Inverting for Near Coastal Bathymetry from Surface Wave Properties

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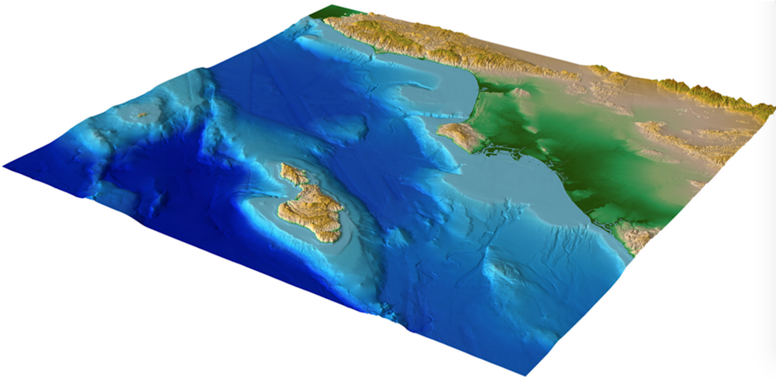
Many coastal processes are affected by bathymetry

Bathymetry is submarine topography

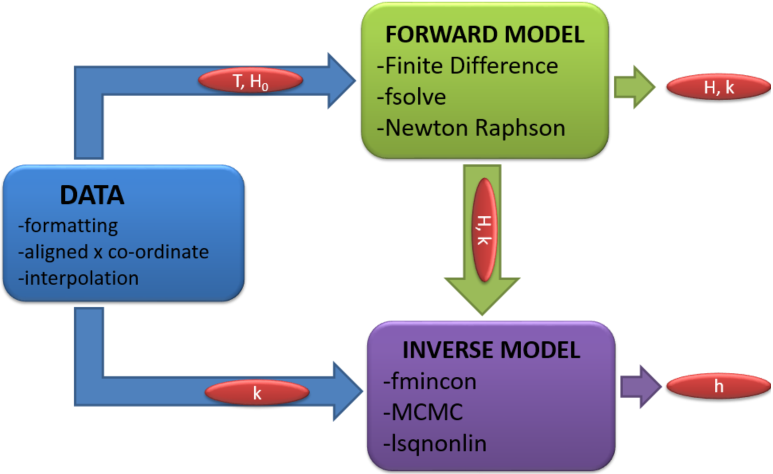


Direct measurements are expensive and challenging

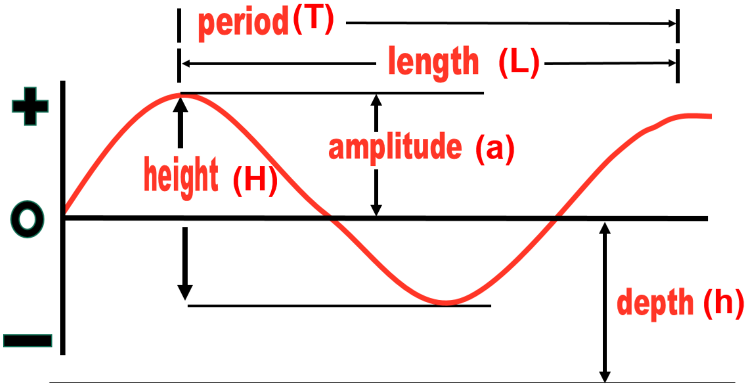
LARC

CRAB

Inverse models estimate depth using data & physics

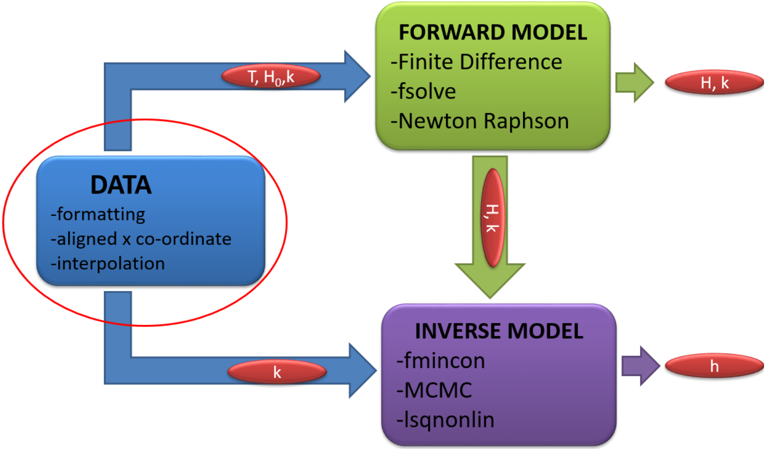


Bathymetry is related to surface wave properties

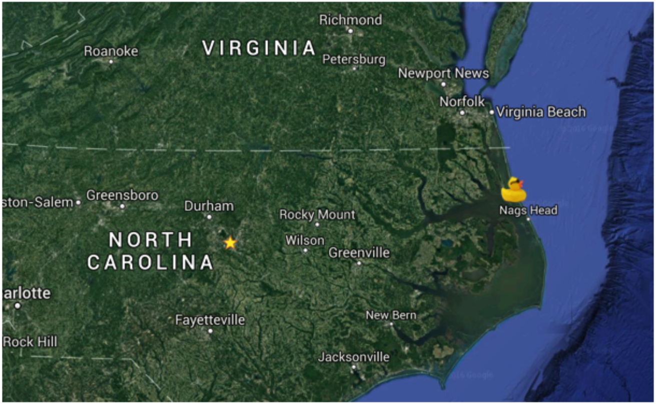


$$k = \frac{2\pi}{L} \tag{1}$$

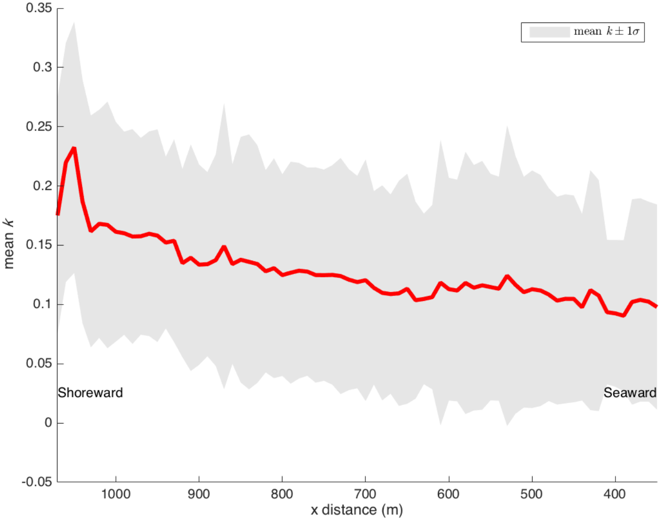
