Inverting for Near Coastal Bathymetry from Surface Wave Properties

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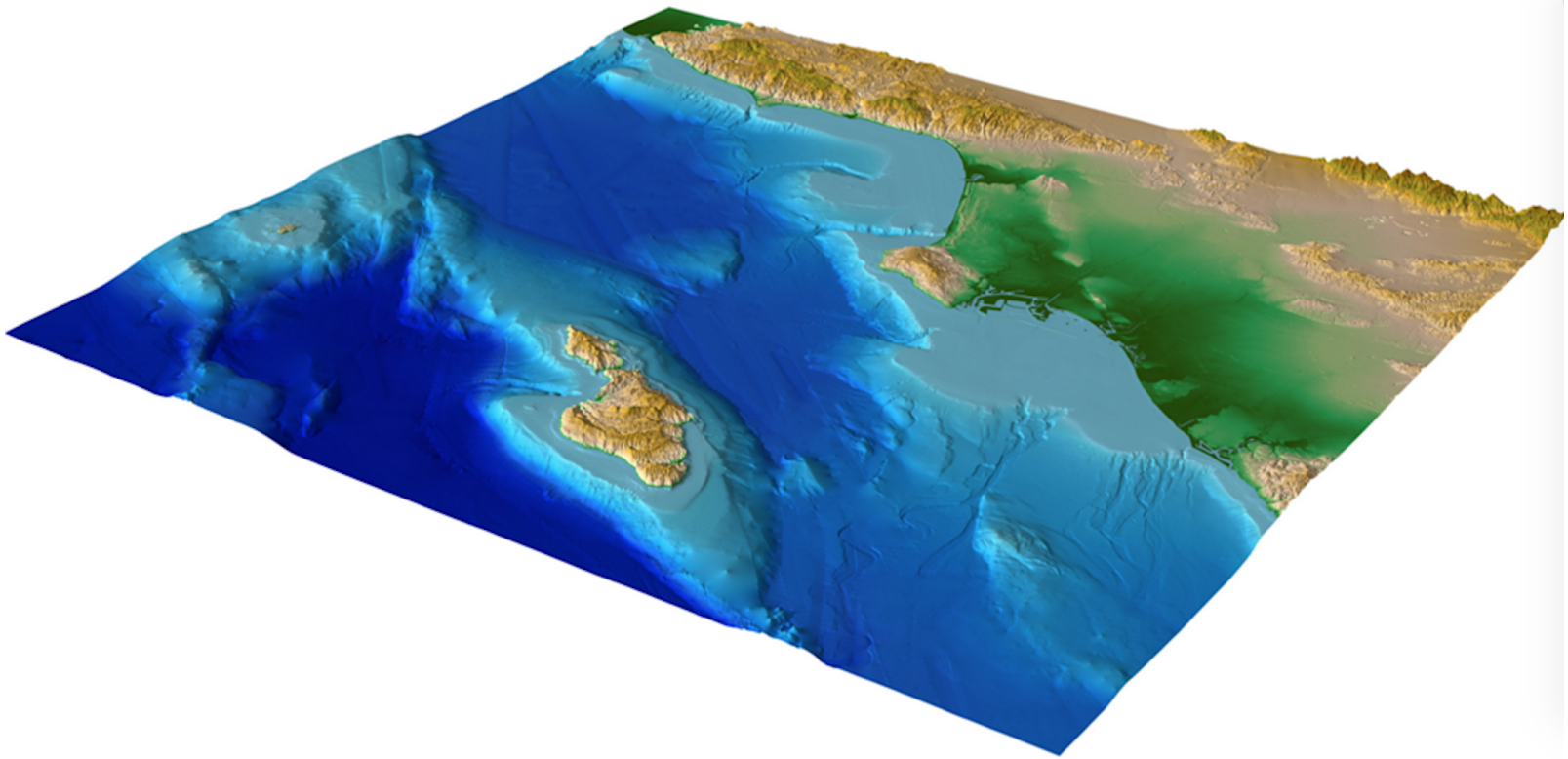
Many coastal processes are affected by bathymetry

