	.	.
1	●	.	.	.	.	.	.	○
	a	b	c	d	e	f	g	h

Table 2.10: Positions around the pieces

8	*	*	*	.	.	.	*	.
7	*	*	*	.	.	.	*	*
6	*	*	*	*	.	*	.	.
5	.	.	.	*	*	*	*	.
4	.	.	.	*	*	*	*	.
3	.	.	*	*	*	*	*	.
2	*	*	*	.	*	.	*	*
1	.	*	*	*	*	.	*	.
	a	b	c	d	e	f	g	h

In the `shift` function code (Listing 4), we can see the use of constants `FileHBB` and `FileABB` (BB for bitboard). These constants allow us to mask certain bits that would end up in incorrect positions after shifting. For example, if we shift the bitboard shown in Table 2.9 in the `EAST` direction, the white pawn on `h1` would end up on `a2`. To avoid this, before shifting, we eliminate elements from column `h` so they don't end up in column `a` (line 8 of Listing 4). The constants `FileABB` and `FileHBB` along with the rank and file enumerations are defined in Listing 5.

---

```

1  enum File : uint8_t {
2      FILE_A, FILE_B, FILE_C, FILE_D, FILE_E, FILE_F, FILE_G, FILE_H,
3      FILE_NB
4  };
5  enum Rank : uint8_t {
6      RANK_1, RANK_2, RANK_3, RANK_4, RANK_5, RANK_6, RANK_7, RANK_8,
7      RANK_NB
8  };
9  constexpr uint64_t FileABB = 0x0101010101010101;
10 // 0x0101010101010101
11 //==
12 ↪ 0b00000000100000001000000010000000100000001000000010000000100000001
13 //      a8      a7      a6      a5      a4      a3      a2
14 ↪ a1
15
16 constexpr uint64_t FileHBB = FileABB << 7;
17 // 0x8080808080808080
18 //==
19 ↪ 0b10000000010000000100000001000000010000000100000001000000010000000
20 //      h8      h7      h6      h5      h4      h3      h2      h1

```

---

## Listing 5: Files and ranks

The function that tests for game over is called `game_over` and is described in Listing 6. Its implementation is straightforward:

- On line 2, we retrieve in `possible` the bitboard with 1s in free positions.
- On line 3, we create the `players` bitboard containing 1s in positions occupied by either player.
- Lines 4 to 8 create the `around_players` bitboard containing 1s at positions around each player's pieces.
- Finally, on line 9, we test whether no free position exists adjacent to either player. The bitwise AND will keep a 1 in a result bit if and only if there is a 1 at that position in both the `possible` and `around_players` bitboards—meaning the corresponding square is both free and accessible by a player. If `around_players & possible` equals 0, it means no position is both adjacent to a player and free.

---

```
1  bool Yolah::game_over() const {
2      uint64_t possible = ~holes & ~black & ~white;
3      uint64_t players  = black | white;
4      uint64_t around_players = shift<NORTH>(players) |
5          shift<SOUTH>(players) | shift<EAST>(players) |
6          shift<WEST>(players) | shift<NORTH_EAST>(players) |
7          shift<NORTH_WEST>(players) | shift<SOUTH_EAST>(players) |
8          shift<SOUTH_WEST>(players);
9      return (around_players & possible) == 0;
10 }
```

---

## Listing 6: Game over test

Now that we know how to test for game over, we will study how to efficiently generate possible moves in a given position.

## 2.3 Generating Possible Piece Moves

Move generation will use a technique called *magic bitboards*, which is a perfect hash function that maps blocker occupancy patterns to unique lookup table indices without collisions (we will see all this in detail later in this chapter). To understand this technique, we will first present perfect hashing through a simple example. Throughout this chapter, we will use the terms *perfect hashing* and *magic bitboards* interchangeably, as they refer to the same technique in this context. We drew inspiration from [4] for this example.

### 2.3.1 Magic Perfect Hashing Function

Suppose we need to find the position of the only 1-bit in an unsigned long integer (`uint64_t`). For example, if we call `position` the function that calculates this position, we get the following results:

```
position(1)           == 0
position(0b1000)      == 3
position(0b10000000) == 7
position(0x8000000000000000) == 63
```

Note that processors have instructions to efficiently compute this function<sup>5</sup>, and the `gcc` compiler allows efficient calculation of this position with the function `__builtin_ctzll(unsigned long long x)`<sup>6</sup>, for example,

```
position(0b1000) == __builtin_ctzll(0b1000)
```

However, we will code this `position` function using a magic perfect hashing function. This function will be useful for move generation later. Using a standard hash table, we could first fill it with 64 values and then simply look up this table, as in the following code:

---

```
1 unordered_map<uint64_t, uint8_t> table {
2     {1, 0}, {0b10, 1}, {0b100, 2}, {0b1000, 3}, //...
3 }
4 int position(uint64_t x) {
5     return table[x];
6 }
```

---

Looking up values in this hash table is far more costly than indexing a simple array<sup>7</sup>. However, we cannot directly use an array because the indexing values span too large a range—for example, the value `0x8000000000000000` (9, 223, 372, 036, 854, 775, 808!). The idea is to find an efficient hash function that transforms each of the 64 values<sup>8</sup>—which we call bitboards—into the range  $[0, 2^6 - 1 = 63]$ . Moreover, we want no collisions—this is called perfect hashing. If there were collisions, positions would be incorrect. For example, if keys `0b100` and `0b1000000` were both transformed to index 42, we would need to store value 2 and 6 in `table[42]`, but we cannot store more than one value at the same location.

The magic perfect hashing function will have the following form (listing 7):

---

<sup>5</sup>`tzcnt` on Intel.

<sup>6</sup>Returns the number of trailing 0-bits in `x`, starting at the least significant bit position. If `x` is 0, the result is undefined.

<sup>7</sup>We will measure the performance gain from magic perfect hashing compared to using a standard hash table at the end of this section.

<sup>8</sup>`1, 0b10, 0b100, 0b1000, 0b10000, ....`

---

```
1 constexpr uint64_t MAGIC = //...
2 constexpr int K = 6;
3 int magic_perfect_hashing(uint64_t bitboard) {
4     return bitboard * MAGIC >> (64 - K);
5 }
```

---

Listing 7: Magic bitboard perfect hashing

In this function:

- On line 2, the constant `K` gives the number of bits for our index. Here with `K == 6`, this gives us a maximum of  $2^K = 2^6 = 64$  possible values in the table.
- On line 4, we see the formula for calculating the table index from the `uint64_t` key parameter. We multiply the key by the `MAGIC` constant and then right-shift to keep only the `K` most significant bits of the result. This operation is very inexpensive, but how do we find this magic constant?

A simple method to find the `MAGIC` constant is to generate it randomly until we find a value that produces no collisions! This approach is used in Stockfish [3] to generate magic bitboards. Listing 8 shows this approach.

---

```
1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 int main() {
6     random_device rd;
7     mt19937_64 mt(rd());
8     uniform_int_distribution<uint64_t> d;
9     unordered_map<uint64_t, int> positions;
10    for (int i = 0; i < 64; i++) {
11        positions[1ULL << i] = i;
12    }
13    constexpr int K = 7;
14    auto index = [](uint64_t magic, int k, uint64_t bitboard) {
15        return bitboard * magic >> (64 - k);
16    };
17    while (true) {
18        uint64_t MAGIC = d(mt);
19        bool found = true;
20        set<uint64_t> seen;
21        for (const auto [bitboard, pos] : positions) {
```

```
22         uint64_t i = index(MAGIC, K, bitboard);
23         if (seen.contains(i)) {
24             found = false;
25             break;
26         }
27         seen.insert(i);
28     }
29     if (found) {
30         cout << format("found magic for K = {}: {:#x}\n", K,
↪ MAGIC);
31         int size = 1 << K;
32         vector<int> table(size, -1);
33         for (const auto [bitboard, pos] : positions) {
34             table[index(MAGIC, K, bitboard)] = pos;
35         }
36         cout << format("uint8_t positions[{}] = {{", size);
37         for (int i = 0; i < size; i++) {
38             cout << table[i] << ', ' ;
39         }
40         cout << "};\n";
41         break;
42     }
43 }
44 }
```

---

Listing 8: Random search for the **MAGIC** constant

- On lines 6 to 8, we set up a random generator to randomly generate **uint64\_t** values.
- On line 9, the lookup table **positions** associates each long integer containing only one 1-bit with its position. This table is initialized on lines 10 to 12.
- On line 13, the constant **K** is the number of bits in our key obtained by magic perfect hashing. **K** must be large enough to represent all values we need to cover. For example, here we have 64 possible values since we have 64 possible positions for the 1-bit in a 64-bit long integer. With **K** == 7, we can cover  $2^7 = 128$  different values. Note that this is more than the 64 values we need, and **K** == 6 would suffice (**K** == 5 would be too small). However, when trying to find the **MAGIC** constant randomly for **K** == 6, this program could not find one in reasonable time (we will see another approach to succeed with **K** == 6).
- On lines 14 to 16, the **index** function calculates the index for key **bitboard** (this bitboard contains only one 1-bit) using the magic perfect hashing formula.
- The **while** loop, lines 17 to 43, loops until it finds a **MAGIC** constant that achieves magic perfect hashing (this loop could potentially run forever).



- On line 18, we create the **MAGIC** value randomly.
- The `set<uint64_t>` on line 20 allows us to remember the indices (obtained via the `index` function) we have already used, to verify we have no collisions.
- On lines 21 to 28, the **for** loop tests all bitboards, finds the index for each using the hash (`index` function), and verifies there are no collisions (line 23) with indices obtained for previous bitboards.
- Finally, on lines 29 to 42, if we found a **MAGIC** constant that creates magic perfect hashing, we display it and then display a C++ array containing the position of the 1-bit for each bitboard. The output of this program for a given execution is shown below.

---

```
found magic for K = 7: 0x65e4d4ee86638416
uint8_t positions[128] = {
    63, -1, 54, -1, 49, 55, 33, -1, 50, -1, -1, 56, 34, -1, 43, -1,
    51, -1, -1, 11, -1, -1, 57, 3, 39, 35, 14, -1, 44, 22, -1, -1,
    52, 31, -1, -1, -1, -1, 12, 20, -1, 18, -1, -1, 58, -1, -1, 4,
    60, 40, 0, 36, -1, 15, -1, -1, 45, -1, 27, 23, 6, -1, -1, -1,
    62, 53, 48, 32, -1, -1, -1, 42, -1, 10, -1, 2, 38, 13, 21, -1,
    30, -1, -1, 19, 17, -1, -1, -1, 59, -1, -1, -1, -1, 26, 5, -1,
    61, 47, -1, 41, 9, 1, 37, -1, 29, -1, 16, -1, -1, -1, 25, -1,
    46, -1, 8, -1, 28, -1, -1, 24, -1, 7, -1, -1, -1, -1, -1, -1, -1
};
```

---

To get the position of the 1-bit for bitboard `0b1000` for example, we need to look up the following value:

---

```
positions[0b1000 * 0x65e4d4ee86638416 >> (64 - 7)]
== positions[0x2f26a774331c20b0 >> 57]
== positions[0x17]
== positions[23]
== 3
```

---

Notice that there are vacant slots in the `positions` array—those containing `-1`—so we wasted space. We would not have wasted any space if we had found a **MAGIC** constant for `K == 6`. But is this possible?

To answer this question, we will proceed differently. Obviously, we do not want to scan through all  $2^{64}$  (9, 223, 372, 036, 854, 775, 808) possible values and test each one for magic perfect hashing. We will use an Satisfiability Modulo Theories (SMT) solver that allows us to set constraints before searching the entire space. Setting constraints first will prune the search space. Note that we have no guarantee the search will be

fast, but for this problem, finding a **MAGIC** constant for  $K == 6$  will be very quick. If you are interested in how an SMT solver works, you can consult [5] and [6].

The code in Listing 9 describes the approach using the Z3 Theorem Prover (Z3) solver [7].

---

```
1  from z3 import *
2
3  positions = {}
4  for i in range(64):
5      positions[1 << i] = i
6
7  solver = Solver()
8  MAGIC = BitVec('magic', 64)
9  K = 6
10 bitboards = list(positions.keys())
11
12 def index(magic, k, bitboard):
13     return magic * bitboard >> (64 - k)
14
15 for i in range(64):
16     index1 = index(MAGIC, K, bitboards[i])
17     for j in range(i + 1, 64):
18         index2 = index(MAGIC, K, bitboards[j])
19         solver.add(index1 != index2)
20
21 if solver.check() == sat:
22     model = solver.model()
23     m = model[MAGIC].as_long()
24     print(f'found magic for K = {K}: {m:#x}')
25     size = 1 << K
26     table = [-1] * size
27     for bitboard, pos in positions.items():
28         table[index(m, K, bitboard) & size - 1] = pos
29     print(f'constexpr uint8_t positions[{size}] = {{{')
30     for i in range(size):
31         print(f'{{table[i]}}, ', end='')
32     print('\n};')
```

---

Listing 9: Searching for the **MAGIC** constant with an SMT solver

- Lines 3 to 5 associate each bitboard containing only one 1-bit with the corresponding position of that bit. We use the `positions` dictionary (line 3) for

this.

- On line 7, we instantiate the Z3 solver.
- On line 8, we declare that the **MAGIC** constant is of type **z3.BitVec**, a type provided by the Z3 solver for representing bit vectors—here we use a 64-bit vector.
- The **index** function defined on lines 12 and 13 calculates the hash based on the **MAGIC** constant and **K**.
- On lines 15 to 19, we give the solver the constraints that hashed values from the **index** function for two different bitboards must not map to the same location. This constraint ensures perfect hashing.
- On line 21, we run the solver, which returns **sat** if it found a **MAGIC** value satisfying the constraints. Note that if the solver returns **unsat** instead of **sat**, it means no 64-bit integer allows perfect hashing. Even though the search space is very large, on my machine, finding a solution takes less than a second!
- Lines 22 to 32 print to standard output the **MAGIC** value and the C++ array of obtained values. This array is shown below.

---

```
found magic for K = 6: 0x2643c51ab9dfa5b
constexpr uint8_t positions[64] = {
    0,  1,  2, 14,  3, 22, 28, 15, 11,  4, 23, 55,  7, 29, 41, 16,
    12, 26, 53,  5, 24, 33, 56, 35, 61,  8, 30, 58, 37, 42, 17, 46,
    63, 13, 21, 27, 10, 54,  6, 40, 25, 52, 32, 34, 60, 57, 36, 45,
    62, 20,  9, 39, 51, 31, 59, 44, 19, 38, 50, 43, 18, 49, 48, 47
};
```

---

Note that in *Python* it was important to perform the operation **index(m, K, bitboard) & size - 1** on line 28 because Python integers have arbitrary precision. For example, we have:

---

```
0x2643c51ab9dfa5b * 0x8000000000000000 >> 58
== 0x4c878a3573bf4b60

(0x2643c51ab9dfa5b * 0x8000000000000000 >> 58) & size - 1
== 0x4c878a3573bf4b60 & 0b111111
== 32
```

---

In C++ we would have:

---

```
    0x2643c51ab9dfa5b * 0x8000000000000000 >> 58
== 32
```

---

Note that we don't have the same problem on line 18 because `MAGIC` is of type `BitVec(64)`.

To test the efficiency of our magic perfect hashing technique, we set up the micro-benchmark shown in Listing 10. We use the *Google Benchmark* library for these measurements [8]. The benchmark execution results are shown in Listing 11. The `BM_magic_positions` function using magic perfect hashing is much faster than the `BM_unordered_map_positions` function using a standard hash table. Of course, the `BM_builtin_ctz_positions` function using a dedicated instruction is the most efficient (but will be of no use for move generation).