Before we invert we need data



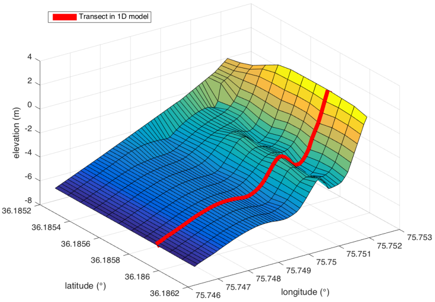
Data was collected by the USACE in Duck, NC

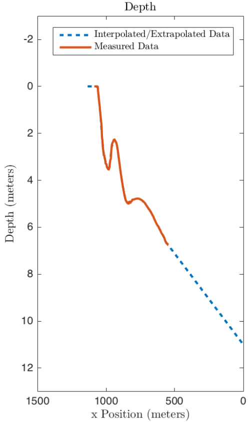


Data includes T, H at offshore boundary, 1D k

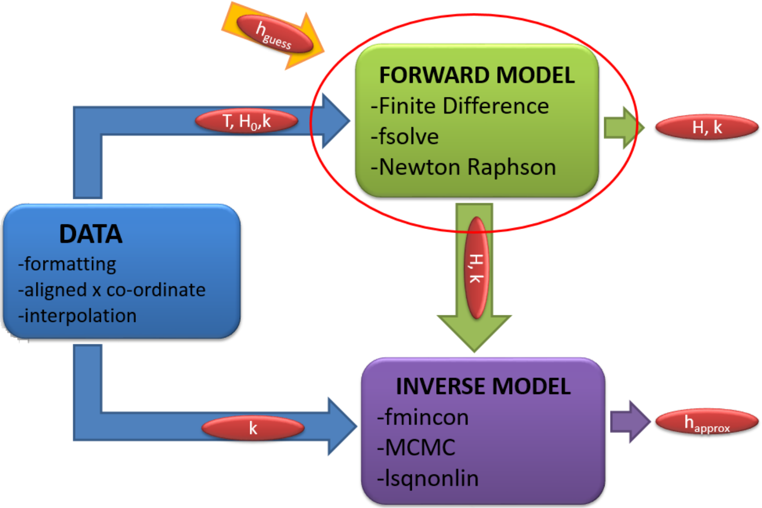


Known bathymetry is used for testing our results





Forward model computes k assuming h_{guess} & BC



1D wave physics is known for near-coastal regions

Assume linear wave theory:

$$\begin{cases} \frac{d}{dx}\left(EC_g\right) = -\delta, \\ \\ \sigma^2 = gk\tanh(kh), \end{cases}$$

$$\frac{d}{dx}\left(\frac{\lambda}{k}\left(1+\frac{2kh}{\sinh(2kh)}\right)H^2\right) = -\delta$$

