

# Inverting for Near Coastal Bathymetry from Surface Wave Properties

Lasith Adhikari, Charnelle Bland,  
Lopamudra (Monty) Chakravarty, Wenbin Dong,  
Olaniyi Samuel Iyiola, Gail Muldoon, Clint Seinen

Supervised by  
Ty Hesser (USACE) & Lea Jenkins (Clemson)

Industrial Mathematical and Statistical Modeling Workshop

July 2016

# Many coastal processes are affected by bathymetry

Bathymetry  
Inversion  
from Waves

Introduction

Data

Forward  
Model

Inverse  
Methods

Discussion



# Bathymetry is submarine topography

Bathymetry  
Inversion  
from Waves

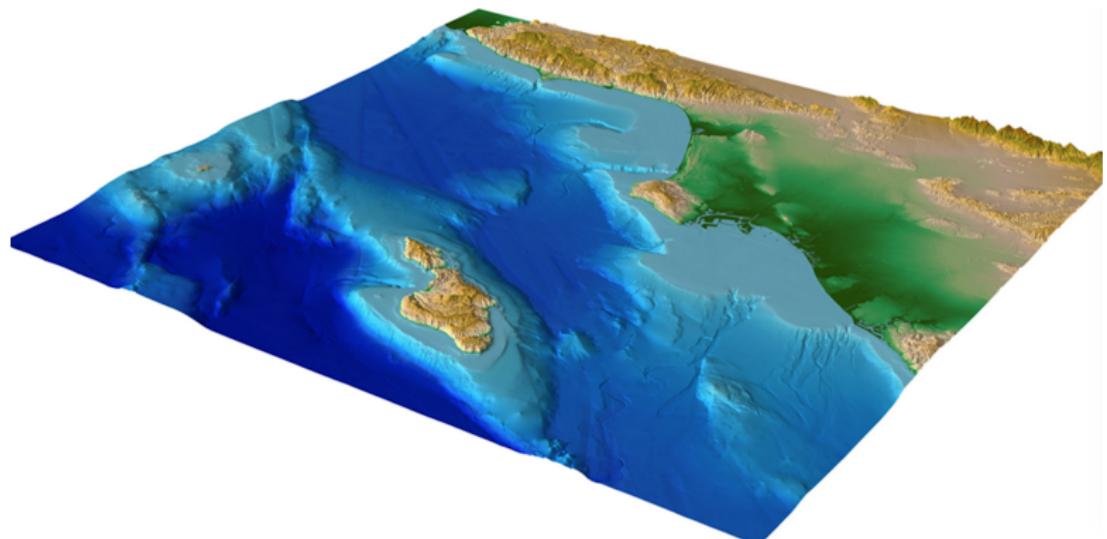
Introduction

Data

Forward  
Model

Inverse  
Methods

Discussion



# Direct measurements are expensive and challenging

Bathymetry  
Inversion  
from Waves

Introduction

Data

Forward  
Model

Inverse  
Methods

Discussion



LARC



CRAB

# Inverse models estimate depth using data & physics

Bathymetry  
Inversion  
from Waves

