Inverting for Near Shore Bathymetry from Surface Wave Properties

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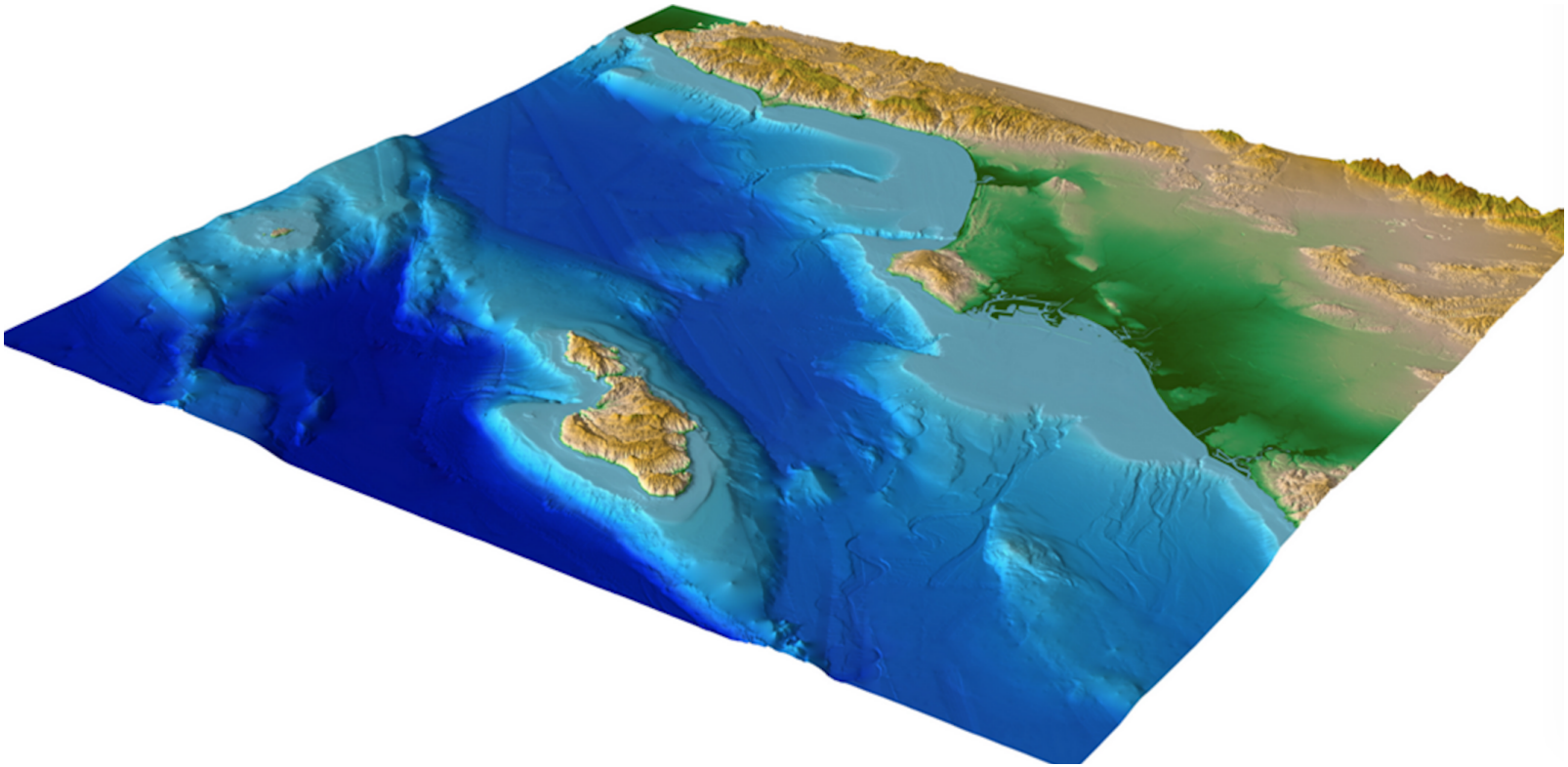
Many coastal processes are affected by bathymetry

