Inverting for Near Shore Bathymetry from Surface Wave Properties

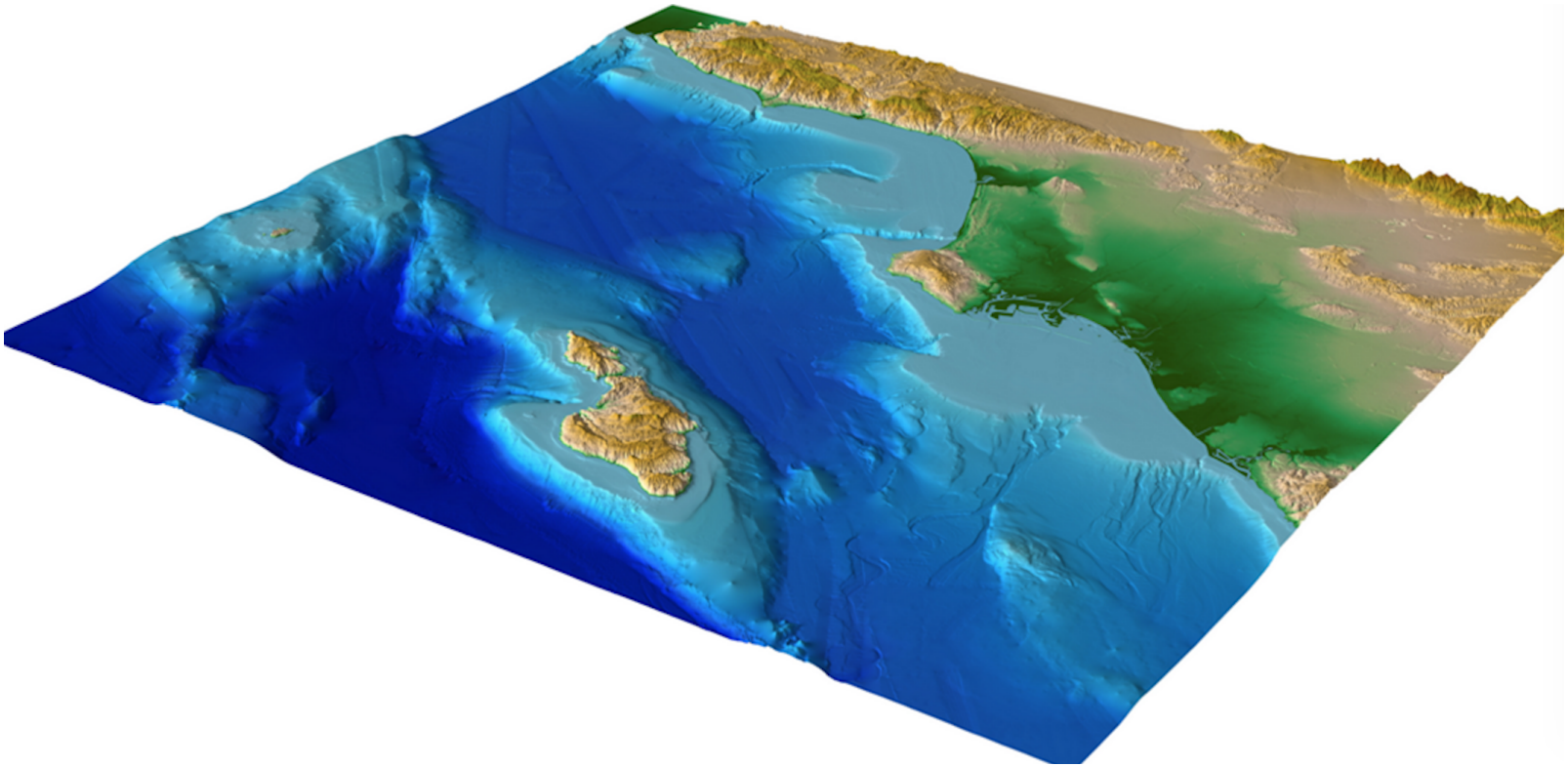
Lasith Adhikari, Charnelle Bland,
Lopamudra Chakravarty, Wenbin Dong,
Olaniyi Samuel Iyiola, Gail Muldoon, Clint Seinen

Supervised by
Ty Hesser (USACE), Matthew Farthing (USACE)
& Lea Jenkins (Clemson)