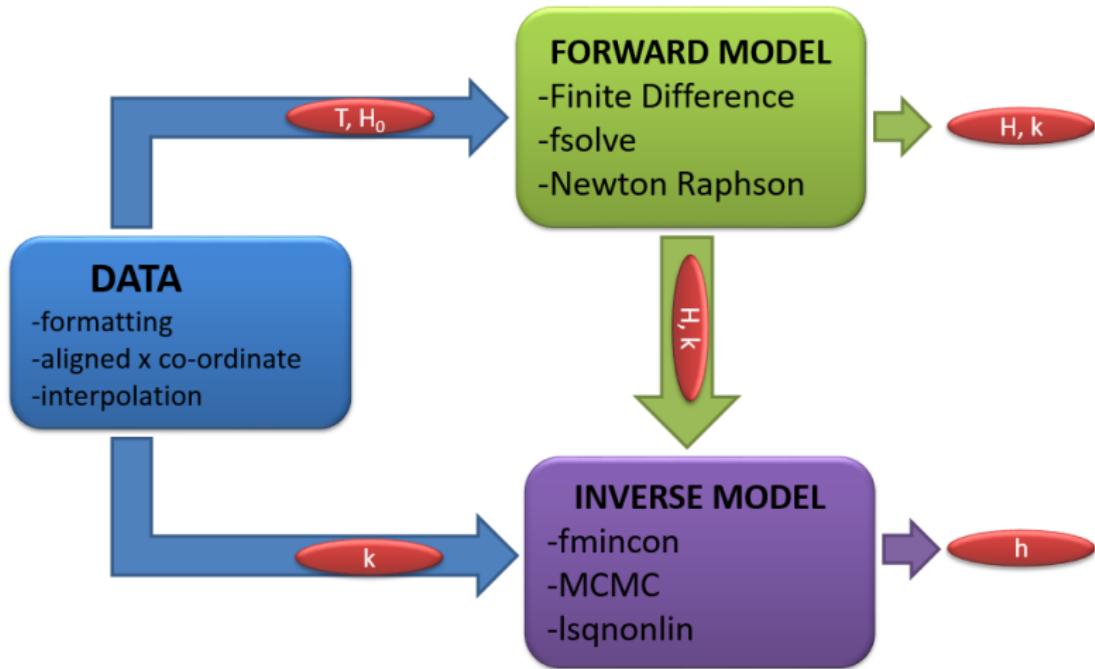
Introduction

Data

Forward  
Model

Inverse  
Methods

Discussion



# Bathymetry is related to surface wave properties

Bathymetry  
Inversion  
from Waves

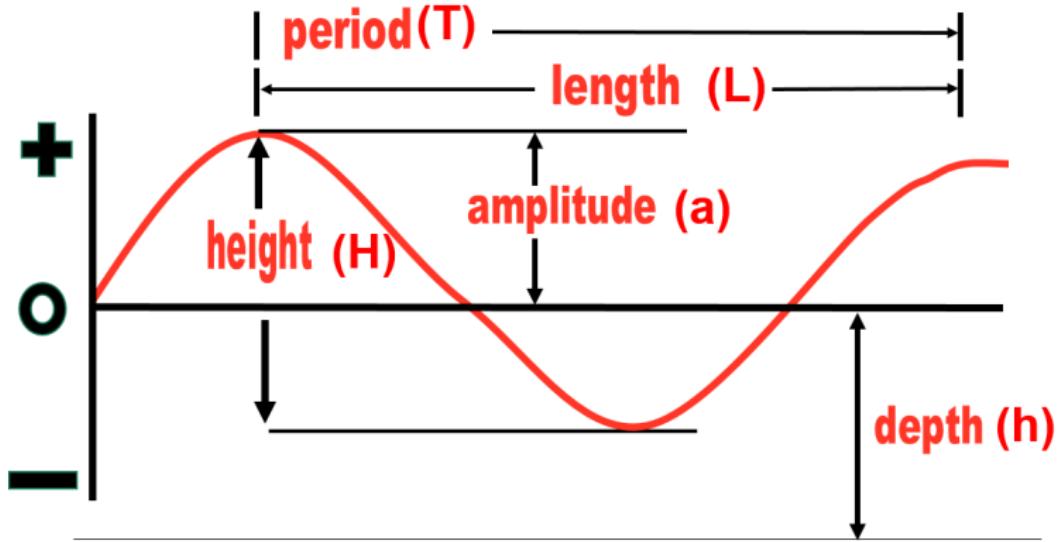
Introduction

Data

Forward  
Model

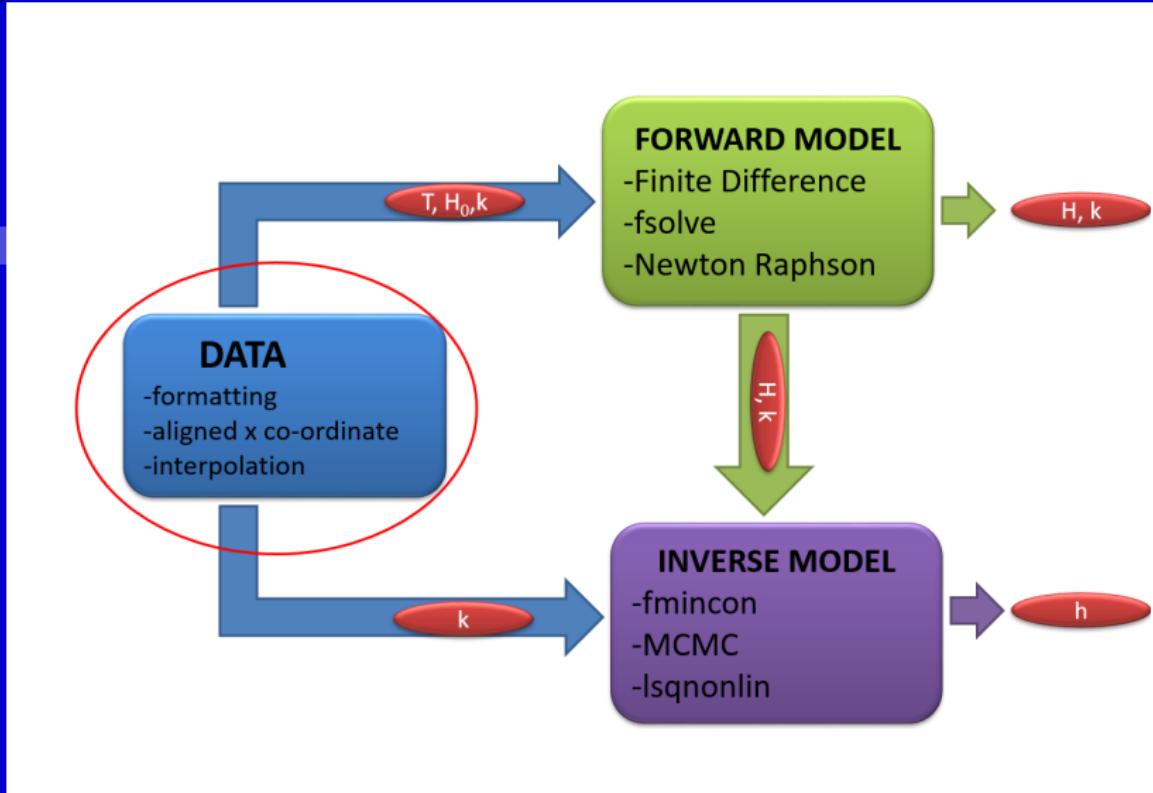
Inverse  
Methods

Discussion



$$k = \frac{2\pi}{L} \quad (1)$$

# Before we invert we need data



# Data was collected by the USACE in Duck, NC

Bathymetry  