Bathymetry is submarine topography

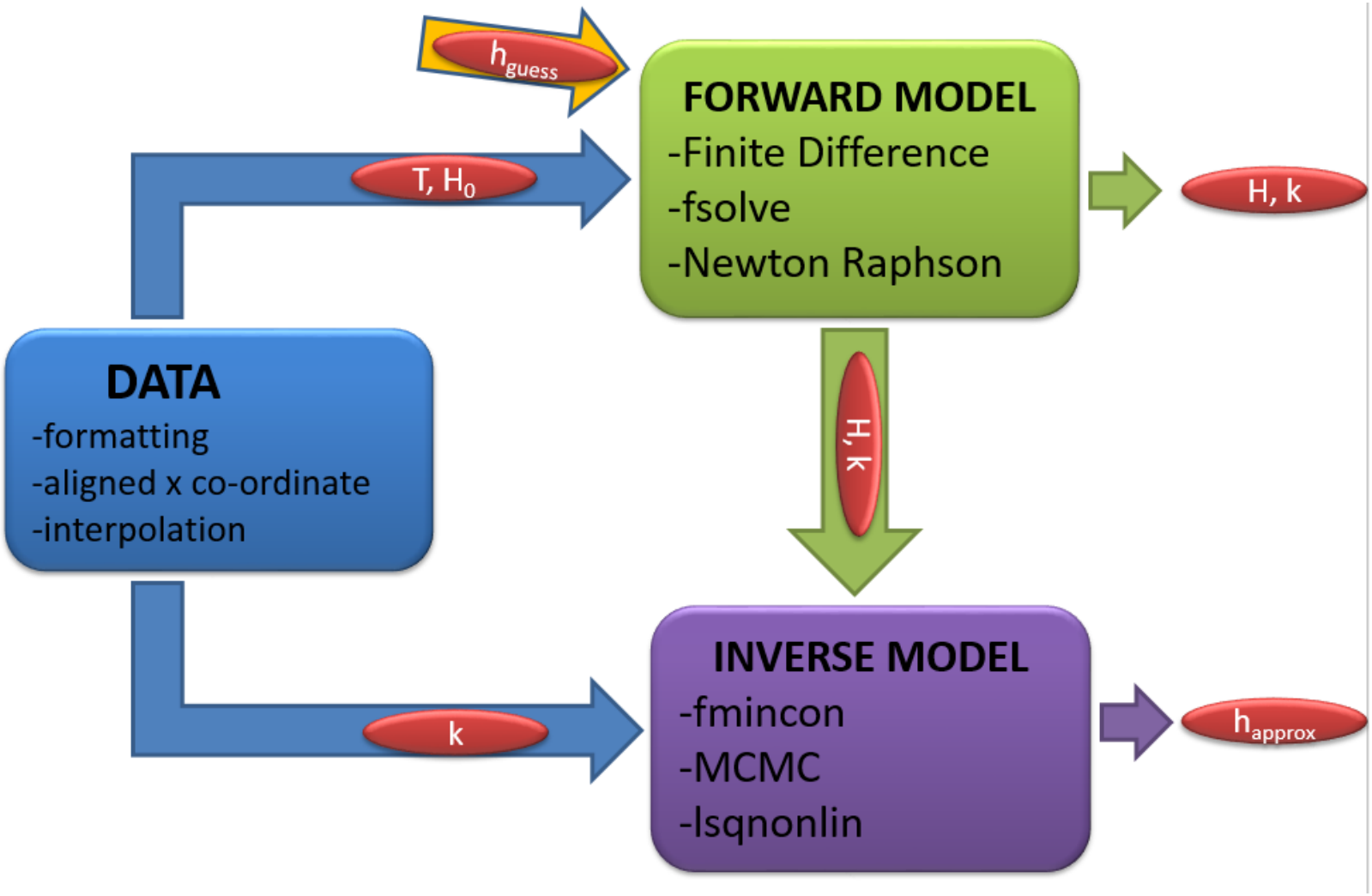


Direct measurements are expensive and challenging

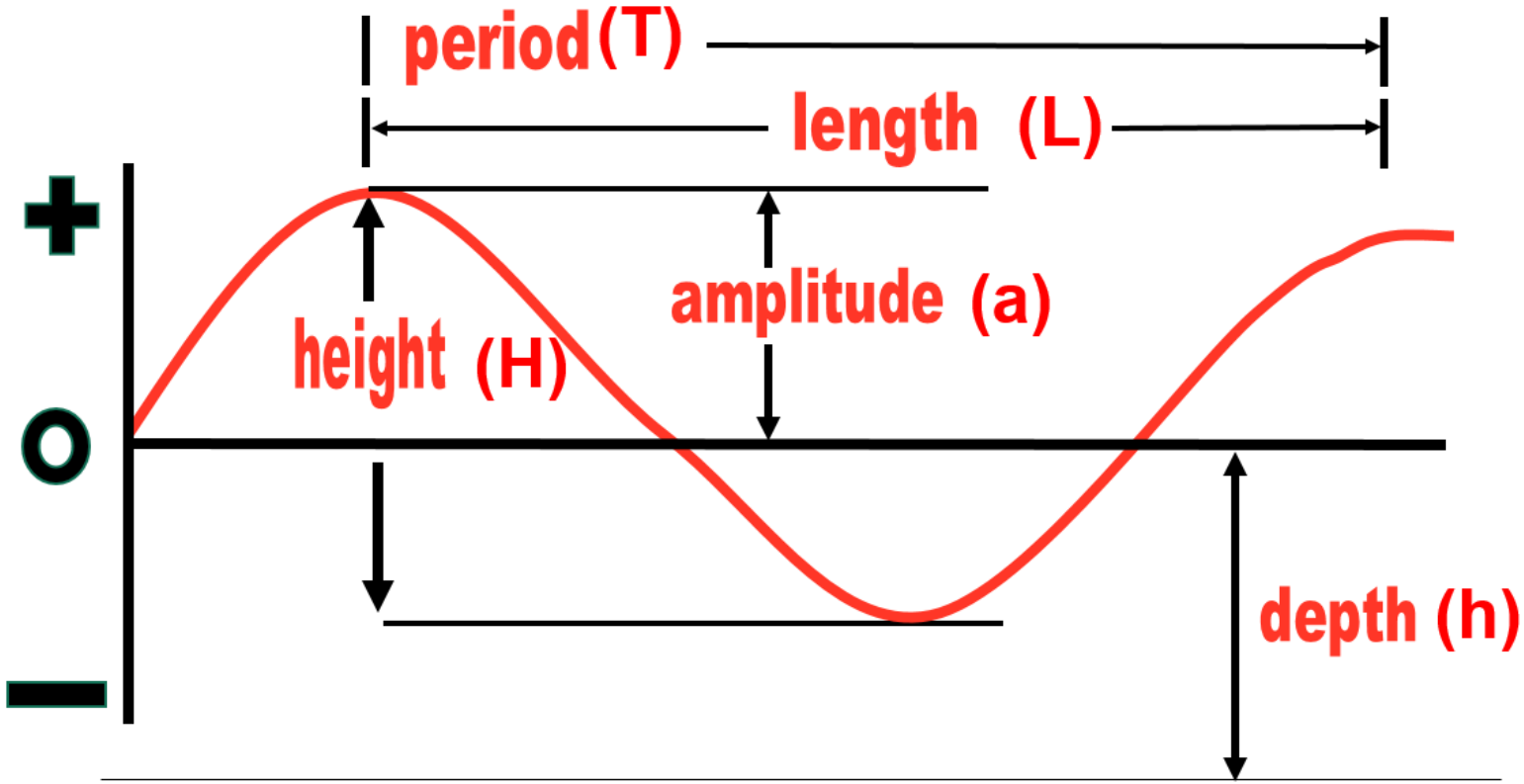
LARC

CRAB

Inverse models estimate depth using data & physics

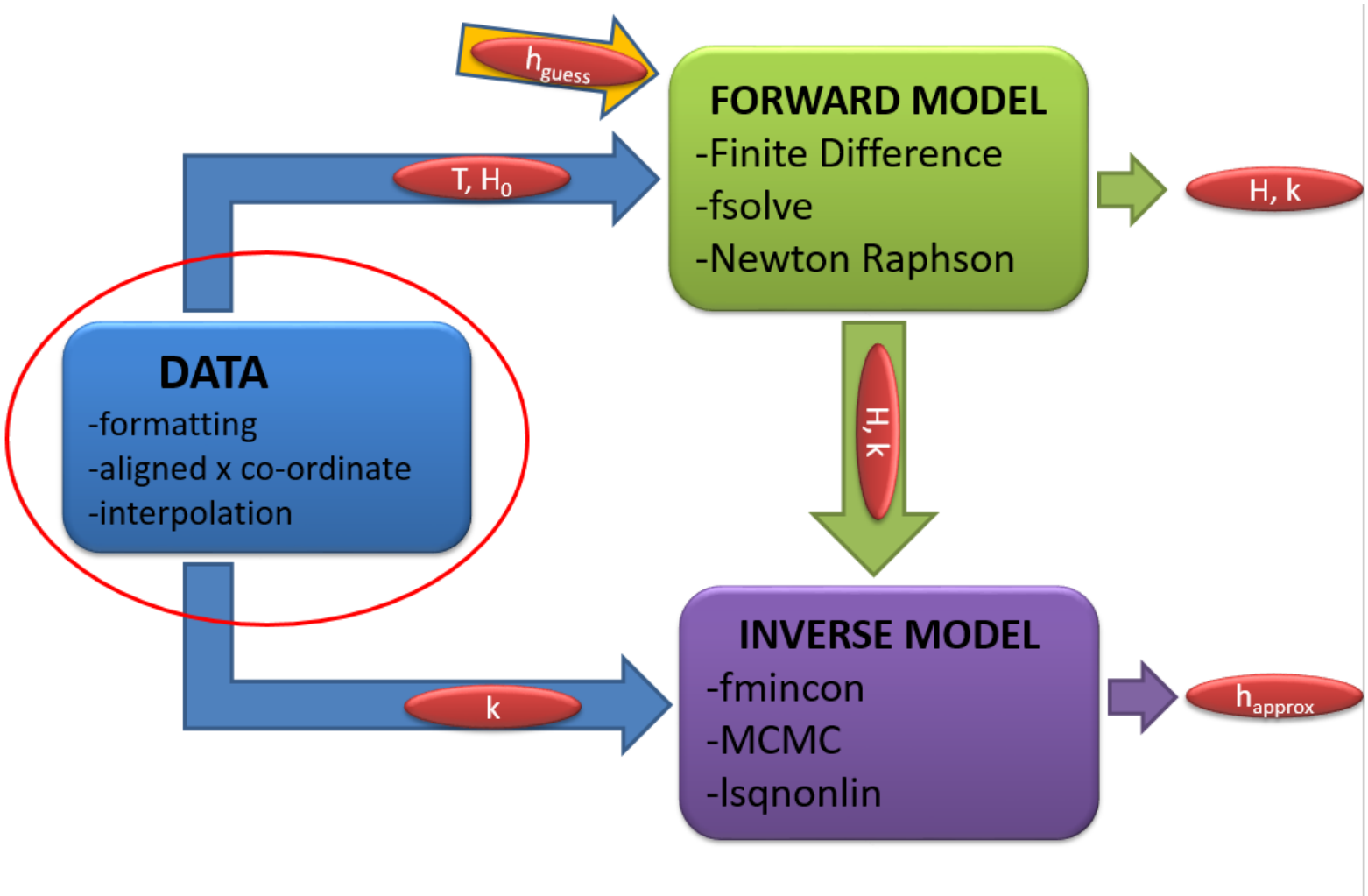


Bathymetry is related to surface wave properties