Industrial Mathematical and Statistical Modeling Workshop

Many coastal processes are affected by bathymetry

Bathymetry is submarine topography

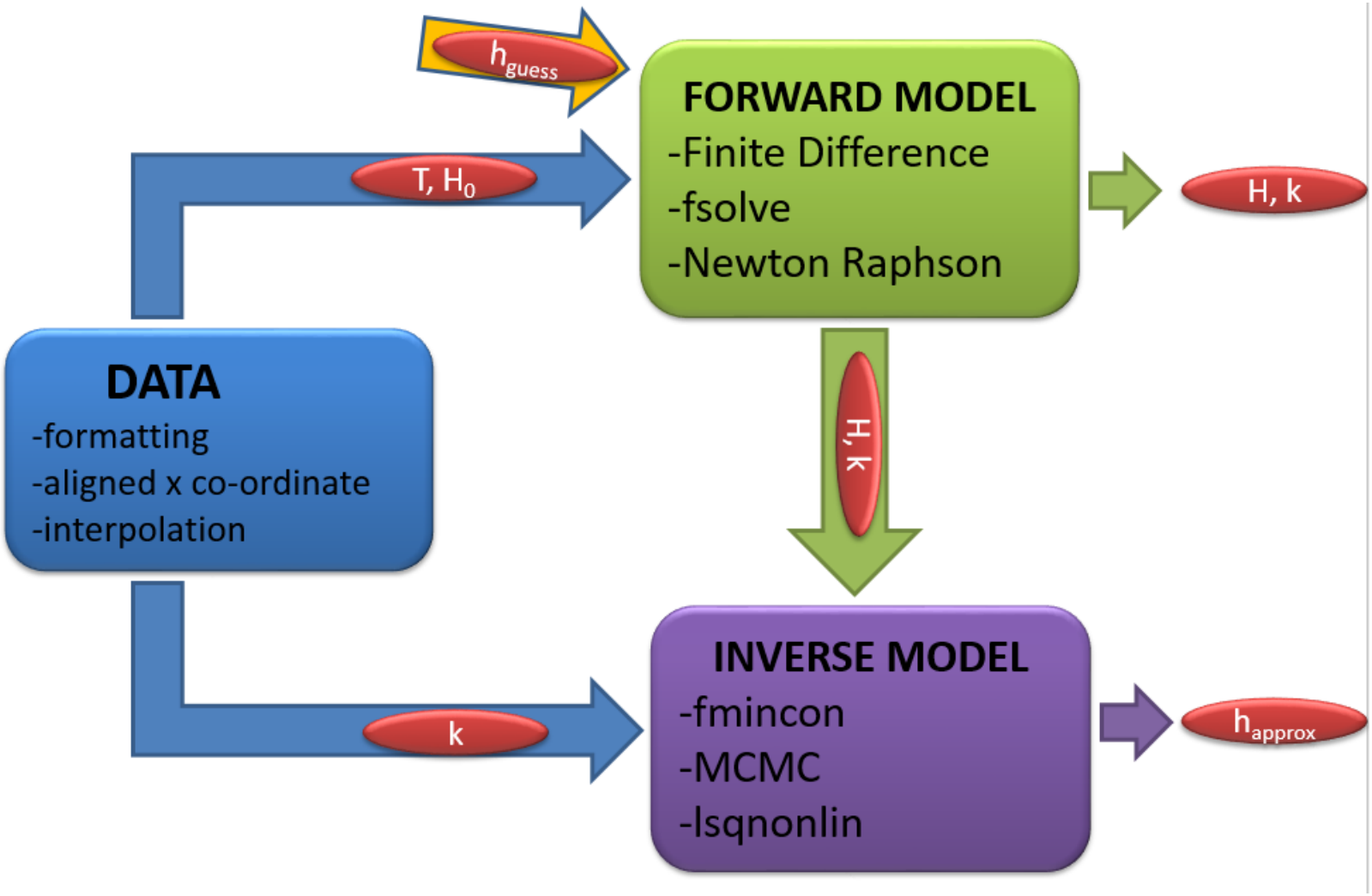


Direct measurements are expensive and challenging

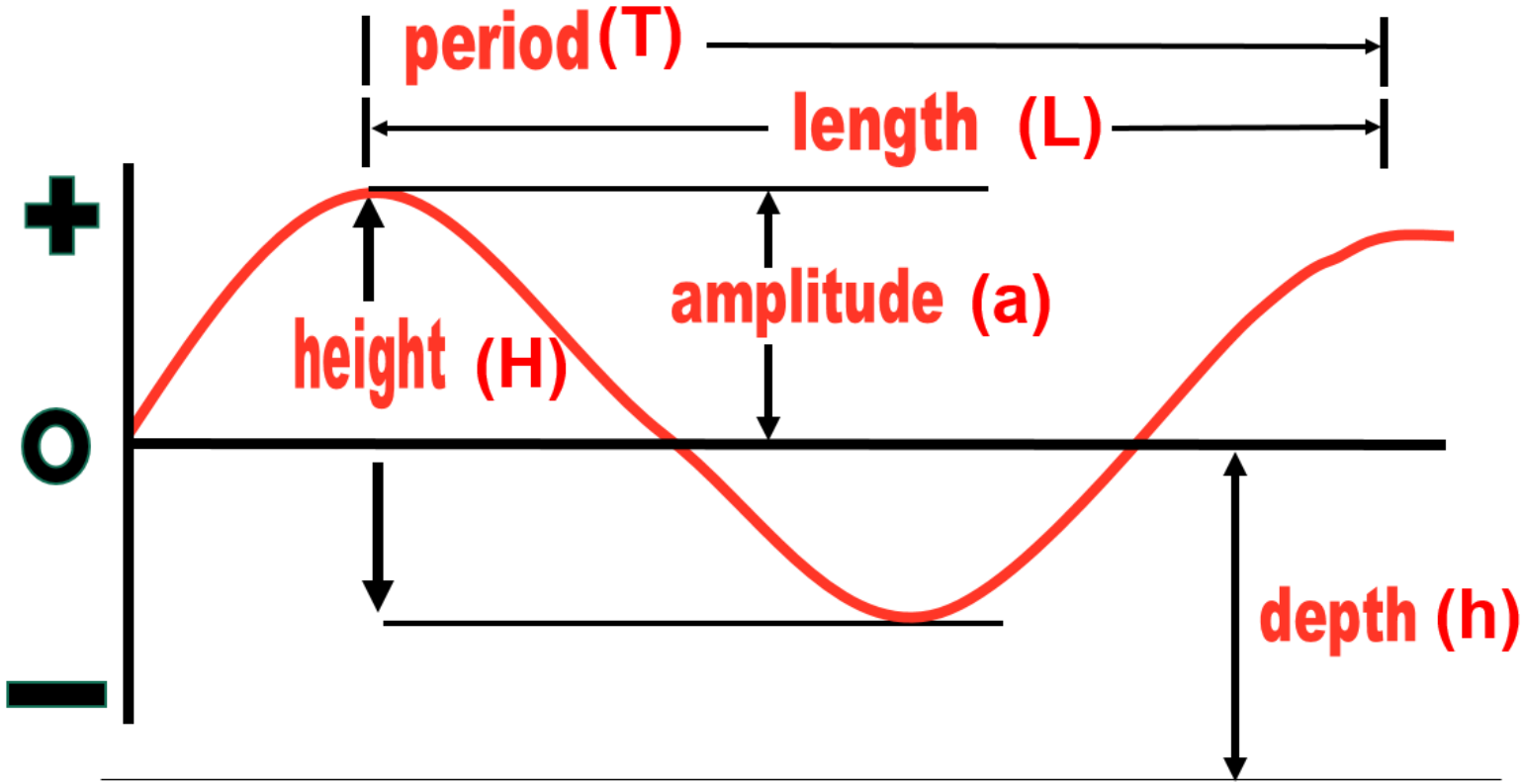
LARC

CRAB

Inverse models estimate depth using data & physics