Inversion  
from Waves

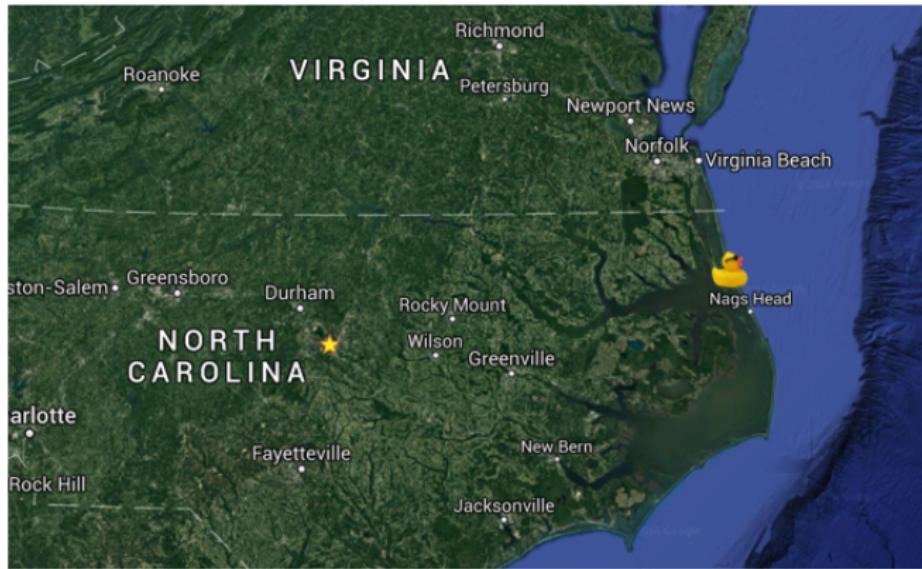
Introduction

Data

Forward  
Model

Inverse  
Methods

Discussion



# Data includes $T$ , $H$ at offshore boundary, 1D $k$

Bathymetry  
Inversion  
from Waves

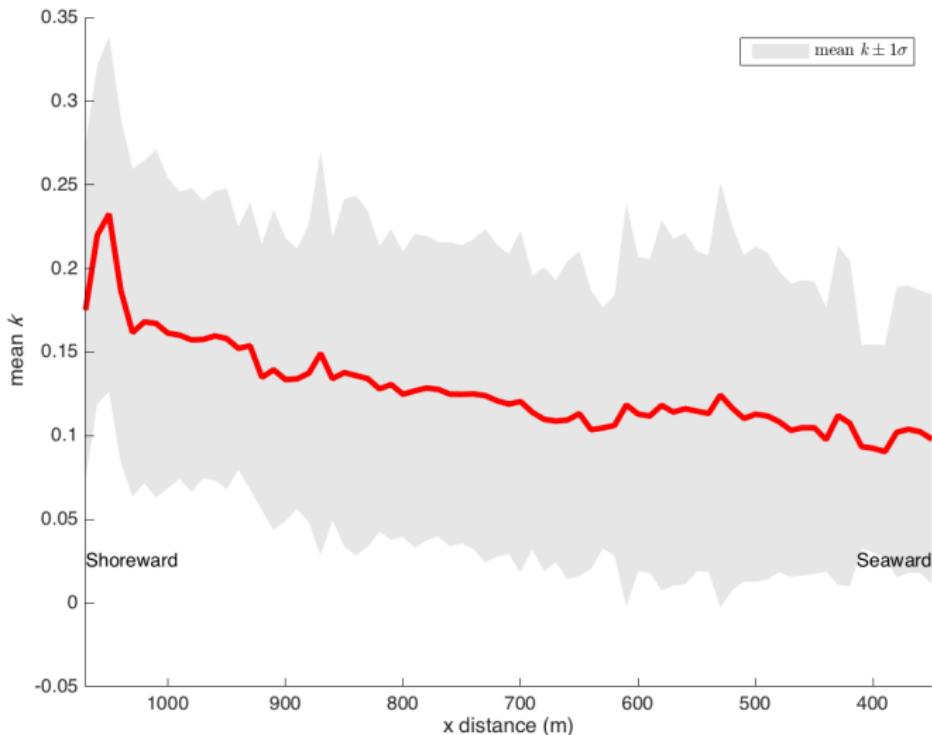
Introduction

Data

Forward  
Model

Inverse  
Methods

Discussion



# Known bathymetry is used for testing our results

## Bathymetry Inversion from Waves

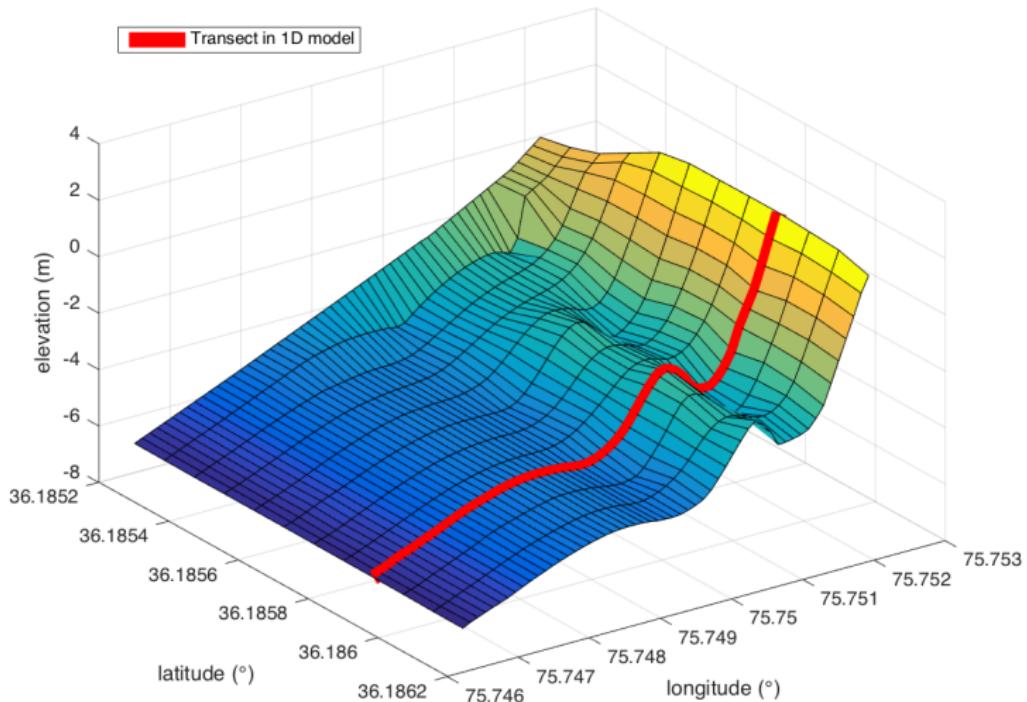