Bathymetry is submarine topography

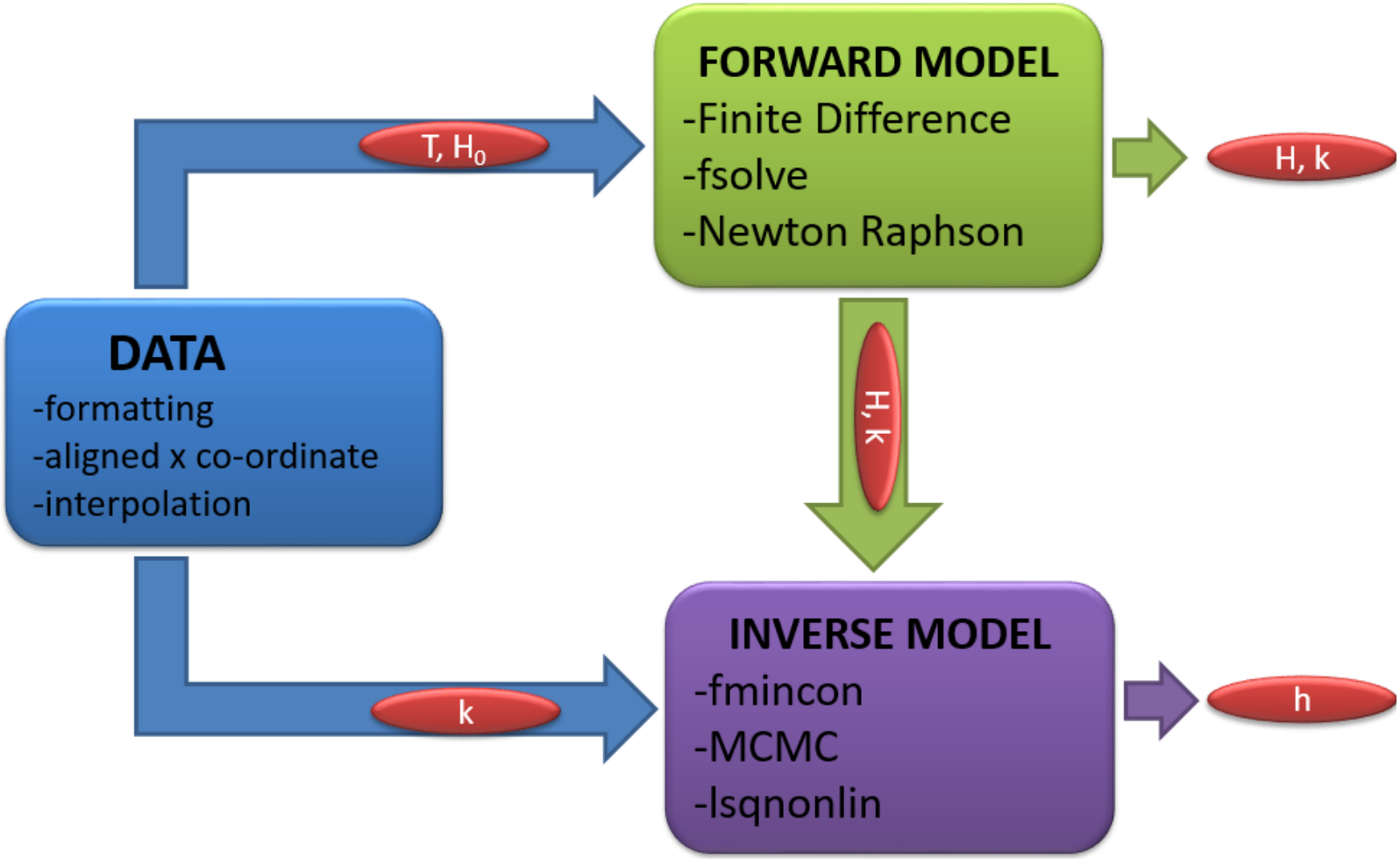


Direct measurements are expensive and challenging