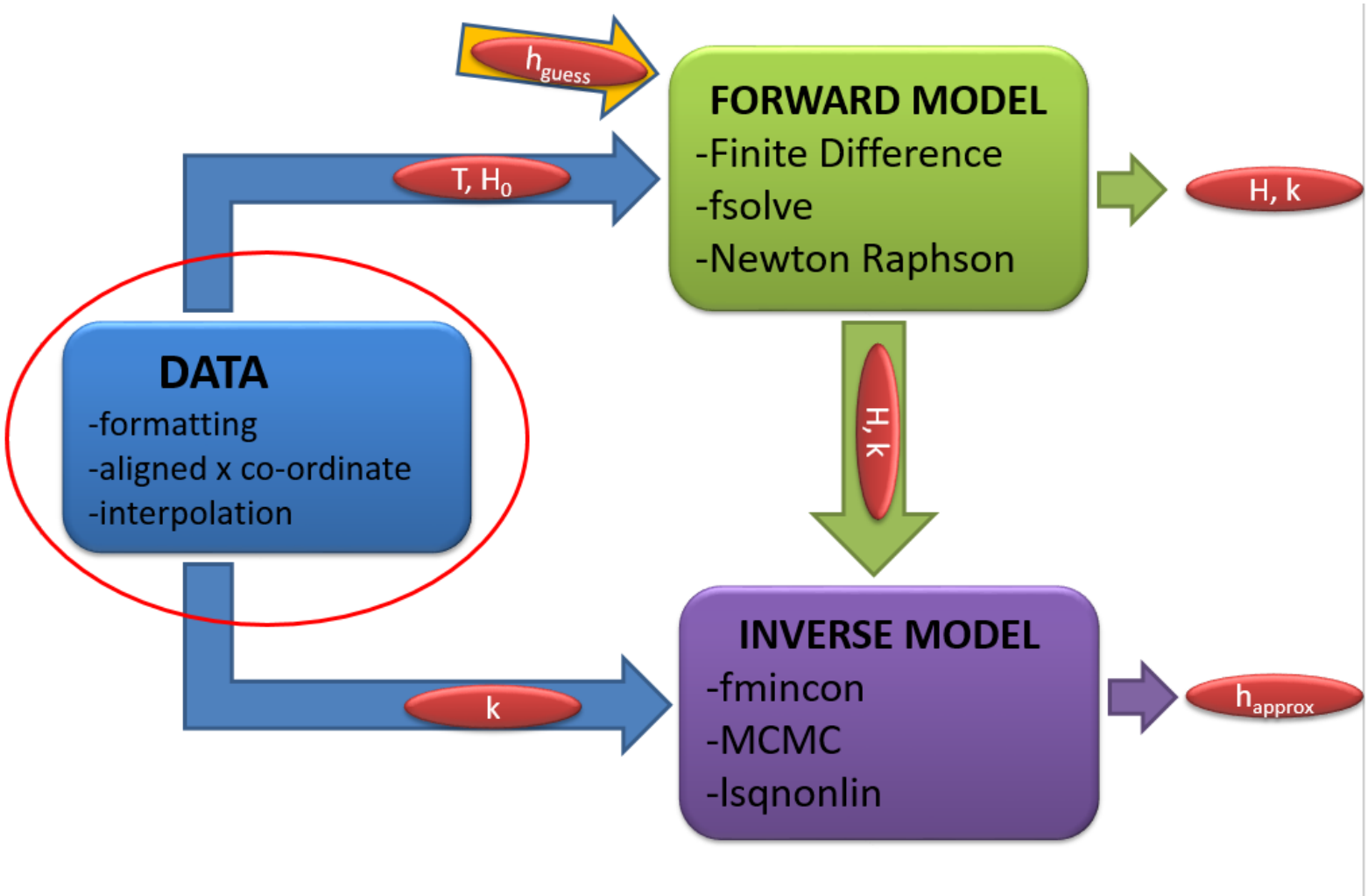


Bathymetry is related to surface wave properties

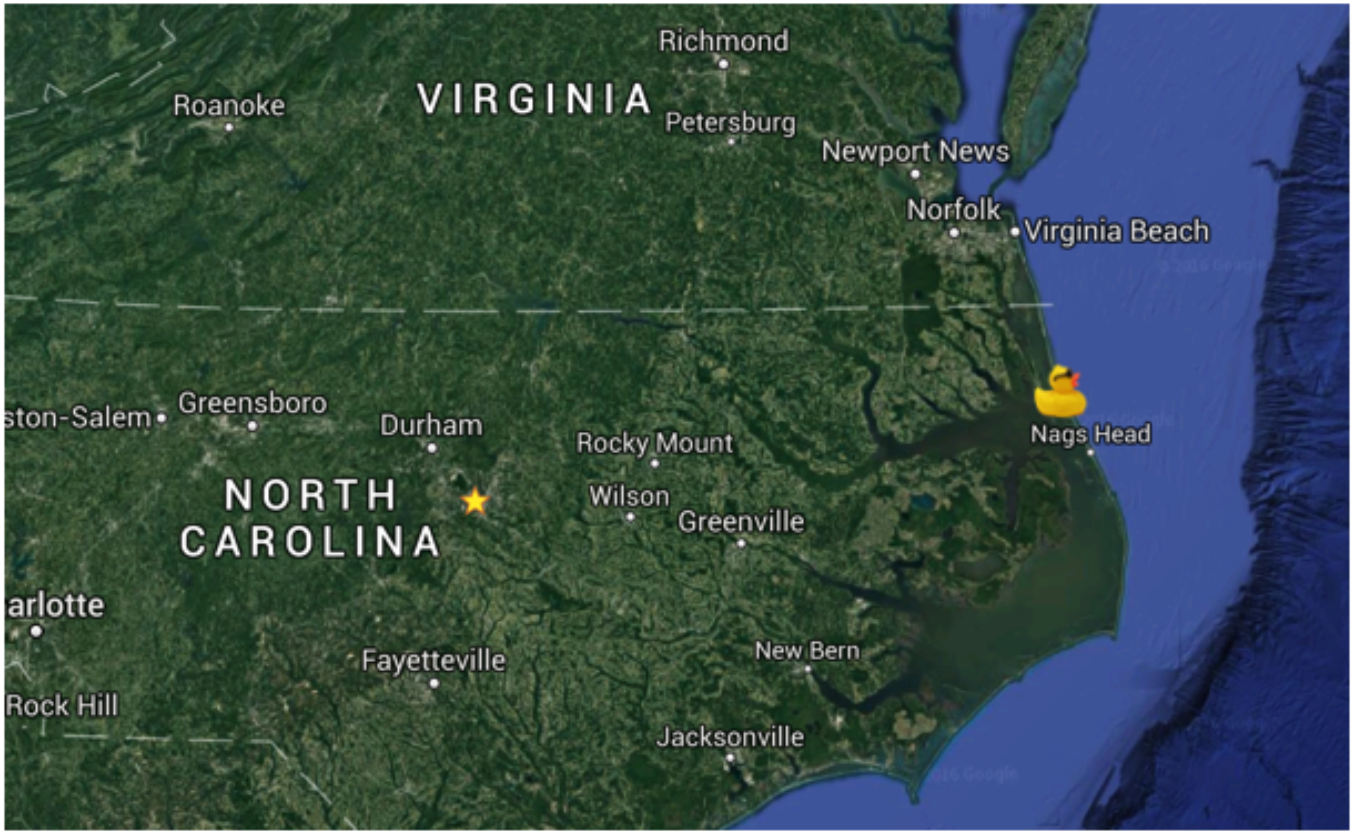


