Massachusetts Institute of Technology Physics Department

Physics 8.01L | IAP 2014

Experiment 4: Work and Energy

Purpose of the Experiment:

In this experiment, you allow a cart to roll down an inclined ramp and run into a spring that is attached to a force sensor. You will measure the position of the cart and the force exerted on it by the spring while they are in contact.

- You will verify the theoretical acceleration of a cart on an incline.
- You will investigate experimentally the work–kinetic energy theorem, how kinetic energy converts to potential energy of gravity.
- You will observe and quantify the effect of non-conservative forces and find the fraction of the energy that is lost due to these forces at various stages of the cart's motion.

When the cart is rolling up the ramp, stopping forces and gravity act in the same direction, producing a greater downward acceleration, $|a_1| > g\sin(\theta)$. When the cart is rolling down the ramp, stopping forces and gravity act in opposite directions, so the acceleration is expected to be less, $|a_2| < g\sin(\theta)$. The average is $(a_1 + a_2)/2 = -g\sin(\theta)$. Your first task is to verify this expectation.

Considering Work and Energy for the motion up and back down the incline, the work done by the stopping force causes a difference in the velocity when the cart gets back down compared to the velocity it started with. If $|a_s|$ is the *magnitude* of the acceleration due to the stopping force, then the work it does is $-M|a_s|d$ going up and the same going down for a total of $-2M|a_s|d$ where d is the distance from where the cart leaves the spring to the point where it stops before coming back down the incline. Equating work to change in kinetic energy (since the cart starts and ends at the same height, the potential energy of gravity is the same at the beginning and end of the motion) leads to the formula: $a_s = (v_2^2 - v_1^2)/4d$. Note that a_s itself (as opposed to $|a_s|$) will be negative. The next test is to see if you can find any evidence for a stopping force in your data.

Considering Work and Energy for the motion up the incline the cart starts with kinetic energy which is mostly converted to potential energy of gravity. When the cart stops, almost all of the energy in now in the potential energy, with the remainder having been lost due to the stopping force. Using the fact that the initial kinetic energy is equal to the final potential energy minus the work done by the stopping force, you can show that $d = (v_1^2)/\{2(g\sin(\theta) + |a_s|)\}$ where d is the distance moved along the track, $d\sin(\theta)$ is the final height, and the work done by the stopping force is $-M|a_s|d$. You will test this prediction.

When it first hits the spring the cart has kinetic energy, which is transformed into potential energy as the spring is compressed. Once the spring is compressed to its maximum value (and the cart is stopped) the force of the spring will accelerate the cart and give it a velocity in the opposite direction. If the spring were perfect, the cart would leave with the same kinetic energy it arrived with, but moving in the opposite direction. You will determine if your data show that the spring behaves in this ideal fashion.

When doing the actual experiment, do NOT follow these instructions! Instead, follow the instructions that appear in the web submission form.

Electronic setup:

1 The Lab Pro interface should be powered on (green light comes on). Connect the motion sensor and the force sensor to DIG/SONIC 1and channel 1 the Lab Pro interface, respectively.

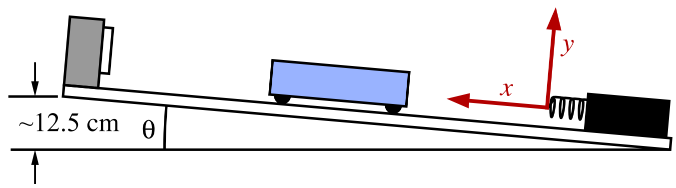
2 Be sure to zero the force sensor before each measurement.

Mechanical setup:

1 Turn the leveling screw all the way up so it does **not** touch the table top.

2 The motion sensor should be mounted with its back end flush with the end of the track and aimed at the center of the car. The slide switch on top of the motion sensor should be set to the cart icon.

3 Raise the end of the track with the motion sensor by placing pieces of 2×4 under the motion sensor to raise the end of the track about 12–13 cm above the table. Your apparatus should look something like this:



4 **Measure** from the table to the **bottom edge of the track** as shown in the diagram. You can calculate the $\sin(\theta)$, which is all you will need in this experiment, using this height and the fact that the track is 122 cm long. Find $g\sin(\theta)$ using $g = 9.81\ m/s^2$.

