

Inverting for Near Shore Bathymetry from Surface Wave Properties

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Many coastal processes are affected by bathymetry

Bathymetry
Inversion
from Waves

Introduction

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Discussion



Bathymetry is submarine topography

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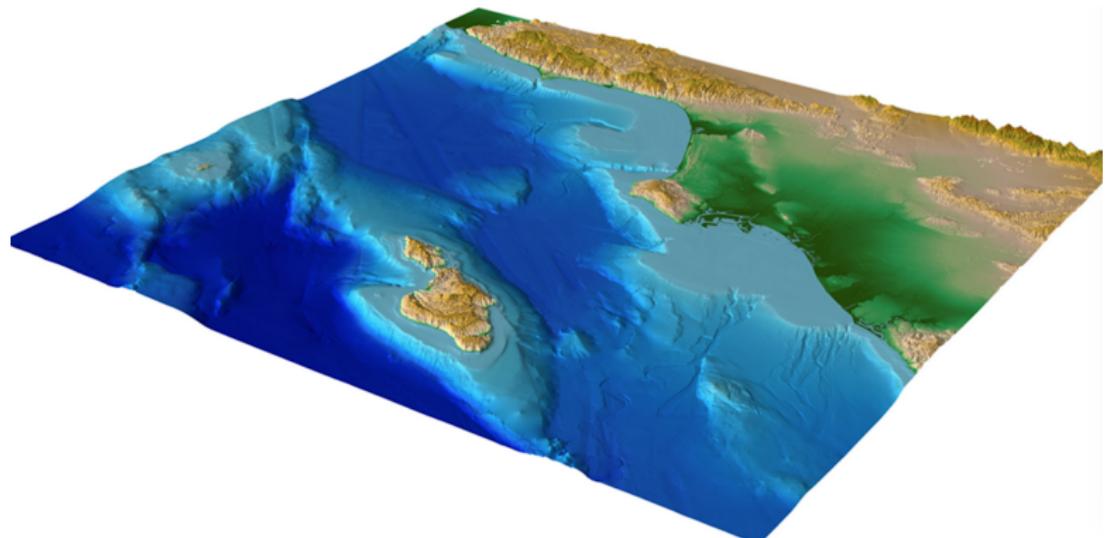
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Direct measurements are expensive and challenging

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LARC



CRAB

Inverse models estimate depth using data & physics

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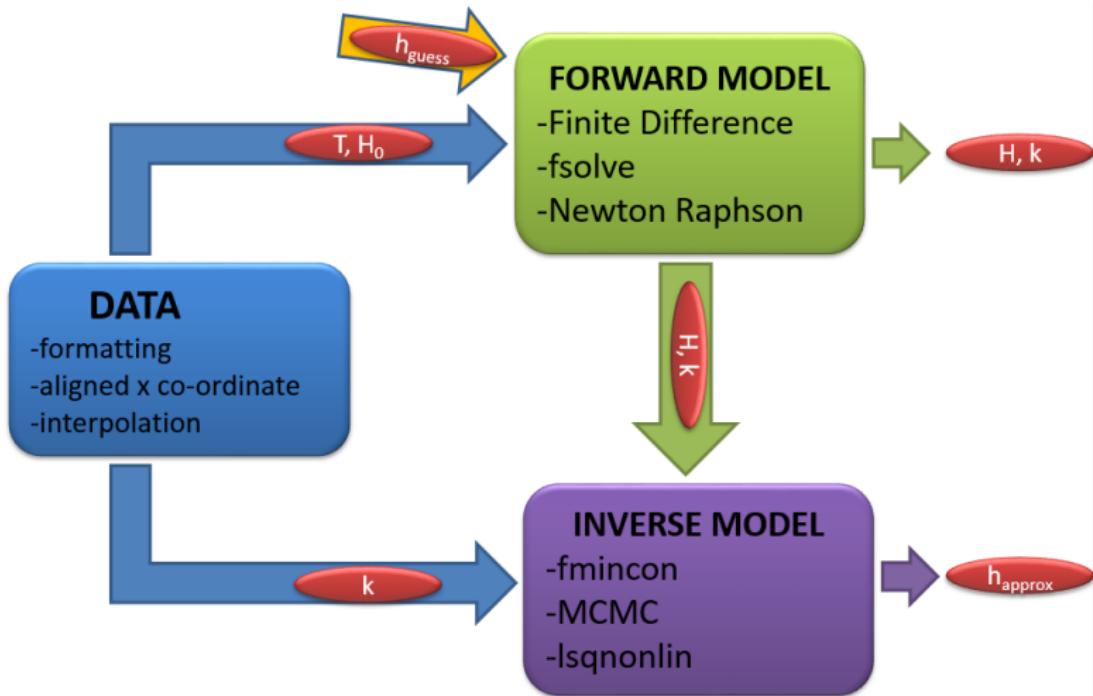
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Bathymetry is related to surface wave properties

