Maths : Asymptotic comparison

1 Definitions :

(u_n) est dominée par (v_n) :

$$u_n = O(v_n) \iff \exists (M, n_0) \in \mathbb{R} \times \mathbb{N} \mid \forall n \in \mathbb{N},\ n \geq n_0 \Rightarrow \left| \frac{u_n}{v_n} \right| \leq M$$

(u_n) est négligeable devant (v_n) :

$$u_n = o(v_n) \iff \frac{u_n}{v_n} \xrightarrow[n\to+\infty]{} 0$$

(u_n) est équivalente à (v_n) :

$$u_n \sim v_n \iff \frac{u_n}{v_n} \xrightarrow[n\to+\infty]{} 1$$

2 Proprieties :

2.1

$$\forall l \in \mathbb{R}^*, \quad u_n \sim l \iff u_n \xrightarrow[n\to+\infty]{} l, \quad (u_n) \in \mathbb{R}^{\mathbb{N}}$$

$$u_n \sim v_n \text{ et } u'_n \sim v'_n \Rightarrow u_n u'_n \sim v_n v'_n$$

$$u_n \sim v_n \iff u_n - v_n = o(v_n)$$

2.2 Sums

$$u_n = o(v_n) \Rightarrow u_n + v_n \sim v_n$$

$$\begin{aligned} \square\ u_n = o(v_n) &\Rightarrow \frac{u_n}{v_n} \xrightarrow[n\to+\infty]{} 0 \\ &\Rightarrow \frac{u_n}{v_n} + 1 \xrightarrow[n\to+\infty]{} 1 \\ &\Rightarrow \frac{u_n + v_n}{v_n} \xrightarrow[n\to+\infty]{} 1 \\ &\Rightarrow u_n + v_n \sim v_n \quad \blacksquare \end{aligned}$$

$$\begin{cases} u_n \sim \lambda w_n \\ v_n \sim \mu w_n \\ \lambda + \mu \neq 0 \end{cases} \Rightarrow u_n + v_n \sim (\lambda + \mu) w_n$$

$$\begin{aligned} \square \begin{cases} u_n \sim \lambda w_n \\ v_n \sim \mu w_n \\ \lambda + \mu \neq 0 \end{cases} &\Rightarrow \begin{cases} u_n = \lambda w_n + o(w_n) \\ v_n = \mu w_n + o(w_n) \\ \lambda + \mu \neq 0 \end{cases} \\ &\Rightarrow \begin{cases} u_n + v_n = (\lambda + \mu) w_n + o(w_n) \\ \lambda + \mu \neq 0 \end{cases} \\ &\Rightarrow u_n + v_n \sim (\lambda + \mu) w_n \quad \blacksquare \end{aligned}$$