5 Place a cart on the track **with the end having the Velcro™ patches facing up the slope** towards the motion sensor.

6 For practice, place the cart about halfway up the track from the force sensor and release it. It will roll down the track, bounce back up, and then repeat the motion.

7 **IMPORTANT** When you do the actual measurements, one member of the group needs to hold the force sensor firmly so the track doesn't move when the cart bounces.

Part One: Acceleration of Cart along Incline

Question 1

Measure and record the height of the bottom edge of the end of the track.

Question 2

Calculate $g\sin(\theta)$.

To take data:

1 Chose the *WorkEnergy* program from the 8.01L Templates folder in Logger Pro.

2 If you get an error message, something is turned off or unplugged, so check your setup.

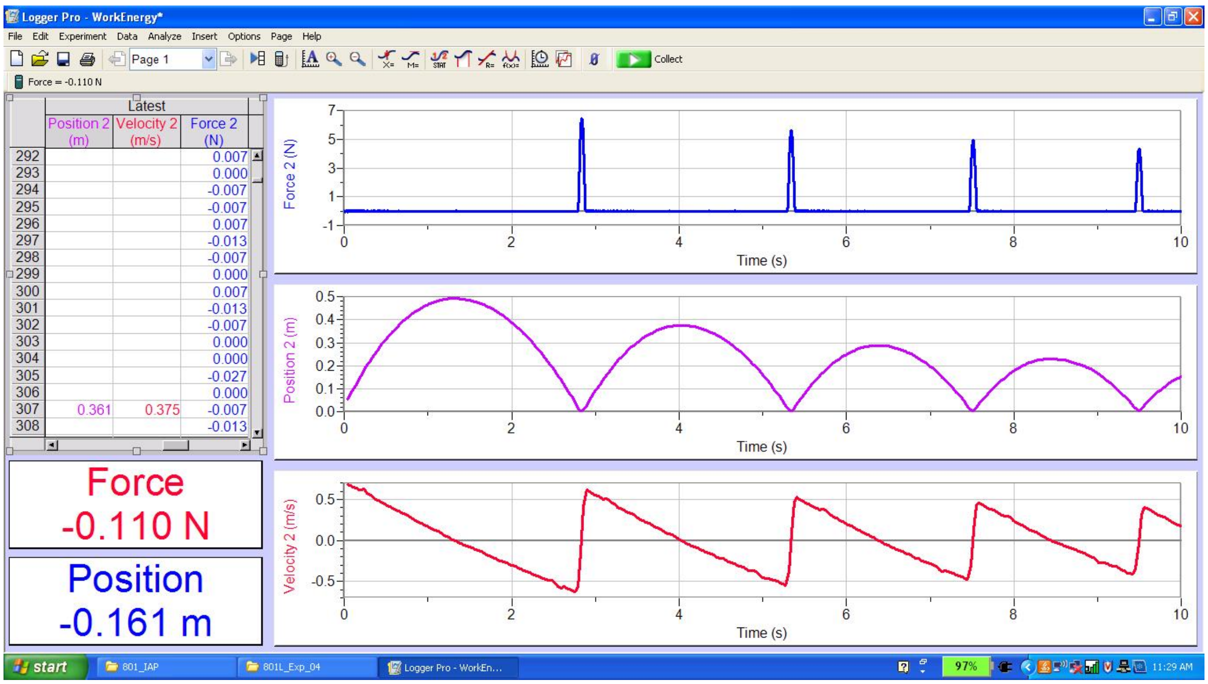
3 First, zero both sensors. Temporarily remove the force sensor and place the cart at the bottom of the track. Zero both sensors by hitting the "Zero" button or going to Experiment>Zero. Return the force sensor to its original position. Before a new measurement, you should zero the force sensor again, but not the motion sensor (uncheck that box in the pop-up menu).

4 When you are ready to measure, hold the cart about 30-40 cm up from the Force sensor.

5 **IMPORTANT:** One member of the group needs to hold the force sensor firmly so the track doesn't move when the cart bounces on the spring.

6 Release the cart and as soon as possible choose "Collect" [green arrow icon]. Don't start collecting before releasing the cart.