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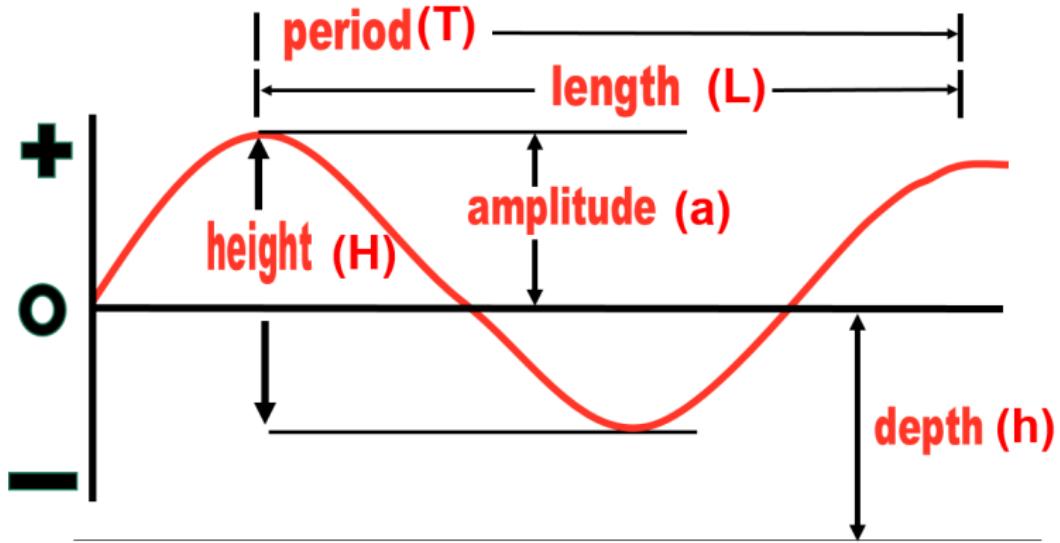
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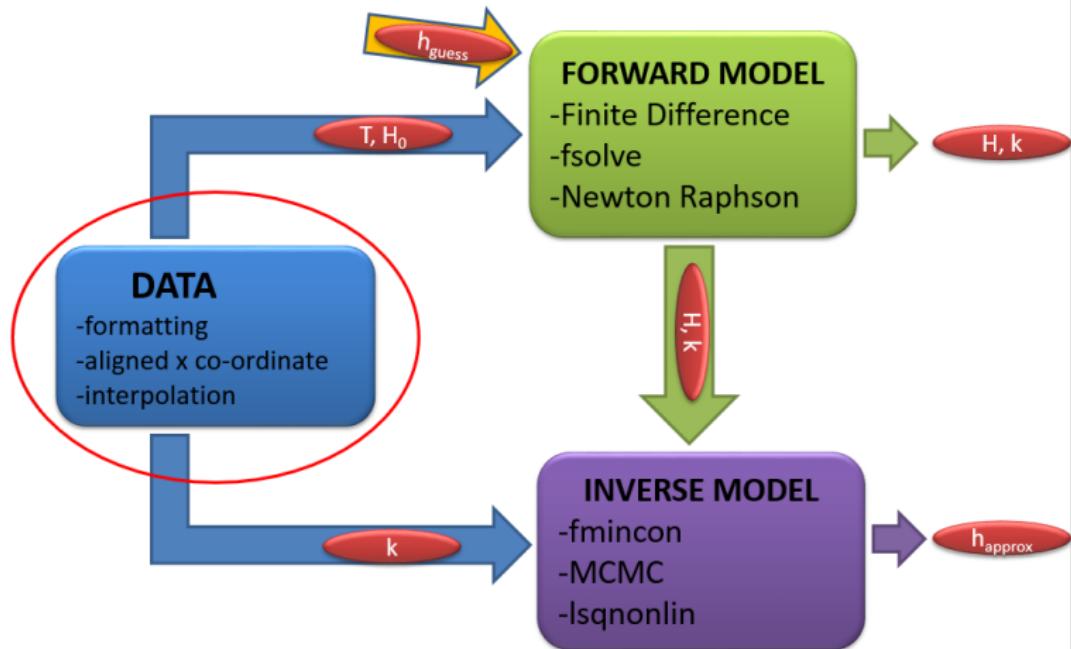
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$$\text{wave number: } k = \frac{2\pi}{L}$$