---

```
1  #include <benchmark/benchmark.h>
2  #include <unordered_map>
3  #include <vector>
4
5  std::vector<uint64_t> generate_isolated_bits() {
6      std::vector<uint64_t> samples;
7      for (int i = 0; i < 64; i++) {
8          samples.push_back(1ULL << i);
9      }
10     return samples;
11 }
12 static void BM_magic_positions(benchmark::State& state) {
13     constexpr uint64_t MAGIC = 0x2643c51ab9dfa5b;
14     constexpr int K = 6;
15     constexpr uint8_t positions[64] = {
16         0, 1, 2, 14, 3, 22, 28, 15, 11, 4, 23, 55, 7, 29, 41,
17         ↪ 16,
18         12, 26, 53, 5, 24, 33, 56, 35, 61, 8, 30, 58, 37, 42, 17,
19         ↪ 46,
20         63, 13, 21, 27, 10, 54, 6, 40, 25, 52, 32, 34, 60, 57, 36,
21         ↪ 45,
22         62, 20, 9, 39, 51, 31, 59, 44, 19, 38, 50, 43, 18, 49, 48,
23         ↪ 47
24     };
25     auto samples = generate_isolated_bits();
26     size_t idx = 0;
```

```
23     for (auto _ : state) {
24         uint64_t bitboard = samples[idx++ & 63];
25         benchmark::DoNotOptimize(positions[bitboard * MAGIC >> (64 -
↪ K)]);
26     }
27     state.SetItemsProcessed(state.iterations());
28 }
29 BENCHMARK(BM_magic_positions);
30 static void BM_unordered_map_positions(benchmark::State& state) {
31     std::unordered_map<uint64_t, uint8_t> map;
32     for (uint8_t i = 0; i < 64; i++) {
33         map[1ULL << i] = i;
34     }
35     auto samples = generate_isolated_bits();
36     size_t idx = 0;
37     for (auto _ : state) {
38         uint64_t bitboard = samples[idx++ & 63];
39         benchmark::DoNotOptimize(map[bitboard]);
40     }
41     state.SetItemsProcessed(state.iterations());
42 }
43 BENCHMARK(BM_unordered_map_positions);
44 static void BM_builtin_ctz_positions(benchmark::State& state) {
45     auto samples = generate_isolated_bits();
46     size_t idx = 0;
47     for (auto _ : state) {
48         uint64_t bitboard = samples[idx++ & 63];
49         benchmark::DoNotOptimize(__builtin_ctzll(bitboard));
50     }
51     state.SetItemsProcessed(state.iterations());
52 }
53 BENCHMARK(BM_builtin_ctz_positions);
54 BENCHMARK_MAIN();
```

---

Listing 10: Micro-benchmark comparing execution times of magic perfect hashing, a standard hash table, and a dedicated machine instruction.

---

```
1 Run on (12 X 4400 MHz CPU s)
2 CPU Caches: L1 Data 32 KiB (x6), L1 Instruction 32 KiB (x6),
3             L2 Unified 256 KiB (x6), L3 Unified 12288 KiB (x1)
4 Load Average: 0.62, 1.33, 2.92
```

5					
6	Benchmark	Time	CPU	Iterations	Throughput
7	-----	-----	-----	-----	-----
8	BM_magic_positions	0.542 ns	0.542 ns	1000000000	1.85 G/s
9	BM_unordered_map_positions	30.2 ns	30.2 ns	23361655	33.1 M/s
10	BM_builtin_ctz_positions	0.480 ns	0.480 ns	1000000000	2.08 G/s

Listing 11: Micro-benchmark execution results from Listing 10

Now that we have seen how the magic perfect hashing technique works through a simple example, we can study generating possible piece moves using this technique.

### 2.3.2 Magic Bitboards for Piece Moves

For each board configuration, we want to be able to provide, for a given square, the list of possible moves for that game configuration. For example, for the board in figure 2.2 where the possible moves for the piece at **d3** are represented by white crosses in figure 2.3, we would like to very efficiently obtain the bitboard shown in table 2.11 indicating the different possible moves.

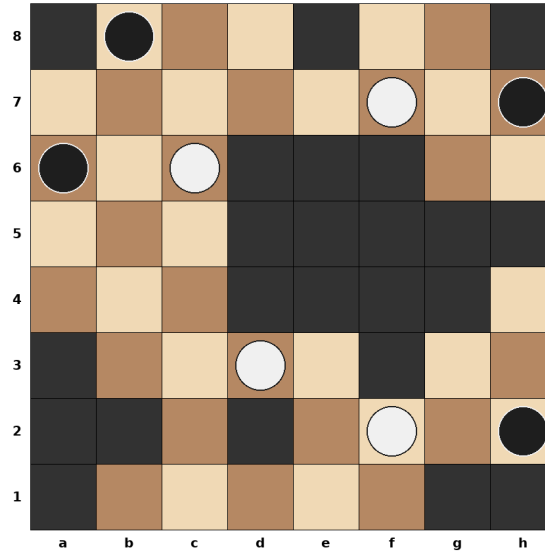


Figure 2.2: Board position example (after move 23 in figure 1.8)

Let's take square **d3** as an example. We first need a mask that will allow us to isolate the board squares reachable by the piece at **d3** without considering obstacles. This mask is represented in table 2.12. Let **mask** be the mask from table 2.12 and **board** be the bitboard from table 2.13 representing the game board from figure 2.2. We obtain the bitboard **occupancies**, giving the obstacles on the possible trajectories of the piece at **d3** by doing: **occupancies** = **mask** & **board**. This bitboard is given in table 2.14. There is a 1 for each square containing a piece or a hole. What we need is to be able to index a table, let's call it **uint64\_t MOVES\_D3[]**, using the bitboard **occupancies**, to obtain the bitboard of possible moves from square **d3**. This bitboard is represented in table 2.11. Then we only need to iterate through each 1 bit in this

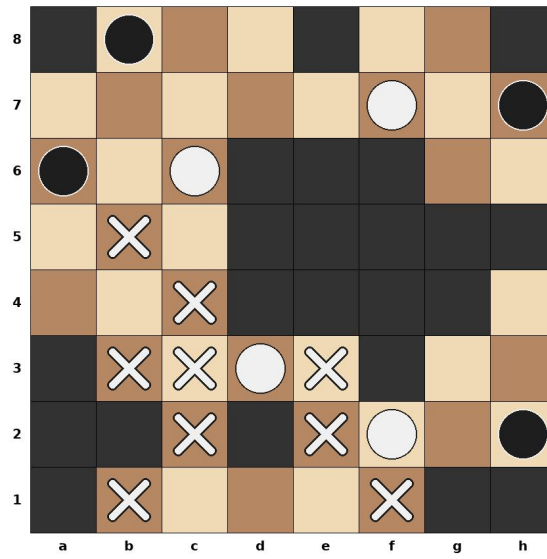


Figure 2.3: Possible moves (white crosses) for the white piece at **d3**

Table 2.11: Bitboard of possible moves for the piece at d3 (see figure 2.3)

8	56 0	57 0	58 0	59 0	60 0	61 0	62 0	63 0
7	48 0	49 0	50 0	51 0	52 0	53 0	54 0	55 0
6	40 0	41 0	42 0	43 0	44 0	45 0	46 0	47 0
5	32 0	33 <b>1</b>	34 0	35 0	36 0	37 0	38 0	39 0
4	24 0	25 0	26 <b>1</b>	27 0	28 0	29 0	30 0	31 0
3	16 0	17 <b>1</b>	18 <b>1</b>	19 0	20 <b>1</b>	21 0	22 0	23 0
2	8 0	9 0	10 <b>1</b>	11 0	12 <b>1</b>	13 0	14 0	15 0
1	0 0	1 <b>1</b>	2 0	3 0	4 0	5 <b>1</b>	6 0	7 0
	a	b	c	d	e	f	g	h

Table 2.12: Masque pour les cases atteignables par la pièce en d3 sans considérer les éventuels obstacles

8	56 0	57 0	58 0	59 <b>1</b>	60 0	61 0	62 0	63 0
7	48 0	49 0	50 0	51 <b>1</b>	52 0	53 0	54 0	55 <b>1</b>
6	40 <b>1</b>	41 0	42 0	43 <b>1</b>	44 0	45 0	46 <b>1</b>	47 0
5	32 0	33 <b>1</b>	34 0	35 <b>1</b>	36 0	37 <b>1</b>	38 0	39 0
4	24 0	25 0	26 <b>1</b>	27 <b>1</b>	28 <b>1</b>	29 0	30 0	31 0
3	16 <b>1</b>	17 <b>1</b>	18 <b>1</b>	19 <b>0</b>	20 <b>1</b>	21 <b>1</b>	22 <b>1</b>	23 <b>1</b>
2	8 0	9 0	10 <b>1</b>	11 <b>1</b>	12 <b>1</b>	13 0	14 0	15 0
1	0 0	1 <b>1</b>	2 0	3 <b>1</b>	4 0	5 <b>1</b>	6 0	7 0
	a	b	c	d	e	f	g	h

Table 2.13: Board bitboard

8	56 <b>1</b>	57 <b>1</b>	58 0	59 0	60 <b>1</b>	61 0	62 0	63 <b>1</b>
7	48 0	49 0	50 0	51 0	52 0	53 <b>1</b>	54 0	55 <b>1</b>
6	40 <b>1</b>	41 0	42 <b>1</b>	43 <b>1</b>	44 <b>1</b>	45 <b>1</b>	46 0	47 0
5	32 0	33 0	34 0	35 <b>1</b>	36 <b>1</b>	37 <b>1</b>	38 <b>1</b>	39 <b>1</b>
4	24 0	25 0	26 0	27 <b>1</b>	28 <b>1</b>	29 <b>1</b>	30 <b>1</b>	31 0
3	16 <b>1</b>	17 0	18 0	19 <b>1</b>	20 0	21 <b>1</b>	22 0	23 0
2	8 <b>1</b>	9 <b>1</b>	10 0	11 <b>1</b>	12 0	13 <b>1</b>	14 0	15 <b>1</b>
1	0 <b>1</b>	1 0	2 0	3 0	4 0	5 0	6 <b>1</b>	7 <b>1</b>
	a	b	c	d	e	f	g	h



Table 2.14: Occupancies bitboard (**mask** & **board**) for d3

8	56	57	58	59	60	61	62	63
	0	0	0	0	0	0	0	0
7	48	49	50	51	52	53	54	55
	0	0	0	0	0	0	0	1
6	40	41	42	43	44	45	46	47
	1	0	0	1	0	0	0	0
5	32	33	34	35	36	37	38	39
	0	0	0	1	0	1	0	0
4	24	25	26	27	28	29	30	31
	0	0	0	1	1	0	0	0
3	16	17	18	19	20	21	22	23
	1	0	0	0	0	1	0	0
2	8	9	10	11	12	13	14	15
	0	0	0	1	0	0	0	0
1	0	1	2	3	4	5	6	7
	0	0	0	0	0	0	0	0
	a	b	c	d	e	f	g	h

bitboard to know which squares we can move to and create the corresponding move. To obtain the index in `MOVES_D3`, we will use the magic bitboard technique we described in the previous section. For square **d3**, we need to find a value **K** and a value **MAGIC** (see listing 7) to be able to obtain the bitboard of possible moves by doing: `MOVES_D3[occupancies * MAGIC >> (64 - K)]`.

To find a **MAGIC** constant for square **d3**, we will need to enumerate all possible obstacle placements on the squares reachable by the piece at **d3**. This means we must enumerate all subsets of 1 bits of the bitboard in table 2.12. Eight subsets from this enumeration are presented in figure 2.4. There are 25 squares on the trajectory of the piece at **d3** (see table 2.12), which means there are  $2^{25} = 33,554,432$  subsets to enumerate!

We would like to reduce this number of possible obstacle configurations on the piece's trajectory. Indeed, as we will see when we detail the code to determine the value of the **MAGIC** constant, the memory space to store the possible moves for a given square will depend on this number of configurations.

To reduce this number of configurations, we will not consider the squares on the board edges to remove the 1 bits on ranks 1 and 8 and files a and h from the mask in table 2.12. We obtain the bitboard in table 2.15. We will be missing information about the edges, but we will assume that if it's possible to play to **d7** for example, it will also be possible to play to **d8**. Of course there might be an obstacle at **d8**, but we can easily eliminate this impossible move as we will see later in this chapter (section 2.3.4). With this reduction, we are left with  $2^{17} = 131,072$  subsets to enumerate. That's much better, but we can still reduce this number further.

Indeed, we can split a piece's moves into diagonal and orthogonal moves. We will

8	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0
	a	b	c	d	e	f	g	h

(a) Empty (0 bits)

8	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0
4	0	0	0	1	0	0	0	0
3	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0
	a	b	c	d	e	f	g	h

(b) 1 bit set

8	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0
5	0	1	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0
3	1	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0
	a	b	c	d	e	f	g	h

(c) 2 bits set

8	0	0	0	1	0	0	0	0
7	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0
4	0	0	1	0	0	0	0	0
3	0	0	0	0	0	1	0	0
2	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0
	a	b	c	d	e	f	g	h

(d) 3 bits set

8	0	0	0	0	0	0	0	0
7	0	0	0	1	0	0	0	1
6	1	0	0	0	0	0	1	0
5	0	1	0	0	0	0	0	0
4	0	0	0	1	1	0	0	0
3	0	1	0	0	0	0	0	1
2	0	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0
	a	b	c	d	e	f	g	h

(e) 9 bits set

8	0	0	0	1	0	0	0	0
7	0	0	0	1	0	0	0	1
6	1	0	0	1	0	0	1	0
5	0	1	0	1	0	1	0	0
4	0	0	1	1	1	0	0	0
3	1	1	1	0	1	1	1	1
2	0	0	1	1	1	0	0	0
1	0	1	0	1	0	1	0	0
	a	b	c	d	e	f	g	h

(f) 25 bits set (full)

 Figure 2.4: Six subsets of the mask for d3 showing different occupancy patterns (0 to 25 bits set) from the  $2^{25}$  possible ones

Table 2.15: Mask for reachable squares for the piece at **d3** excluding board edges

8	56	57	58	59	60	61	62	63
	0	0	0	0	0	0	0	0
7	48	49	50	51	52	53	54	55
	0	0	0	1	0	0	0	0
6	40	41	42	43	44	45	46	47
	0	0	0	1	0	0	1	0
5	32	33	34	35	36	37	38	39
	0	1	0	1	0	1	0	0
4	24	25	26	27	28	29	30	31
	0	0	1	1	1	0	0	0
3	16	17	18	19	20	21	22	23
	0	1	1	0	1	1	1	0
2	8	9	10	11	12	13	14	15
	0	0	1	1	1	0	0	0
1	0	1	2	3	4	5	6	7
	0	0	0	0	0	0	0	0
	a	b	c	d	e	f	g	h

obtain two tables: `MOVES_D3_DIAG` and `MOVES_D3_ORTH0`. We will need to consult two tables instead of one to obtain the possible moves at **d3** (or any other square), but we will save a lot of memory space. We will see how to combine the values from these two tables later in this chapter. Instead of the mask from table 2.15, we will now have the two masks from figure 2.5.

For the orthogonal mask in figure 2.5a, we have  $2^{10} = 1,024$  subsets and for the diagonal mask in figure 2.5b, we have  $2^7 = 128$  subsets, so we go from  $2^{17} = 131,072$  to  $2^{10} + 2^7 = 1,152$  obstacle configurations to consider for the piece at **d3**!

