Discrete Image Reconstruction using Parallel Beam Geometry

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Introduction

For the final project our team will implement the algorithm for computerized axial tomography (CAT or CT) based on parallel beam geometry. The problem aims to solve an ill-posed inverse problem in medical imaging where X-ray beams are passed through an object and the intensity of the beams at the input and output are measured. The aim is to reconstruct an image of the internal structure of an object from the intensity variations arising from beams passing through heterogeneous material.

Mathematical model The absorption coefficient function $f(x, y)$ can be reconstructed using the *modified filtered back-projection formula* [1],

$$f(x,y) \approx \frac{1}{2}\mathscr{B}(\mathscr{F}^{-1}A * \mathscr{R}f)(x,y). \tag{1}$$

Here, A is a low-pass filter function, $\mathscr{B}$ is the Back-projection transform, $\mathscr{F}^{-1}$ is the inverse Fourier transform. The Radon transform $\mathscr{R}$ is given by

$$\mathscr{R}f = \int_C f(x,y)\mathrm{d}\ell = ln\left(\frac{I_0}{I_1}\right). \tag{2}$$

This is the ratio of the intensity at input and output which is the integral of $f(x, y)$ along C, which forms the input data provided. We leave the derivation of Equations (1) and (2) along with detailed descriptions of filters and interpolation choices to forthcoming submissions.

Computational considerations From a computational point-of-view, we plan to implement three choices of filters consisting of (1) the Ram-Lak filter, (2) the Shepp-Logan filter and (3) the Low-pass Cosine filter. The back-projection which requires an interpolation of $f(x, y)$ on the mesh grid will be implemented using (1) Nearest neighbor and (2) a linear interpolation operator.

Due to the lack of real data, it is common practice to test the efficiency and viability of algorithms on phantom images. We will use several such images to compare and contrast the choices of filters and interpolation which lead to obtaining an optimal solution for the problem. We note that in practice data collected from experiments involves noise. The standard method which takes this into consideration involves adding Poisson noise to the image data. We will implement multiple test scenarios where Poisson noise added to the image is used for various choices of filters and interpolation schemes thereby testing their efficiency.

In order to make the project an open-source package, we will host a github repository for the project. We will also try to ensure that the code will be conformant with existing libraries we will ensure that the classes are usable through OpenCV and possibly OpenGL.

References

[1] T.G. Feeman. *The Mathematics of Medical Imaging: A Beginners Guide*. Springer Undergraduate Texts in Mathematics and Technology. Springer, 2009.