POLI 5003: Problem Set # 1, Team A

The Partido Revolucionario Institucional (PRI) maintained authoritarian rule over Mexico for more than seventy years, from the end of the Mexican Revolution until after the July 2000 elections. The dataset accompanying this assignment (mex2000.dta) is drawn from a survey conducted during that electoral campaign. You will use it to examine the predictors of Mexicans' attitudes towards the PRI and its opponents at that critical time in the country's history.

```
> # Setup
> require(foreign)
> mex <- read.dta("mex2000.dta")</pre>
> var.labels <- attr(mex, "var.labels")</pre>
> data.key <- data.frame(var.name=names(mex),var.labels)</pre>
> data.key
    var.name
1
     PRIfeel
2
     PANfeel
3
     PRDfeel
4
     prefPRI
5
     prefPAN
6
    rightide
7
    econpers
8
     econnat
9
     corrupt
10
       crime
11
      female
12
         ses
13 churchatt
                                                                            var.labels
                            What is your opinion of the PRI? O=very bad 10=very good
1
2
                            What is your opinion of the PAN? O=very bad 10=very good
3
                            What is your opinion of the PRD? O=very bad 10=very good
4
                  PRIfeel - feeling toward best-liked opposition party (PAN or PRD)
5
                                                                     PANfeel - PRIfeel
                                      Political ideology, 0=very left, 10=very right
6
7
   Change in personal economic situation, 1 yr. 1=much worse now, 5=much better now
8
   View of national economic sit. over past yr. 1=much worse now, 5=much better now
                View of gov't corruption, past yr. 1=much less now, 5=much more now
9
                      View of crime over past year, 1=much less now, 5=much more now
10
11
                                                                   Female? 0=no, 1=yes
12
                                       Socioeconomic status, 1=very low, 6=very high
13
      Church attendance: 1=never, 2=occationally, 3=monthly, 4=weekly, 5=more often
```

1. During its long rule, the PRI worked to present itself as the party of all Mexicans and was therefore something of an ideological chameleon. Nevertheless, we might hypothesize that people who leaned more to the right would hold more favorable views of this authoritarian party (Americanists may recall V.O. Key's writings about the one-party South), and suppose we want to control for their assessments of the recent performance of the national economy as well as of their personal characteristics. Is this ideology hypothesis supported by a regression of PRIfeel using Empirical Bayes and the other default settings of MCMCpack? How do you know? Describe the estimated effect of ideology on attitudes toward the PRI.

```
> require(MCMCpack)
> ipak <- function(pkg){</pre>
       new.pkg <- pkg[!(pkg %in% installed.packages()[, "Package"])]</pre>
       if (length(new.pkg))
           install.packages(new.pkg, dependencies = TRUE)
       sapply(pkg, require, character.only = TRUE)
   }
> packages <- c("ggplot2", "RCurl", "MCMCpack", "inline", "Rcpp")
> ipak(packages)
 ggplot2
            RCurl MCMCpack
                             inline
                                         Rcpp
    TRUE
             TRUE
                      TRUE
                               TRUE
                                         TRUE
> m1<-MCMCregress(PRIfeel~rightide+econnat+female+ses+churchatt,data=mex,
      burnin = 1000, mcmc = 10000,
      thin = 1, verbose = 0, seed = 19880201, beta.start = NA,
      b0 = 0, B0 = 0, c0 = 0.001, d0 = 0.001, sigma.mu = NA, sigma.var = NA,
      marginal.likelihood =("none") )
> summary(m1)
Iterations = 1001:11000
Thinning interval = 1
Number of chains = 1
Sample size per chain = 10000
1. Empirical mean and standard deviation for each variable,
```

 Empirical mean and standard deviation for each variable, plus standard error of the mean:

```
MeanSDNaive SETime-series SE(Intercept)1.678650.402820.00402820.0040282rightide0.289610.026820.00026820.0002678econnat0.792250.090870.00090870.0009087
```

```
female 0.53840 0.16174 0.0016174 0.0016174 ses -0.23964 0.07272 0.0007272 0.0007095 churchatt 0.03758 0.07191 0.0007191 0.0007191 sigma2 10.38477 0.37170 0.0037170 0.0036131
```