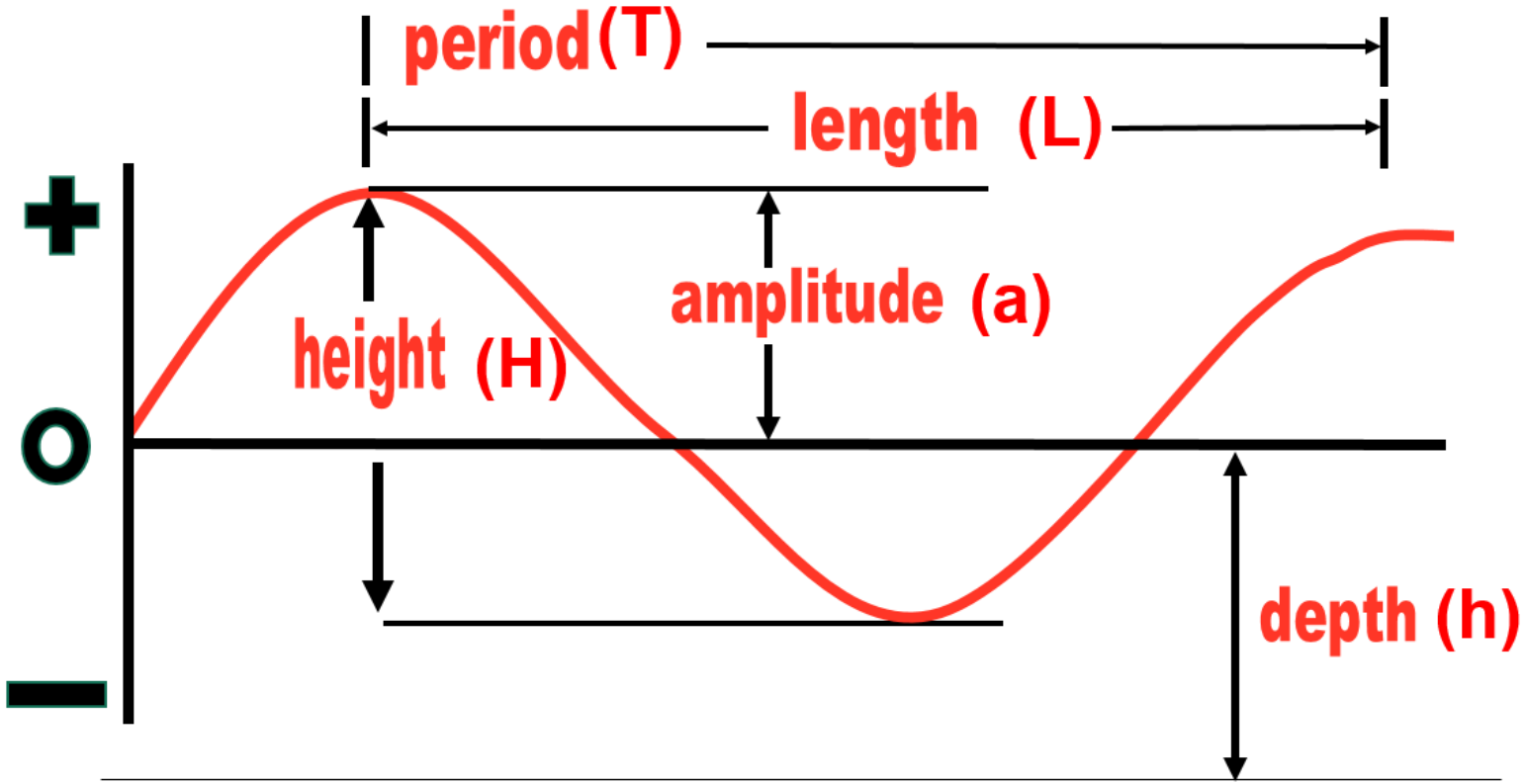
LARC

CRAB

Inverse models estimate depth using data & physics

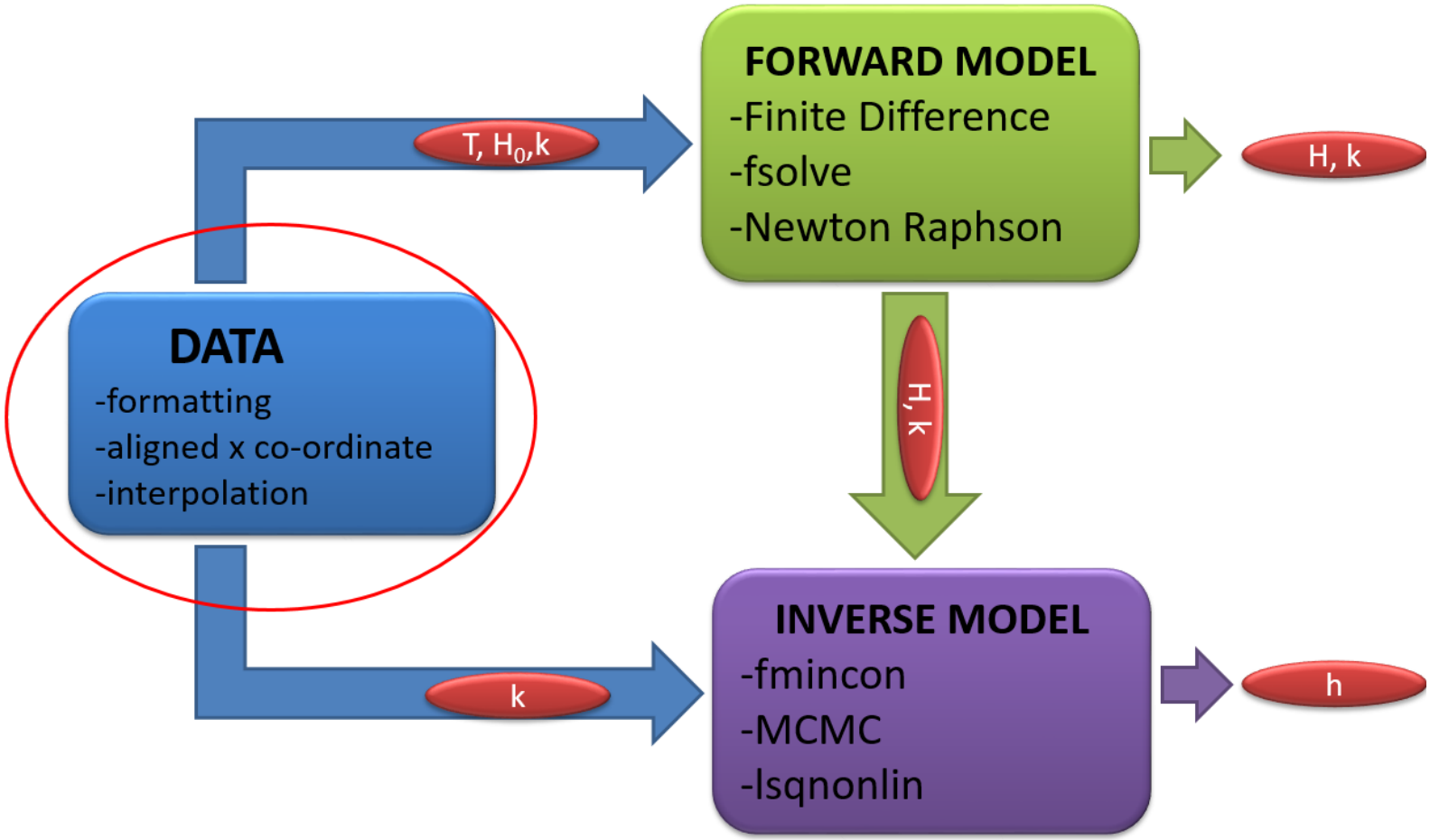