$$\text{wave number: } k = \frac{2\pi}{L}$$

Before we invert we need data



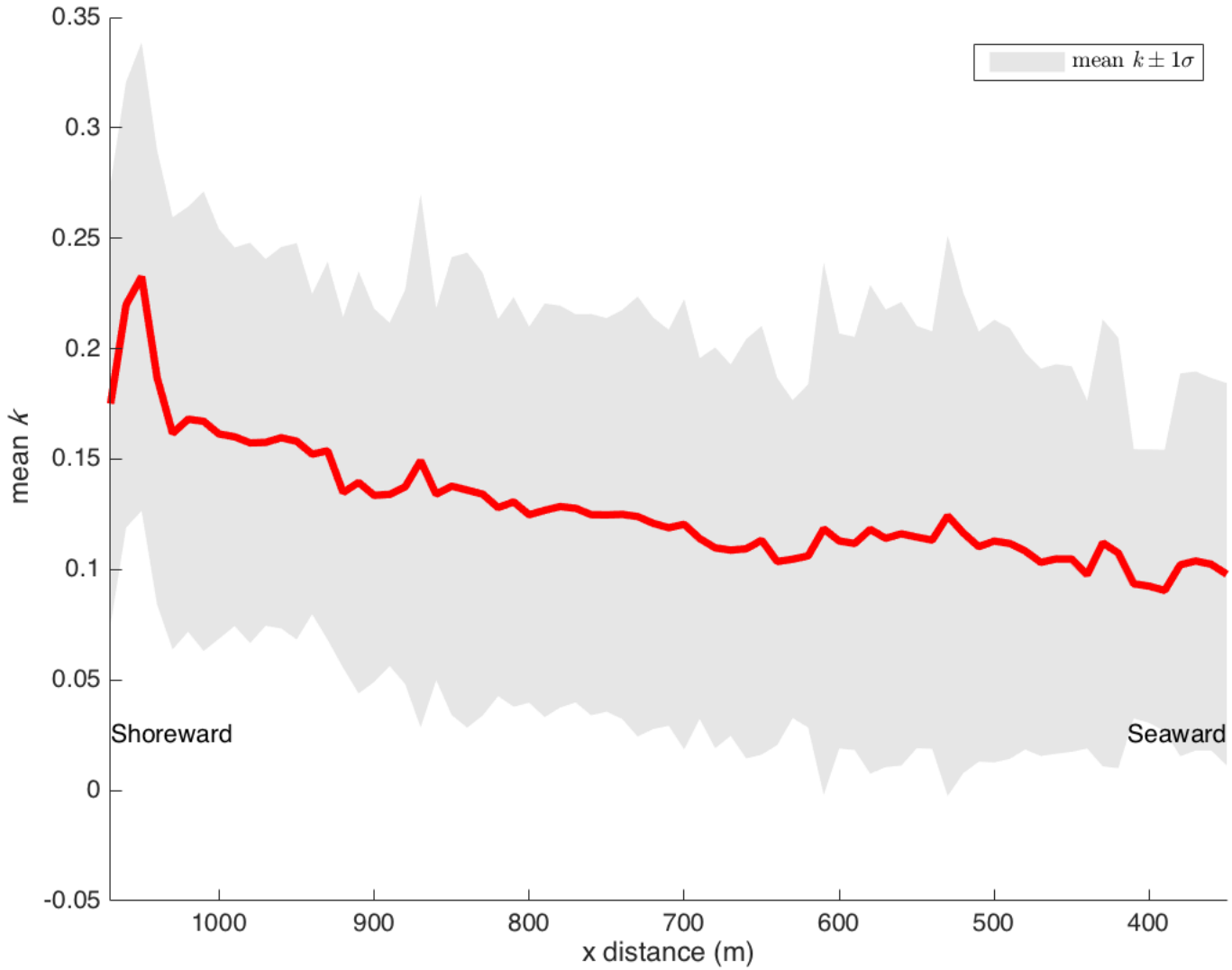
Data was collected by the USACE in Duck, NC