2. Quantiles for each variable:

>

```
50%
               2.5%
                         25%
                                           75%
                                                  97.5%
(Intercept)
             0.8814
                    1.40950 1.67825
                                       1.94583
                                               2.46143
rightide
             0.2370
                    0.27159 0.28963
                                       0.30782
                                               0.34258
econnat
             0.6148
                    0.73190 0.79161
                                       0.85248
                                               0.97128
female
             0.2197
                    0.43032 0.53729 0.64594
                                               0.86219
            -0.3817 -0.28873 -0.24061 -0.19132 -0.09585
ses
churchatt
            -0.1027 -0.01148 0.03721 0.08571
                                               0.17929
             9.6841 10.12877 10.37587 10.63067 11.12679
sigma2
```

As we can see in the section 2 of the summary, the 95% confidence interval of righttide ideology falls between 0.2370 and 0.34258 (both larger than 0). Thus we conclude that it is statistically significant that righttide ideology has a 0.28961 coefficient effect on

people's preference to PRI: with one unit increase in righttide leaning, the person's

preference to PRI increases by 0.28961 (section 1).

2. Suppose the literature further suggests that the effect of ideology on respondents' feelings about the PRI would be weaker among those who held more positive assessments of the national economy's recent performance. Assess this conditional hypothesis using Empirical Bayes and the other default settings of MCMCpack.

```
> m2<-MCMCregress(PRIfeel~rightide+econnat+rightide:econnat+female+ses+churchatt,out)
+ thin = 1, verbose = 0, seed = 19880201, beta.start = NA,
+ b0 = 0, B0 = 0, c0 = 0.001, d0 = 0.001, sigma.mu = NA, sigma.var = NA,
+ marginal.likelihood =("none") )
> summary(m2)

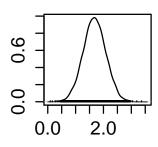
Iterations = 1001:11000
Thinning interval = 1
```

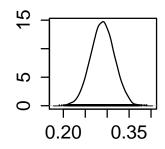
Number of chains = 1

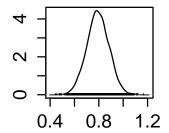
Sample size per chain = 10000

1. Empirical mean and standard deviation for each variable, plus standard error of the mean:

Density of (Interce) Density of rightid Density of econna



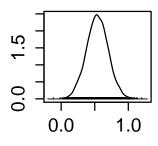


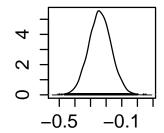


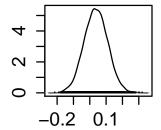
= 10000 Bandwidth = 0.= 10000 Bandwidth = 0. (= 10000 Bandwidth = 0.

Density of female

Density of ses Density of churcha

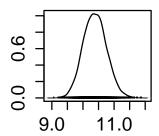






= 10000 Bandwidth = 0.= 10000 Bandwidth = 0.= 10000 Bandwidth = 0.

Density of sigma2



= 10000 Bandwidth = 0.

Figure 1: Density of Independent Variable

```
SD Naive SE Time-series SE
                     Mean
(Intercept)
                  1.41214 0.62440 0.0062440
                                                   0.0062440
rightide
                  0.33330 0.08212 0.0008212
                                                   0.0008076
econnat
                  0.89115 0.20099 0.0020099
                                                   0.0020099
female
                  0.53641 0.16154 0.0016154
                                                   0.0015916
ses
                  -0.23862 0.07374 0.0007374
                                                   0.0007374
churchatt
                  0.03804 0.07145 0.0007145
                                                   0.0007279
rightide:econnat -0.01598 0.02851 0.0002851
                                                   0.0002851
sigma2
                  10.39155 0.36606 0.0036606
                                                   0.0035053
```

25%

2. Quantiles for each variable:

2.5%

label="T:res"