Bathymetry is related to surface wave properties

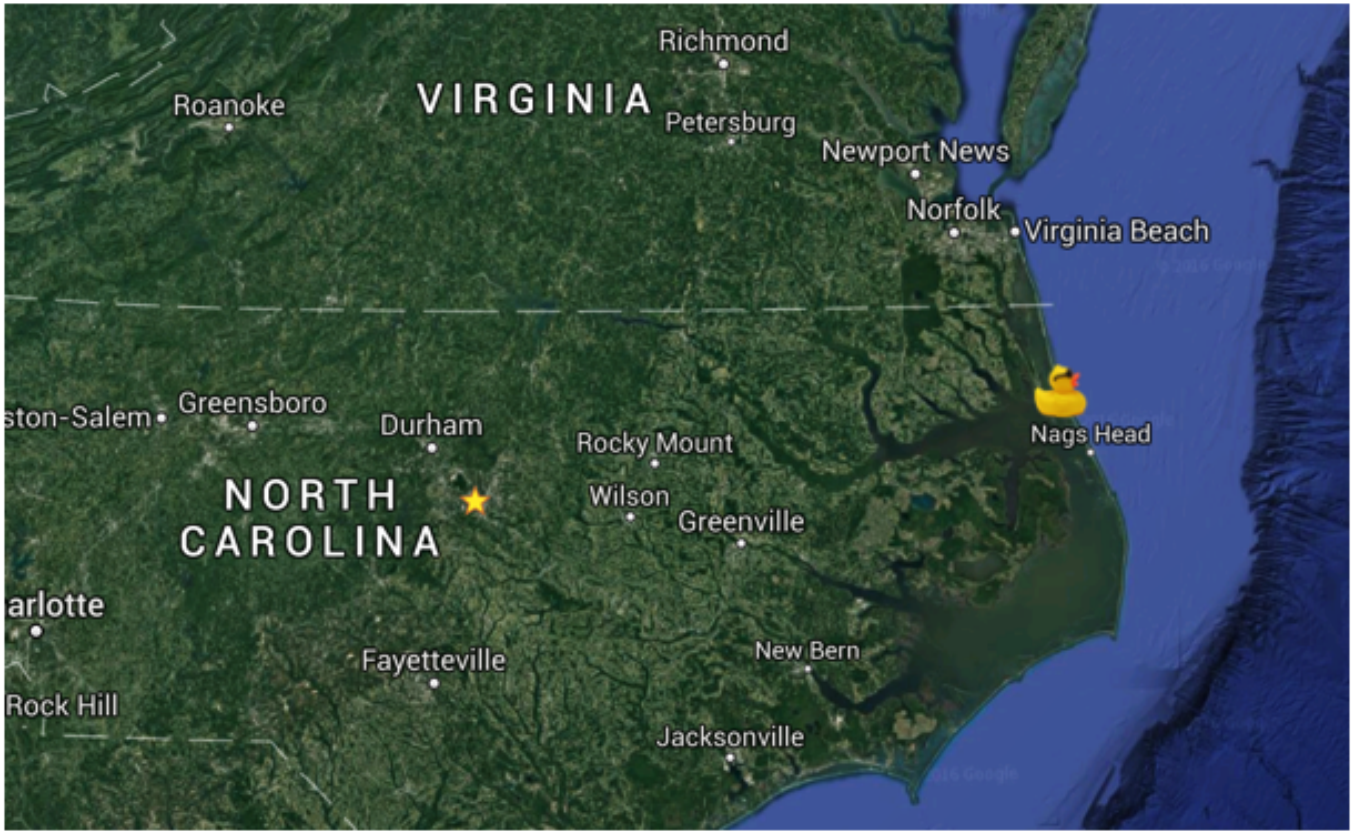


$$k = \frac{2\pi}{L} \tag{1}$$