We can now study how to find the **MAGIC** and **K** constants for the orthogonal and diagonal moves of each square on the board. The code is given in listing 12<sup>9</sup>. An execution of this program<sup>10</sup> is given in listing 13<sup>11</sup>.

---

```

1  constexpr uint64_t FileABB = 0x0101010101010101;
2  constexpr uint64_t FileHBB = FileABB << 7;
3  constexpr uint64_t Rank1BB = 0xFF;
4  constexpr uint64_t Rank8BB = Rank1BB << (8 * 7);
5
6  constexpr bool is_ok(Square s) { return s >= SQ_A1 && s <= SQ_H8; }
7
8  constexpr File file_of(Square s) { return File(s & 7); }
```

---

<sup>9</sup>We tried to use the Z3 solver to obtain these constants but without success.

<sup>10</sup>Given the random generator and its initialization, the program may find different constants for different executions.

<sup>11</sup>The output is reformatted for readability.

8	56	57	58	59	60	61	62	63
	0	0	0	0	0	0	0	0
7	48	49	50	51	52	53	54	55
	0	0	0	1	0	0	0	0
6	40	41	42	43	44	45	46	47
	0	0	0	1	0	0	0	0
5	32	33	34	35	36	37	38	39
	0	0	0	1	0	0	0	0
4	24	25	26	27	28	29	30	31
	0	0	0	1	0	0	0	0
3	16	17	18	19	20	21	22	23
	0	1	1	0	1	1	1	0
2	8	9	10	11	12	13	14	15
	0	0	0	1	0	0	0	0
1	0	1	2	3	4	5	6	7
	0	0	0	0	0	0	0	0
	a	b	c	d	e	f	g	h

(a) Orthogonal mask

8	56	57	58	59	60	61	62	63
	0	0	0	0	0	0	0	0
7	48	49	50	51	52	53	54	55
	0	0	0	0	0	0	0	0
6	40	41	42	43	44	45	46	47
	0	0	0	0	0	0	1	0
5	32	33	34	35	36	37	38	39
	0	1	0	0	0	1	0	0
4	24	25	26	27	28	29	30	31
	0	0	1	0	1	0	0	0
3	16	17	18	19	20	21	22	23
	0	0	0	0	0	0	0	0
2	8	9	10	11	12	13	14	15
	0	0	1	0	1	0	0	0
1	0	1	2	3	4	5	6	7
	0	0	0	0	0	0	0	0
	a	b	c	d	e	f	g	h

(b) Diagonal mask

Figure 2.5: Orthogonal and diagonal masks for d3 without edge squares

```

9
10 constexpr Rank rank_of(Square s) { return Rank(s >> 3); }
11
12 constexpr uint64_t rank_bb(Rank r) { return Rank1BB << (8 * r); }
13
14 constexpr uint64_t rank_bb(Square s) { return rank_bb(rank_of(s)); }
15
16 constexpr uint64_t file_bb(File f) { return FileABB << f; }
17
18 constexpr uint64_t file_bb(Square s) { return file_bb(file_of(s)); }
19
20 constexpr uint64_t square_bb(Square s) {
21     return uint64_t(1) << s;
22 }
23
24 constexpr Square operator+(Square s, Direction d) {
25     return Square(int(s) + int(d));
26 }
27
28 int manhattan_distance(Square sq1, Square sq2) {
29     int d_rank = std::abs(rank_of(sq1) - rank_of(sq2));
30     int d_file = std::abs(file_of(sq1) - file_of(sq2));
31     return d_rank + d_file;

```

```

32 }
33
34 enum MoveType {
35     ORTHOGONAL,
36     DIAGONAL
37 };
38
39 uint64_t reachable_squares(MoveType mt, Square sq, uint64_t occupied)
40 ↪ {
41     uint64_t moves = 0;
42     Direction o_dir[4] = {NORTH, SOUTH, EAST, WEST};
43     ↪ Direction d_dir[4] =
44     ↪ {NORTH_EAST, SOUTH_EAST, SOUTH_WEST, NORTH_WEST};
45     for (Direction d : (mt == ORTHOGONAL ? o_dir : d_dir)) {
46         Square s = sq;
47         while (true) {
48             Square to = s + d;
49             if (!is_ok(to) || manhattan_distance(s, to) > 2) break;
50             uint64_t bb = square_bb(to);
51             if ((square_bb(to) & occupied) != 0) break;
52             moves |= bb;
53             s = to;
54         }
55     }
56     return moves;
57 }
58
59 std::pair<int, uint64_t> magic_for_square(MoveType mt, Square sq) {
60     using namespace std;
61     uint64_t edges = ((Rank1BB | Rank8BB) & ~rank_bb(sq)) |
62                     ((FileABB | FileHBB) & ~file_bb(sq));
63     uint64_t moves_bb = reachable_squares(mt, sq, 0) & ~edges;
64     vector<uint64_t> occupancies;
65     vector<uint64_t> possible_moves;
66     uint64_t b = 0;
67     int size = 0;
68     do {
69         occupancies.push_back(b);
70         possible_moves.push_back(reachable_squares(mt, sq, b));
71         size++;
72         b = (b - moves_bb) & moves_bb;
73     } while (b);

```

```
72     int k = popcount(moves_bb);
73     int shift = 64 - k;
74     random_device rd;
75     mt19937_64 twister(rd());
76     uniform_int_distribution<uint64_t> d;
77     vector<uint32_t> seen(1 << k);
78     vector<uint64_t> moves(1 << k);
79     for (uint32_t cnt = 0;; cnt++) {
80         uint64_t magic = d(twister) & d(twister) & d(twister);
81         bool found = true;
82         for (size_t j = 0; j < occupancies.size(); j++) {
83             uint64_t occ = occupancies[j];
84             int index = magic * occ >> shift;
85             if (seen[index] == cnt && moves[index] !=
↪ possible_moves[j])
86                 {
87                     found = false;
88                     break;
89                 }
90             seen[index] = cnt;
91             moves[index] = possible_moves[j];
92         }
93         if (found) {
94             return {k, magic};
95         }
96     }
97     unreachable();
98 }
99
100 int main() {
101     using namespace std;
102     for (MoveType mt : {ORTHOGONAL, DIAGONAL}) {
103         stringstream ss_k, ss_magic;
104         ss_k << format("int {}_K[64] = {{",
105                        mt == ORTHOGONAL ? "H" : "D");
106         ss_magic << format("uint64_t {}_MAGIC[64] = {{",
107                           mt == ORTHOGONAL ? "H" : "D");
108         for (int sq = SQ_A1; sq <= SQ_H8; sq++) {
109             const auto [k, magic] = magic_for_square(mt, Square(sq));
110             ss_k << dec << k << ',';
111             ss_magic << showbase << hex << magic << ',';
112         }
```

```

113         cout << ss_k.str() << "};\n";
114         cout << ss_magic.str() << "};\n\n";
115     }
116 }

```

---

Listing 12: Program to find the **K** and **MAGIC** constants from listing 7 for the obstacle configurations of a given move type and square

In listing 12,

- At line 39, the function `uint64_t reachable_squares(MoveType mt, Square sq, uint64_t occupied)` will allow us to create the mask of squares reachable from square `sq` for the move type `mt`, which will be either orthogonal (**ORTHOGONAL** at line 35) or diagonal (**DIAGONAL** at line 36). The created mask will be similar to the masks in figure 2.5.
  - At lines 43 to 53, we iterate through each direction according to the move type, starting from the starting square (`sq`) until we encounter an obstacle.
  - At line 47, we test whether we go off the board or wrap around, for example, if we are at square `SQ_A8` and the direction is **EAST**, `SQ_H8 + EAST == SQ_B1`, the test with Manhattan distance will allow us to exclude this kind of case (see line 28 for the definition of Manhattan distance and listing 5 for the definitions of **File** and **Rank**).
  - At line 48, we transform the destination square `to` into a bitboard that contains a single 1, on the bit corresponding to the square.
  - At line 49, we test whether there is an obstacle on the square we want to move to using the bitboard we just created and the bitboard `occupied` containing the obstacles on the board.
  - At line 50, we add the new square as a possible move.
  - At line 51, we move one square in direction `d`.
- At line 57, the function `pair<int, uint64_t> magic_for_square(MoveType mt, Square sq)` will return a pair whose first element will be the value of **K**, and whose second element will be the **MAGIC** constant (see listing 7), for the move type `mt` (**ORTHOGONAL** or **DIAGONAL**) and square `sq`.
  - At lines 59 to 60, we create a mask that will cover the board edges but being careful not to exclude squares that must remain accessible by the piece. For example, if `sq == SQ_A1`, we obtain the bitboard from figure 2.6a and if `sq == SQ_C2`, we obtain the one from figure 2.6b.
  - At line 61, we create the bitboard `moves_bb` of squares accessible by the piece from square `sq`. The parameter `occupied` is equal to zero to indicate that there are no obstacles. We remove the edges with `& ~edges` as we saw in paragraph 2.3.2.
  - At lines 62 to 71, we will create all obstacle configurations on the squares accessible by the piece at `sq`, without considering the squares in `edge`, by enumerating all subsets of the bitboard `moves_bb`.

8	56	57	58	59	60	61	62	63
	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>
7	48	49	50	51	52	53	54	55
	0	0	0	0	0	0	0	<b>1</b>
6	40	41	42	43	44	45	46	47
	0	0	0	0	0	0	0	<b>1</b>
5	32	33	34	35	36	37	38	39
	0	0	0	0	0	0	0	<b>1</b>
4	24	25	26	27	28	29	30	31
	0	0	0	0	0	0	0	<b>1</b>
3	16	17	18	19	20	21	22	23
	0	0	0	0	0	0	0	<b>1</b>
2	8	9	10	11	12	13	14	15
	0	0	0	0	0	0	0	<b>1</b>
1	0	1	2	3	4	5	6	7
	<b>0</b>	0	0	0	0	0	0	<b>1</b>
	a	b	c	d	e	f	g	h

(a) Edge mask for square a1

8	56	57	58	59	60	61	62	63
	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>
7	48	49	50	51	52	53	54	55
	<b>1</b>	0	0	0	0	0	0	<b>1</b>
6	40	41	42	43	44	45	46	47
	<b>1</b>	0	0	0	0	0	0	<b>1</b>
5	32	33	34	35	36	37	38	39
	<b>1</b>	0	0	0	0	0	0	<b>1</b>
4	24	25	26	27	28	29	30	31
	<b>1</b>	0	0	0	0	0	0	<b>1</b>
3	16	17	18	19	20	21	22	23
	<b>1</b>	0	0	0	0	0	0	<b>1</b>
2	8	9	10	11	12	13	14	15
	<b>1</b>	0	<b>0</b>	0	0	0	0	<b>1</b>
1	0	1	2	3	4	5	6	7
	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>
	a	b	c	d	e	f	g	h

(b) Edge mask for square c2

Figure 2.6: Edge masks excluding the rank and file of the piece

\* At lines 62 and 63, the vector **occupancies** will contain all obstacle configurations on the piece's trajectory, without considering the squares in **edge**, and for each of these configurations, the vector **possible\_moves** will contain the bitboard of possible moves for that configuration. For example, we could have as an obstacle configuration the bitboard from figure 2.7a and the associated possible moves would be represented by the bitboard from figure 2.7b.

8	56	57	58	59	60	61	62	63
	0	0	0	0	0	0	0	0
7	48	49	50	51	52	53	54	55
	0	0	0	0	0	0	0	0
6	40	41	42	43	44	45	46	47
	0	0	0	<b>1</b>	0	0	0	0
5	32	33	34	35	36	37	38	39
	0	0	0	<b>1</b>	0	<b>1</b>	0	0
4	24	25	26	27	28	29	30	31
	0	0	0	<b>1</b>	<b>1</b>	0	0	0
3	16	17	18	19	20	21	22	23
	0	0	0	<b>0</b>	0	<b>1</b>	0	0
2	8	9	10	11	12	13	14	15
	0	0	0	<b>1</b>	0	0	0	0
1	0	1	2	3	4	5	6	7
	0	0	0	0	0	0	0	0
	a	b	c	d	e	f	g	h

(a) Occupancy configuration example

8	56	57	58	59	60	61	62	63
	0	0	0	0	0	0	0	0
7	48	49	50	51	52	53	54	55
	0	0	0	0	0	0	0	0
6	40	41	42	43	44	45	46	47
	<b>1</b>	0	0	0	0	0	0	0
5	32	33	34	35	36	37	38	39
	0	<b>1</b>	0	0	0	0	0	0
4	24	25	26	27	28	29	30	31
	0	0	<b>1</b>	0	0	0	0	0
3	16	17	18	19	20	21	22	23
	<b>1</b>	<b>1</b>	<b>1</b>	<b>0</b>	<b>1</b>	0	0	0
2	8	9	10	11	12	13	14	15
	0	0	<b>1</b>	0	<b>1</b>	0	0	0
1	0	1	2	3	4	5	6	7
	0	<b>1</b>	0	0	0	<b>1</b>	0	0
	a	b	c	d	e	f	g	h

(b) Possible moves for this occupancy

Figure 2.7: Example of occupancy configuration and corresponding possible moves



- \* The loop between lines 66 and 71 will allow us to enumerate all subsets of the bitboard `moves_bb`. To do this, we use the technique called *Carry-Rippler*<sup>12</sup> at line 70. Let's take a fictitious example with `moves_bb = 0b1011`. We then obtain

---

```

1  uint64_t b = 0;                // b == 0
2  b = (0 - 0b1011) & 0b1011;    // b == 0b0001
3  b = (0b0001 - 0b1011) & 0b1011; // b == 0b0010
4  b = (0b0010 - 0b1011) & 0b1011; // b == 0b0011
5  b = (0b0011 - 0b1011) & 0b1011; // b == 0b1000
6  b = (0b1000 - 0b1011) & 0b1011; // b == 0b1001
7  b = (0b1001 - 0b1011) & 0b1011; // b == 0b1010
8  b = (0b1010 - 0b1011) & 0b1011; // b == 0b1011
9  b = (0b1011 - 0b1011) & 0b1011; // b == 0

```

---

- \* At line 68, we create the possible moves for the obstacle configuration `b`. Note that since `b` does not cover the squares in `edge`, if no obstacle in `b` blocks the piece's moves to a square in `edge`, it will appear in the possible moves.
- At line 72, we count the number of 1 bits in the bitboard `moves_bb`. We set `k` to this value, which means that for square `sq` and move type `mt`, the space occupied by our perfect hash table<sup>13</sup> will be  $2^k$ . Note that some obstacle configurations produce the same possible moves, so it might be possible to obtain a smaller value of `k`. But this approach allows us to find the different **MAGIC** constants very quickly and the memory consumed is relatively small (less than 110 KiB in total).
- At lines 74 to 76, we set up the random generator that will allow us to create the **MAGIC** constants.
- At line 77, the array `seen` will allow us to memorize the indices, obtained by the formula at line 84, that we have already encountered.
- At line 78, the array `moves` will allow us to memorize the possible moves for a given index. It may happen that two different obstacle configurations lead to the same possible moves, this array will allow us not to consider as collisions these two different configurations leading to the same index but having the same possible moves.
- The loop from line 79 to 96 will search for the **MAGIC** constant that allows creating the perfect hash for square `sq` and value `k`. Note that this loop is potentially infinite. We inform the compiler, at line 97, that this part of the code will never be reached thanks to the function `std::unreachable`.
- At line 80, we generate a **MAGIC** candidate. We generate three random numbers and perform a bitwise AND between them to obtain a random

---

<sup>12</sup>If you are interested in this kind of bit manipulation tricks, you will love the book *Hacker's Delight*[9].