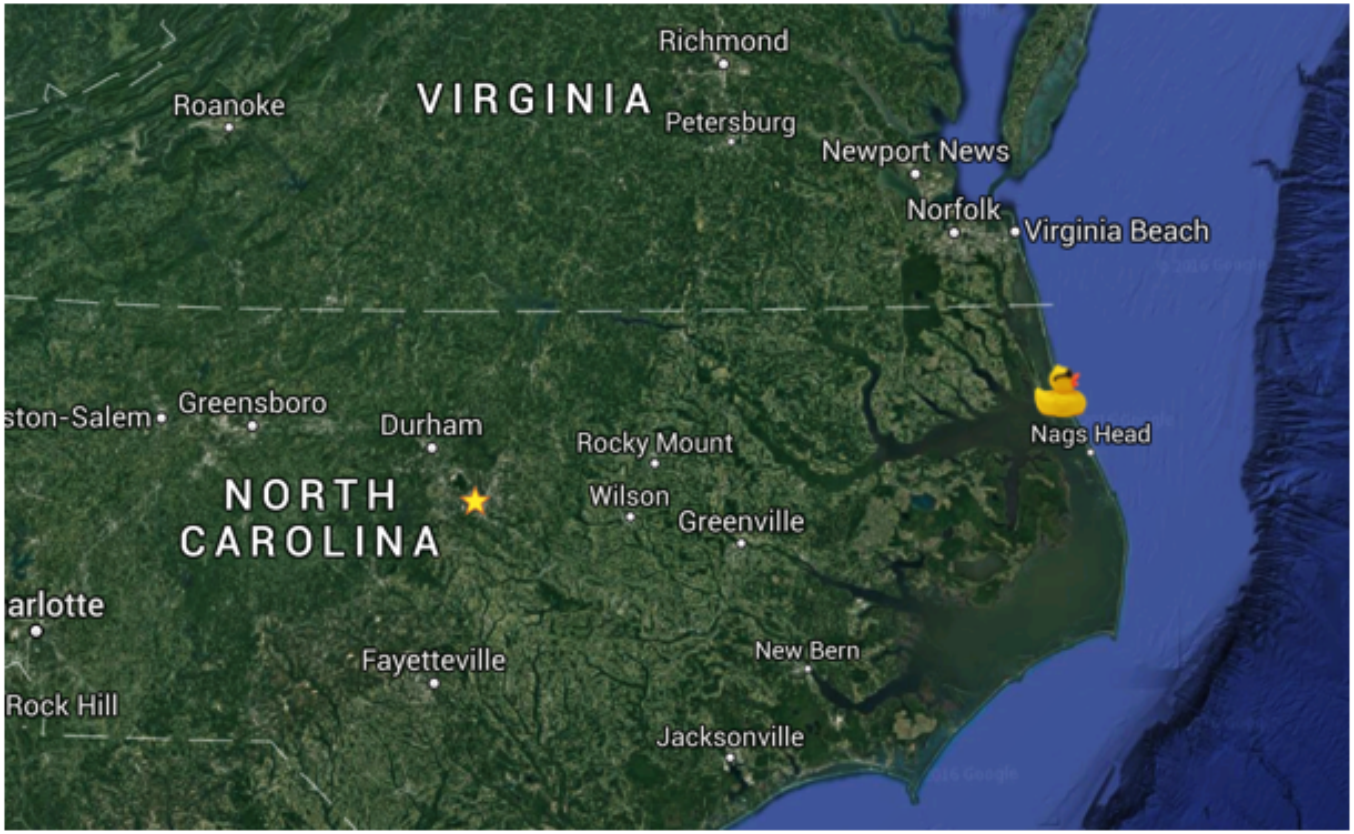


$$\text{wave number: } k = \frac{2\pi}{L}$$

Before we invert we need data



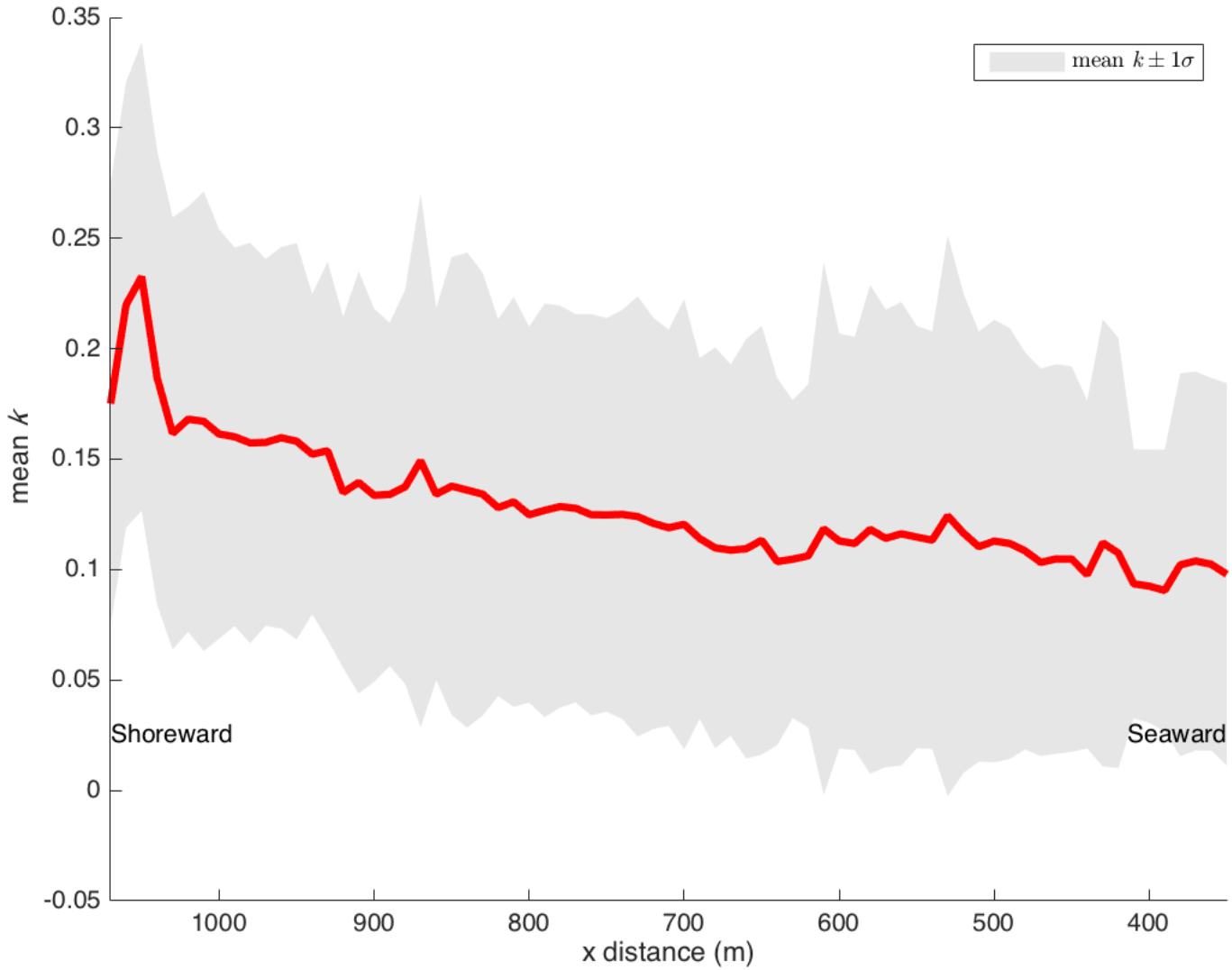
Data was collected by the USACE in Duck, NC



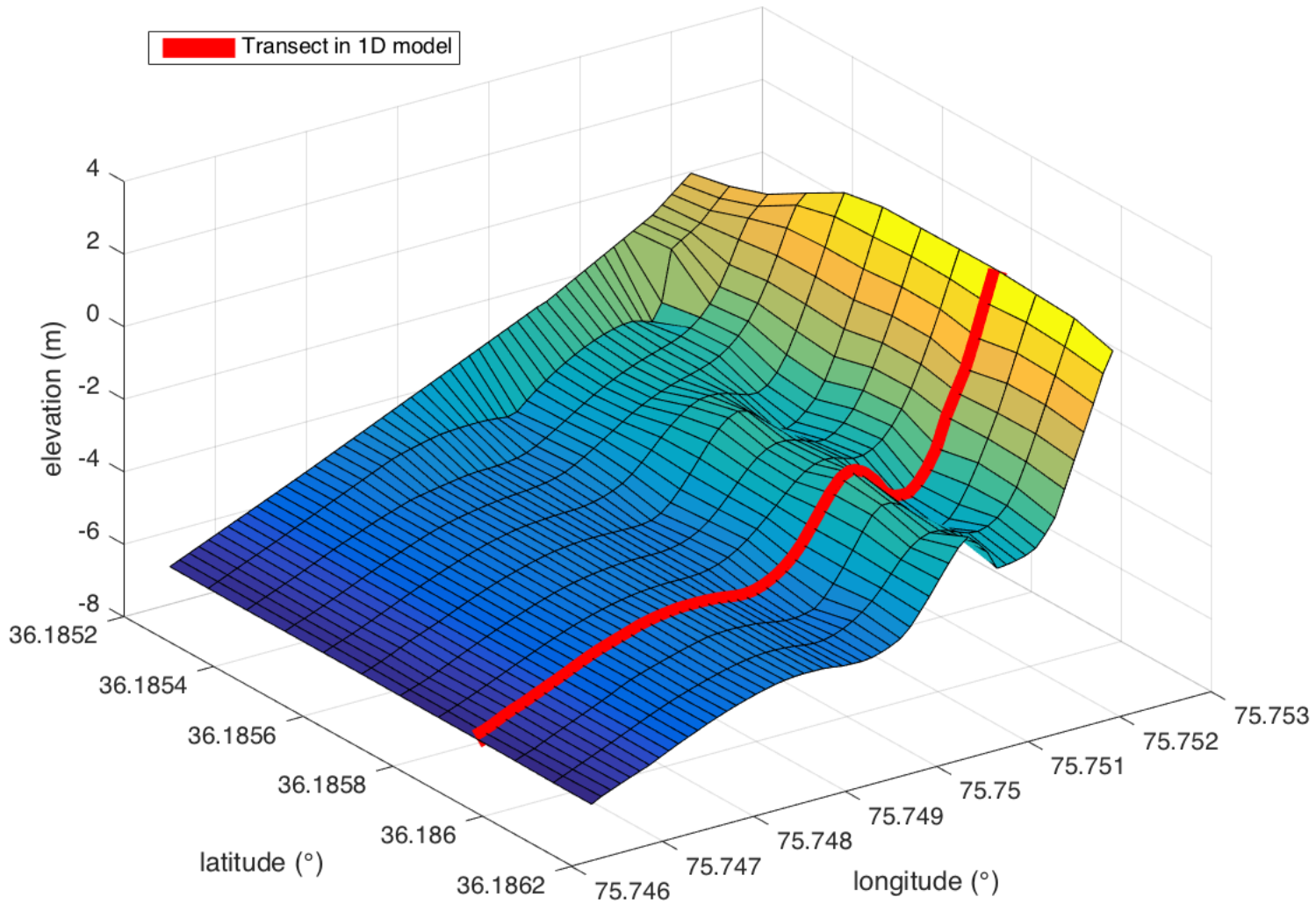
Model coordinate system has $x = 0\ m$ offshore