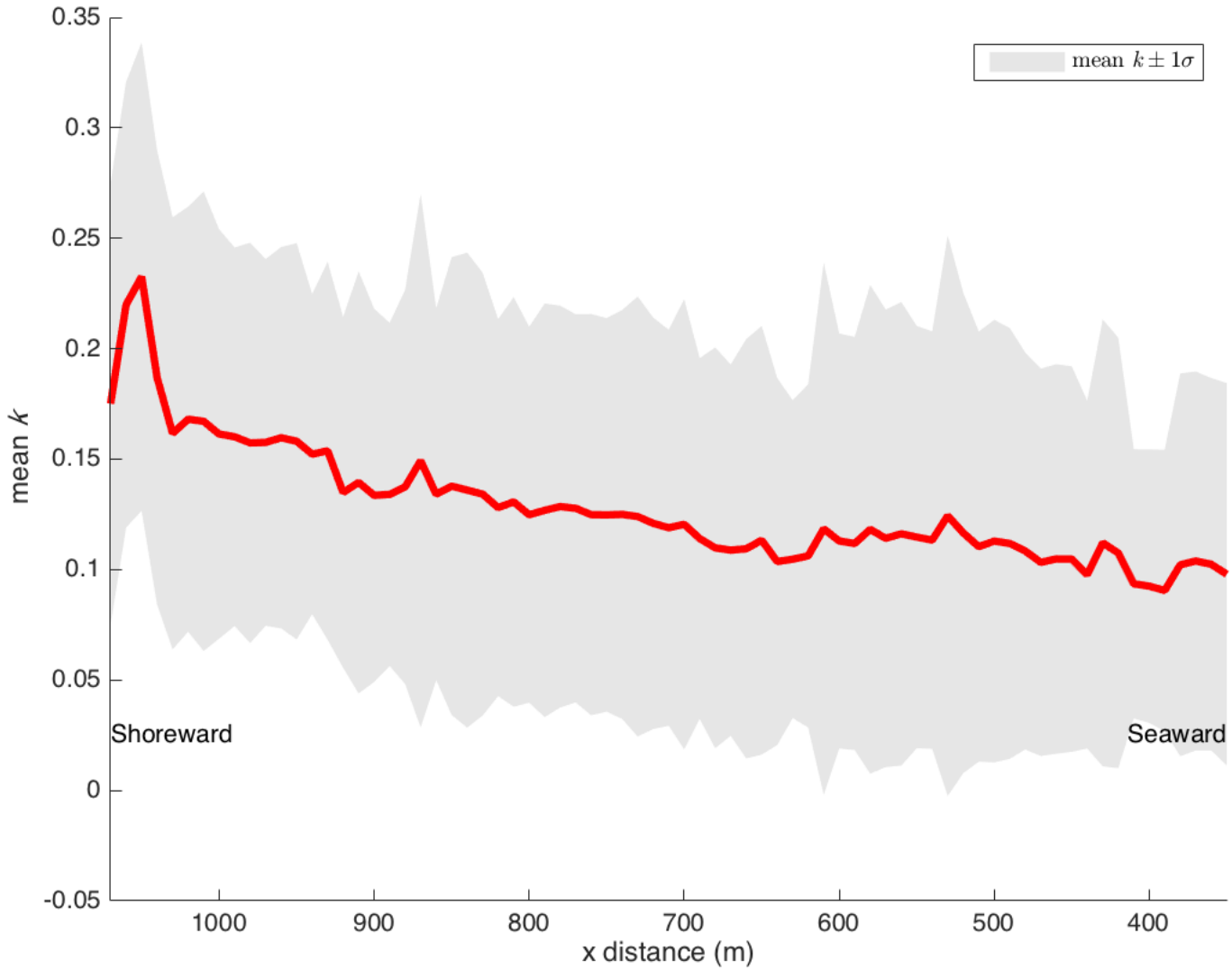
Before we invert we need data



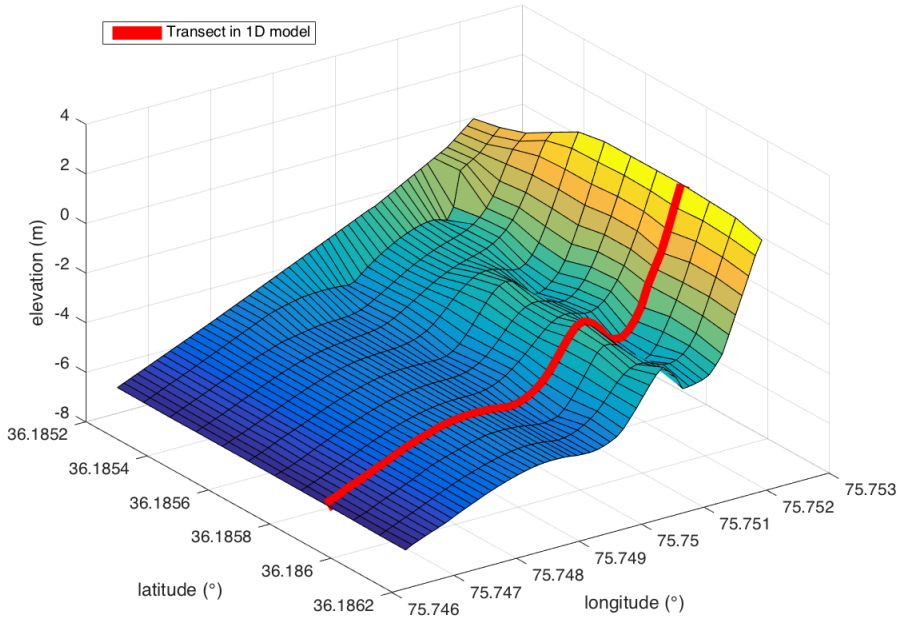
Data was collected by the USACE in Duck, NC