Before we invert we need data



Data was collected by the USACE in Duck, NC

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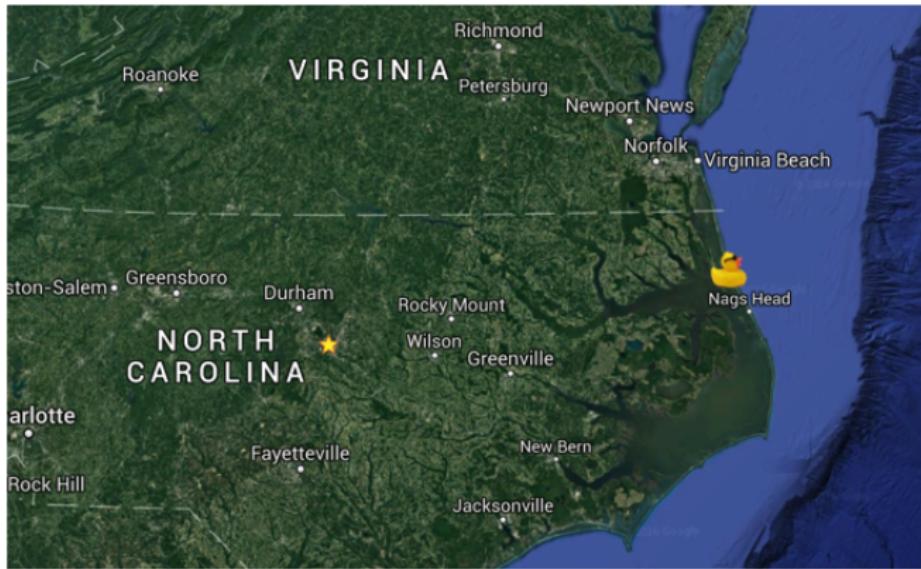
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Model coordinate system has $x = 0$ m offshore

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Remote sensing of surface properties are possible

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Data includes T , H at offshore boundary; 1D k

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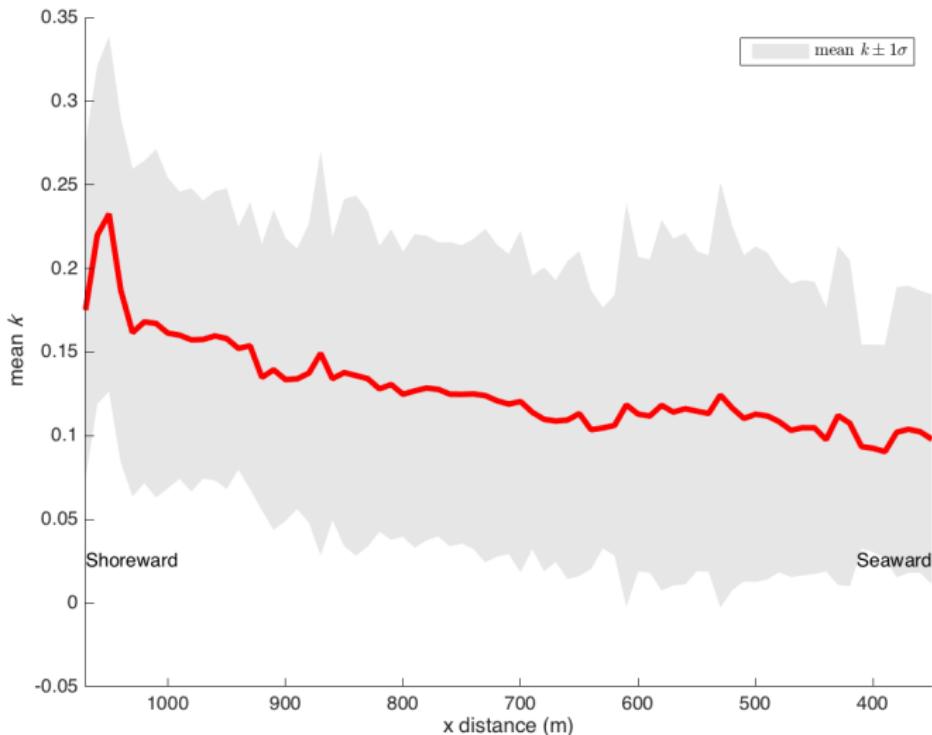
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Known bathymetry is used for testing our results