Remote sensing of surface properties are possible

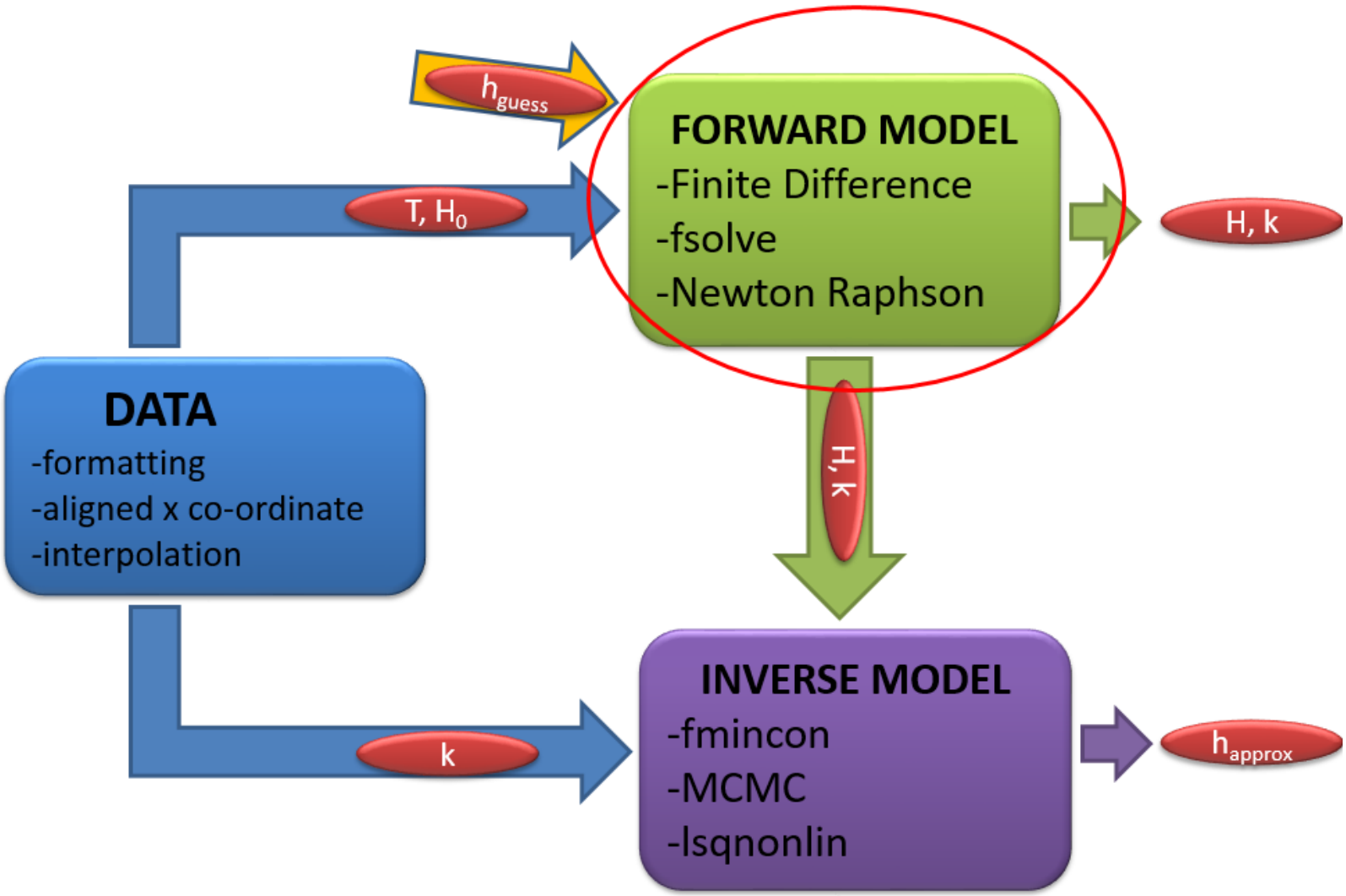
Data includes T, H at offshore boundary; 1D k



Known bathymetry is used for testing our results



Forward model computes k assuming h_{guess} & BC



Wave dispersion relationship relates k to h

Dispersion Relation [1]:

$$\sigma^2 = gk\tanh(kh) \Longleftrightarrow \left(\frac{2\pi}{T}\right)^2 = g\left(\frac{2\pi}{L}\right) tanh\left(\frac{2\pi h}{L}\right)$$

- Relates wave number (k) and Period (T)
- Wave length (L) varies with depth (h)
- Period (T) remains constant

1D forward model relates H and h

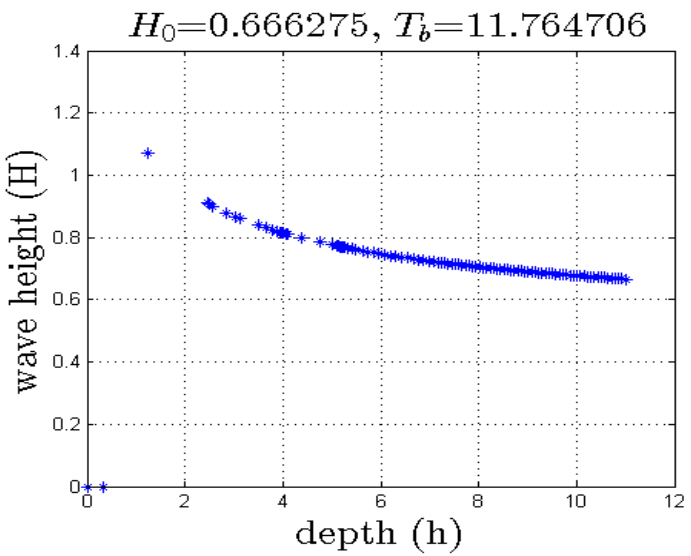
Energy Flux Method [2]:

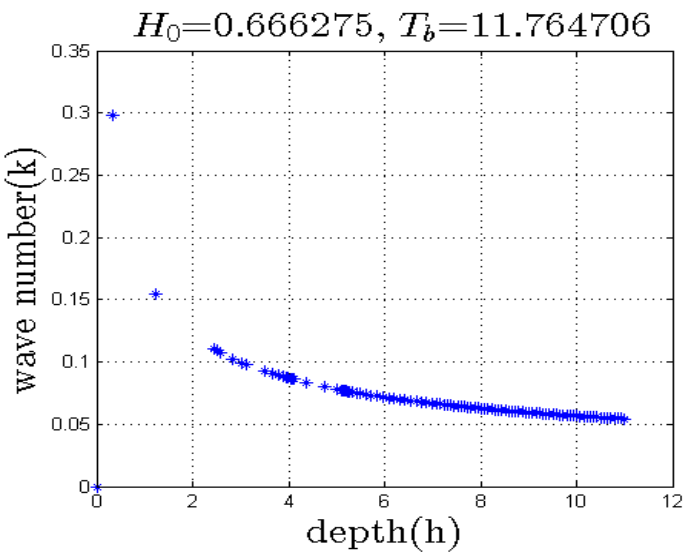
$$\frac{d}{dx}\left(EC_g\right) = -\delta$$