7 After 10 sec, you should see a graph of raw data something like this.

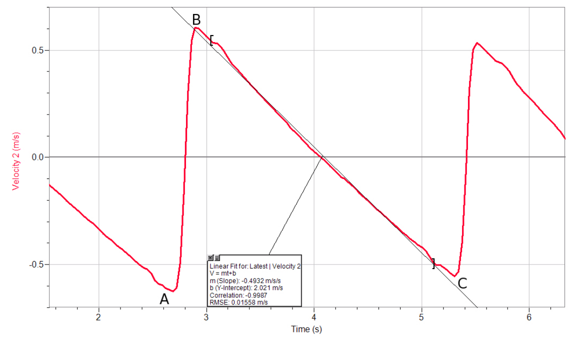


8 The top curve is the output of the force sensor. The peaks in the force curve occur when the cart bounces off the spring. The middle curve is the position of the cart as measured by the echo delay of the ultrasonic pulse between the motion sensor and the cart. The position curve shows that the cart bounces back up to a lower height each time (this will have been very obvious as you watched the bouncing). The bottom curve is the velocity plot, where positive velocity corresponds to motion *up* the inclined track. **Your data might have more peaks than shown in this example. If it looks different in any other way, check the steps (especially Mechanical Setup steps 1 & 2) and try again. If that fails, ask for help right away.**

9 You will take only one set of data in this experiment so it is critical that your data be high quality. If in doubt, ask for the opinion of the staff.

To analyze data:

First, select the plot of velocity vs. time; it should look like a sawtooth plot, an expanded section of which is shown here. The sharp change from point A to point B is when the cart bounces off the spring. The less steep change from point B to point C is the cart going up and coming back down the incline. Note where the zero velocity point is and also note that *positive* velocity is *up* the incline. If you have forgotten how to manipulate the graph and the marker lines, see "Logger Pro Hints".



The first step is to fit the velocity versus time going up and down the incline (B to C) with a straight line to measure the average acceleration. With your cursor, select a data range that begins a bit to the right of point B and ends a bit to the left of point C. When fitting these data, make sure your selection is within the linear region, not in the curved regions at the ends. Hit the "Linear Fit" button or use Analyze>Linear Fit. On your graph, a solid line will automatically appear along with a box with the results of the linear fit. The slope of the fit corresponds to the average acceleration going up and down the incline.

Question 3

What is your average acceleration?

Question 4

Compare the average acceleration you measured to the $-g\sin(\theta)$ you calculated from the track height. Careful students typically get values within 5% of the expected one. How well did you do?

Part Two: Is There a Stopping Force?

Move the cursor to find the largest value of the velocity near the two points B and C. The box on the plot gives you the X and Y position of the marker line.

Question 5

What maximum velocities did you measure at B and C?

Now, go to the position vs. time plot. The first downward spike corresponds to the time when the mass first hits the spring. The curve following the spike corresponds to the mass going up and down the incline. The second downward spike is when the cart hits the spring for the second time. Move the cursor to find the position (the Y coordinate) at the bottom of the first spike and at the top of the first curved section. You will use these numbers to calculate the distance the cart moved up the incline as described below. Note that the "Calculated Column" for Position2 has an approximate offset entered so that the bottom of the spikes should be close to zero. If you want, you can double-click on this column and adjust the offset so the bottom of the spikes are zero.

Question 6

What are the position along the track at the bottom of the first spike and the position along the track at the top of the first bounce after the first spike and their difference (the distance the cart moved up the track)?

Work/energy predicts that the speed after a trip up and down the incline will be smaller if there is a stopping force.

Question 7

Using your speeds at point B (v_1) and C (v_2) and the distance d found from the difference between the positions in Question 6, calculate the stopping acceleration $a_s = (v_2^2 - v_1^2)/4d$. Express your answer both in m/s^2 and also as a percentage of the acceleration down the incline, $-g\sin(\theta)$.

Question 8

Does your data show evidence for a non-zero stopping force?

Part Three: Energy Conservation Up the Incline

Work and energy considerations also predict that the distance up the incline to the stopping point is given by $d = (v_1^2)/\{2(g\sin(\theta) + a_s)\}$.

Question 9

Use this formula, your speed at point B (v_1) and a_s (if non-zero) from Part Two to predict how far the cart should go up the incline before stopping. **Note:** For this part, it is typically more accurate to use the acceleration you actually measured in Part One for $g\sin(\theta)$ instead of your value using the height of the end of the track.