)

```
(Intercept)
                  0.17666
                           0.988293
                                     1.41492
                                              1.832663
                                                         2.63241
rightide
                  0.16944
                           0.278760
                                     0.33312 0.389432
                                                        0.49443
econnat
                  0.49626
                           0.755669
                                     0.89095
                                              1.028299
                                                        1.28377
female
                  0.21728
                          0.427948
                                     0.53707
                                              0.645899
                                                         0.85682
                 -0.38200 -0.288061 -0.23851 -0.189034 -0.09362
ses
                 -0.10125 -0.009644
                                     0.03751
                                              0.086170
churchatt
                                                         0.17828
rightide:econnat -0.07107 -0.035582 -0.01618
                                              0.003339
                                                         0.04018
                  9.69708 10.137559 10.37946 10.629708 11.13262
sigma2
> # library(stargazer)
   stargazer(m1, m2, title="MCMC regression Results",
             dep.var.labels="Feeling about the PRI",
> #
             covariate.labels=c("Ideology", "National Economy", "Female", "SES",
             align=TRUE,
```

50%

75%

97.5%

> ##Kellen&Erico: stargazer pack does not support making table for MCMC results

According to m2, the effect of postivive national economy assessment on righttide respondents" feeling about the PRI is sum of the coefficient of econnat and the coefficient of interaction term. As we can see from the section 1 of the output, "econnat (beta3)=0.89115" and "rightide:econnat(beta7)= -0.01598", the influence of positive economy performance on preference to PRI is 0.89115+(-0.01598). Since the coefficient of rightide:econnat is negative, it means the positive assessments of the national economy weakens the effect of ideology on feeling about the PRI. Thus the result does support the conditional hypothesis.

omit.stat=c("adj.rsq", "f", "ser"),

3. Suppose a "helpful" reviewer points out that Empirical Bayes imposes a very strong set of priors, and that noninformative priors may yield a different result. Going well beyond the call of duty, this reviewer even offers some code for you to study and then try

out. Is the reviewer correct that your conclusions regarding the conditional hypothesis change with a different prior?

```
> require(rstan)
> # First we have to define the model
> PRIfeel.code <- '
      data {
          int<lower=0> N;
          vector[N] PRIfeel;
          vector[N] rightide;
          vector[N] econnat;
          vector[N] female;
          vector[N] ses;
          vector[N] churchatt;
      }
      transformed data {
          vector[N] rightideXeconnat;
          rightideXeconnat <- rightide .* econnat;</pre>
      parameters {
          real beta1;
                                   // coef for constant (default prior is uniform,
          real beta2;
          real beta3;
          real beta4;
          real beta5;
          real beta6;
          real beta7;
          real<lower=0> sigma;
      }
      model {
          PRIfeel ~ normal(beta1 + beta2 * rightide + beta3 * econnat +
                                beta4 * female + beta5 * ses +
                                beta6 * churchatt + beta7 * rightideXeconnat, sigma
      }
> # Then put the data into the expected format
> mex.data <- list(N = nrow(mex), PRIfeel = mex$PRIfeel, rightide = mex$rightide,
                     econnat = mex$econnat, female = mex$female,
                     ses = mex$ses, churchatt = mex$churchatt)
> # Now we can run it
> set.seed(324)
> m2.stan <- stan(model_code = PRIfeel.code, data = mex.data,
```

TRANSLATING MODEL 'PRIfeel.code' FROM Stan CODE TO C++ CODE NOW. COMPILING THE C++ CODE FOR MODEL 'PRIfeel.code' NOW. cygwin warning:

MS-DOS style path detected: C:/PROGRA~1/R/R-30~1.2/etc/x64/Makeconf Preferred POSIX equivalent is: /cygdrive/c/PROGRA~1/R/R-30~1.2/etc/x64/Makeconf CYGWIN environment variable option "nodosfilewarning" turns off this warning. Consult the user's guide for more details about POSIX paths:

http://cygwin.com/cygwin-ug-net/using.html#using-pathnames C:/Users/Dong/Documents/R/win-library/3.0/rstan/include//stansrc/stan/agrad/rev/var C:/Users/Dong/Documents/R/win-library/3.0/rstan/include//stansrc/stan/agrad/rev/cha SAMPLING FOR MODEL 'PRIfeel.code' NOW (CHAIN 1).

```
Iteration:
             1 / 1000 [ 0%]
                               (Warmup)
Iteration: 100 / 1000 [ 10%]
                               (Warmup)
Iteration: 200 / 1000 [ 20%]
                               (Warmup)
Iteration: 300 / 1000 [ 30%]
                               (Warmup)
Iteration: 400 / 1000 [ 40%]
                               (Warmup)
Iteration: 500 / 1000 [ 50%]
                               (Warmup)
Iteration: 600 / 1000 [ 60%]
                               (Sampling)
Iteration: 700 / 1000 [ 70%]
                               (Sampling)
Iteration: 800 / 1000 [ 80%]
                               (Sampling)
Iteration: 900 / 1000 [ 90%]
                               (Sampling)
Iteration: 1000 / 1000 [100%]
                                (Sampling)
Elapsed Time: 24.736 seconds (Warm-up)
              21.666 seconds (Sampling)
              46.402 seconds (Total)
```

SAMPLING FOR MODEL 'PRIfeel.code' NOW (CHAIN 2).

```
Iteration:
             1 / 1000 [ 0%]
                               (Warmup)
Iteration: 100 / 1000 [ 10%]
                               (Warmup)
Iteration: 200 / 1000 [ 20%]
                               (Warmup)
Iteration: 300 / 1000 [ 30%]
                               (Warmup)
Iteration: 400 / 1000 [ 40%]
                               (Warmup)
Iteration: 500 / 1000 [ 50%]
                               (Warmup)
Iteration: 600 / 1000 [ 60%]
                               (Sampling)
Iteration: 700 / 1000 [ 70%]
                               (Sampling)
Iteration: 800 / 1000 [ 80%]
                               (Sampling)
Iteration: 900 / 1000 [ 90%]
                               (Sampling)
Iteration: 1000 / 1000 [100%]
                                (Sampling)
Elapsed Time: 21.216 seconds (Warm-up)
```

```
21.647 seconds (Sampling) 42.863 seconds (Total)
```

SAMPLING FOR MODEL 'PRIfeel.code' NOW (CHAIN 3).

```
1 / 1000 [ 0%]
Iteration:
                              (Warmup)
Iteration: 100 / 1000 [ 10%] (Warmup)
Iteration: 200 / 1000 [ 20%]
                              (Warmup)
Iteration: 300 / 1000 [ 30%]
                              (Warmup)
Iteration: 400 / 1000 [ 40%] (Warmup)
Iteration: 500 / 1000 [ 50%]
                              (Warmup)
Iteration: 600 / 1000 [ 60%]
                              (Sampling)
Iteration: 700 / 1000 [ 70%] (Sampling)
Iteration: 800 / 1000 [ 80%]
                              (Sampling)
Iteration: 900 / 1000 [ 90%]
                              (Sampling)
Iteration: 1000 / 1000 [100%]
                               (Sampling)
Elapsed Time: 23.968 seconds (Warm-up)
              17.652 seconds (Sampling)
             41.62 seconds (Total)
```

> print(m2.stan)

Inference for Stan model: PRIfeel.code.
3 chains, each with iter=1000; warmup=500; thin=1;
post-warmup draws per chain=500, total post-warmup draws=1500.