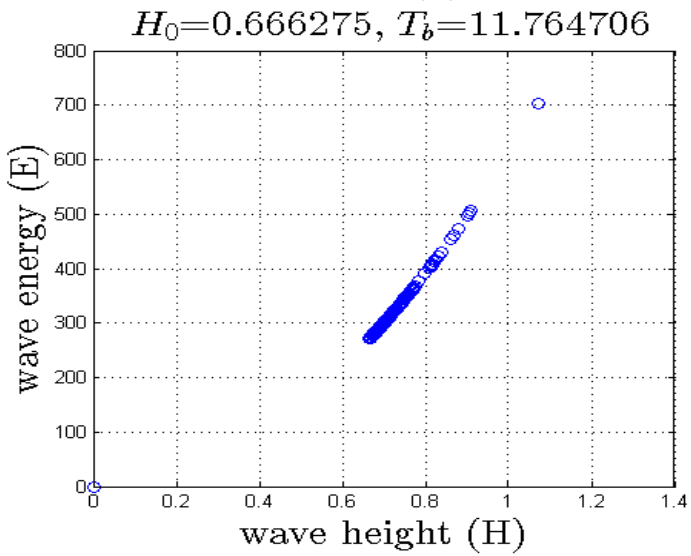
- Relates wave height (H) and wave number (k)
- $E(\rho, g, \mathbf{H})$: Wave Energy
- $C_g(\rho, \mathbf{k}, h)$: Group celerity
- $\delta(\rho, g, T, \mathbf{H})$: [3]Wave energy dissipation function

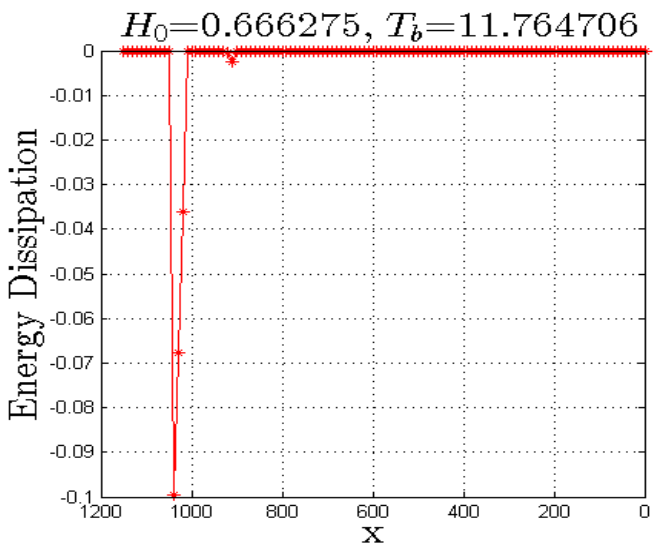
[1] Dean et al, 2000 (pg 64), [2] Edward et al, 1983 (pg 2) and [3] Alex et al, 2008 (pg 2).