### Introduction

### Data

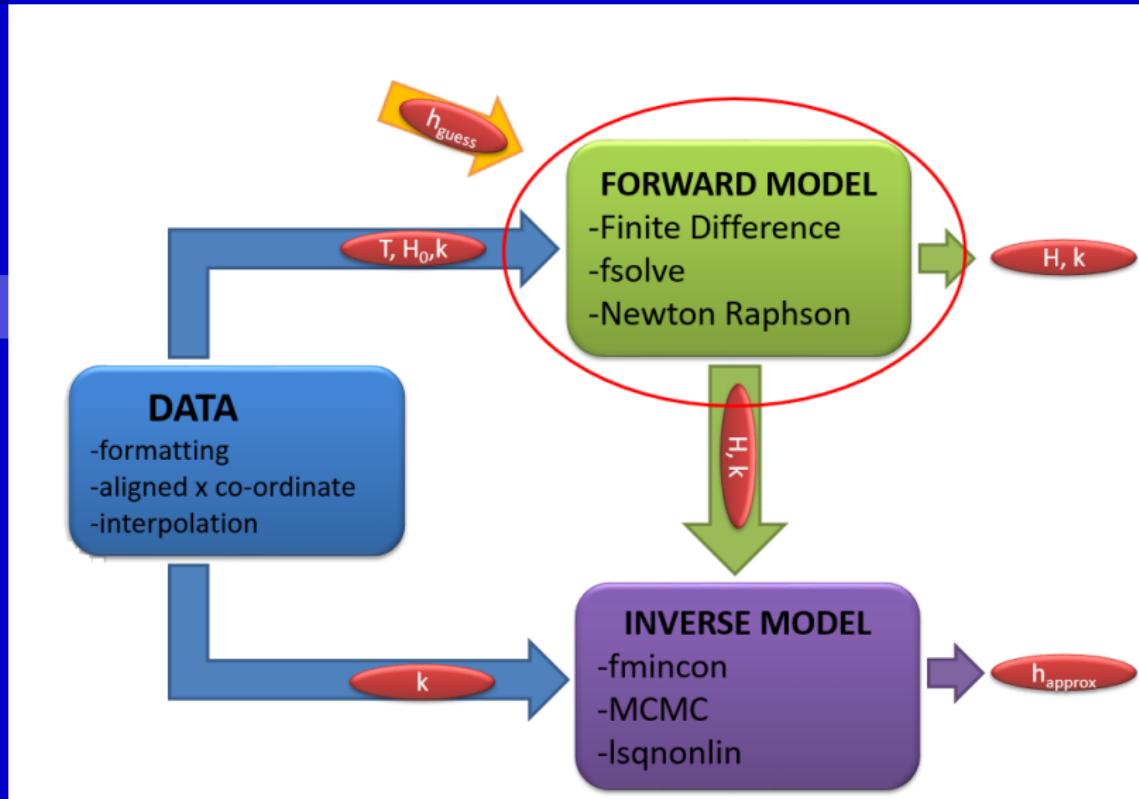
### Forward Model

### Inverse Methods

### Discussion



# Forward model computes $k$ assuming $h_{guess}$ & BC



# 1D wave physics is known for near-coastal regions

Bathymetry  
Inversion  
from Waves

Introduction

Data

Forward  
Model

Inverse  
Methods

Discussion

Assume linear wave theory:

$$\begin{cases} \frac{d}{dx} (EC_g) = -\delta, \\ \sigma^2 = gk \tanh(kh), \end{cases}$$