Question 10

Compare your predicted value of d from Question 9 to the measured value as found in Question 6.

Part Four: Energy Conservation in the Spring

If the interaction with the spring is perfectly elastic, the cart should leave the spring with the same speed it hits the spring. The correction due to a_s is tiny in this case because the distances involved are very small. Measure the speed for two pairs of points A and B (i.e. A1, B1, A2, B2) separated by several bounces.

Question 11A

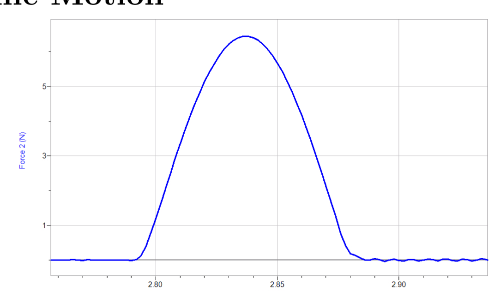
Calculate the ratio of speeds (magnitudes of velocities) and the ratio of kinetic energies for point B divided by point A in the two cases.

Question 11B

Does the spring remove a roughly constant fraction of velocity or of kinetic energy or of neither?

Part Five: Testing Predictions from Simple Harmonic Motion

Go to the Force vs. time plot (see example). The sharp spikes indicate where the cart hits the spring. Record the *time* where the Force first starts climbing steeply and also at the center of the top of the peak where the Force reaches its maximum value. The maximum force occurs when the spring is most compressed, i.e. when the cart stops.



Also, record the maximum force. Notice what happens to the force after the cart leaves the spring, it goes down to zero but then bounces up and down with a small amplitude for a while.

Question 12

Record the time when force starts to rise, the time when the force reaches its peak value, and the value of the maximum force.

The distance a mass on a spring moves from the moment it hits until it stops (i.e. the maximal distance the spring is compressed) is $2v\Delta t/\pi$, where v is the speed when it hits the spring and Δt is the time it takes to stop.

Question 13

Use this formula, the speed at point A, and the time for the spring to stop the cart (subtract the two times listed in the previous step) to find the spring compression.

Question 14

The maximum force divided by the compression distance gives k. What is your spring constant?

Using Work/energy, you can show that for an object hitting a horizontal spring, the object stops where $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$. Note that this formula ignores the potential energy of gravity and the work done by the stopping force since the distance moved is very small.

Question 15

Using your value for the spring constant and the spring compression, calculate the left side of this equation.

Question 16

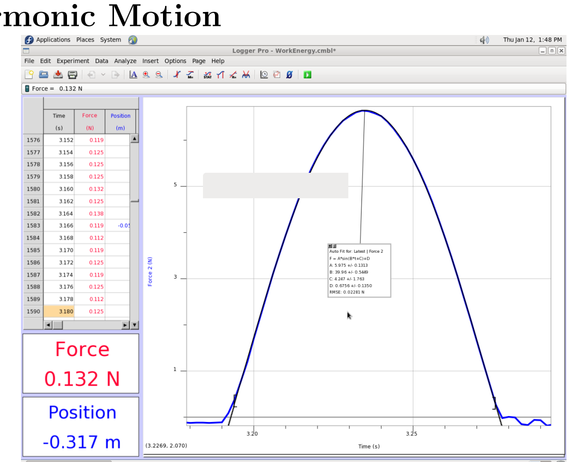
Using 0.25 kg for the mass of the cart and your value for velocity right before the cart hit the spring, calculate the left side of this equation.

Question 17

Do your data agree with the predictions from conservation of energy?

Part Six: More Detailed Testing of Simple Harmonic Motion

Finally, simple harmonic motion predicts that the position versus time should be a sine function. Since, for a spring, force varies linearly with position, Force versus time should also be a sine function. Blow up one of the force spikes and fit it with a sine function as shown. The coefficient B in the fit is the angular frequency ω which should equal $\omega = \sqrt{\frac{k}{m}}$.



Question 18

What did you measure for ω and what would you expect based on the measured value of k from Question 14 and the known value of m?

Question 19

The bottom of the first few downward spikes in the position plot will be lower than those later on. Why is that?