1D forward model Results

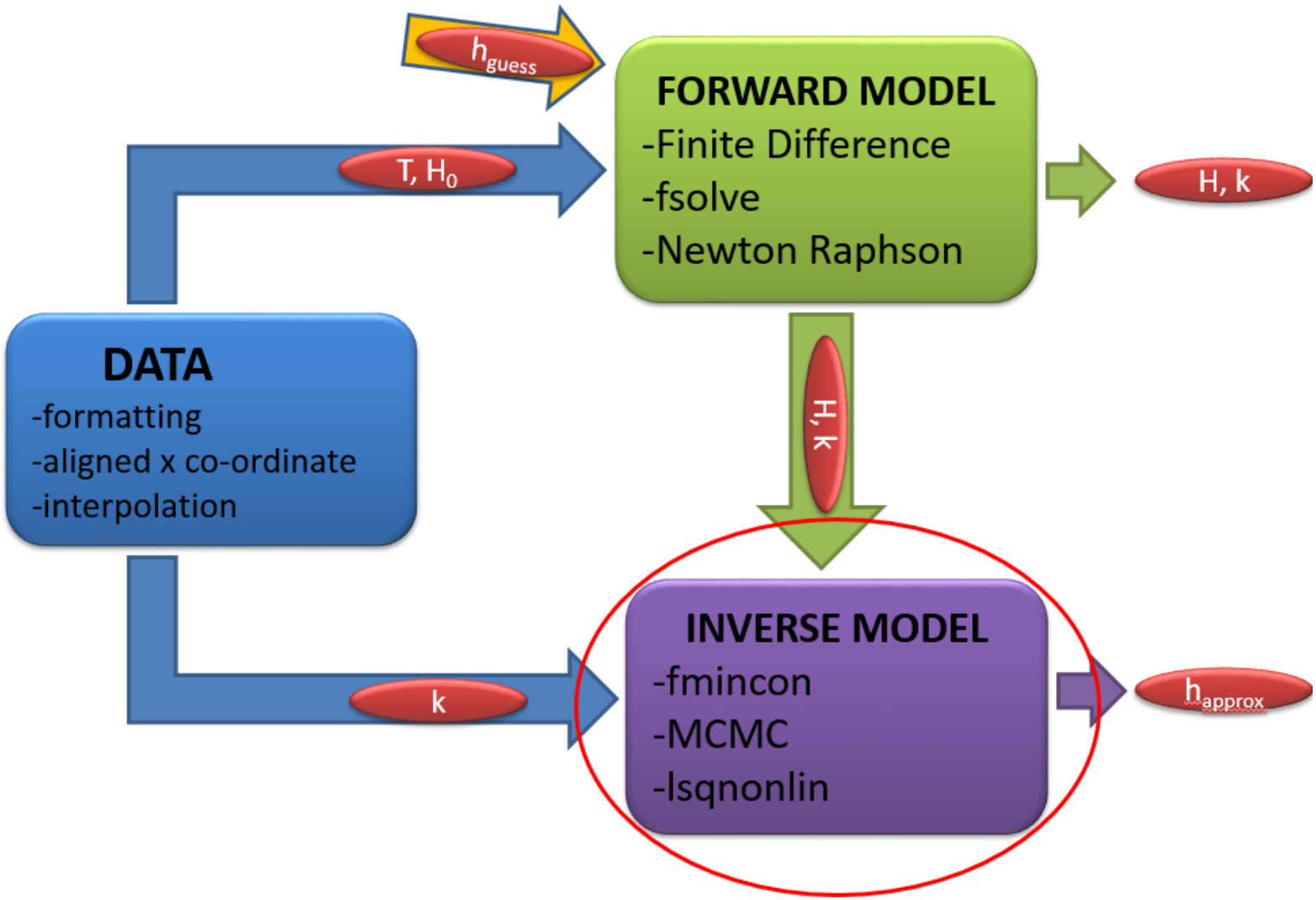








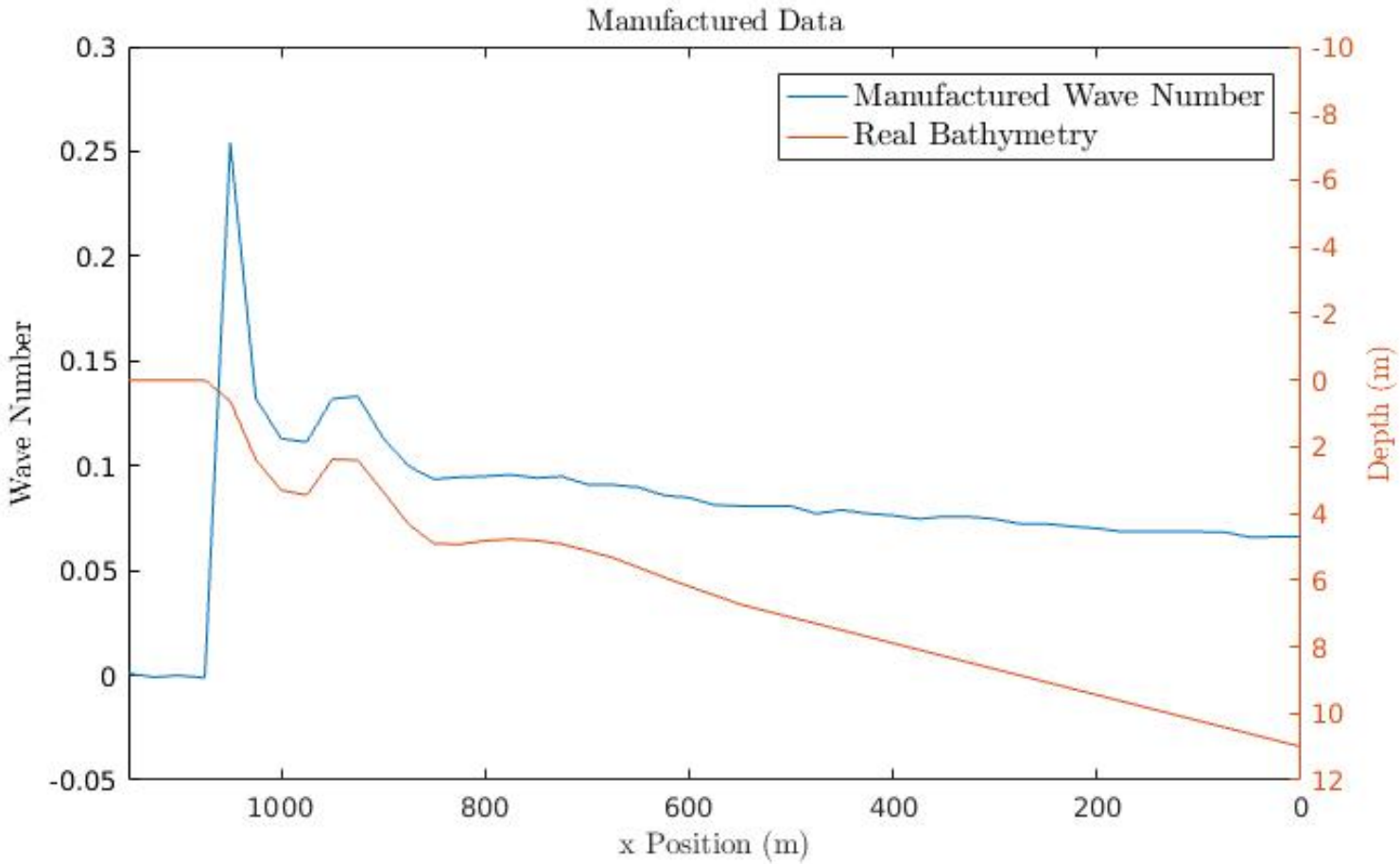
Invert for bathymetry given surface data & physics



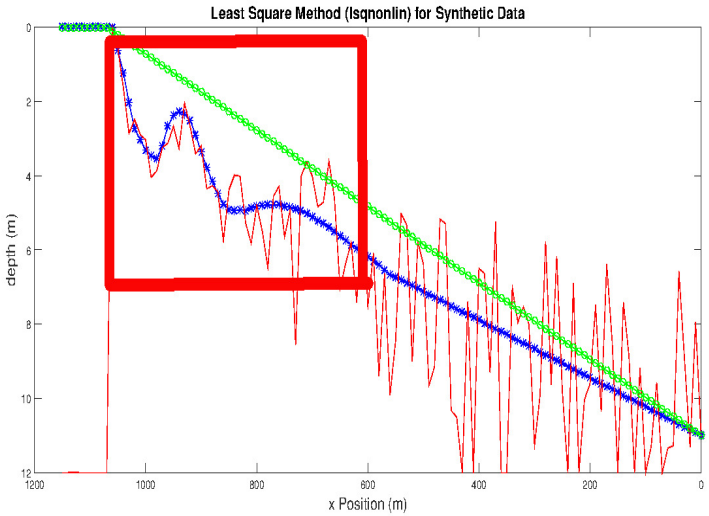
Solutions are computed using 3 inversion methods

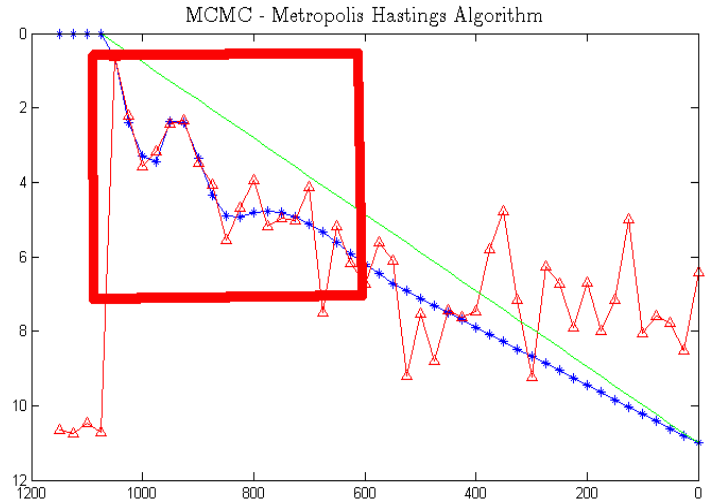
1. Nonlinear Least Squares (lsqnonlin)
   - Logical place to start
2. Bayesian MCMC (Metropolis)
   - Gives a distribution of depth estimates
3. Tikhonov Regularization (fmincon)
   - Can deal with instabilities caused by noise in the measurements

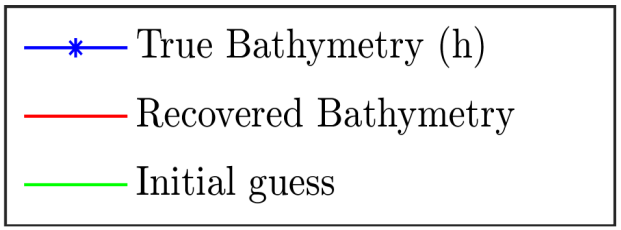
Manufactured "data" is used to test our algorithms