$$\delta = \frac{1}{4h} B \rho g f H_{rms} \left[ (R^3 + \frac{3}{2}R)e^{-R^2} + \frac{3}{4}\sqrt{\pi}(1 - erf(R)) \right],$$

where

$E$  : Wave Energy,  $C_g$  : Group celerity,

$c$  : Wave celerity,  $\sigma$  : Angular frequency,

$g$  : Gravitational acceleration,  $k$  : Wave number

# Invert for bathymetry given surface data & physics

Bathymetry  
Inversion  
from Waves

Introduction

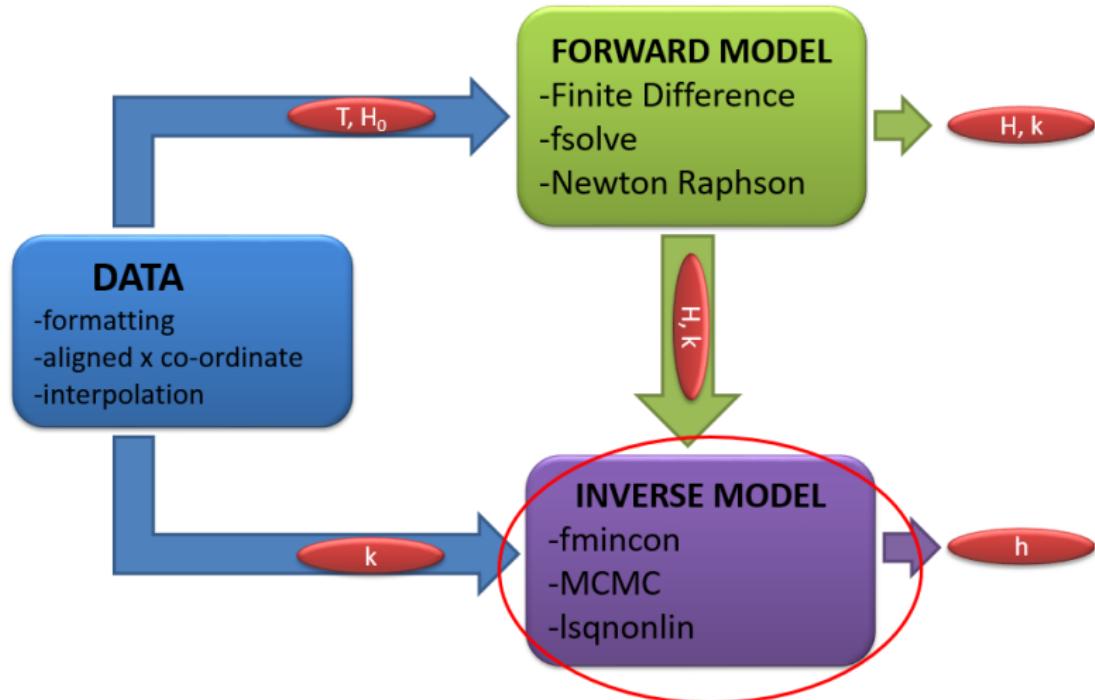
Data

Forward  
Model

Inverse  
Methods

Manufactured  
Real

Discussion



# Solutions are computed using 3 inversion methods

- ① Nonlinear Least Squares
  - Logical place to start
- ② Bayesian MCMC
  - Gives a distribution of depth estimates
- ③ Tikhonov Regularization
  - Bounded-constraint multivariate problem