	mean	se_mean	sd	2.5%	25%	50%	75%	97.5%	n_eff	Rhat
beta1	1.5	0.0	0.6	0.3	1.0	1.4	1.9	2.7	472	1
beta2	0.3	0.0	0.1	0.2	0.3	0.3	0.4	0.5	522	1
beta3	0.9	0.0	0.2	0.5	0.7	0.9	1.0	1.3	552	1
beta4	0.5	0.0	0.2	0.2	0.4	0.5	0.6	0.8	1143	1
beta5	-0.2	0.0	0.1	-0.4	-0.3	-0.2	-0.2	-0.1	924	1
beta6	0.0	0.0	0.1	-0.1	0.0	0.0	0.1	0.2	968	1
beta7	0.0	0.0	0.0	-0.1	0.0	0.0	0.0	0.0	523	1
sigma	3.2	0.0	0.1	3.1	3.2	3.2	3.3	3.3	1155	1
lp	-2713.0	0.1	2.0	-2717.6	-2714.1	-2712.7	-2711.6	-2710.2	562	1

Samples were drawn using NUTS(diag_e) at Thu Mar 13 11:18:40 2014. For each parameter, n_eff is a crude measure of effective sample size, and Rhat is the potential scale reduction factor on split chains (at convergence, Rhat=1).

```
> m2.stan.sim <- as.data.frame(m2.stan)
> HDI.posterior <- function(data = NULL, mass = .95) {</pre>
```

```
n.var \leftarrow dim(data)[2]-2
+
      results.HDI <- matrix(rep(NA, 3*n.var), nrow=n.var, ncol=3)
      for (var in 1:n.var) {
          post <- data[,var]</pre>
          sorted.post <- sort(post)</pre>
          ci.idx <- floor(mass * length(sorted.post))</pre>
          n.ci <- length(sorted.post) - ci.idx</pre>
          ci.width <- rep(0, n.ci)</pre>
          for (i in 1:n.ci) {
               ci.width[i] <- sorted.post[i+ci.idx] - sorted.post[i]</pre>
          }
          HDI.min <- sorted.post[which.min(ci.width)]</pre>
          HDI.max <- sorted.post[which.min(ci.width)+ci.idx]</pre>
          mean.post <- mean(post)</pre>
          results.HDI[var,] <- c(mean.post, HDI.min, HDI.max)
      }
      results.HDI <- as.data.frame(results.HDI)
      names(results.HDI) \leftarrow c("b", "lb", "ub")
      return(results.HDI)
+ }
> HDI.posterior(data=m2.stan.sim)
                        lb
                                     ub
  1.45680481 0.20047082 2.65509789
1
2 0.32983824 0.16562228 0.49026322
3 0.88123471 0.49915704 1.28159359
4 0.52589903 0.20198199 0.83908045
5 -0.24496189 -0.38062972 -0.10160730
6 0.04090839 -0.09469561 0.19153111
7 -0.01489438 -0.07351261 0.04247551
```

As we can see from the output, beta2=0.3 (MCMC rightide:0.333), beta3=0.9 (MCMC econnat:0.891), beta4=0.5 (MCMC female=0.536), beta5=-0.2 (MCMC ses=-0.239),beta6=0.0 (MCMC churchatt=0.038), beta7=0.0 (MCMC rightide:econnat=-0.016), the stan result with weak priors (uniform distribution) do not yield a very different result.