<sup>13</sup>This is the approach used in the Stockfish software[3].

number with fewer 1 bits. This trick is very important because without it we couldn't find the **MAGIC** constants in a reasonable time!<sup>14</sup>

- At lines 82 to 92, we verify that there are no collisions for the constant being considered. If this is the case, we return the value `k` and the magic constant found.
  - \* At line 84, for a given obstacle configuration, we calculate its index `index` using the perfect hash function: `(magic * occ >> (64 - k))`.
  - \* At line 85, we test whether there is a collision. We use a classic little trick to avoid having to reinitialize `seen` to zero each time we test a new `magic` candidate. The variable `cnt` allows us to know whether the value contained in `seen` for position `index` is a value updated for an old `magic` candidate or is for the current candidate. Indeed, if `seen[index]` is less than `cnt`, the value is no longer current. Now, the part `(moves[index] != possible_moves[j])` allows us to ignore a collision that produces the same possible moves.
- At lines 100 to 116, the `main` will produce and display on standard output all values of `k` and all **MAGIC** constants for each square on the board and for both move types. These constants will soon be used to initialize the table of all possible moves for each of the obstacle configurations. We can see a possible output in listing 13.

```
int O_K[64] = {
    12, 11, 11, 11, 11, 11, 11, 12,
    11, 10, 10, 10, 10, 10, 10, 11,
    11, 10, 10, 10, 10, 10, 10, 11,
    11, 10, 10, 10, 10, 10, 10, 11,
    11, 10, 10, 10, 10, 10, 10, 11,
    11, 10, 10, 10, 10, 10, 10, 11,
    11, 10, 10, 10, 10, 10, 10, 11,
    12, 11, 11, 11, 11, 11, 11, 12,
};
uint64_t O_MAGIC[64] = {
    0x80011040002082,    0x40022002100040,    0x1880200081181000,
    0x2080240800100080,    0x8080024400800800,    0x4100080400024100,
    0xc080028001000a00,    0x80146043000080,
    0x8120802080034004,    0x8401000200240,    0x202001282002044,
    0x81010021000b1000,    0x808044000800,    0x300080800c000200,
    0x8c000268411004,    0x810080058020c100,
    0xc248608010400080,    0x30024040002000,    0x9001010042102000,
    0x210009001002,    0xa0061d0018001100,    0x2410808004000600,
    0x6400240008025001,    0xc10600010340a4,
```

---

<sup>14</sup>Note that using this trick for listing 8 slowed down obtaining the **MAGIC** constant.

```

    0x628080044011,    0x4810014040002000,    0x380200080801000,
    0x10018580080010,    0x101040080180180,    0x9208020080040080,
    0x10400a21008,      0x6800104200010484,
    0x21400280800020,    0x9400402008401001,    0x8430006800200400,
    0x8104411202000820,    0x8010171000408,      0x1202000402001008,
    0x881100904002208,    0x15a0800a49802100,
    0x224001808004,      0x4420201002424000,    0xc04500020008080,
    0x2503009004210008,    0x42801010010,        0x2000400090100,
    0x8080011810040002,    0x44401c008046000d,
    0x4000800521104100,    0x82000b080400080,    0x10821022420200,
    0x9488a82104100100,    0x1004800041100,      0x81600a0034008080,
    0xa00056210280400,    0x5124088200,
    0x4210410010228202,    0x1802230840001081,    0x1002102000400901,
    0x1100c46010000901,    0x281000408001003,    0xc001001c00028809,
    0x10020008008c4102,    0x280005008c014222,
};

int D_K[64] = {
    6, 5, 5, 5, 5, 5, 5, 6,
    5, 5, 5, 5, 5, 5, 5, 5,
    5, 5, 7, 7, 7, 7, 5, 5,
    5, 5, 7, 9, 9, 7, 5, 5,
    5, 5, 7, 9, 9, 7, 5, 5,
    5, 5, 7, 7, 7, 7, 5, 5,
    5, 5, 5, 5, 5, 5, 5, 5,
    6, 5, 5, 5, 5, 5, 5, 6,
};

uint64_t D_MAGIC[64] = {
    0x811100100408200,    0x412100401044020,    0x404044c00408002,
    0xa0c070200010102,    0x104042001400008,    0x8802013008080000,
    0x1001008860080080,    0x20220044202800,
    0x2002610802080160,    0x4080800808610,      0x91c2800a10a0132,
    0x400242401822000,    0x8530040420040001,    0x142010c210048,
    0x8841820801241004,    0x804212084108801,
    0x2032402094100484,    0x40202110010210a2,    0x8010000800202020,
    0x800240421a800,      0x62200401a00444,      0x224082200820845,
    0x106021492012000,    0x8481020082849000,
    0x40a110c59602800,    0x10020108020400,      0x208c020844080010,
    0x2000480004012020,    0x8001004004044000,    0xa044104128080200,
    0x1108008015cc1400,    0x8284004801844400,
    0x8180a020c2004,      0x9101004080100,      0x8840264108800c0,
    0xc004200900200900,    0x8040008020020020,    0x20010802e1920200,
};

```

```
0x80204000480a0,    0xc0a80a100008400,  
0x4018808114000,    0x90092200b9000,    0x80020c0048000400,  
0x6018005500,        0x80a0204110a00,    0x4018808407201,  
0x6050040806500280,  0x108208400c40180,  
0x803081210840480,  0x201210402200200,  0x200010400920042,  
0x902000a884110010,  0x851002021004,    0x43c08020120,  
0x6140500501010044,  0x200a04440400c028,  
0x14a002084046000,  0x10002409041040,  0x100022020500880b,  
0x1000000000460802,  0x21084104410,     0x8000001053300104,  
0x4000182008c20048,  0x112088105020200,  
};
```

Listing 13: Example output from one execution of the program from listing 12

---

```
1  uint64_t orthogonalTable[102400];  
2  uint64_t diagonalTable[5248];  
3  Magic orthogonalMagics[SQUARE_NB];  
4  Magic diagonalMagics[SQUARE_NB];
```

---

Listing 14: The tables allowing us to generate possible moves from a given square and an obstacle configuration

Now that we have the **MAGIC** constants, we will be able to use them to initialize the tables described in listing 14, which will store the necessary information allowing us to generate possible moves from a given square and an obstacle configuration.

- At line 1, the **orthogonalTable** array will contain the bitboards of possible moves for all orthogonal moves, for all squares and all obstacle configurations. The value 102400 for the size of this table is obtained by calculating

---

```
size_t size = 0;  
for (int i = 0; i < 64; i++) {  
    size += 1 << O_K[i];  
}
```

---

- At line 2, the **diagonalTable** array does the same but for diagonal moves.
- At line 3, the **orthogonalMagics** array will define for each square of the board the different information allowing us to apply the perfect hashing and retrieve the possible moves in the **orthogonalTable** array.
- At line 4, the **diagonalMagics** array does the same for diagonal moves.

---

```
1 struct Magic {
2     uint64_t mask;
3     uint64_t magic;
4     uint64_t* moves;
5     uint32_t shift;
6
7     uint32_t index(uint64_t occupied) const {
8         return uint32_t( ((occupied & mask) * magic) >> shift );
9     }
10 };
```

---

Listing 15: Data structure allowing us to store the useful information to find possible moves for a given square and obstacle configuration

Listing 15 describes the structure allowing us to store the information for applying the perfect hashing for a given square.

- At line 2, the `mask` attribute is the bitboard allowing us to isolate the part of the board that contains the squares affected by the piece's movement for the considered square and movement type. This mask corresponds to the `moves_bb` variable at line 61 of listing 12.
- At line 3, the `magic` attribute is the `MAGIC` constant that we found for the considered square and movement. This attribute will take the value of one of the constants from the `O_MAGIC` or `D_MAGIC` tables from listing 13.
- At line 4, the `moves` attribute is a pointer to the part of the `orthogonalTable` or `diagonalTable`, depending on the movement type, that will contain the possible moves for the considered square.
- At line 5, the `shift` attribute is the shift (`64 - K`) in the formula from listing 7.
- At lines 7 to 9, the function `uint32_t index(uint64_t occupied) const` calculates the position in `moves` where to find the bitboard of possible moves. We find the formula from listing 7 with the mask applied to the obstacles to only consider those in the piece's trajectory.

---

```
1 uint64_t moves_bb(Square sq, uint64_t occupied) {
2     uint32_t idx_omoves = orthogonalMagics[sq].index(occupied);
3     uint32_t idx_dmoves = diagonalMagics[sq].index(occupied);
4     return orthogonalMagics[sq].moves[idx_omoves] |
5         diagonalMagics[sq].moves[idx_dmoves];
6 }
```

---

Listing 16: The `moves_bb` function uses the elements from listings 14 and 15 to efficiently compute the bitboard of possible moves for a given square and obstacle configuration

The function `uint64_t moves_bb(Square sq, uint64_t occupied)` from listing 16 will use the tables from listing 14 to compute the bitboard of possible moves, both orthogonal and diagonal, for the square `sq` and the obstacle configuration `occupied`. This function will be used for move generation in section 2.3.4. To do this,

- At line 2, we compute the index `idx_omoves` using the perfect hashing, by using the `Magic` structure for the square `sq` and orthogonal moves.
- At line 3, we compute the index `idx_dmoves` using the perfect hashing, by using the `Magic` structure for the square `sq` and diagonal moves.
- At lines 4 and 5, we can retrieve the bitboard of possible orthogonal moves (line 4) and create the union of these with the diagonal moves by combining this bitboard using a bitwise OR with the bitboard of possible diagonal moves (line 5). We thus obtain the bitboard of all possible moves for the piece at `sq`, taking into consideration all obstacles from the bitboard `occupied`.

---

```
1 void init_all_magics() {  
2     init_magics(ORTHOGONAL, orthogonalTable, orthogonalMagics);  
3     init_magics(DIAGONAL, diagonalTable, diagonalMagics);  
4 }
```

---

Listing 17: Initialisation of all the magic bitboards

The function `void init_all_magics()` from listing 17 will initialize once and for all, at the beginning of the program, the different tables. We simply call at lines 2 and 3 the function `void init_magics(MoveType mt, uint64_t table[], Magic magics[])` described in listing 18 for orthogonal and diagonal moves with their associated tables.

---

```
1 Square& operator++(Square& d) { return d = Square(int(d) + 1); }  
2  
3 void init_magics(MoveType mt, uint64_t table[], Magic magics[]) {  
4     static constexpr uint64_t O_MAGIC[64] = { 0x80011040002082, //  
5     ↪ ...  
6     static constexpr uint64_t D_MAGIC[64] = { 0x811100100408200, //  
7     ↪ ...  
8     using namespace std;  
9     int32_t size = 0;  
10    vector<uint64_t> occupancies;  
11    vector<uint64_t> possible_moves;
```

```
10     for (Square sq = SQ_A1; sq <= SQ_H8; ++sq) {
11         occupancies.clear();
12         possible_moves.clear();
13         Magic& m = magics[sq];
14         uint64_t edges = ((Rank1BB | Rank8BB) & ~rank_bb(sq)) |
15                         ((FileABB | FileHBB) & ~file_bb(sq));
16         uint64_t moves_bb = reachable_squares(mt, sq, 0) & ~edges;
17         m.mask = moves_bb;
18         m.shift = 64 - popcount(m.mask);
19         m.magic = (mt == ORTHOGONAL ? 0_MAGIC : D_MAGIC)[sq];
20         m.moves = table + size;
21         uint64_t b = 0;
22         do {
23             occupancies.push_back(b);
24             possible_moves.push_back(reachable_squares(mt, sq, b));
25             b = (b - moves_bb) & moves_bb;
26             size++;
27         } while (b);
28         for (size_t j = 0; j < occupancies.size(); j++) {
29             int32_t index = m.index(occupancies[j]);
30             m.moves[index] = possible_moves[j];
31         }
32     }
33 }
```

---

Listing 18: TO DO

The function `void init_magics(MoveType mt, uint64_t table[], Magic magics[])` will initialize the different tables for the considered movement type `mt`. It is very similar to the function from listing 12 which allowed us to find the **MAGIC** constants.

- Lines 4 and 5 use the magic constants that we found by running the program from listing 12, whose execution output was given in listing 13.
- At line 8, the `occupancies` array will contain all obstacle configurations for the considered square for each iteration of the loop at line 10. The `possible_moves` array will contain the bitboard of possible moves for a given obstacle configuration. Note that we could have placed these two arrays starting from line 11, since they must be reinitialized at each iteration of the loop at line 10. However, it is more efficient to call the `clear` method at lines 11 and 12, which will avoid unnecessary allocations.
- The loop comprising lines 10 to 32 will update the `magics` parameter for all squares of the board and store the possible moves in a region of the `table` parameter.

- We can recognize, from lines 14 to 27, the enumeration of obstacle subsets that we studied in listing 12. We will update the `Magic` structure `m` from line 13, corresponding to the square `sq`, with
  - At line 17, the mask of squares to consider for the movement of the piece at `sq` and the movement type `mt`.
  - At line 18, the right shift to perform after the multiplication: `(occupied & mask) * magic` (see listing 15).
  - At line 19, the magic constant to use for the movement type `mt` and the considered square `sq`.
- At line 20, we compute the address of the beginning of the location in the `table` array that will allow us to store the possible moves for each obstacle configuration. To do this, we use the `size` variable which counts the number of obstacle configurations we have processed since the beginning of the `for` loop.
- Finally, at lines 28 to 31, for each obstacle configuration, we compute its index using the perfect hashing and we store the possible moves in the region of the `table` array reserved for this purpose.

Now that we have at our disposal the function `moves_bb` from listing 16, it will be quite easy to generate the list of possible moves in a given position of the game. But before doing that, we will first describe how to represent a move.

### 2.3.3 Move Representation

We will represent a move by a pair, whose first element is the departure square and the second element is the arrival square. For example, the game shown in figures 1.7, 1.8, and 1.9 will be textually represented by the following moves:

```
a1:a2  h1:f3  h8:f6  a8:c6  d5:d6  d4:b2  d6:e6  e5:g5  f6:f5  f3:g4
f5:h7  g5:h5  e4:f4  h5:f7  e6:e8  g4:g1  e8:b8  b2:d2  a2:a3  g1:f2
a3:a6  d2:d3  f4:h2  d3:b5  a6:a4  b5:b3  a4:c4  b3:c3  h2:g2  f2:f1
b8:c7  c3:a5  c4:c5  c6:d7  g2:g3  f7:g7  c5:b4  a5:b6  g3:h4  b6:b7
h7:g8  d7:d8  c7:c8  f1:b1  g8:f8  d8:e7  h4:h3  a1:a1  b1:c1  a1:a1
c1:e1  a1:a1  e1:e3  a1:a1  e3:e2  a1:a1  e2:d1  a1:a1  d1:c2  a1:a1
g7:g6  a1:a1  g6:h6  a1:a1  b7:a7
```

---

```
1  class Move {
2      uint16_t data;
3  public:
4      Move() = default;
5      constexpr explicit Move(uint16_t d) : data(d) {}
6      constexpr explicit Move(Square from,
7                              Square to) : data((to << 6) + from) {}
8      constexpr Square from_sq() const {
```



```
9         return Square(data & 0x3F);
10     }
11     constexpr Square to_sq() const {
12         return Square((data >> 6) & 0x3F);
13     }
14     static constexpr Move none() { return Move(0); }
15     constexpr bool operator==(const Move& m) const {
16         return data == m.data;
17     }
18     constexpr bool operator!=(const Move& m) const {
19         return data != m.data;
20     }
21     constexpr explicit operator bool() const { return data != 0; }
22     constexpr uint16_t raw() const { return data; }
23 };
24
25 std::ostream& operator<<(std::ostream& os, const Move& m) {
26     static constexpr std::string_view square2string[SQUARE_NB] = {
27         "a1", "b1", "c1", "d1", "e1", "f1", "g1", "h1",
28         "a2", "b2", "c2", "d2", "e2", "f2", "g2", "h2",
29         "a3", "b3", "c3", "d3", "e3", "f3", "g3", "h3",
30         "a4", "b4", "c4", "d4", "e4", "f4", "g4", "h4",
31         "a5", "b5", "c5", "d5", "e5", "f5", "g5", "h5",
32         "a6", "b6", "c6", "d6", "e6", "f6", "g6", "h6",
33         "a7", "b7", "c7", "d7", "e7", "f7", "g7", "h7",
34         "a8", "b8", "c8", "d8", "e8", "f8", "g8", "h8",
35     };
36     os << square2string[m.from_sq()] << ':' <<
↪ square2string[m.to_sq()];
37     return os;
38 }
```

---

Listing 19: Move representation, the departure and arrival squares are each stored on 6 bits

The `Move` structure, presented in listing 19, represents a move using an unsigned 16-bit integer: `uint16_t` `data`. We need 6 bits to represent the departure square and 6 other bits to represent the arrival square.