# Manufactured “data” is used to test our algorithms

## Bathymetry Inversion from Waves

### Introduction

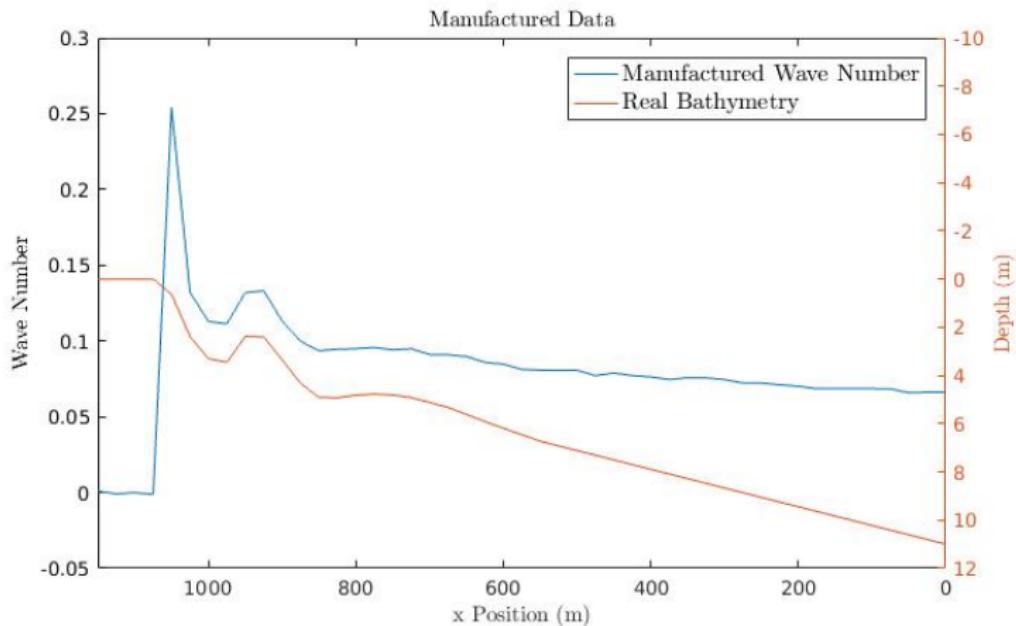
### Data

### Forward Model

### Inverse Methods

### Manufactured Real

### Discussion



# Results from manufactured test

Bathymetry  
Inversion  
from Waves

Introduction

Data

Forward  
Model

Inverse  
Methods

Manufactured  
Real

Discussion

# Real data is used in final solutions

## Bathymetry Inversion from Waves

Introduction

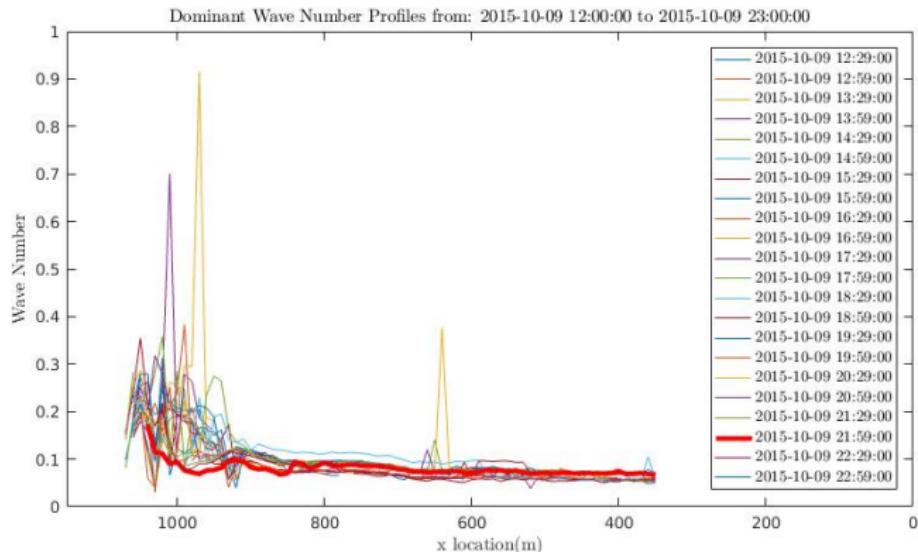
Data

Forward  
Model

Inverse  
Methods

Manufactured  
Real

Discussion



# All bathymetry estimates reasonably locate sandbar

Bathymetry  
Inversion  
from Waves

Introduction

Data

Forward  
Model

Inverse  
Methods

Manufactured  
Real

Discussion

# [This one] gave the best bathymetry estimate

## Bathymetry Inversion from Waves

Introduction

Data

Forward  
Model

Inverse  
Methods

Manufactured  
Real

Discussion

# Future Directions

Bathymetry  
Inversion  
from Waves

Introduction

Data

Forward  
Model

Inverse  
Methods

Discussion

- Inclusion of observed wave height,  $H$ , along the profile
- Further regularization methods
- Expand to using 2D wave physics

## Bathymetry Inversion from Waves

Introduction

Data

Forward  
Model

Inverse  
Methods

Discussion

**THANK YOU  
Questions?**