276: Paint Fence

Input = n , k

n = no of ~~fences~~ posts in a fence
k = no. of colors

→ Each post can be colored with only color.
→ There cannot be three or more consecutive post with same color.

Find no. of ways fence can be painted.

Brute force approach

Recursive approach, trying all possible combinations and pruning the tree whenever condition is invalidated.
Look at the brute-force code.

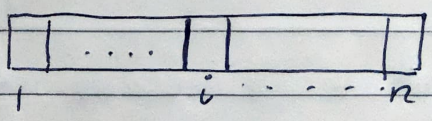
Time complexity :- $O(k^n)$ exponential

⇒ Simple approach

With above approach memorization doesn't work, as tree becomes huge and goes out of memory.

And it is also very difficult to convert above approach to Tabulation.

Start thinking directly from dp way. (Tabulative way)



Assume that you have answer upto i-1

dp[i-1] → number of ways you can paint i-1 posts with k colors.

Now how can we use previous values to calculate new values.

To paint post i, we have two options

→ Paint it in a different color to (i-1), and number of ways to do that is

$= (k-1) \times dp[i-1]$

→ Paint it in same color as (i-1), but that is only possible if (i-1) and (i-2) have different colors.

No. of ways (i-1) post can be different from (i-2)

$= (k-1) \times dp[i-2]$

∴ $dp[i] = (k-1) \times (dp[i-1] + dp[i-2])$

Base case :-

$dp[0] = k$, $dp[1] = k \times k$

Time complexity $= O(n)$, space $= O(n)$

We can improve the space complexity by state reduction as we can see ~~it is~~ the recurrence relation is only dependent on last two values. space - $O(1)$