- At lines 6-7, we construct the move from the departure square `from` and the arrival square `to`. We place the departure square number in the 6 least significant bits of `data` and the arrival square starting from bit 6.
- At line 8, the function `Square from_sq()` retrieves the square whose number is located in bits 0 to 5 of `data`. This corresponds to the departure square.

- At line 11, the function `Square to_sq()` retrieves the square whose number is located in bits 6 to 11 of `data`. This corresponds to the arrival square.
- We represent passing one's turn, which occurs when a player can no longer move but the opponent still has moves available, by the fictitious move `a1:a1`. The function `Move none()`, described at line 14, returns this move. The departure square and arrival square are both `SQ_A1 == 0`.
- At line 25, the function `ostream& operator<<(std::ostream& os, const Move& m)` displays a move in the format: `departure square:arrival square`.

### 2.3.4 List of Moves for a Given Board Configuration

The list of possible moves in a given game configuration is represented by the `MoveList` class presented in listing 20.

- At line 1, the constant `MAX_NB_MOVES` is an upper bound on the maximum number of possible moves for the players. In the starting configuration, the black player has a choice of  $14 \times 4 = 56$  moves. Note that this number of possible moves does not necessarily decrease; for example, in the configuration shown in figure 2.8, the black player has a choice of 60 moves. To obtain this upper bound, we ran many random games and observed the maximum number of available moves for players during these games.

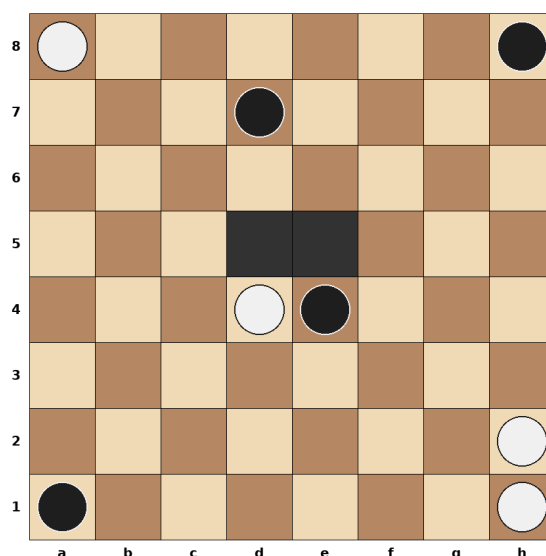
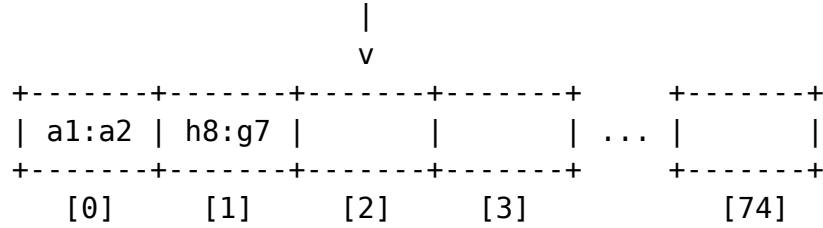


Figure 2.8: In this position, the black player has a choice of 60 moves

- At line 3, moves will be stored in the `move_list` array which, in our usage as we will see later, will be placed on the stack. The `last` pointer will point to the cell just after the last move (which is more convenient for implementation). For example, if the current player only has the moves `Move(SQ_A1, SQ_A2)` and `Move(SQ_H8, SQ_G7)`, we will have the following configuration:

`last`



- At lines 6 to 9, the `begin` and `end` functions allow us to iterate over a `MoveList` object. For example, we can write:

```

MoveList list;
// ...
for (Move m : list) {
    // ...
}

```

- The remaining functions allow us to retrieve the size (`size` at line 10), index the list like an array (`operator[]` at lines 11 to 14), and directly obtain the pointer to the beginning of the `move_list` array (`data` at line 15).

---

```

1  static constexpr uint16_t MAX_NB_MOVES = 75;
2  class MoveList {
3      Move move_list[MAX_NB_MOVES], *last;
4  public:
5      constexpr MoveList() noexcept : last(move_list) {}
6      constexpr const Move* begin() const noexcept { return move_list;
↪  }
7      constexpr const Move* end() const noexcept { return last; }
8      constexpr Move* begin() noexcept { return move_list; }
9      constexpr Move* end() noexcept { return last; }
10     constexpr size_t size() const noexcept { return last - move_list;
↪  }
11     constexpr const Move& operator[](size_t i) const {
12         return move_list[i];
13     }
14     constexpr Move& operator[](size_t i) { return move_list[i]; }
15     constexpr Move* data() noexcept { return move_list; }
16     friend class Yolah;
17 };

```

---

Listing 20: Class representing a list of moves

The function `void Yolah::moves(uint8_t player, MoveList& moves)` presented at line 19 of listing 21, will store in the `moves` parameter of type `MoveList` the possible

moves in the current game configuration for the player passed as parameter (**player**) (which is either **BLACK** or **WHITE**)<sup>15</sup>. In this function,

- The loop between lines 23 and 29 iterates over each piece of player **player**, whose positions are given in the bitboard **bb**, and generates all possible moves for each of them.
  - At line 24, we retrieve the square corresponding to the position of the rightmost 1 bit in the bitboard **bb**. We use the function `Square pop_lsb(uint64_t& b)` defined at line 4 for this purpose. This function returns the square corresponding to the position of the rightmost 1 bit in the bitboard **b** and, additionally, sets that bit to 0. For example,

```
uint64_t bb = 0b1000100100010000;
Square sq = pop_lsb(bb);
// sq == SQ_E1
// bb == 0b1000100100000000
```
  - At line 25, we use our magic bitboard function `moves_bb` to generate the bitboard containing all possible moves for this position and obstacle configuration. We mentioned in paragraph 2.3.2 on page 31 that we would need to handle potential obstacles on the edges that we had ignored. Remember that to limit the number of obstacle configurations to consider, we had ignored the board edges. As a consequence, we might allow moves to edge squares even when there are obstacles there. It is very easy to eliminate these potentially illegal moves by removing moves that land on occupied squares using: `& ~occupied`.
  - At lines 26 to 28, we iterate through the bitboard **b** of possible moves and again use the `pop_lsb` function to retrieve the destination squares of the various moves (the starting position being the **from** position of the piece under consideration), in order to construct a `Move` object that we place in the `moves.move_list` array.
- At lines 30 to 32, we check whether the player had at least one move available, by checking if the `move_list` pointer still points to the beginning of the `moves.move_list` array. If so, which is not the most frequent occurrence<sup>16</sup>, we add the pass move (`Move::none()`) as the only possible move for the player.
- Finally, at line 33, we set the end pointer `moves.last` to the cell just after the one containing the last move.

---

```
1 constexpr Square lsb(uint64_t b) {
2     return Square(std::countr_zero(b));
3 }
4 Square pop_lsb(uint64_t& b) {
```

---

<sup>15</sup>It will prove useful later in this book to be able to retrieve a player's possible moves even when it is not their turn to play.

<sup>16</sup>We indicate this using the C++ `[[unlikely]]` attribute.

```
5     const Square s = lsb(b);
6     b &= b - 1;
7     return s;
8 }
9 class Yolah {
10     uint64_t black = BLACK_INITIAL_POSITION;
11     uint64_t white = WHITE_INITIAL_POSITION;
12     uint64_t holes = 0;
13     uint8_t black_score = 0;
14     uint8_t white_score = 0;
15     uint8_t ply = 0;
16 public:
17     // ...
18 };
19 void Yolah::moves(uint8_t player, MoveList& moves) const noexcept {
20     Move* move_list = moves.move_list;
21     uint64_t occupied = black | white | holes;
22     uint64_t bb = player == BLACK ? black : white;
23     while (bb) {
24         Square from = pop_lsb(bb);
25         uint64_t b = moves_bb(from, occupied) & ~occupied;
26         while (b) {
27             *move_list++ = Move(from, pop_lsb(b));
28         }
29     }
30     if (move_list == moves.move_list) [[unlikely]] {
31         *move_list++ = Move::none();
32     }
33     moves.last = move_list;
34 }
35 constexpr uint8_t Yolah::current_player() const noexcept {
36     return ply & 1;
37 }
38 void Yolah::moves(MoveList& moves) const noexcept {
39     this->moves(current_player(), moves);
40 }
```

---

Listing 21: Generating possible moves for a player in the current game position

The function `uint8_t current_player()` at line 35 of listing 21 returns the current player. To do this, it tests whether the number of moves played since the beginning of the game is even or odd. The black player plays in positions where `ply` equals 0, 2, 4, 6, ..., while the white player plays in positions where `ply` equals 1, 3, 5, 7, .... We test parity by examining the least significant bit of `ply`; using `(ply & 1)` yields: 0 when

even (which corresponds to **BLACK**) and 1 when odd (which corresponds to **WHITE**).

### 2.3.5 Making and Unmaking Moves

To complete the Yolah implementation, we still need to be able to apply a move to the game. The function **void play(Move m)** in listing 22 will modify the board state by playing the move **m** passed as parameter.

- At line 2, we check that the player is not passing their turn<sup>17</sup>; we use the `[[likely]]` attribute because it is much more common for the player to be able to play. Whether they pass or not, we increment, at line 14, the `ply` attribute which we use to count the number of moves played in the game and to determine the current player.
- At line 3, `pos1` is the bitboard corresponding to the move's source square.
- At line 4, `pos2` is the bitboard corresponding to the move's destination square.
- At line 5, we test if it is white's turn to play.
  - If so, we toggle, at line 6, using XOR, the bits corresponding to the source and destination squares of the move in the `white` bitboard which gives the position of the white player's pieces. For example,

```
uint64_t white = 0b1000100100010000;
Move m = Move(SQ_E1, SQ_G1);
uint64_t pos1 = square_bb(m.from_sq());
// pos1 == 0b010000;
uint64_t pos2 = square_bb(m.to_sq());
// pos2 == 0b100000;
// pos1 | pos2 == 0b110000;
// white == 0b1000100100010000;
white ^= pos1 | pos2;
// white == 0b1000100100100000;
```

Then at line 7, we increase the white player's score because they have just made a move.

- If it is black's turn to play, we proceed similarly, but for the black player's `black` bitboard at line 9, and we increase their score at line 10.
- At line 12, we eliminate the source square of the move by adding a hole to the `holes` bitboard at the position corresponding to the move's source square.

---

```
1 void Yolah::play(Move m) {
2     if (m != Move::none()) [[likely]] {
3         uint64_t pos1 = square_bb(m.from_sq());
4         uint64_t pos2 = square_bb(m.to_sq());
```

---

<sup>17</sup>Which they must do if and only if they have no valid move available.

```
5         if (ply & 1) {
6             white ^= pos1 | pos2;
7             white_score++;
8         } else {
9             black ^= pos1 | pos2;
10            black_score++;
11        }
12        holes |= pos1;
13    }
14    ply++;
15 }
```

---

Listing 22: The move `m` is played in the current game position

It will prove useful for several of our AIs to be able to undo the previously played move. The function `void undo(Move m)` in listing 23 will perform the inverse operation of that performed by the function `void play(Move m)` in order to restore the game board to the state it was in before playing move `m`. Typically, we will use `undo` as follows:

```
/* return type */ recursive(Yolah& yolah, /* other parameters */) {
    // ...
    MoveList moves;
    yolah.moves(moves);
    for (Move m : moves) {
        yolah.play(m);
        recursive(yolah, /* ... */);
        yolah.undo(m);
    }
    // ...
    return // ...
}
```

In the previous listing, it is more efficient to use the `undo` function than to save the complete game state before the recursive call and then restore it afterward. The benchmark in listing 24 tests this assertion. The function `count_nodes_with_undo` at line 1 tests state restoration using the `undo` function and makes a number of recursive calls controlled by the `depth` parameter.

The function `count_nodes_with_restore` at line 15 tests restoration by saving the complete game state at line 23. The previous state is restored via the assignment at line 28.