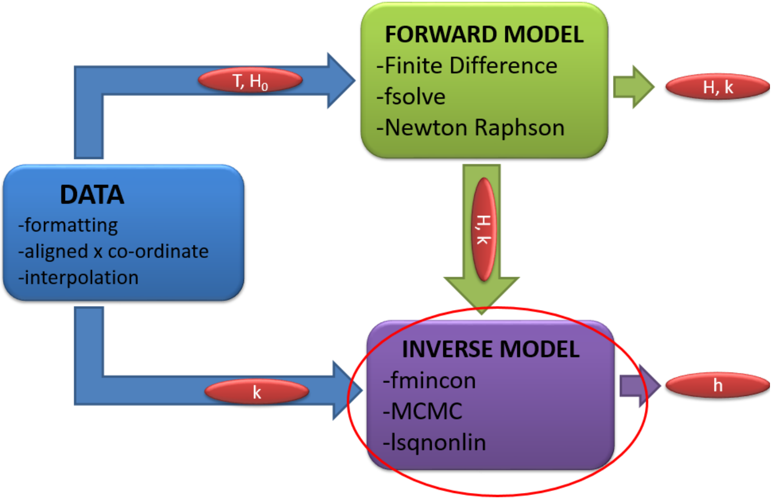
where

$$E : Wave\,Energy,\, C_g : Group\,celerity,$$

$$c : Wave\,celerity,\, \sigma : Angular\,frequency,$$

$$g : Gravitational\ acceleration, k : Wave\,number$$

Invert for bathymetry given surface data & physics



Solutions are computed using 3 inversion methods

1. Nonlinear Least Squares
   - Logical place to start
2. Bayesian MCMC
   - Gives a distribution of depth estimates
3. Tikhonov Regularization
   - Ill-posed problem

Manufactured “data” is used to test our algorithms

Results from manufactured test

Real data is used in final solutions

All bathymetry estimates reasonably locate sandbar

[This one] gave the best bathymetry estimate

Future Directions

- Inclusion of observed wave height, H, along the profile
- Further regularization methods
- Expand to using 2D wave physics

THANK YOU

Questions?

Additive Gaussian Noise Model

Gaussian noise $\boldsymbol{\epsilon}$ corrupted measurements $\mathbf{d}$ with variance ν is given by

$$\mathbf{d} = \mathbf{A}\mathbf{h}_t + \boldsymbol{\epsilon}.$$

$$\begin{aligned} \mathbf{d} &= \text{a vector of measurements,} \\ \mathbf{A} &= \text{a linear forward operator,} \\ \mathbf{h}_t &= \text{the true bathymetry.} \end{aligned}$$

fmincon: Tikhonov Method

Uses a regularized solution with prior information

$$\hat{\mathbf{h}} = \arg\min_{\mathbf{h}\in\mathbb{R}^n} \|\mathbf{A}\mathbf{h} - \mathbf{d}\|_2^2 + \alpha\|\mathbf{h} - \mathbf{h}_p\|_2^2,$$

Bayesian Markov Chain Monte Carlo (MCMC) Method

The MCMC method creates a posterior distribution of depth profiles, given wave number by using the Bayes relationship

$$P(h|k) \propto \Pi(h)L(h|k), \tag{2}$$

MCMC Method: Metropolis Algorithm

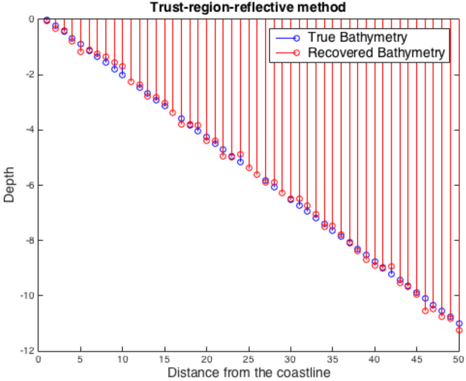
- Prior and likelihood are combined to compute an initial posterior probability distribution of h

$$P(h|k) = log(\Pi(h)) + log(L(h|k)) \tag{3}$$

- Uses a markov chain random walk to arrive at a posterior distribution of h profiles

Nonlinear Least Squares: Trusted Region-Reflective Method

$$\hat{\mathbf{h}} = \underset{\mathbf{h}\in\mathbb{R}^n}{\arg\min} \quad f(\mathbf{h}) = \|\mathbf{A}\mathbf{h} - \mathbf{d}\|_2^2, \tag{4}$$



MCMC Method: Log-Likelihood Function

Loglikelihood function compares simulated and observed k

$$\log L(h|k) = log[exp(-\frac{\sum_{i=1}^{n}(k_{m,i} - k_{d,i})^2}{2\sigma_k^2})] \tag{5}$$