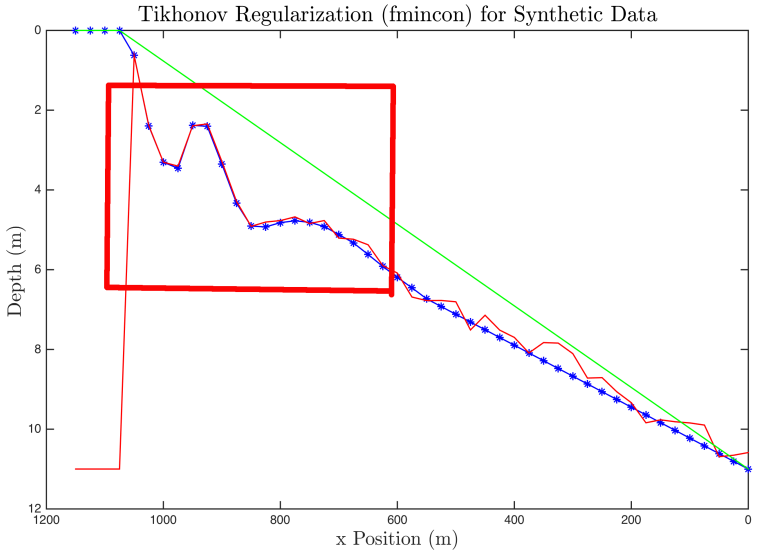


All methods capture the sandbar well

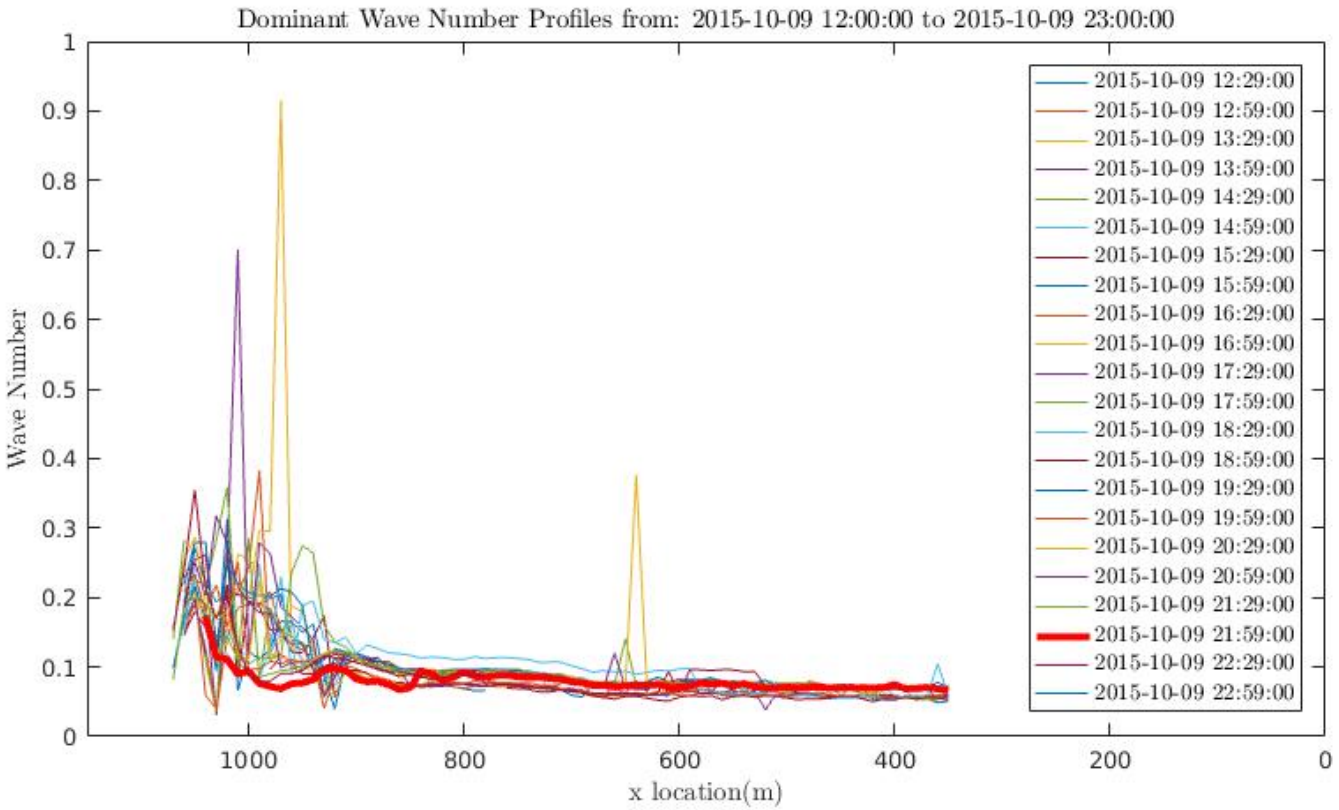




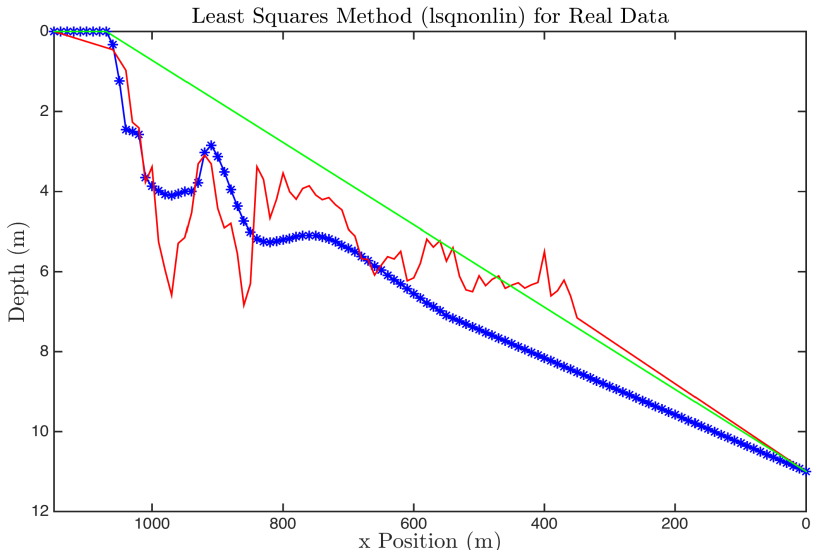


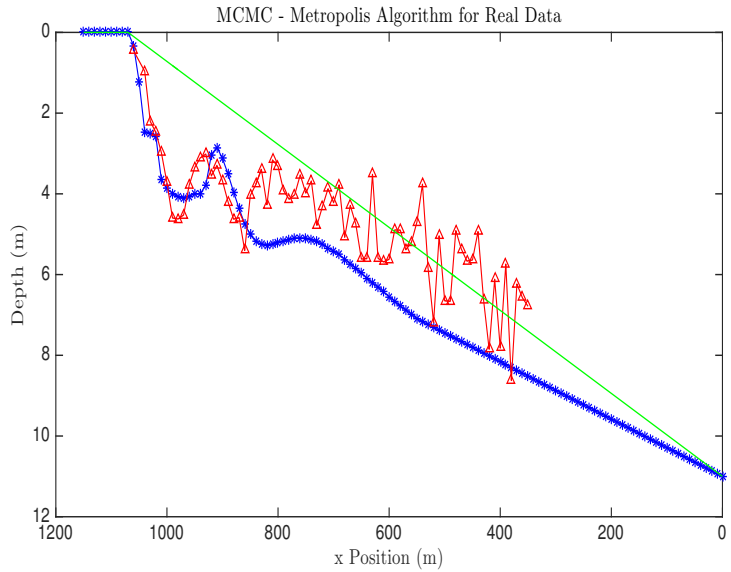


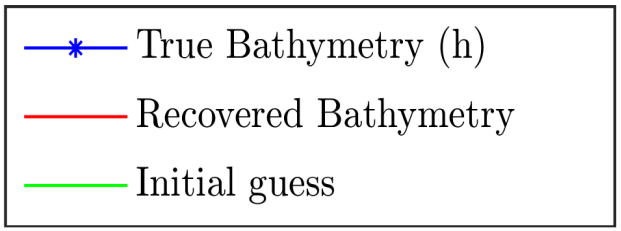
Real k data is selected for a period with low noise