The results are given in listing 25. The performance gain would be even more pronounced for a game with a larger state than `Yolah`'s.

```
1 void Yolah::undo(Move m) {
2     ply--;
3     if (m != Move::none()) [[likely]] {
4         uint64_t pos1 = square_bb(m.from_sq());
5         uint64_t pos2 = square_bb(m.to_sq());
6         if (ply & 1) {
7             white ^= pos1 | pos2;
8             white_score--;
9         } else {
10            black ^= pos1 | pos2;
11            black_score--;
12        }
13        empty ^= pos1;
14    }
15 }
```

---

Listing 23: The last played move `m` is undone

```
1 uint64_t count_nodes_with_undo(Yolah& yolah, int depth) {
2     if (depth == 0 || yolah.game_over()) {
3         return 1;
4     }
5     uint64_t count = 0;
6     MoveList moves;
7     yolah.moves(moves);
8     for (const Move& m : moves) {
9         yolah.play(m);
10        count += count_nodes_with_undo(yolah, depth - 1);
11        yolah.undo(m);
12    }
13    return count;
14 }
15 uint64_t count_nodes_with_restore(Yolah& yolah, int depth) {
16     if (depth == 0 || yolah.game_over()) {
17         return 1;
18     }
19     uint64_t count = 0;
20     MoveList moves;
21     yolah.moves(moves);
22     // Save the complete state
```



```

23     Yolah saved = yolah;
24     for (const Move& m : moves) {
25         yolah.play(m);
26         count += count_nodes_with_restore(yolah, depth - 1);
27         // Restore the complete state
28         yolah = saved;
29     }
30     return count;
31 }
32 static void BM_recursive_with_undo(benchmark::State& state) {
33     int depth = state.range(0);
34     for (auto _ : state) {
35         Yolah yolah;
36         uint64_t nodes = count_nodes_with_undo(yolah, depth);
37         benchmark::DoNotOptimize(nodes);
38     }
39     // Calculate nodes for reporting
40     Yolah yolah;
41     uint64_t nodes = count_nodes_with_undo(yolah, depth);
42     state.SetItemsProcessed(state.iterations() * nodes);
43     state.counters["nodes"] = nodes;
44 }
45 static void BM_recursive_with_restore(benchmark::State& state) {
46     int depth = state.range(0);
47     for (auto _ : state) {
48         Yolah yolah;
49         uint64_t nodes = count_nodes_with_restore(yolah, depth);
50         benchmark::DoNotOptimize(nodes);
51     }
52     Yolah yolah;
53     uint64_t nodes = count_nodes_with_restore(yolah, depth);
54     state.SetItemsProcessed(state.iterations() * nodes);
55     state.counters["nodes"] = nodes;
56 }
57 // Test at various depths
58 BENCHMARK(BM_recursive_with_undo)->Arg(3)->Arg(4)->Arg(5);
59 BENCHMARK(BM_recursive_with_restore)->Arg(3)->Arg(4)->Arg(5);
60 int main(int argc, char** argv) {
61     init_all_magics();
62     benchmark::Initialize(&argc, argv);
63     benchmark::RunSpecifiedBenchmarks();
64 }

```

Listing 24: Benchmark to test the assertion that the `undo` function provides a performance gain over saving the complete game state

Run on (12 X 4400 MHz CPU s)

CPU Caches: L1 Data 32 KiB (x6), L1 Instruction 32 KiB (x6),  
L2 Unified 256 KiB (x6), L3 Unified 12288 KiB (x1)

Benchmark	Time (ms)	Throughput
-----		
BM_recursive_with_undo/3	1.09	150.4 M/s
BM_recursive_with_undo/4	56.50	153.1 M/s
BM_recursive_with_undo/5	3059.00	149.3 M/s
BM_recursive_with_restore/3	1.24	132.3 M/s
BM_recursive_with_restore/4	70.00	123.6 M/s
BM_recursive_with_restore/5	3492.00	130.8 M/s

Listing 25: Benchmark results from listing 24

## 2.4 Testing with Random Games

Nous présentons deux manières de tester notre implémentation de Yolah. Ces deux façons de faire se basent sur le déroulement de parties aléatoires. Dans un premier temps nous inspectons visuellement si tout semble correct, et une fois les éventuels bugs "faciles" détectés, nous passons à une batterie de tests importante, toujours en générant des parties aléatoires, mais cette fois-ci en testant des propriétés du jeu qui devraient être respectées par notre implémentation et en comparant la génération des coups à base de magic bitboards avec une implémentation beaucoup plus simple (mais bien moins efficace) pour vérifier que les deux implémentations différentes donnent les mêmes résultats. Une fois cette batterie de tests passée avec succès, nous profilons le code en générant un ensemble de parties aléatoires, et en se basant sur cette analyse, nous améliorons le temps d'exécution du programme.

### 2.4.1 Tests par observation de parties aléatoires

Le code pour tester visuellement notre implémentation est donné dans le listing 26. Un début d'exécution de ce code est donné dans le listing 27.

- Aux lignes 4 et 5, nous initialisons un générateur pseudo-aléatoire, soit avec une graine donnée par l'utilisateur (paramètre `seed`) pour pouvoir reproduire le test, ou un nombre aléatoire en utilisant `random_device`<sup>18</sup>.

---

<sup>18</sup>Ce générateur va générer des nombres aléatoires de haute qualité, en utilisant par exemple

- La boucle des lignes 5 à 24 va jouer `nb_games` parties aléatoires.
  - À la ligne 6, on initialise une nouvelle partie.
  - Aux lignes 7 à 22, on joue la partie aléatoire.
    - \* Aux lignes 8 et 9, on récupère la liste des coups possibles dans la position actuelle du jeu et on les trie pour rendre l’affichage des coups possibles plus lisible.
    - \* Aux lignes 10 à 13, on affiche le nombre de coups possibles puis la liste de tous les coups. Vous pouvez voir le résultat dans le listing 27.
    - \* À la ligne 15, on affiche le plateau avec les coups représentés par des croix. Cela permet de voir facilement si des coups manquent ou si des coups sont de trop.
    - \* Aux lignes 16 à 18, on choisit un coup aléatoirement et on l’affiche.
    - \* Aux lignes 19 à 20, on attend l’appuie sur la touche entrée pour continuer la boucle. Cela nous laisse le temps d’analyser les sorties.

---

```
1 void play_random_games(size_t nb_games,
2                         optional<uint64_t> seed = nullopt) {
3     MoveList moves;
4     random_device rd;
5     mt19937 mt(seed.value_or(rd()));
6     for (size_t i = 0; i < nb_games; i++) {
7         Yolah yolah;
8         while (!yolah.game_over()) {
9             yolah.moves(moves);
10            sort(begin(moves), end(moves));
11            cout << format("# moves: {}\n", moves.size());
12            for (const auto& m : moves) {
13                cout << m << ' ';
14            }
15            cout << "\n\n";
16            cout << YolahWithMoves(yolah, moves) << '\n';
17            uniform_int_distribution<uint64_t> d(0, moves.size()-1);
18            Move m = moves[d(mt)];
19            cout << m << '\n';
20            std::string _;
21            std::getline(std::cin, _);
22            yolah.play(m);
23        }
```

---

`/dev/urandom` pour Linux. Par contre, l’obtention du nombre aléatoire va être assez lente comparée à l’utilisation d’un générateur pseudo-aléatoire comme `mt19937` par exemple. Un des usages typique de `random_device` est d’initialiser un générateur pseudo-aléatoire comme nous le faisons ici.

```

24         cout << yolah << '\n';
25     }
26 }
27 int main() {
28     init_all_magics();
29     play_random_games(42);
30 }

```

Listing 26: Playing random games, displaying the list of moves and the board to check the engine visually

Black player

8	○	.	.	.	.	.	.	●
7	.	.	.	.	.	.	.	.
6	.	.	.	.	.	.	.	.
5	.	.	.	●	○	.	.	.
4	.	.	.	○	●	.	.	.
3	.	.	.	.	.	.	.	.
2	.	.	.	.	.	.	.	.
1	●	.	.	.	.	.	.	○
	a	b	c	d	e	f	g	h

score: 0/0

# moves: 56

a1:b1 a1:c1 a1:d1 a1:e1 a1:f1 a1:g1 a1:a2 a1:b2 a1:a3 a1:c3 a1:a4 a1:a5  
a1:a6 a1:a7 e4:b1 e4:e1 e4:c2 e4:e2 e4:g2 e4:d3 e4:e3 e4:f3 e4:f4 e4:g4  
e4:h4 e4:f5 e4:g6 e4:h7 d5:a2 d5:b3 d5:c4 d5:a5 d5:b5 d5:c5 d5:c6 d5:d6  
d5:e6 d5:b7 d5:d7 d5:f7 d5:d8 d5:g8 h8:h2 h8:h3 h8:h4 h8:h5 h8:f6 h8:h6  
h8:g7 h8:h7 h8:b8 h8:c8 h8:d8 h8:e8 h8:f8 h8:g8

Black player

8	○	X	X	X	X	X	X	●
7	X	X	.	X	.	X	X	X
6	X	.	X	X	X	X	X	X
5	X	X	X	●	○	X	.	X

4	X	.	X	○	●	X	X	X
3	X	X	X	X	X	X	.	X
2	X	X	X	.	X	.	X	X
1	●	X	X	X	X	X	X	○
	a	b	c	d	e	f	g	h

score: 0/0

d5:a2

White player

8	○	.	.	.	.	.	.	●
7	.	.	.	.	.	.	.	.
6	.	.	.	.	.	.	.	.
5	.	.	.		○	.	.	.
4	.	.	.	○	●	.	.	.
3	.	.	.	.	.	.	.	.
2	●	.	.	.	.	.	.	.
1	●	.	.	.	.	.	.	○
	a	b	c	d	e	f	g	h

score: 1/0

# moves: 55

h1:b1 h1:c1 h1:d1 h1:e1 h1:f1 h1:g1 h1:g2 h1:h2 h1:f3 h1:h3 h1:h4 h1:h5  
h1:h6 h1:h7 d4:d1 d4:g1 d4:b2 d4:d2 d4:f2 d4:c3 d4:d3 d4:e3 d4:a4 d4:b4  
d4:c4 d4:c5 d4:b6 d4:a7 e5:h2 e5:g3 e5:f4 e5:f5 e5:g5 e5:h5 e5:d6 e5:e6  
e5:f6 e5:c7 e5:e7 e5:g7 e5:b8 e5:e8 a8:a3 a8:a4 a8:a5 a8:a6 a8:c6 a8:a7  
a8:b7 a8:b8 a8:c8 a8:d8 a8:e8 a8:f8 a8:g8

White player

8	○	x	x	x	x	x	x	●
7	x	x	x	.	x	.	x	x
6	x	x	x	x	x	x	.	x

5	x	.	x		o	x	x	x
4	x	x	x	o	●	x	.	x
3	x	.	x	x	x	x	x	x
2	●	x	.	x	.	x	x	x
1	●	x	x	x	x	x	x	o
	a	b	c	d	e	f	g	h

score: 1/0

h1:f1

Listing 27: Début du déroulement du programme du listing 26

## 2.4.2 Tests différentiels et tests basés sur les propriétés

L'observation des parties aléatoires précédentes nous permet de détecter rapidement des bugs éventuels. Cette façon de faire est bien pratique pour voir en détail le fonctionnement de notre programme. Mais nous voulons aussi une manière de tester automatiquement notre jeu. Pour ce faire, nous allons tester le jeu en testant des propriétés qu'il doit vérifier. Nous allons aussi générer les coups possibles d'une manière simple (et peu efficace) dans laquelle nous avons une grande confiance, justement car elle est basique à implémenter, et pouvoir comparer les coups générés par notre jeu avec ceux générés simplement. Tous ces tests se feront sur des parties générées aléatoirement. Nous allons ainsi pouvoir produire assez facilement une très grande quantité de tests. Cette façon de tester s'appelle les *tests basés sur les propriétés* [10] et les *tests différentiels* [11]. Les tests basés sur les propriétés consistent à vérifier que certaines propriétés invariantes sont toujours respectées lors de l'exécution de scénarios générés aléatoirement. Les tests différentiels comparent les résultats de deux implémentations différentes du même algorithme : une implémentation optimisée (notre générateur de coups avec magic bitboards) et une implémentation de référence simple mais manifestement correcte.

Le code du listing 28 illustre cette approche des tests.

- Aux lignes 8 à 12, la structure **TestResult** va contenir le résultat d'un test donné. L'attribut **passed** est à vrai lorsque le test a réussi et à faux sinon. L'attribut **message** stocke le diagnostic qui sera utilisé en cas d'erreur (voir la fonction **run\_test** à la ligne 80).
- À la ligne 13, la fonction **pass** rend un **TestResult** qui indique que le test c'est déroulé correctement.
- À la ligne 14, la fonction **fail(string msg)** rend un **TestResult** qui indique que le test a échoué avec le diagnostic d'erreur **msg**.

- Aux lignes 17 à 44 est définie la fonction `slow_moves_generation` qui permet d'obtenir la liste des coups possibles dans la configuration du jeu `yolah` passée en paramètre (ligne 87). Cette fonction va nous permettre de faire les tests différentiels. Comme nous le disions, cette façon de générer les coups est bien plus simple que celle à base de bitboards magiques. Nous avons une grande confiance dans l'exactitude de `slow_moves_generation`. Les boucles aux lignes 19 et 20, vont parcourir toutes les cases du plateau. On construit la case de départ du coup `from` à la ligne 21, puis on vérifie à la ligne 22 que cette case contient un pion du joueur pour lequel on crée la liste de coups. Les fonctions `square_of` et `get_from` sont définies précisément dans le listing ??, mais `Square square_of(int rank, int file)` va créer une case à partir des coordonnées `(rank, file)` et `uint8_t Yolah::get_from(Square sq)` retourne le type d'élément (un des joueurs, un trou ou un emplacement libre) sur la case `sq`. Les boucles aux lignes 23 et 24 vont parcourir toutes les directions possibles pour la pièce, on fait attention à filtrer le déplacement nul à la ligne 25. Enfin, la boucle de la ligne 28 à 37 va se déplacer dans la direction choisie `(di, dj)` tant qu'on ne sort pas du plateau et qu'il n'y a pas d'obstacles. Si on sort du plateau à la ligne 29, ou qu'un obstacle est détecté à la ligne 33, on sort de la boucle pour essayer une nouvelle direction. On n'oublie pas à la ligne 42 d'ajouter le coup `Move::none()` si aucun coup n'était possible. Cette façon de construire tous les coups possibles est très intuitive (mais inefficace).
- Aux lignes 46 à 53, la fonction `check_move_count` est notre premier test différentiel. Le premier paramètre `fast` est la liste des coups obtenue grâce aux bitboards magiques, et le deuxième paramètre `expected` est la liste de coups obtenue avec la fonction précédente. Nous vérifions dans ce test que les nombres de coups produits par les deux approches sont bien identiques.
- Aux lignes 55 à 84, la fonction `check_move_lists_equal` est notre deuxième et dernier test différentiel. On vérifie que les listes de coups sont identiques, en les triant tout d'abord aux lignes 58 et 59, puis en vérifiant que les deux listes de coups sont maintenant identiques aux lignes 60 et 61. Le reste de la fonction crée un message d'erreur en calculant les différences entre les deux listes.
- Aux lignes 86 à 92, la fonction `check_undo` va vérifier la propriété suivante. Soit `yolah` une configuration quelconque du jeu et `m` un coup possible dans cette position. Posons `Yolah before = yolah;`, puis effectuons les actions suivantes.

```
yolah.play(m);  
yolah.undo(m);  
Yolah after = yolah;
```

On vérifie que l'annulation d'un coup fonctionne correctement en testant si `before == after` à la ligne 87.

- La deuxième propriété que l'on va tester est donnée dans la fonction `check_game_over_move` aux lignes 94 à 101. On y teste que le seul coup possible dans une position terminale du jeu est le coup `Move::none()`. En effet, les joueurs n'ont plus de coups à disposition et ils doivent donc passer leur tour. À la ligne 96, on vérifie qu'il n'y a qu'un seul coup et que celui-ci est bien le coup `Move::none()`.