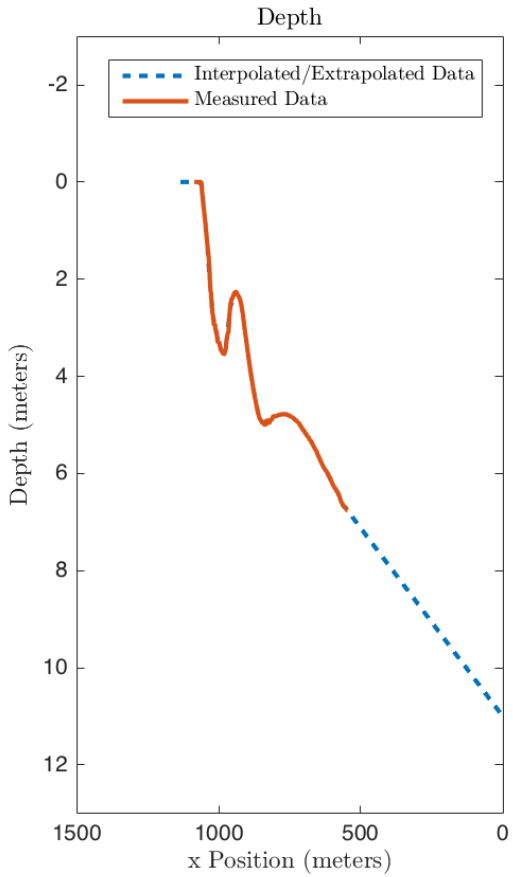


Data includes T, H at offshore boundary, 1D k

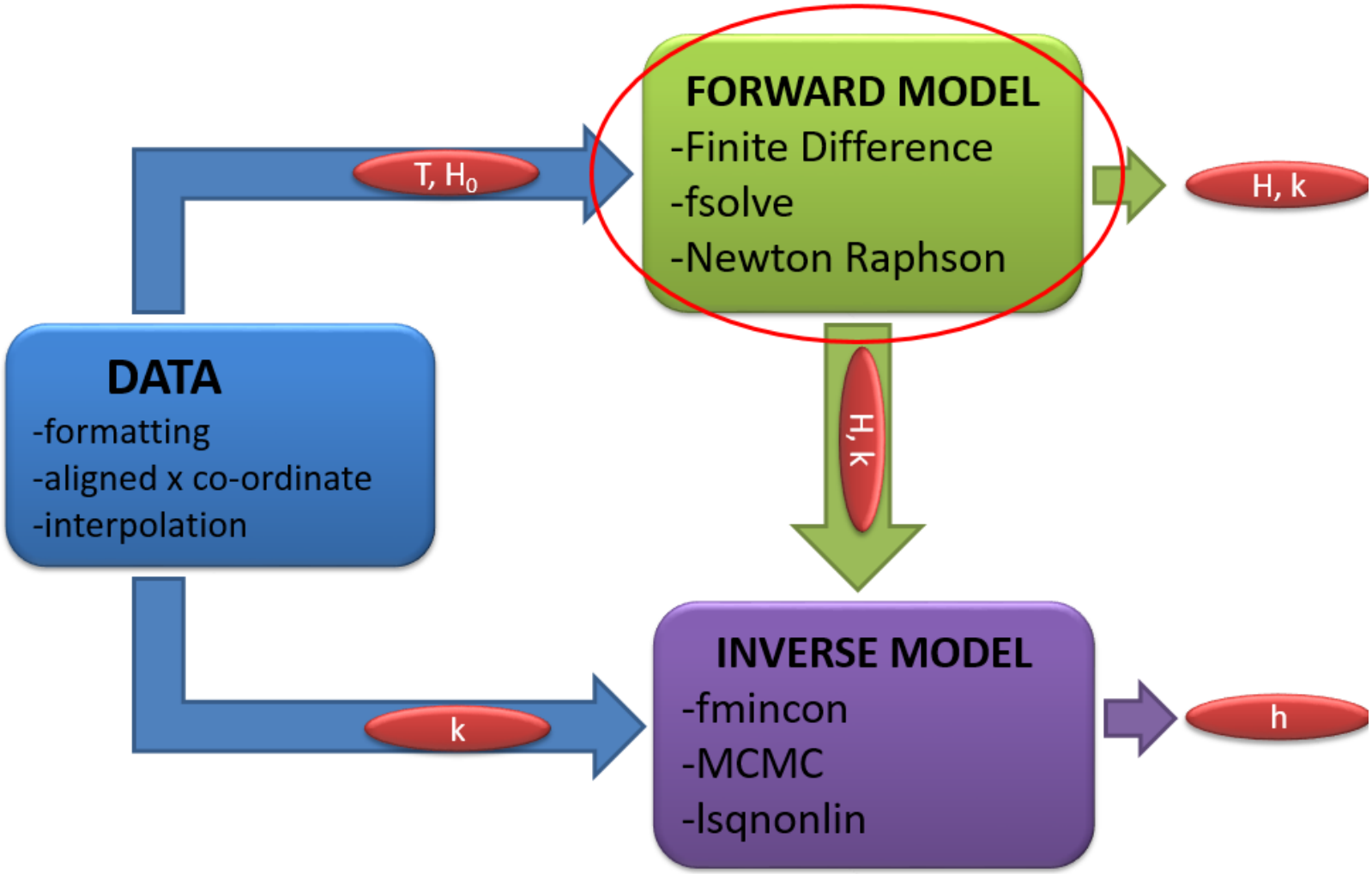


Known bathymetry is used for testing our results





Forward model computes k assuming h_{guess} & BC



1D wave physics is known for near-coastal regions

Assume linear wave theory:

$$\begin{cases} \frac{d}{dx}\left(EC_g\right) = -\delta, \\ \\ \sigma^2 = gk\tanh(kh), \end{cases}$$

$$\frac{d}{dx}\left(\frac{\lambda}{k}\left(1+\frac{2kh}{\sinh(2kh)}\right)H^2\right) = -\delta$$