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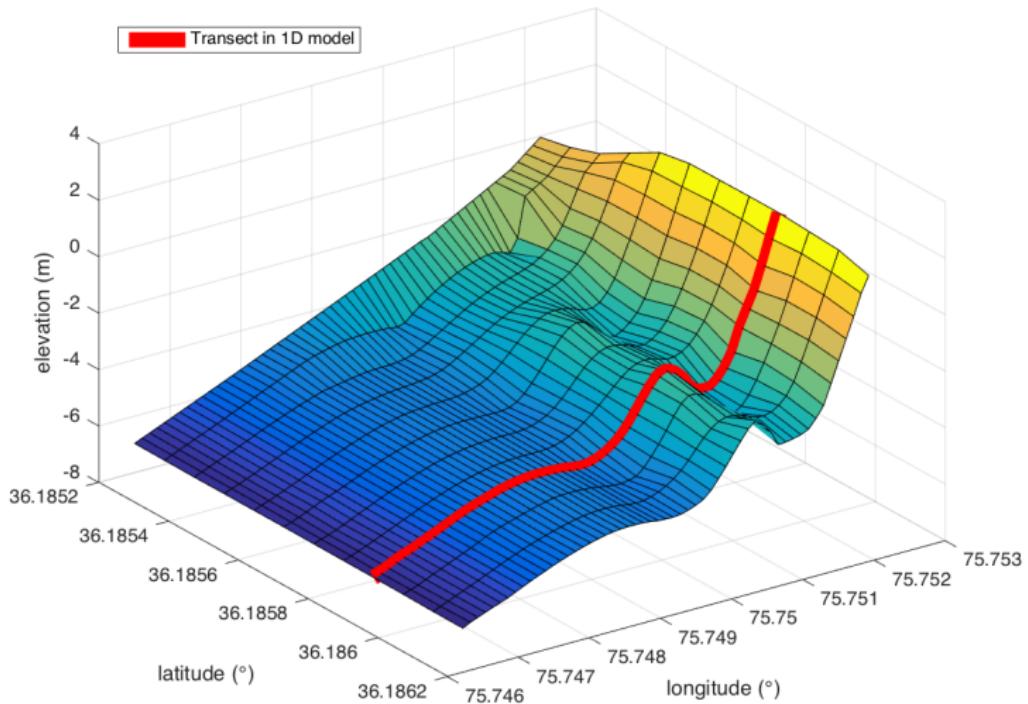
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Forward model computes k assuming h_{guess} & BC

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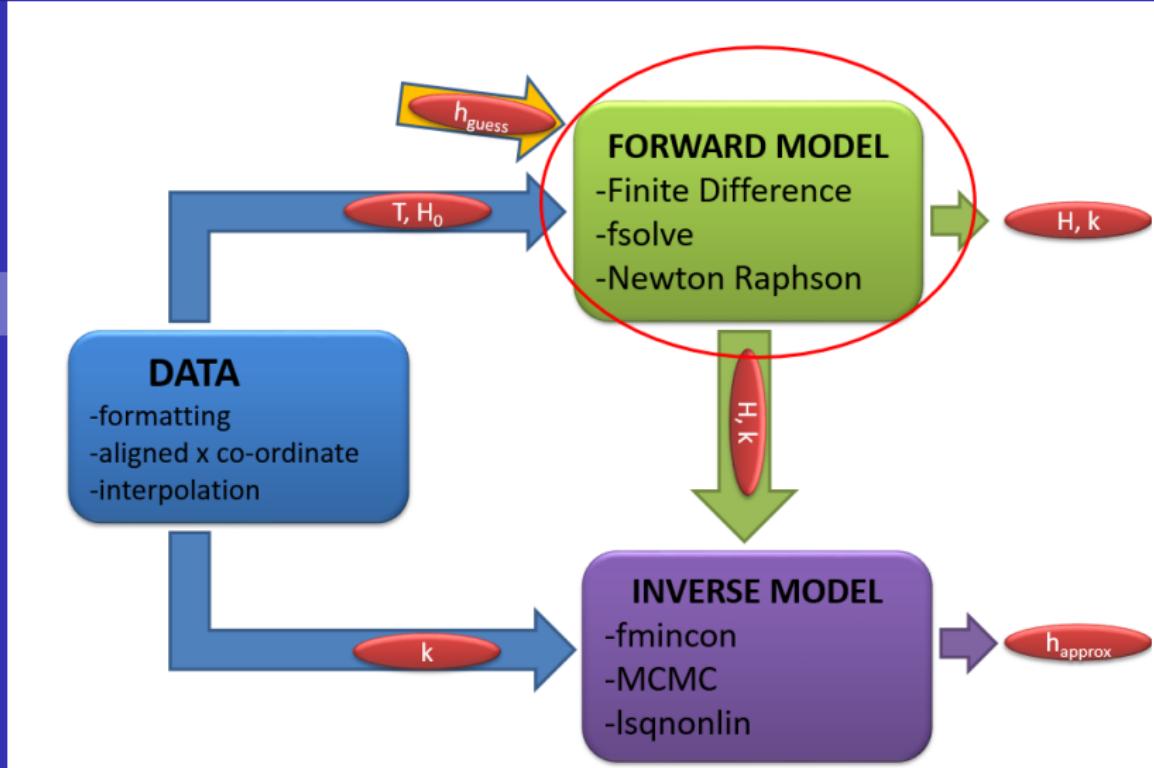
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Wave dispersion relationship relates k to h

Dispersion Relation [1]:

$$\sigma^2 = gk \tanh(kh) \iff \left(\frac{2\pi}{T}\right)^2 = g \left(\frac{2\pi}{L}\right) \tanh\left(\frac{2\pi h}{L}\right)$$

- Relates wave number (k) and Period (T)
- Wave length (L) varies with depth (h)
- Period (T) remains constant

1D forward model relates H and h

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Energy Flux Method [2]:

$$\frac{d}{dx} (EC_g) = -\delta$$

- Relates wave height (H) and wave number (k)
- $E(\rho, g, \mathbf{H})$: Wave Energy
- $C_g(\rho, \mathbf{k}, h)$: Group celerity
- $\delta(\rho, g, T, \mathbf{H})$: ³Wave energy dissipation function

¹Dean et al, 2000 (pg 64), ²Edward et al, 1983 (pg 2) and ³Alex et al, 2008 (pg 2).