- Aux lignes 103 à 119, la fonction `check_none_move_execution` va tester la propriété que le coup `Move::none()` doit seulement incrémenter le nombre de coups dans la partie. Pour ce faire, la boucle à la ligne 106 va tester à la ligne 107, que le contenu de chaque case n'a pas changé après l'exécution du coup nul. Ensuite, à la ligne 115, on teste si le nombre de coups a bien été incrémenté.
- Aux lignes 121 à 156, la fonction `check_regular_move_execution` va tester les propriétés que doit vérifier un coup normal. Aux lignes 126 à 132, on vérifie que les cases qui ne sont pas concernées par le coup n'ont pas changé. Aux lignes 133 à 137, on vérifie que le nombre de coups a été incrémenté, qu'un trou est apparu sur la case de départ du coup et enfin que la pièce se trouve maintenant sur la case d'arrivée du coup.
- Aux lignes 158 à 234, la fonction `random_games` va lancer tous les tests sur `nb_games` parties aléatoires aux lignes 178 à 219. 221 à 233 Les lignes 190 à 202 affichent un résumé des tests.
- Le listing 29 donne le résultat de l'exécution du `main`. Même si, comme le disait Dijkstra, "Program testing can be used to show the presence of bugs, but never to show their absence," et qu'il faille toujours rester prudent, le grand nombre de configurations du jeu explorées nous donne néanmoins une bonne confiance dans la validité de notre implémentation de Yolah. Maintenant que nous avons testé notre implémentation, nous allons étudier les performances de celle-ci.

```
1 namespace test {
2 constexpr string_view RED    = "\033[1;31m";
3 constexpr string_view GREEN = "\033[1;32m";
4 constexpr string_view YELLOW = "\033[1;33m";
5 constexpr string_view RESET = "\033[0m";
6 constexpr string_view BOLD   = "\033[1m";
7
8 struct TestResult {
9     bool passed;
10    string message;
11    operator bool() const { return passed; }
12 };
13
14 TestResult pass() { return {true, ""}; }
15 TestResult fail(string msg) { return {false, std::move(msg)}; }
16
17 vector<Move> slow_moves_generation(const Yolah& yolah) {
18     vector<Move> res;
19     for (int i = 0; i < 8; i++) {
20         for (int j = 0; j < 8; j++) {
21             Square from = square_of(i, j);
```



```

22         if (yolah.get(from) != yolah.current_player()) continue;
23         for (int di = -1; di <= 1; di++) {
24             for (int dj = -1; dj <= 1; dj++) {
25                 if (di == 0 && dj == 0) continue;
26                 int ii = i + di;
27                 int jj = j + dj;
28                 for(;;) {
29                     if (ii < 0 || ii >= 8 || jj < 0 || jj >= 8) {
30                         break;
31                     }
32                     Square to = square_of(ii, jj);
33                     if (yolah.get(to) != FREE) break;
34                     res.emplace_back(from, to);
35                     ii += di;
36                     jj += dj;
37                 }
38             }
39         }
40     }
41 }
42 if (res.empty()) res.push_back(Move::none());
43 return res;
44 };
45
46 TestResult check_move_count(const MoveList& fast,
47                             const vector<Move>& expected) {
48     if (fast.size() != expected.size()) {
49         return fail(format("# of moves: expected {} got {}",
50                             expected.size(), fast.size()));
51     }
52     return pass();
53 }
54
55 TestResult check_move_lists_equal(MoveList& fast,
56                                   vector<Move>& expected,
57                                   const Yolah& yolah) {
58     sort(begin(fast), end(fast));
59     sort(begin(expected), end(expected));
60     if (equal(begin(fast), end(fast),
61               begin(expected), end(expected))) {
62         return pass();
63     }

```

```
64     ostreamstream oss;
65     oss << "move lists differ\n" << yolah << '\n';
66     vector<Move> only_in_fast, only_in_expected;
67     set_difference(begin(fast), end(fast),
68                   begin(expected), end(expected),
69                   back_inserter(only_in_fast));
70     set_difference(begin(expected), end(expected),
71                   begin(fast), end(fast),
72                   back_inserter(only_in_expected));
73     if (!only_in_expected.empty()) {
74         oss << "  Only in expected: ";
75         for (const auto& m : only_in_expected) oss << m << ' ';
76         oss << '\n';
77     }
78     if (!only_in_fast.empty()) {
79         oss << "  Only in fast: ";
80         for (const auto& m : only_in_fast) oss << m << ' ';
81         oss << '\n';
82     }
83     return fail(oss.str());
84 }
85
86 TestResult check_undo(const Yolah& before, const Yolah& after) {
87     if (before == after) return pass();
88     ostreamstream oss;
89     oss << "undo failed\n  Previous state:\n" << before
90         << "\n  State after undo:\n" << after << '\n';
91     return fail(oss.str());
92 }
93
94 TestResult check_game_over_moves(const Yolah& yolah,
95                                 const MoveList& moves) {
96     if (moves.size() == 1 && moves[0] == Move::none()) return pass();
97     ostreamstream oss;
98     oss << "only Move::none() should be available when game is over\n"
99         << YolahWithMoves(yolah, moves) << '\n';
100    return fail(oss.str());
101 }
102
103 TestResult check_none_move_execution(const Yolah& before,
104                                     const Yolah& after) {
105     bool ok = true;
```

```

106     for (Square sq = SQ_A1; sq <= SQ_H8; ++sq) {
107         if (before.get(sq) != after.get(sq)) {
108             ok = false;
109             break;
110         }
111     }
112     if (!ok) {
113         return fail("Move::none() must not change the board content");
114     }
115     if (before.nb_plies() + 1 != after.nb_plies()) {
116         return fail("Move::none() must increment the number of plies");
117     }
118     return pass();
119 }
120
121 TestResult check_regular_move_execution(const Yolah& before,
122                                       const Yolah& after, Move m) {
123     Square from = m.from_sq();
124     Square to = m.to_sq();
125     bool ok = true;
126     for (Square sq = SQ_A1; sq <= SQ_H8; ++sq) {
127         if (sq == from || sq == to) continue;
128         if (before.get(sq) != after.get(sq)) {
129             ok = false;
130             break;
131         }
132     }
133     if (ok && before.nb_plies() + 1 == after.nb_plies() &&
134         after.get(from) == HOLE &&
135         after.get(to) == before.current_player()) {
136         return pass();
137     }
138     ostream oss;
139     oss << "Move execution incorrect\n" << after
140         << "\n Move: " << m << '\n';
141     if (!ok) {
142         oss << " Squares not concerned by the Move must not changed";
143     }
144     if (after.get(from) != HOLE) {
145         oss << " From square should be hole\n";
146     }
147     if (after.get(to) != before.current_player()) {

```

```
148         oss << " To square should contain a piece from "
149         << (before.current_player() == BLACK ? "black" : "white")
150         << " player\n";
151     }
152     if (before.nb_plies() + 1 != after.nb_plies()) {
153         oss << " The number of plies must be incremented\n";
154     }
155     return fail(oss.str());
156 }
157
158 void random_games(size_t nb_games, optional<uint64_t> seed) {
159     MoveList fast_moves;
160     random_device rd;
161     mt19937 mt(seed.value_or(rd()));
162     size_t total_tests = 0;
163     size_t passed_tests = 0;
164
165     auto run_test = [&](TestResult result) -> bool {
166         total_tests++;
167         if (result) {
168             passed_tests++;
169             return true;
170         }
171         cout << format("{}FAIL:{} {}\n", RED, RESET, result.message);
172         return false;
173     };
174
175     cout << format("{}\n=== Running Random Games Tests ===\n{}",
176         BOLD, RESET);
177
178     for (size_t i = 0; i < nb_games; i++) {
179         Yolah yolah;
180         while (!yolah.game_over()) {
181             yolah.moves(fast_moves);
182             vector<Move> expected_moves = slow_moves_generation(yolah);
183
184             if (!run_test(
185                 check_move_count(fast_moves, expected_moves))) break;
186             if (!run_test(
187                 check_move_lists_equal(fast_moves,
188                     expected_moves, yolah))) break;
189         }
```

```

190     uniform_int_distribution<int> d(0, fast_moves.size() - 1);
191     Move m = fast_moves[d(mt)];
192     Yolah before = yolah;
193     yolah.play(m);
194
195     if (m == Move::none()) {
196         if (!run_test(
197             check_none_move_execution(before, yolah))) break;
198     } else {
199         if (!run_test(
200             check_regular_move_execution(before,
201                 yolah, m))) break;
202     }
203
204     yolah.undo(m);
205     if (!run_test(check_undo(before, yolah))) break;
206     yolah.play(m);
207 }
208
209 if (!yolah.game_over()) continue;
210
211 yolah.moves(fast_moves);
212 if (!run_test(
213     check_game_over_moves(yolah, fast_moves))) continue;
214
215 yolah.play(Move::none());
216 yolah.moves(fast_moves);
217 if (!run_test(
218     check_game_over_moves(yolah, fast_moves))) continue;
219 }
220
221 cout << format("\n{ }=== Test Summary ==={ }\n", BOLD, RESET);
222 cout << format("Total tests: { }\n", total_tests);
223 cout << format("Passed: { }{ }{ }\n", GREEN, passed_tests, RESET);
224 cout << format("Failed: { }{ }{ }\n",
225     (passed_tests == total_tests ? GREEN : RED),
226     total_tests - passed_tests, RESET);
227 if (passed_tests == total_tests) {
228     cout << format("{ }All tests passed!{ }\n", GREEN, RESET);
229 } else {
230     double pass_rate = 100.0 * passed_tests / total_tests;
231     cout << format("{ }Pass rate: {:.2f}%{ }\n", YELLOW, pass_rate,

```

```
232             RESET);  
233     }  
234 }}  
235  
236 int main() {  
237     init_all_magics();  
238     test::random_games(10000, 42);  
239 }
```

Listing 28: Tests basés sur les propriétés et différentiels utilisés pour valider notre implémentation.

```
=== Running Random Games Tests ===  
  
=== Test Summary ===  
Total tests: 2223036  
Passed: 2223036  
Failed: 0  
All tests passed!
```

Listing 29: Résultats de l'exécution du programme du listing 28

### 2.4.3 Tests de performance

Nous allons analyser les performances de notre implémentation de Yolah en utilisant le programme du listing 30 qui ressemble au programme du listing 26. Mais contrairement à ce dernier, le programme du listing 30 n'arrête pas le déroulement des parties et affiche quelques statistiques sur les parties.

- À la ligne 6, `black_wins` va comptabiliser le nombre de victoires du joueur noir tout au long des `nb_games` parties. Aux lignes 28 à 30, on affiche le nombre de victoires du joueur noir et le pourcentage que cela représente par rapport à toutes les parties jouées.
- À la ligne 7, `white_wins` va comptabiliser le nombre de victoires du joueur blanc. On affiche aux lignes 31 à 33 les statistiques associées.
- À la ligne 8, on procède de même pour le nombre de parties nulles grâce à la variable `draws`.
- Aux lignes 11 à 16, on joue une partie aléatoire, en obtenant la liste des coups disponibles à la ligne 12, puis en sélectionnant un coup aléatoire à la ligne 14 puis en jouant ce coup à la ligne 15.
- Nous allons jouer un million de parties comme nous pouvons le voir dans le `main` à la ligne 40.

```
1 void play_random_games(size_t nb_games,
2                         optional<uint64_t> seed = nullopt) {
3     MoveList moves;
4     random_device rd;
5     mt19937 mt(seed.value_or(rd()));
6     size_t black_wins = 0;
7     size_t white_wins = 0;
8     size_t draws = 0;
9     for (size_t i = 0; i < nb_games; i++) {
10        Yolah yolah;
11        while (!yolah.game_over()) {
12            yolah.moves(moves);
13            uniform_int_distribution<int> d(0, moves.size() - 1);
14            Move m = moves[d(mt)];
15            yolah.play(m);
16        }
17        auto [black_score, white_score] = yolah.score();
18        if (black_score > white_score) {
19            black_wins++;
20        } else if (white_score > black_score) {
21            white_wins++;
22        } else {
23            draws++;
24        }
25    }
26    cout << format("\n=== Game Statistics ===\n");
27    cout << format("Total games: {}\n", nb_games);
28    cout << format("Black wins: {} ({:.1f}%)\n",
29                  black_wins,
30                  100.0 * black_wins / nb_games);
31    cout << format("White wins: {} ({:.1f}%)\n",
32                  white_wins,
33                  100.0 * white_wins / nb_games);
34    cout << format("Draws: {} ({:.1f}%)\n",
35                  draws,
36                  100.0 * draws / nb_games);
37 }
38 int main() {
39     init_all_magics();
40     play_random_games(1000000, 42);
41 }
```

Listing 30: Nous jouons un million de parties aléatoires pour évaluer les performances de notre implémentation de Yolah

Nous allons d’abord analyser les traces d’appels des fonctions pour déterminer les parties du code les plus coûteuses en temps. Nous utilisons l’outil `perf` [12] pour collecter les données de performance et `stackcollapse-perf.pl`, issu du projet `FlameGraph` [13], pour analyser les traces d’appels. La commande que nous utilisons est la suivante.

```
$ sudo perf record -g ./chapter02
$ sudo perf script | stackcollapse-perf.pl > profile.txt
```

Nous avons demandé à Claude [1] de générer un script Python permettant de visualiser les traces d’appels sous la forme d’un arbre qui est représenté dans le listing 31.

- La fonction `play_random_games` compte pour 96.79% du temps d’exécution du programme.
- La suite des appels de fonctions se lit en suivant les branches de l’arbre. Par exemple, on peut voir ci-dessous les différents appels de fonctions qui ont amené à l’exécution de `Magic::index`.

```
play_random_games
├─ Yolah::moves
│   └─ moves_bb
│       └─ Magic::index                8.87%
```

on voit de plus que l’exécution de `Magic::index` via cette suite d’appels représente 8.87% du temps d’exécution du programme.

- Notons que l’ordre des fonctions ne représente pas l’ordre du listing, les feuilles de l’arbre sont triées selon le pourcentage du temps d’exécution.
- Remarquons aussi que l’on peut déduire facilement le temps passé dans une fonction en dehors des appels de fonctions qu’elle a effectué. Par exemple, pour la fonction `play_random_games`, son temps d’exécution compte pour 96.79% du temps d’exécution du programme. Si on soustrait de ce nombre la somme (82.25% + 5.33% + 3.89% + 3.86%), qui représente les pourcentages du temps d’exécution du programme des fonctions `Yolah::moves`, `std::uniform_int_distribution`, `Yolah::game_over` et `Yolah::play`, on obtient 1.46%, qui donne le pourcentage du temps d’exécution total passé dans le reste de la fonction `play_random_games`. La proportion du temps passé dans le reste de la fonction est donc de  $1.46/96.79 = 1.5\%$ .
- L’arbre nous permet de voir rapidement que si l’on souhaite améliorer le temps d’exécution de notre programme, il faut se concentrer sur la fonction `Yolah::moves` appelée par `play_random_games`. En effet, le temps d’exécution de cette fonction représente 82.25% du temps d’exécution total du programme.



play_random_games	96.79%
└─ Yolah::moves	82.25%
└─ moves_bb	33.07%
└─ Magic::index	8.87%
└─ Move::Move	13.86%
└─ pop_lsb	13.85%
└─ lsb	13.18%
└─ std::uniform_int_distribution<...	5.33%
└─ Yolah::game_over	3.89%
└─ shift<>	2.14%
└─ Yolah::play	3.86%

Listing 31: Traces d'appels des fonctions permettant d'identifier les fonctions les plus coûteuses

Grâce au listing 31, nous savons que nous devons concentrer nos efforts sur la fonction `Yolah::moves`. Nous allons de nouveau utiliser l'outil `perf` [12] mais cette fois-ci pour accéder aux compteurs de performance matériel (PMU - Performance Monitoring Unit) [14] du processeur. Ces compteurs mesurent des événements tels que les défauts de prédiction de branchement, les défauts de cache, etc. Ces compteurs vont nous guider afin de trouver les faiblesses de notre implémentation<sup>19</sup>. Nous utilisons la commande suivante pour analyser le comportement de notre programme.

```
$ perf stat -r 10 -e cycles,instructions,branches,branch-misses,\
L1-dcache-loads,L1-dcache-load-misses,LLC-loads,LLC-load-misses,\
l1d_pend_miss.pending,l1d_pend_miss.pending_cycles ./chapter02
```

Le résultat de cette commande est présenté dans le listing 32.

- L'option `-r 10` de la commande `perf` ci-dessus permet de lancer le programme dix fois, pour faire la moyenne et calculer l'écart-type<sup>20</sup> des résultats obtenus.
- Le compteur `cycles` indique le nombre de cycles effectués par le processeur durant l'exécution de notre programme<sup>21</sup>.
- Le compteur `instructions` mesure le nombre d'instructions exécutées par notre programme. Le nombre d'instructions exécutées par cycle Instructions Per Cycle (IPC) est de 1.38. Les processeurs modernes sont super-scalaires, ils ont la capacité d'exécuter plusieurs instructions par cycle. Plus ce nombre est élevé, et plus les unités de calculs du processeur sont utilisées. Nous pourrions comparer cet IPC avec celui des versions optimisées que nous développerons par la suite.
- Le compteur `branches` va comptabiliser le nombre total de branchements exécutés (conditionnels, inconditionnels, indirects, appels et retours de fonction).