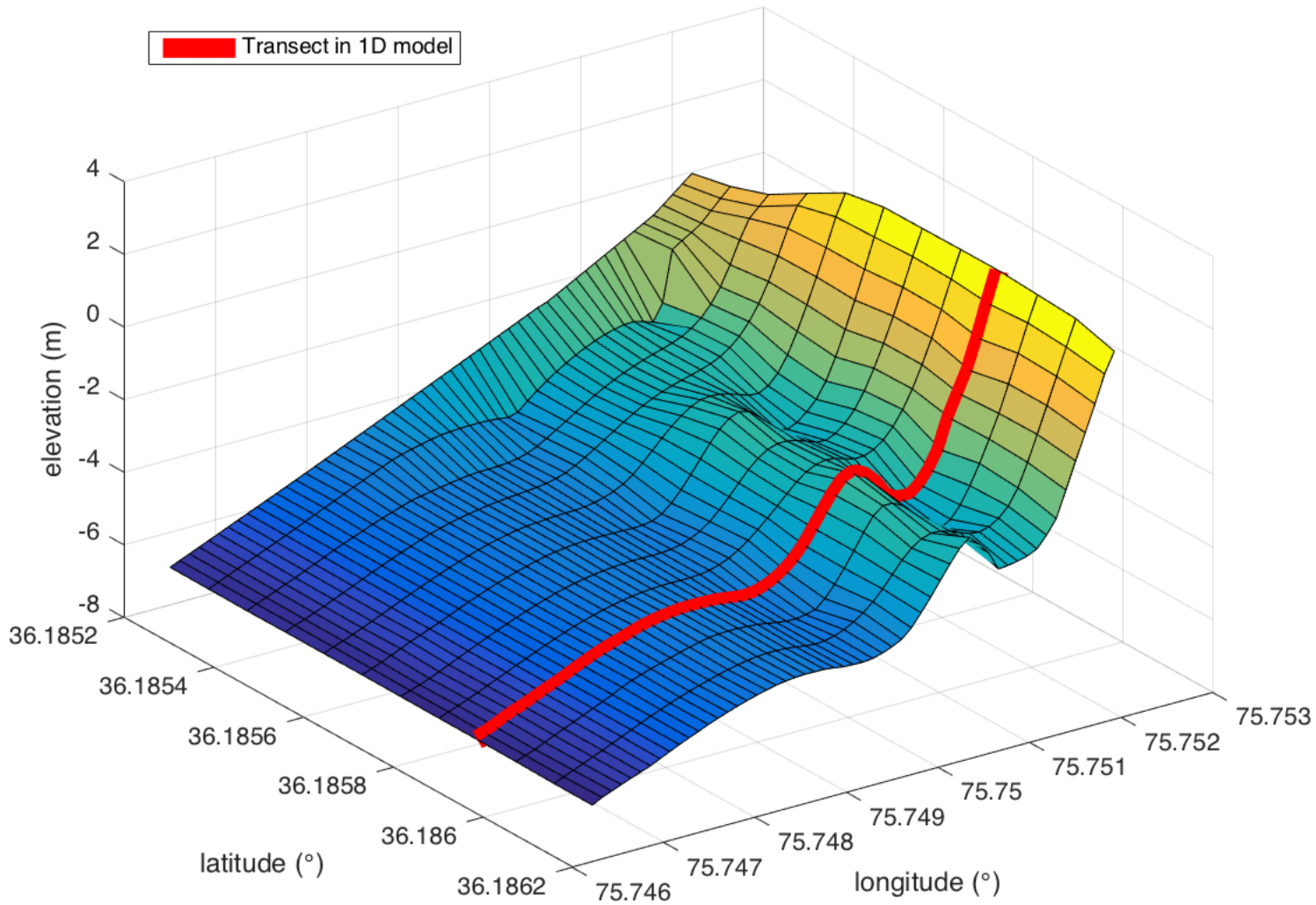
Model coordinate system has $x = 0\ m$ offshore