1D forward model Results

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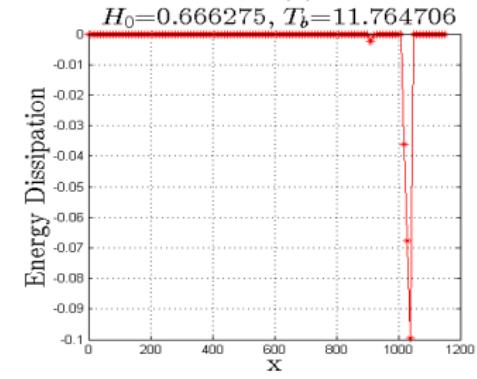
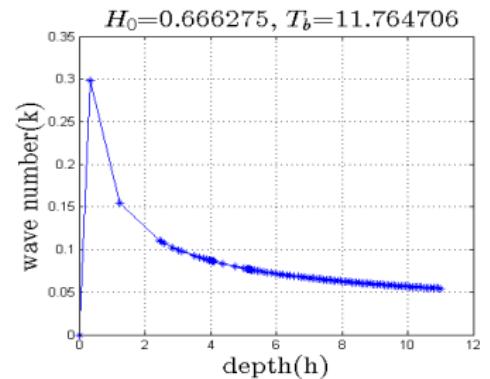
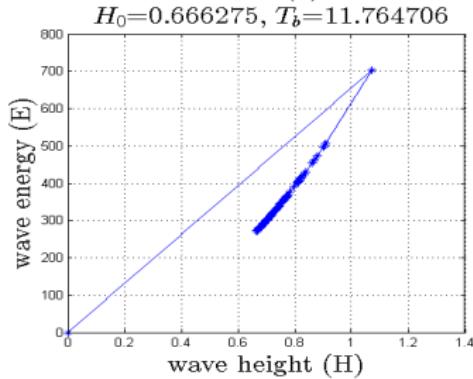
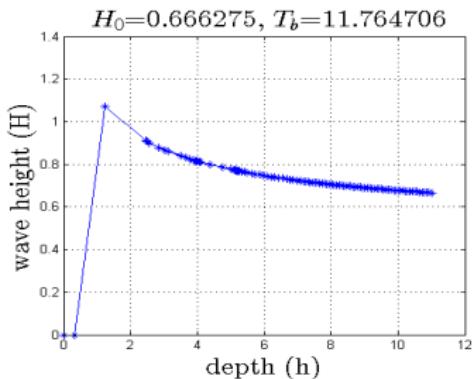
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Invert for bathymetry given surface data & physics

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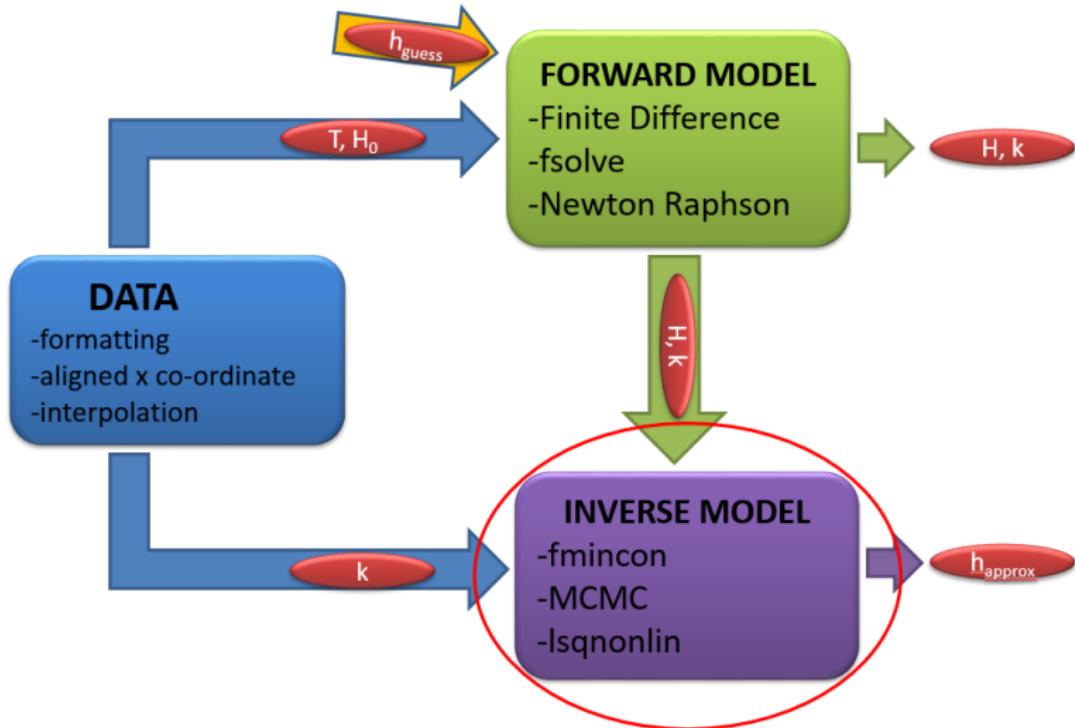
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Manufactured
Real