<sup>19</sup>Pour approfondir le sujet de l'optimisation sur processeurs modernes, vous pouvez consulter [15, 16, 17, 18].

<sup>20</sup>Plus précisément, l'écart-type en pourcentage de la moyenne.

<sup>21</sup>En mode utilisateur et en mode noyau. Les compteurs sont activés quand un processus s'exécute et désactivés lorsqu'il perd la main.

- Le compteur **branch-misses** indique le nombre de fois où le prédicteur de branchement s'est trompé. Les processeurs modernes, pour ne pas laisser leurs unités de calcul sans travail, exécutent des instructions de manière spéculative, c'est-à-dire avant de connaître le résultat des branchements conditionnels. Lorsque la prédiction est correcte, le processeur a gagné du temps. En cas d'erreur, les instructions exécutées spéculativement sont annulées, et le processeur reprend l'exécution du programme au bon endroit après le branchement. Ces erreurs de branchements sont coûteuses. Dans notre cas, le prédicteur de branchement se trompe dans 11.30% des cas. Ce taux est élevé et c'est la première chose que nous allons essayer d'améliorer.

Performance counter stats for './chapter02' (10 runs):

```

17 675 748 046      cycles                      ( +-  0.14% )
24 357 812 457      instructions    # 1.38 IPC      ( +-  0.03% )
 2 429 931 966      branches                      ( +-  0.03% )
 274 551 866      branch-misses    # 11.30%      ( +-  0.03% )
 3 132 479 051      L1-dcache-loads          ( +-  0.03% )
 147 715 205      L1-dcache-load-misses # 4.71%   ( +-  0.16% )
 53 089 459      LLC-loads                      ( +-  1.24% )
 24 898      LLC-load-misses    # 0.05%      ( +- 31.02% )
 2 413 363 382      lld_pend_miss.pending        ( +-  0.78% )
 2 202 499 896      lld_pend_miss.pending_cycles  ( +-  0.70% )

```

4.23 +- 0.01 seconds time elapsed

Listing 32: Compteurs de performances pour le programme du listing 30.

### Listing 33

```

1  void moves(uint8_t player, MoveList& moves) const noexcept {
2      Move* move_list = moves.move_list;
3      uint64_t occupied = black | white | holes;
4      uint64_t bb = player == BLACK ? black : white;
5
6      while (bb) {
7          Square from = pop_lsb(bb);
8          uint64_t b = moves_bb(from, occupied) & ~occupied;
9          while (b) {
10             *move_list++ = Move(from, pop_lsb(b));
11         }
12     }
13
14     if (move_list == moves.move_list) [[unlikely]] {

```

```
15         *move_list++ = Move::none();
16     }
17
18     moves.last = move_list;
19 }
```

Listing 34

```
1 void moves(uint8_t player, MoveList& moves) const noexcept {
2     Move* move_list = moves.move_list;
3     uint64_t occupied = black | white | holes;
4     uint64_t bb = player == BLACK ? black : white;
5
6     Square from0 = pop_lsb(bb);
7     Square from1 = pop_lsb(bb);
8     Square from2 = pop_lsb(bb);
9     Square from3 = pop_lsb(bb);
10
11     uint64_t b0 = moves_bb(from0, occupied) & ~occupied;
12     uint64_t b1 = moves_bb(from1, occupied) & ~occupied;
13     uint64_t b2 = moves_bb(from2, occupied) & ~occupied;
14     uint64_t b3 = moves_bb(from3, occupied) & ~occupied;
15
16     while (b0) {
17         *move_list++ = Move(from0, pop_lsb(b0));
18     }
19     while (b1) {
20         *move_list++ = Move(from1, pop_lsb(b1));
21     }
22     while (b2) {
23         *move_list++ = Move(from2, pop_lsb(b2));
24     }
25     while (b3) {
26         *move_list++ = Move(from3, pop_lsb(b3));
27     }
28
29     if (move_list == moves.move_list) [[unlikely]] {
30         *move_list++ = Move::none();
31     }
32
33     moves.last = move_list;
```

34 }

Listing 35

```

play_random_games          95.17%
├── Yolah::moves            73.06%
│   ├── moves_bb           26.47%
│   │   └── Magic::index    6.30%
│   ├── Move::Move         15.48%
│   └── pop_lsb            11.93%
│       └── lsb            11.68%
├── std::uniform_int_distribution<... 8.61%
├── Yolah::play            5.87%
└── Yolah::game_over       5.02%
    └── shift<>           2.67%

```

Listing 36

```

=== Game Statistics ===
Total games: 1000000
Black wins:  499124 (49.9%)
White wins:  395468 (39.5%)
Draws:       105408 (10.5%)

Performance counter stats for './chapter02' (100 runs):

12 832 947 780      cycles                #    4,021 GHz                ( +- 0,00% )
23 589 523 540      instructions         #    1,82  insn per cycle     ( +- 0,00% )
 2 328 267 849      branches            # 729,484 M/sec               ( +- 0,00% )
 236 977 678        branch-misses        # 10,17% of all branches      ( +- 0,00% )
 3 513 433 988      L1-dcache-loads       #    1,101 G/sec              ( +- 0,00% )
 145 286 847        L1-dcache-load-misses #    4,13% of all L1-dcache accesses ( +- 0,00% )
 54 964 849         LLC-loads            # 17,221 M/sec                ( +- 0,00% )
 38 307             LLC-load-misses        #    0,07% of all LL-cache accesses ( +- 0,00% )

3,19340 +- 0,00975 seconds time elapsed ( +- 0,31% )

```

Listing 37

```

1 Move random_move(mt19937& mt) const noexcept {
2     uint64_t occupied = black | white | holes;
3     uint64_t player_bb = current_player() == BLACK ? black : white;
4     Square from0 = pop_lsb(player_bb);
5     Square from1 = pop_lsb(player_bb);
6     Square from2 = pop_lsb(player_bb);
7     Square from3 = pop_lsb(player_bb);

```

```

8      uint64_t b0 = moves_bb(from0, occupied) & ~occupied;
9      uint64_t b1 = moves_bb(from1, occupied) & ~occupied;
10     uint64_t b2 = moves_bb(from2, occupied) & ~occupied;
11     uint64_t b3 = moves_bb(from3, occupied) & ~occupied;
12     uint32_t n0 = popcount(b0);
13     uint32_t n1 = popcount(b1);
14     uint32_t n2 = popcount(b2);
15     uint32_t n3 = popcount(b3);
16     uint32_t n = n0 + n1 + n2 + n3;
17     if (n == 0) [[unlikely]] {
18         return Move::none();
19     }
20     uniform_int_distribution<uint32_t> d(0, n - 1);
21     uint32_t bit = d(mt);
22     int bb_index = (bit >= n0) + (bit >= n0 + n1) + (bit >= n0 + n1 +
↪ n2);
23     bit -= (bb_index > 0) * n0 + (bb_index > 1) * n1 + (bb_index > 2)
↪ * n2;
24     Square from = (Square[]){ from0, from1, from2, from3 }[bb_index];
25     uint64_t bb = (uint64_t[]){ b0, b1, b2, b3 }[bb_index];
26     Square to = Square(std::countr_zero(_pdep_u64(1ULL << bit, bb)));
27     return Move(from, to);
28 }
29 void play_random_games_fast(size_t nb_games, optional<uint64_t> seed =
↪ nullopt) {
30     MoveList moves;
31     random_device rd;
32     mt19937 mt(seed.value_or(rd()));
33     size_t black_wins = 0;
34     size_t white_wins = 0;
35     size_t draws = 0;
36     size_t max_nb_moves = 0;
37     for (size_t i = 0; i < nb_games; i++) {
38         Yolah yolah;
39         while (!yolah.game_over()) {
40             Move m = yolah.random_move(mt);
41             yolah.play(m);
42         }
43         auto [black_score, white_score] = yolah.score();
44         if (black_score > white_score) {
45             black_wins++;
46         } else if (white_score > black_score) {

```

```
47         white_wins++;
48     } else {
49         draws++;
50     }
51 }
52 cout << format("\n=== Game Statistics ===\n");
53 cout << format("Total games: {} \n", nb_games);
54 cout << format("Black wins:  {} ( {:.1f} %) \n", black_wins, 100.0 *
↪ black_wins / nb_games);
55 cout << format("White wins:  {} ( {:.1f} %) \n", white_wins, 100.0 *
↪ white_wins / nb_games);
56 cout << format("Draws:      {} ( {:.1f} %) \n", draws, 100.0 * draws
↪ / nb_games);
57 }
```

Listing 38

```
play_random_games_fast          96.01%
├─ Yolah::random_move           81.29%
│   ├── moves_bb                47.14%
│   │   └─ Magic::index         5.99%
│   ├── std::uniform_int_distributi... 12.88%
│   ├── pop_lsb                 4.36%
│   │   └─ lsb                  4.11%
│   └─ std::popcount            2.79%
├─ Yolah::game_over             9.53%
│   └─ shift<>                  5.96%
└─ Yolah::play                  3.95%
```

Listing 39

```
=== Game Statistics ===
Total games: 1000000
Black wins:  498503 (49.9%)
White wins:  395975 (39.6%)
Draws:       105522 (10.6%)

Performance counter stats for './chapter02' (100 runs):

  6 446 405 052      cycles                #    4,085 GHz                ( +-
15 466 860 279      instructions           #    2,41  insn per cycle     ( +-
   793 399 286      branches                # 502,748 M/sec              ( +-
    2 206 340      branch-misses            #    0,28% of all branches   ( +-
  3 605 550 330      L1-dcache-loads         #    2,285 G/sec              ( +-
   145 697 914      L1-dcache-load-misses    #    4,03% of all L1-dcache accesses ( +
    55 012 701      LLC-loads                #   34,860 M/sec              ( +-

```

11 412	LLC-load-misses	#	0,02% of all LL-cache accesses (
1,57951 +- 0,00265 seconds time elapsed ( +- 0,17% )			

Listing 40

```

1  constexpr uint32_t NB_ITERATIONS = 1000;
2  constexpr uint64_t SEED = 42;
3
4  class PRNG {
5      uint64_t s;
6      uint64_t rand64() {
7          s ^= s >> 12, s ^= s << 25, s ^= s >> 27;
8          return s * 2685821657736338717ULL;
9      }
10     public:
11         PRNG(uint64_t seed) :
12             s(seed) {
13         }
14         uint64_t seed() {
15             return s;
16         }
17         template<typename T>
18         T rand() {
19             return T(rand64());
20         }
21     };
22
23     constexpr uint32_t reduce(uint32_t x, uint32_t N) {
24         return ((uint64_t) x * (uint64_t) N) >> 32;
25     }
26
27     uint32_t random_prng_modulo(uint32_t nb_iterations) {
28         uint32_t res = 0;
29         PRNG prng(SEED);
30         for (uint32_t i = 1; i <= nb_iterations; i++) {
31             res += prng.rand<uint32_t>() % i;
32         }
33         return res;
34     }
35
36     static void BM_random_prng_modulo(benchmark::State& state) {

```

```
37     for (auto _ : state) {
38         benchmark::DoNotOptimize(random_prng_modulo(NB_ITERATIONS));
39     }
40     state.SetItemsProcessed(state.iterations());
41 }
42 BENCHMARK(BM_random_prng_modulo);
43
44 uint32_t random_prng_reduce(uint32_t nb_iterations) {
45     uint32_t res = 0;
46     PRNG prng(SEED);
47     for (uint32_t i = 1; i <= nb_iterations; i++) {
48         res += reduce(prng.rand<uint32_t>(), i);
49     }
50     return res;
51 }
52
53 static void BM_random_prng_reduce(benchmark::State& state) {
54     for (auto _ : state) {
55         benchmark::DoNotOptimize(random_prng_reduce(NB_ITERATIONS));
56     }
57     state.SetItemsProcessed(state.iterations());
58 }
59 BENCHMARK(BM_random_prng_reduce);
60
61 uint32_t random_mt19937(uint32_t nb_iterations) {
62     uint32_t res = 0;
63     mt19937 mt(SEED);
64     for (uint32_t i = 1; i <= nb_iterations; i++) {
65         res += uniform_int_distribution<uint32_t>{0, i - 1}(mt);
66     }
67     return res;
68 }
69
70 static void BM_random_mt19937(benchmark::State& state) {
71     for (auto _ : state) {
72         benchmark::DoNotOptimize(random_mt19937(NB_ITERATIONS));
73     }
74     state.SetItemsProcessed(state.iterations());
75 }
76 BENCHMARK(BM_random_mt19937);
77
78 uint32_t random_mt19937_modulo(uint32_t nb_iterations) {
```



```

79     uint32_t res = 0;
80     mt19937 mt(SEED);
81     for (uint32_t i = 1; i <= nb_iterations; i++) {
82         res += uniform_int_distribution<uint32_t>{}(mt) % i;
83     }
84     return res;
85 }
86
87 static void BM_random_mt19937_modulo(benchmark::State& state) {
88     for (auto _ : state) {
89
90         ↪ benchmark::DoNotOptimize(random_mt19937_modulo(NB_ITERATIONS));
91     }
92     state.SetItemsProcessed(state.iterations());
93 }
94 BENCHMARK(BM_random_mt19937_modulo);
95
96 uint32_t random_mt19937_reduce(uint32_t nb_iterations) {
97     uint32_t res = 0;
98     mt19937 mt(SEED);
99     for (uint32_t i = 1; i <= nb_iterations; i++) {
100         res += reduce(uniform_int_distribution<uint32_t>{}(mt), i);
101     }
102     return res;
103 }
104
105 static void BM_random_mt19937_reduce(benchmark::State& state) {
106     for (auto _ : state) {
107
108         ↪ benchmark::DoNotOptimize(random_mt19937_reduce(NB_ITERATIONS));
109     }
110     state.SetItemsProcessed(state.iterations());
111 }
112 BENCHMARK(BM_random_mt19937_reduce);
113
114 BENCHMARK_MAIN();

```

Listing 41

Run on (12 X 4400 MHz CPU s)  
CPU Caches:  
L1 Data 32 KiB (x6)

```
L1 Instruction 32 KiB (x6)
L2 Unified 256 KiB (x6)
L3 Unified 12288 KiB (x1)
Load Average: 0.79, 0.91, 1.05
```

```
-----
Benchmark                                Time                CPU    Iterations UserCounters...
-----
BM_random_prng_modulo                    2651 ns              2651 ns      258011 items_per_second=37
BM_random_prng_reduce                    1667 ns              1667 ns      417675 items_per_second=59
BM_random_mt19937                        5114 ns              5114 ns      122426 items_per_second=19
BM_random_mt19937_modulo                 6069 ns              6068 ns      116983 items_per_second=16
BM_random_mt19937_reduce                 4868 ns              4868 ns      146061 items_per_second=20
```

Listing 42

## 2.5 What's Next

## 2.6 Complete Commented Game Board Code

# Chapter 3

## AI Players



## Chapter 4

### Monte Carlo Player



# Chapter 5

## MCTS Player





# Chapter 6

## Minmax Player



## Chapter 7

# Minmax with Neural Network Player



# Chapter 8

## AI Tournament



# Chapter 9

## Conclusion





# Acronymes

**IPC** Instructions Per Cycle. 71

**SMT** Satisfiability Modulo Theories. 23, 24

**Z3** Z3 Theorem Prover. 24, 25, 33



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