4. Assess the original, unconditional hypothesis using noninformative priors. Include a graph of the posterior distribution for the effect of ideology on feelings toward the PRI. Also graph the highest density intervals of the posteriors of the effects of all variables in the model, remembering to standardize the continuous variables by dividing them by twice their standard deviations.

```
> # First define the model for the original, unconditional hypothesis > PRIfeel.code.1 <- '
```

```
data {
+
          int<lower=0> N;
          vector[N] PRIfeel;
          vector[N] rightide;
          vector[N] econnat;
          vector[N] female;
          vector[N] ses;
          vector[N] churchatt;
      }
      parameters {
          real beta1;
                                  // coef for constant (default prior is uniform,
          real beta2;
          real beta3;
          real beta4;
          real beta5;
          real beta6;
          real<lower=0> sigma;
      }
      model {
          PRIfeel ~ normal(beta1 + beta2 * rightide + beta3 * econnat +
                                beta4 * female + beta5 * ses +
                                beta6 * churchatt, sigma);
      }
> # Then put the data into the expected format
> mex.data <- list(N = nrow(mex), PRIfeel = mex$PRIfeel, rightide = mex$rightide,
                     econnat = mex$econnat, female = mex$female,
                     ses = mex$ses, churchatt = mex$churchatt)
> #run it
> set.seed(740)
> m1.stan <- stan(model_code = PRIfeel.code.1, data = mex.data,</pre>
                  iter = 1000, chains = 3)
TRANSLATING MODEL 'PRIfeel.code.1' FROM Stan CODE TO C++ CODE NOW.
COMPILING THE C++ CODE FOR MODEL 'PRIfeel.code.1' NOW.
cygwin warning:
 MS-DOS style path detected: C:/PROGRA~1/R/R-30~1.2/etc/x64/Makeconf
 Preferred POSIX equivalent is: /cygdrive/c/PROGRA~1/R/R-30~1.2/etc/x64/Makeconf
  CYGWIN environment variable option "nodosfilewarning" turns off this warning.
 Consult the user's guide for more details about POSIX paths:
    http://cygwin.com/cygwin-ug-net/using.html#using-pathnames
C:/Users/Dong/Documents/R/win-library/3.0/rstan/include//stansrc/stan/agrad/rev/var
```

```
C:/Users/Dong/Documents/R/win-library/3.0/rstan/include//stansrc/stan/agrad/rev/cha
SAMPLING FOR MODEL 'PRIfeel.code.1' NOW (CHAIN 1).
Iteration:
             1 / 1000 [ 0%]
                               (Warmup)
Iteration: 100 / 1000 [ 10%]
                               (Warmup)
Iteration: 200 / 1000 [ 20%]
                               (Warmup)
Iteration: 300 / 1000 [ 30%]
                              (Warmup)
Iteration: 400 / 1000 [ 40%]
                               (Warmup)
Iteration: 500 / 1000 [ 50%]
                               (Warmup)
Iteration: 600 / 1000 [ 60%]
                              (Sampling)
Iteration: 700 / 1000 [ 70%]
                               (Sampling)
                               (Sampling)
Iteration: 800 / 1000 [ 80%]
Iteration: 900 / 1000 [ 90%]
                               (Sampling)
Iteration: 1000 / 1000 [100%]
                                (Sampling)
Elapsed Time: 12.351 seconds (Warm-up)
              16.77 seconds (Sampling)
              29.121 seconds (Total)
SAMPLING FOR MODEL 'PRIfeel.code.1' NOW (CHAIN 2).
                               (Warmup)
Iteration:
             1 / 1000 [ 0%]
Iteration: 100 / 1000 [ 10%]
                               (Warmup)
Iteration: 200 / 1000 [ 20%]
                               (Warmup)
Iteration: 300 / 1000 [ 30%]
                               (Warmup)
Iteration: 400 / 1000 [ 40%]
                               (Warmup)
Iteration: 500 / 1000 [ 50%]
                              (Warmup)
Iteration: 600 / 1000 [ 60%]
                               (Sampling)
Iteration: 700 / 1000 [ 70%]
                               (Sampling)
```

(Sampling)

(Sampling)

(Warmup)

(Warmup)

(Warmup)

(Warmup)

(Warmup)

(Sampling)

Iteration: 800 / 1000 [80%]

Iteration: 900 / 1000 [90%]

Iteration: 1000 / 1000 [100%]

Iteration: 100 / 1000 [10%]

Iteration: 200 / 1000 [20%]

Iteration: 300 / 1000 [30%]

Iteration: 400 / 1000 [40%]

Iteration:

Elapsed Time: 15.817 seconds (Warm-up)

1 / 1000 [0%]

12.943 seconds (Sampling) 28.76 seconds (Total)

SAMPLING FOR MODEL 'PRIfeel.code.1' NOW (CHAIN 3).