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Solutions are computed using 3 inversion methods

- ① Nonlinear Least Squares (lsqnonlin)
 - Logical place to start
- ② Bayesian MCMC (Metropolis)
 - Gives a distribution of depth estimates
- ③ Tikhonov Regularization (fmincon)
 - Bounded-constraint multivariate problem

Manufactured “data” is used to test our algorithms

Bathymetry Inversion from Waves

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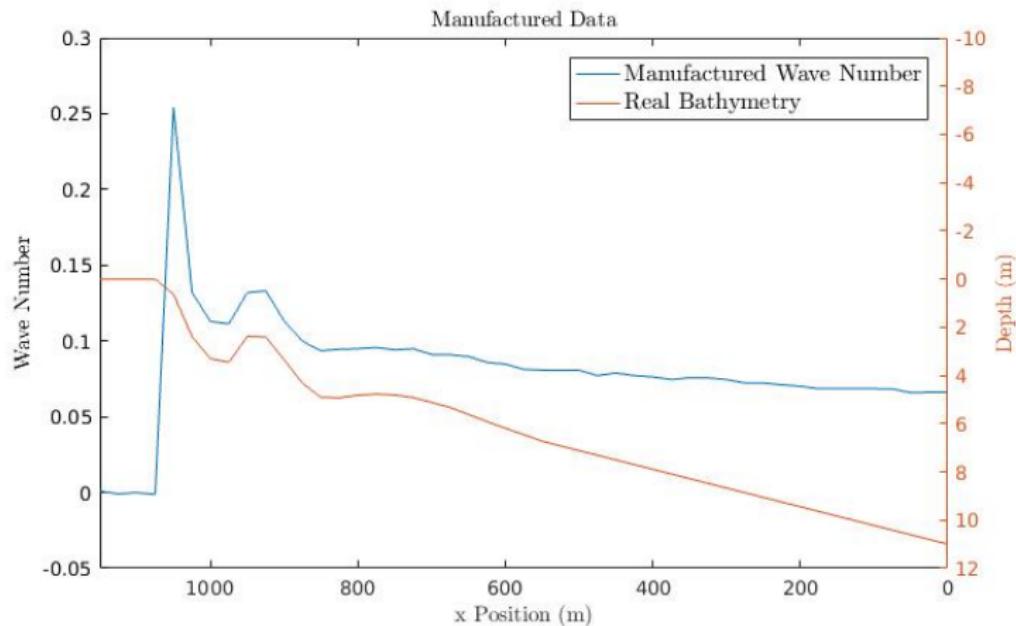
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Manufactured Real

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All methods capture the sandbar well

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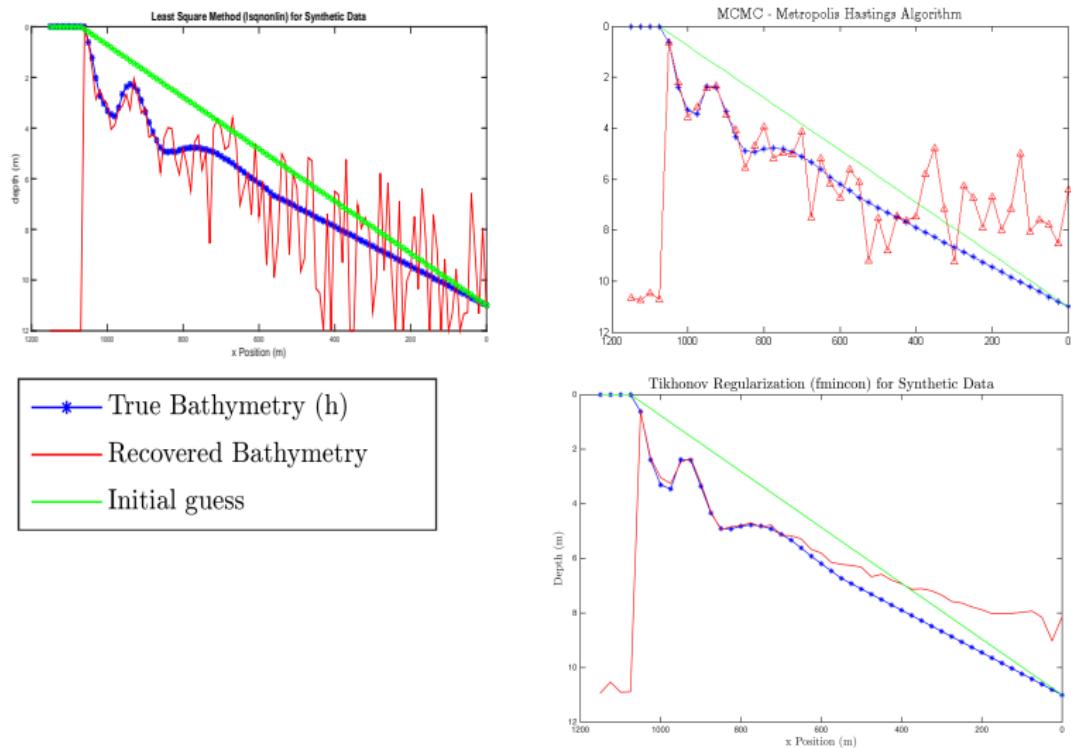
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Real k data is selected for a period with low noise