Remote sensing of surface properties are possible

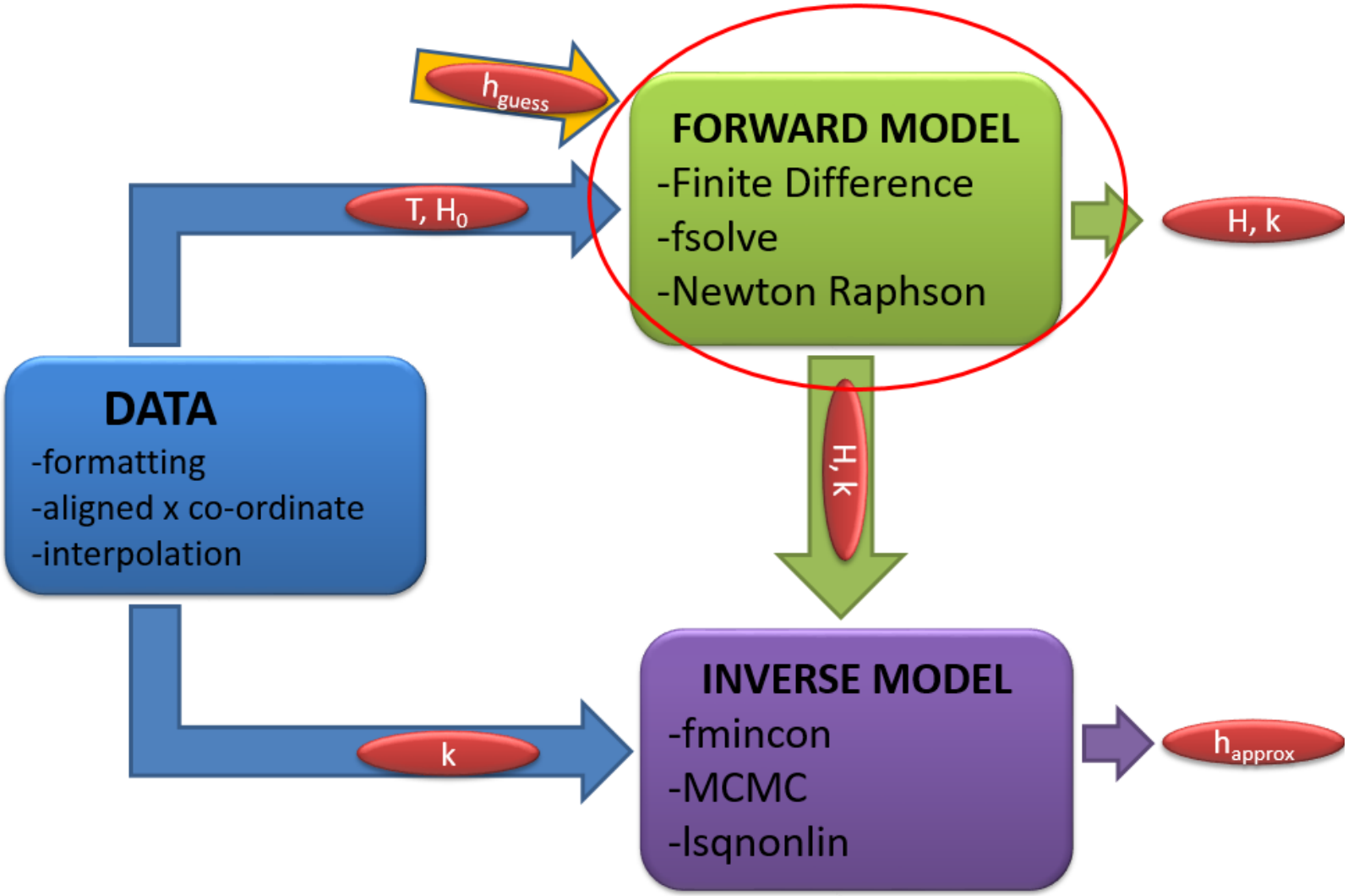
Data includes T, H at offshore boundary; 1D k



Known bathymetry is used for testing our results



Forward model computes k assuming h_{guess} & BC



Wave dispersion relationship relates k to h

Dispersion Relation:

$$\sigma^2 = gk\tanh(kh) \Longleftrightarrow \left(\frac{2\pi}{T}\right)^2 = g\left(\frac{2\pi}{L}\right) tanh\left(\frac{2\pi h}{L}\right)$$