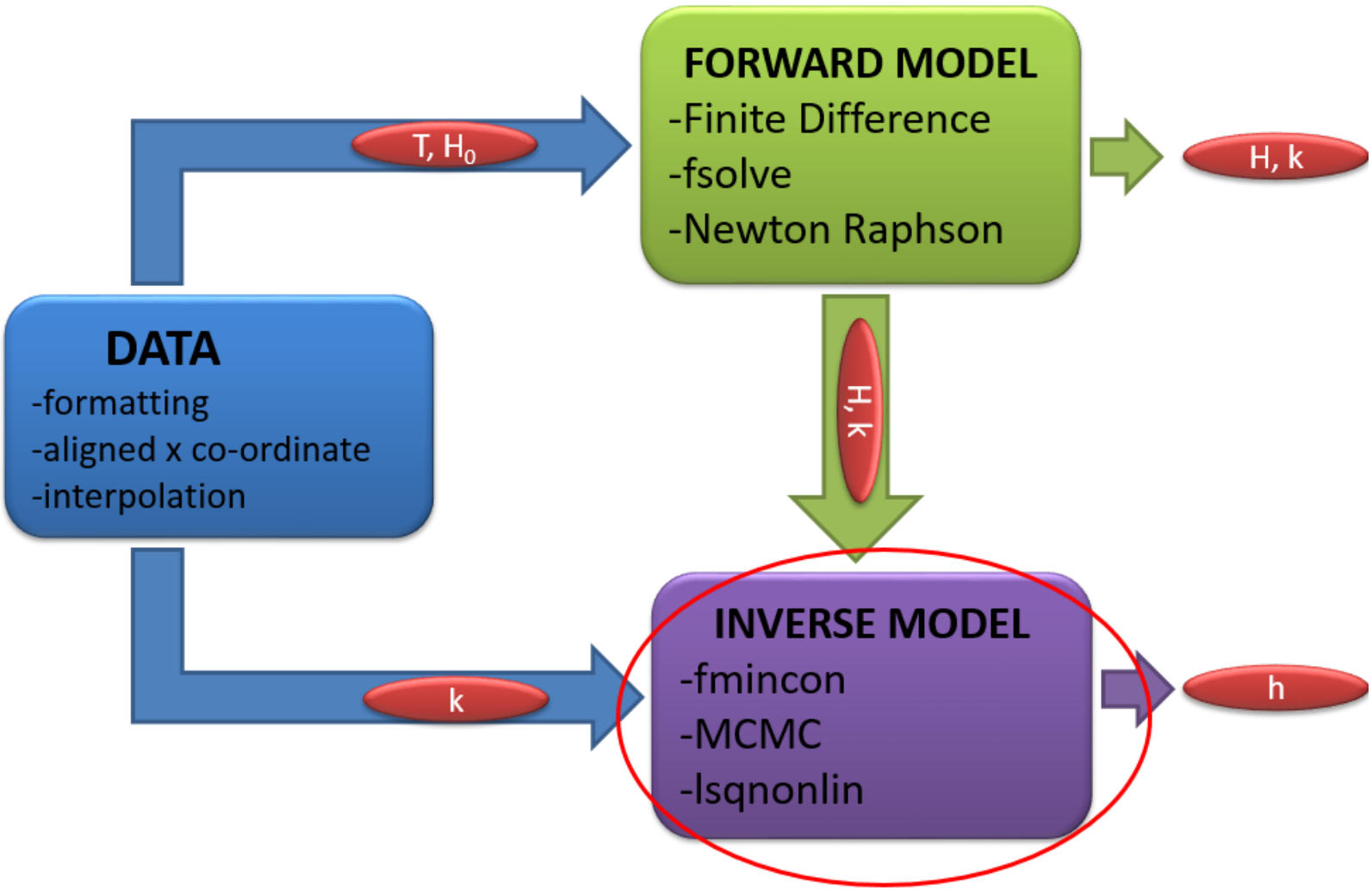
where,

$$E : Wave\, Energy,\ C_g : Group\, celerity,$$

$$c : Wave\, celerity,\ \sigma : Angular\ frequency,$$

$$g : Gravitational\ acceleration, k : Wave\, number$$

We invert for bathymetry given the surface data and physics



Inversion Methods

- Nonlinear Least Squares
- MCMC
- fmincon

Synthetic Data

Synthetic Results plots for each

Real data for runs

Real Results plots for each

Future Directions

- Use of wave heights
- Regularization methods
- 2D problem

THANK YOU!