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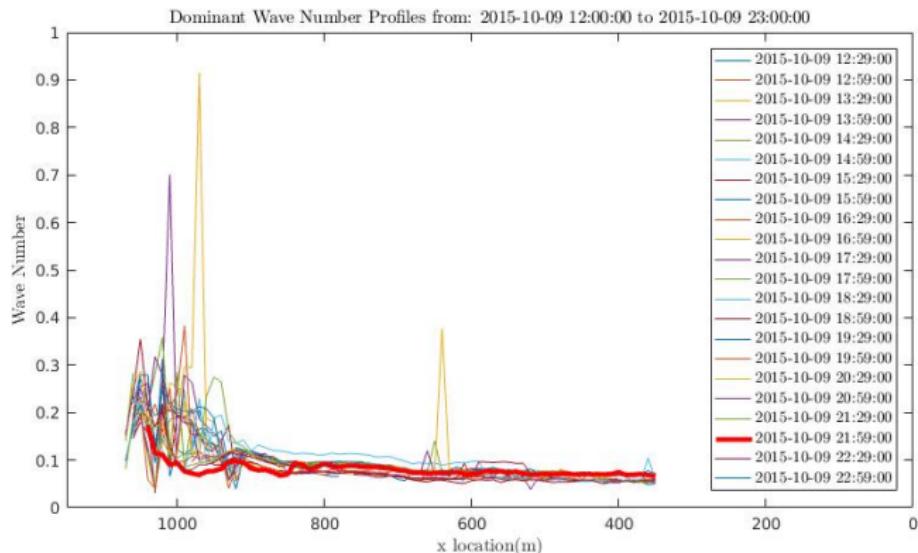
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Bathymetry estimates perform well in shallow water

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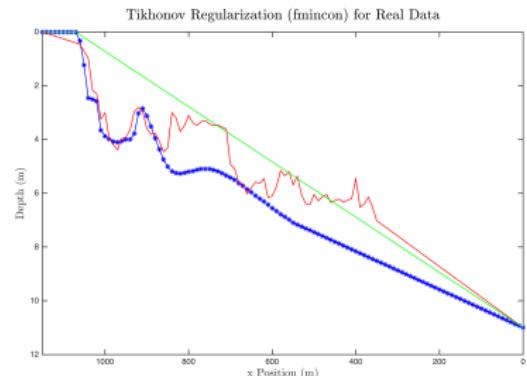
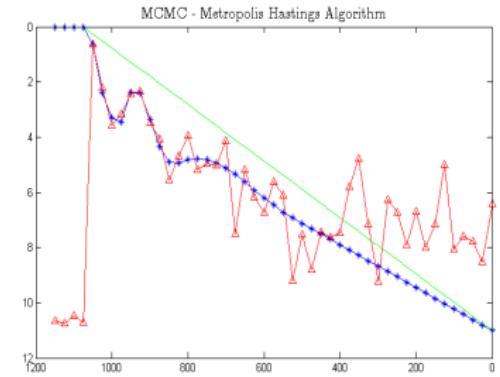
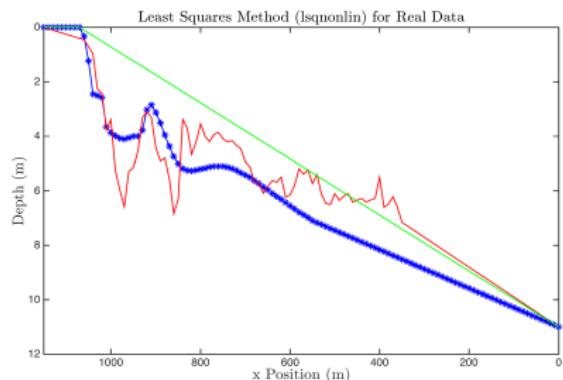
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Bathymetry estimates are limited by noisy k data

