1. THE SUM

$$z = \big(l(x)+l(y),\ u(x)+u(y)\big)$$

$$w_z = \frac{u(x)+u(y)-(l(x)+l(y))}{2}$$

$$w_z = \frac{u(x)-l(x)}{2} + \frac{u(y)-l(y)}{2}$$

$$w_z = w_x + w_y$$

2. THE DIFFERENCE

$$z = \big(l(x)-u(y),\ u(x)-l(y)\big)$$

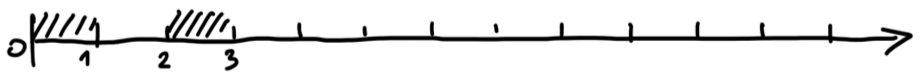
$$w_z = \frac{u(x)-l(y) - l(x)+u(y)}{2}$$

$$w_z = \frac{u(x)-l(x)}{2} + \frac{u(y)-l(y)}{2}$$

$$w_z = w_x + w_y$$

```scheme
(define (mul-interval x y)
  (let ((p1 (* (lower-bound x) (lower-bound y)))
        (p2 (* (lower-bound x) (upper-bound y)))
        (p3 (* (upper-bound x) (lower-bound y)))
        (p4 (* (upper-bound x) (upper-bound y))))
    (make-interval (min p1 p2 p3 p4)
                   (max p1 p2 p3 p4))))
```

ex. 1. $x = \langle 0, 1\rangle$ & $y = \langle 2, 3\rangle$



$w_x = 0.5$, $w_y = 0.5$

$z = x \cdot y = \langle 0, 3\rangle$

$w_z = 1.5$

ex. 2. $x = \langle 0, 1\rangle$ & $y = \langle 1, 2\rangle$

$w_x = 0.5$, $w_y = 0.5$

$z = x \cdot y = \langle 0, 2\rangle$

$w_z = 1$

if w_z would be a function of only w_x, w_y then it would produce the same result for the same arguments and the examples above shows otherwise

```scheme
(define (div-interval x y)
  (mul-interval
   x
   (make-interval (/ 1.0 (upper-bound y))
                  (/ 1.0 (lower-bound y)))))
```

The same ex. holds for division:

1. $x = \langle 0, 1\rangle$, $y = \langle 2, 3\rangle$

$\frac{1}{y} = \langle \frac{1}{3}, \frac{1}{2}\rangle$

$z = x \cdot \frac{1}{y} = \langle 0, \frac{1}{2}\rangle$

$w_z = \frac{1}{4}$

2. $x = \langle 0, 1\rangle$, $y = \langle 1, 2\rangle$

$\frac{1}{y} = \langle \frac{1}{2}, 1\rangle$

$z = x \cdot \frac{1}{y} = \langle 0, 1\rangle$

$w_z = 0.5$