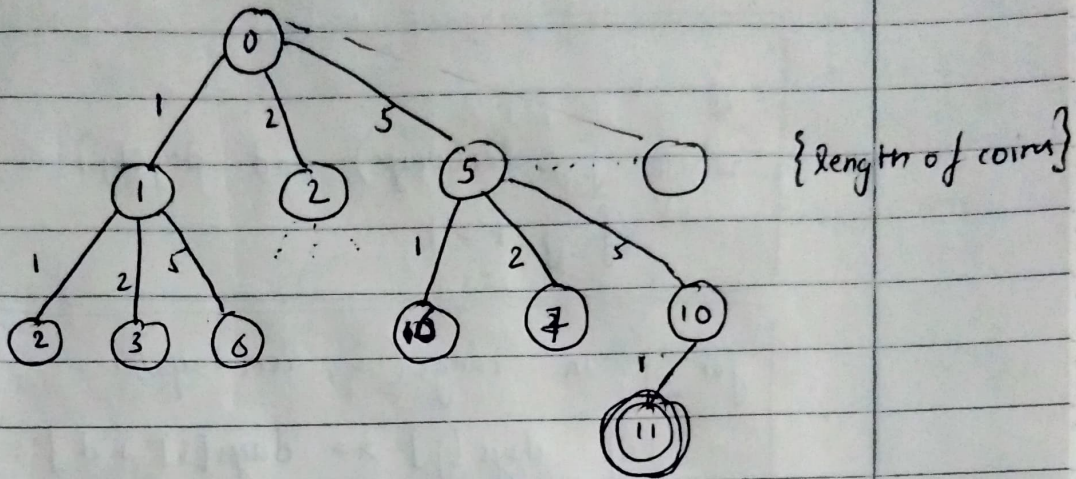


322 - Coin Change Problem

Start with brute force:- Decision tree is best to understand

Ex:- coins = [1, 2, 5], amount = 11



In the above brute force approach the time complexity is exponential

worst case complexity: length^{amount} $\therefore (L^a)$

But with memorization we can stop the repeated calculations.

Tabulation method can be used as well:

DP =

0	1										1
0	1	2	3	4	5	6	7	8	9	10	11

As seen in memorization decision tree, we are adding {1, 2, and 5} at each step to reach the target.

Here we will build bottom up $DP[i]$ to make the next.

$$\begin{aligned} & \forall i \in [0, 11] \\ & \forall j \in \text{costs coins} \\ & DP[i] = \left\{ \min(DP[i], DP[i - \text{costs}[j]] + 1) \mid i \geq \text{costs}[j] \right\} \end{aligned}$$

In the tabulation steps:

We derived the mathematical formula that can be translated to code:

Example to understand

$DP[2] =$ ~~for j=1~~ float('inf')

$$\text{for } j=1 \quad DP[2] = \min(DP[2], \underbrace{DP[2-1] + 1}_{\uparrow})$$

This means we have used 1 coin of ① and what's left now amount=1, that can be done by $DP[\text{amount}]$

$$DP[2] = \min('inf', 1+1) = 2$$

$$\begin{aligned} \text{for } j=2 \quad DP[2] &= \min(DP[2], DP[2-2] + 1) \\ &= \min(2, 0+1) = 1 \end{aligned}$$

Using this we can reach $DP[11]$