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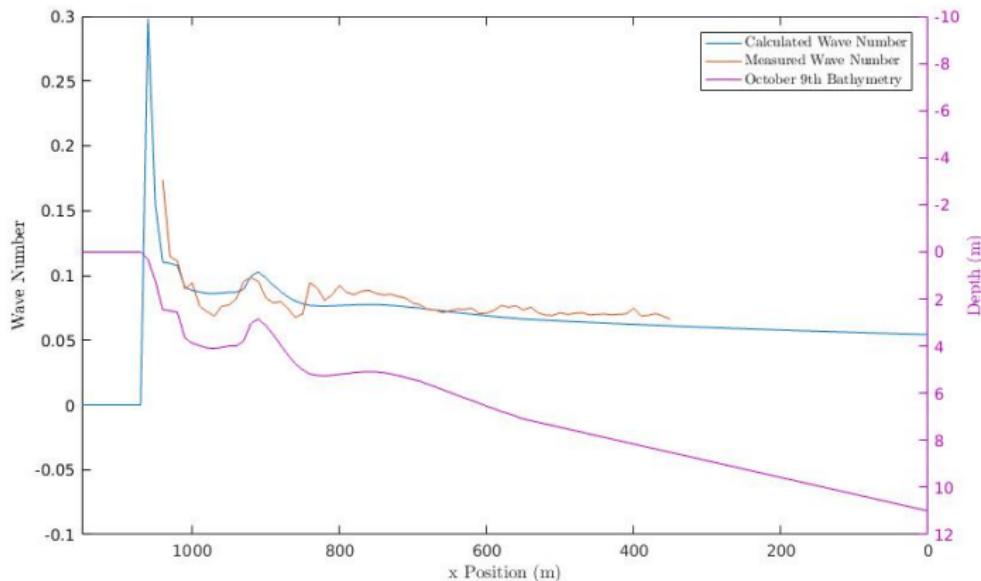
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Missing data & deep water cause poor h offshore

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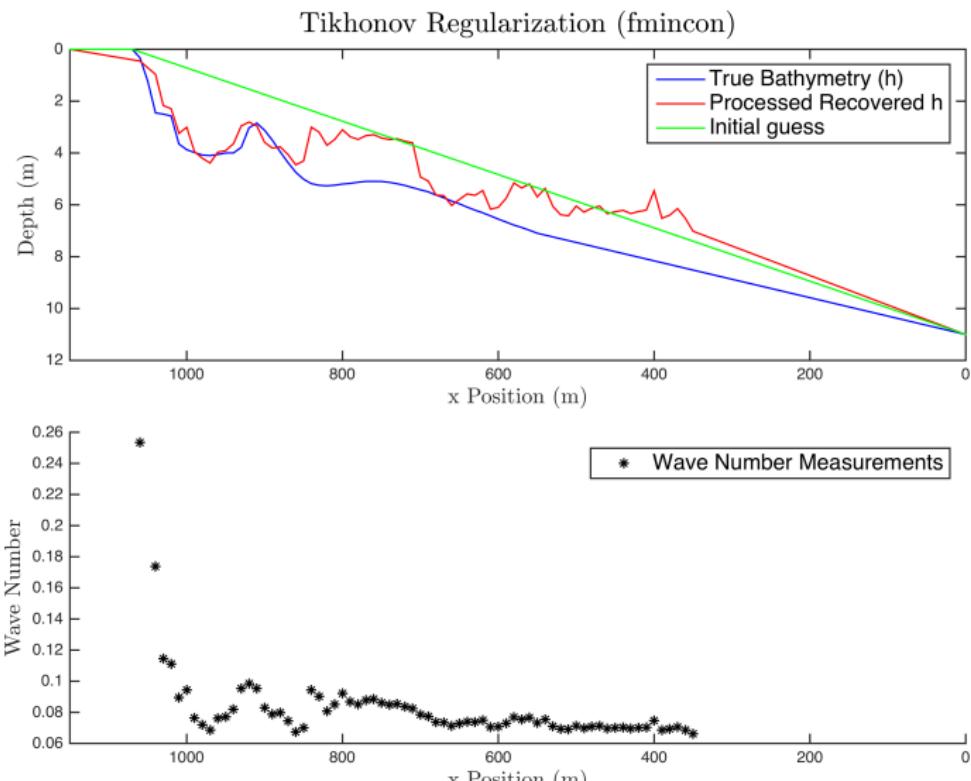
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Future Directions

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- Non-linear wave theory
 - Linear wave theory is just a starting point!
- Inclusion of observed wave height, H , or other measured variables, along the profile
 - this method will allow for assimilation of other measured variables
- Application of further regularization methods
 - we heuristically “tuned” our regularization
 - perhaps incorporate prior knowledge?
- Incorporate uncertainty in measurements
- Expansion to 2D wave physics

**Thank you!
Questions?**

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