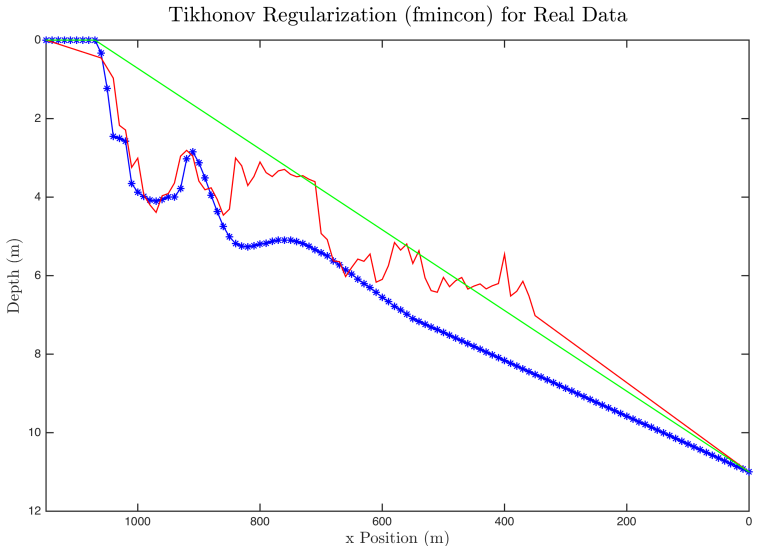


Bathymetry estimates perform well in shallow water

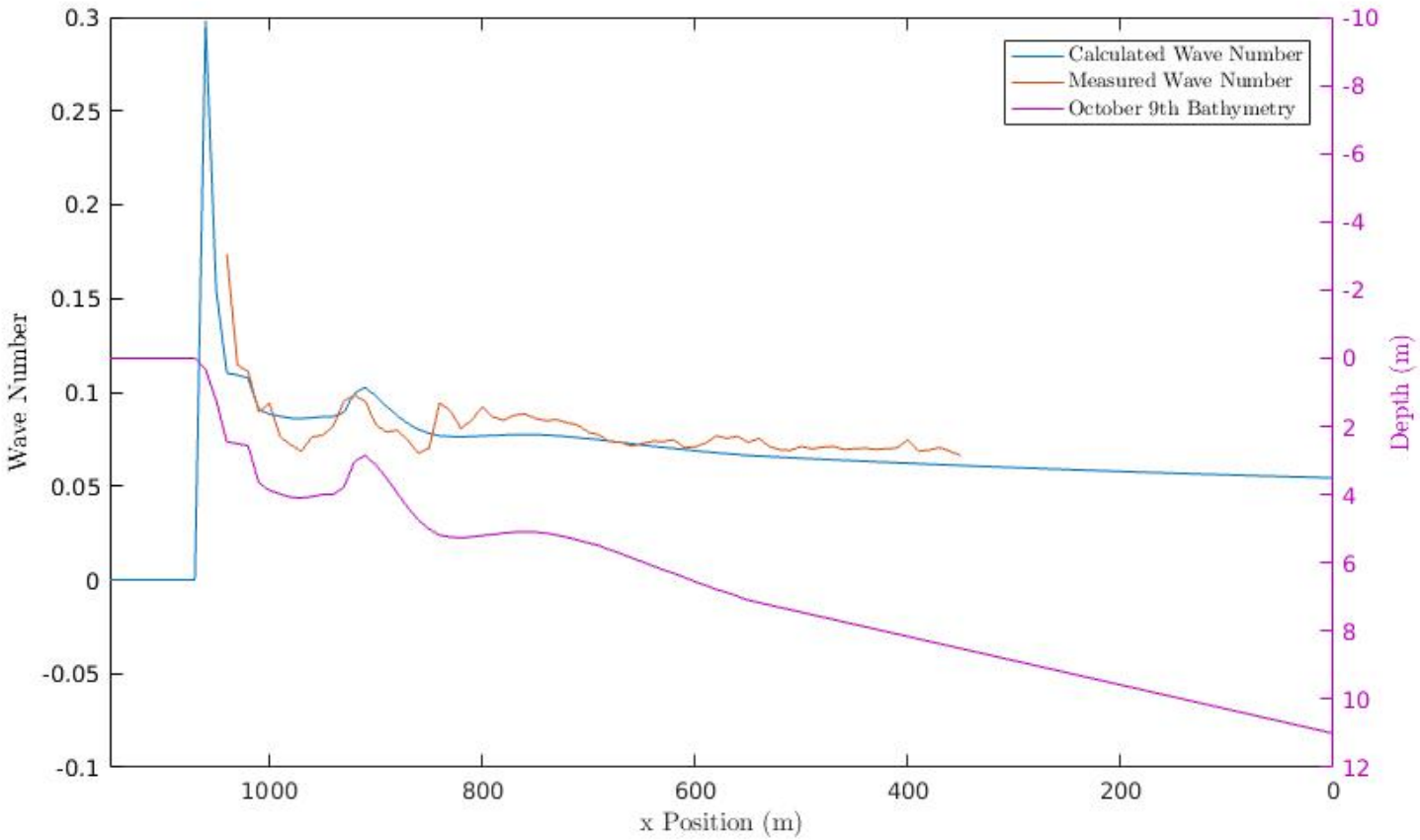




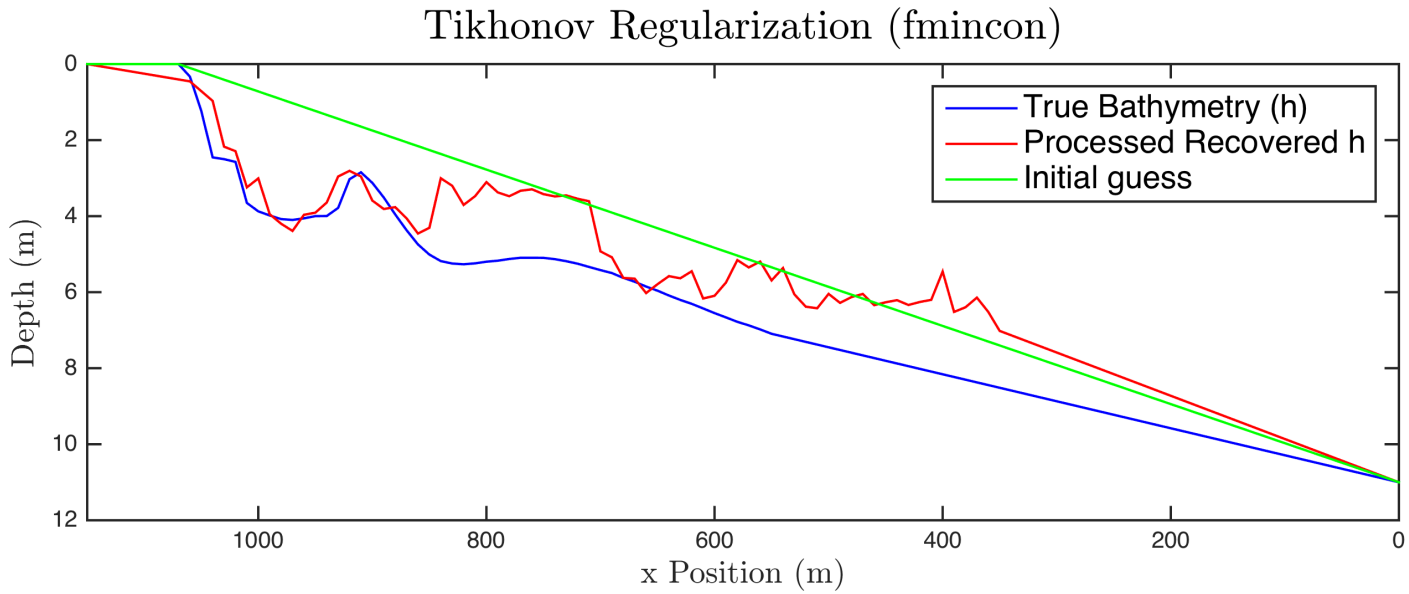


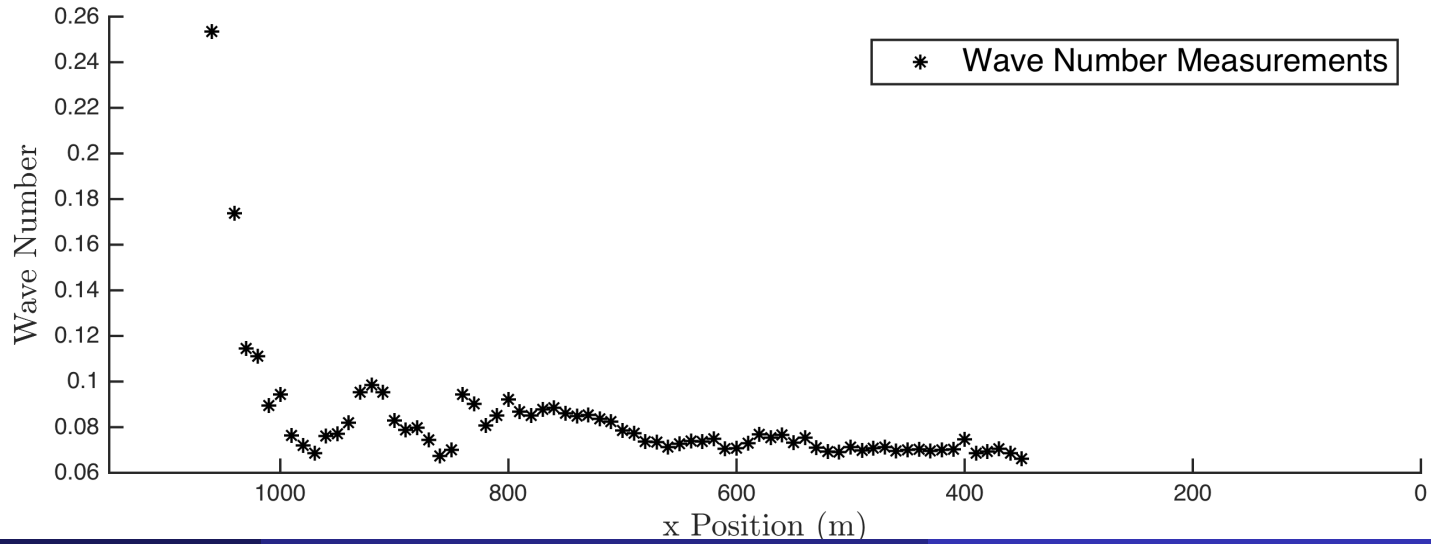


Bathymetry estimates are limited by noisy k data



Missing data & deep water cause poor h offshore





Future Directions

- Non-linear wave theory
  - Linear wave theory is just a starting point!
- Inclusion of observed wave height, H, or other measured variables, along the profile
  - this method will allow for assimilation of other measured variables
- Application of further regularization methods
  - we heuristically "tuned" our regulariztion
  - mayhaps incorporate prior knowledge?
- Incorporate uncertainty in measurements
- Expansion to 2D wave physics

Thank you!
Questions?

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