> print(m1.stan)

Inference for Stan model: PRIfeel.code.1.
3 chains, each with iter=1000; warmup=500; thin=1;
post-warmup draws per chain=500, total post-warmup draws=1500.

	mean	se_mean	sd	2.5%	25%	50%	75%	97.5%	n_eff	Rhat
beta1	1.7	0.0	0.4	0.9	1.4	1.7	1.9	2.4	898	1
beta2	0.3	0.0	0.0	0.2	0.3	0.3	0.3	0.3	1388	1
beta3	0.8	0.0	0.1	0.6	0.7	0.8	0.9	1.0	1144	1
beta4	0.5	0.0	0.2	0.2	0.4	0.5	0.7	0.9	1500	1
beta5	-0.2	0.0	0.1	-0.4	-0.3	-0.2	-0.2	-0.1	1298	1
beta6	0.0	0.0	0.1	-0.1	0.0	0.0	0.1	0.2	1178	1
sigma	3.2	0.0	0.1	3.1	3.2	3.2	3.3	3.3	1370	1
lp	-2712.9	0.1	1.9	-2717.7	-2713.9	-2712.6	-2711.4	-2710.2	535	1

Samples were drawn using NUTS(diag_e) at Thu Mar 13 11:20:49 2014. For each parameter, n_eff is a crude measure of effective sample size, and Rhat is the potential scale reduction factor on split chains (at convergence, Rhat=1).

> m1.stan.sim <- as.data.frame(m1.stan)</pre>

As we can see in the output of stan model m1 (as well as Figure 2), the 95% confidence interval of righttide ideology falls between 0.2 and 0.3 (both larger than 0). Thus we conclude that it is statistically significant that righttide ideology has a 0.3 coefficient effect on people's preference to PRI: with one unit increase in righttide leaning, the person's preference to PRI increases by 0.3.

	b	lb	ub
1	1.66748537	0.91114533	2.39840570
2	0.28935858	0.23719339	0.34214825
3	0.79270565	0.61712673	0.96336336

```
4 0.54017318 0.21638730 0.90287772
5 -0.23899217 -0.37716589 -0.09583156
6 0.03951507 -0.09586657 0.19696646
```

Figure 2: Posterior Distribution of the Effect of Ideology

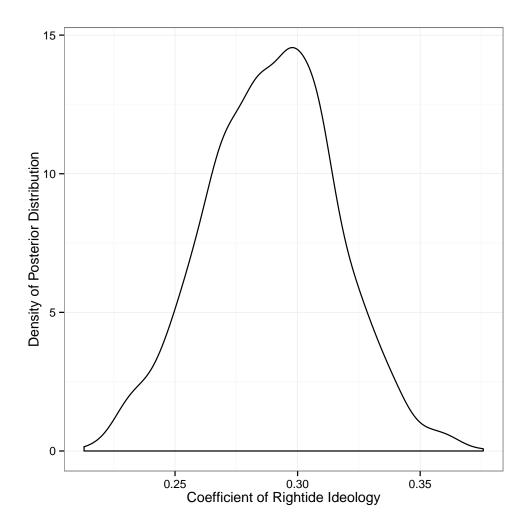
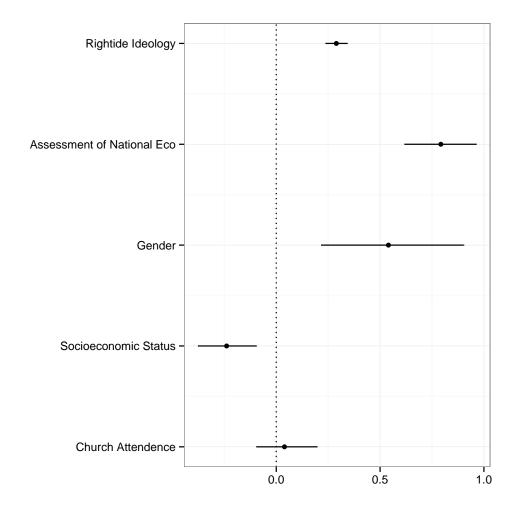


Figure 3: Posterior Distribution of the Effect of Ideology



 $\it Note$: Continuous variables rescaled by dividing by twice their standard deviations per ?.