Additive Gaussian Noise Model

Gaussian noise $\boldsymbol{\epsilon}$ corrupted measurements $\mathbf{d}$ with variance ν is given by

$$\mathbf{d} = \mathbf{A}\mathbf{h}_t + \boldsymbol{\epsilon}.$$

$\mathbf{d}$ = a vector of measurements,
$\mathbf{A}$ = a linear forward operator,
$\mathbf{h}_t$ = the true bathymetry.

fmincon: Tikhonov Method

Uses a regularized solution with prior information

$$\hat{\mathbf{h}} = \underset{\mathbf{h}\in\mathbb{R}^n}{\arg\min} \|\mathbf{A}\mathbf{h} - \mathbf{d}\|_2^2 + \alpha\|\mathbf{h} - \mathbf{h}_p\|_2^2,$$

Bayesian Markov Chain Monte Carlo (MCMC) Method

The MCMC method creates a posterior distribution of depth profiles, given wave number by using the Bayes relationship

$$P(h|k) \propto \Pi(h)L(h|k), \tag{2}$$

MCMC Method: Metropolis Algorithm

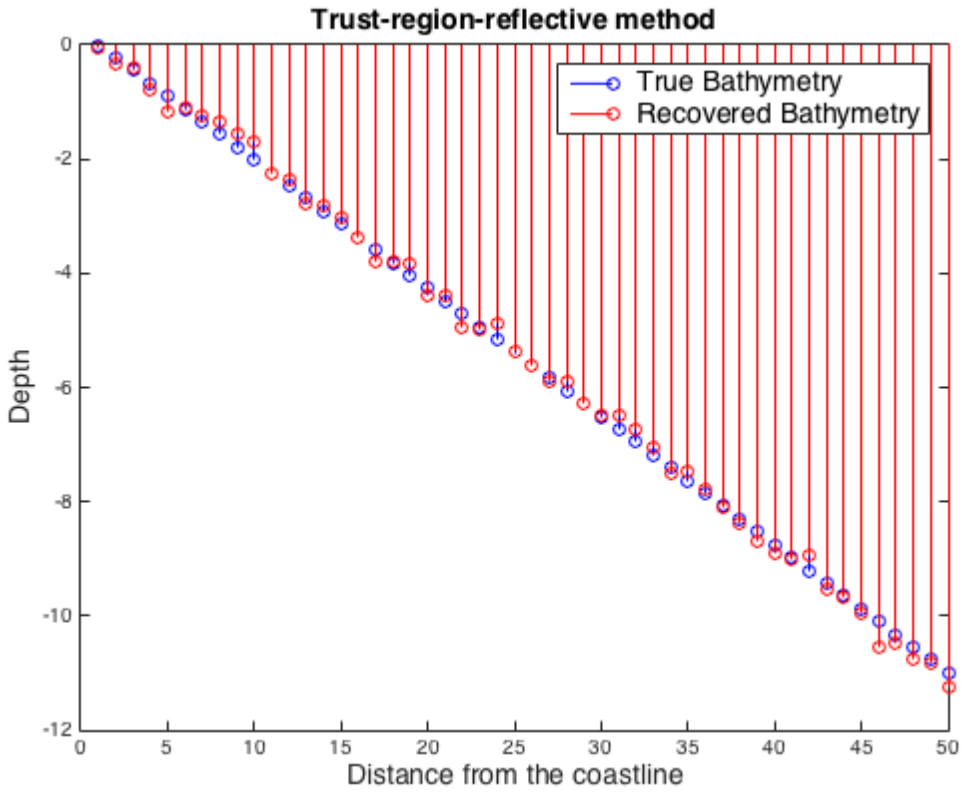
- Prior and likelihood are combined to compute an initial posterior probability distribution of h

$$P(h|k) = log(\Pi(h)) + log(L(h|k)) \tag{3}$$

- Uses a markov chain random walk to arrive at a posterior distribution of h profiles

Nonlinear Least Squares: Trusted Region-Reflective Method

$$\hat{\mathbf{h}} = \underset{\mathbf{h}\in\mathbb{R}^n}{\arg\min} \quad f(\mathbf{h}) = \|\mathbf{A}\mathbf{h} - \mathbf{d}\|_2^2, \tag{4}$$



Results

MCMC Method: Log-Likelihood Function

Loglikelihood function compares simulated and observed k values

$$\log L(h|k) = loge^{-\frac{\sum_{i=1}^{n}(k_{m,i}-k_{d,i})^2}{2\sigma_d^2}} \tag{5}$$