- Relates wave number (k) and Period (T)
- Wave length (L) varies with depth (h)
- Period (T) remains constant

1D forward model relates H and h

Energy Flux Method:

$$\frac{d}{dx}\left(EC_g\right) = -\delta$$

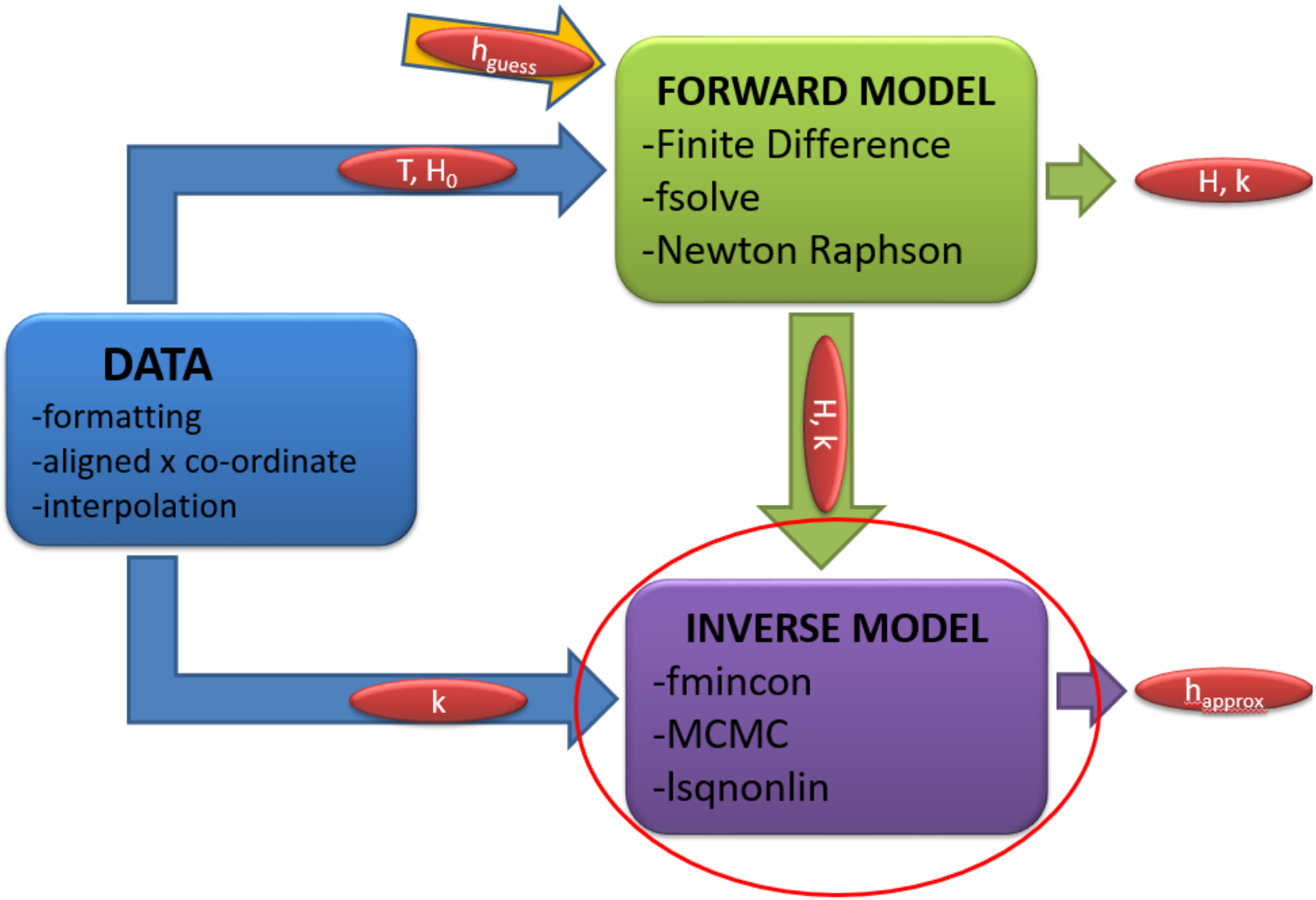
$$\delta = \tfrac{1}{4h} B\rho g f H_{rms}^3 \left[(R^3 + \tfrac{3}{2}R)e^{-R^2} + \tfrac{3}{4}\sqrt{\pi}(1 - erf(R))\right]$$

(Janssen and Battjes, 2007)

where $R = \frac{H_b}{H_{rms}}$, $H_b = 0.78h$, $H_{rms} = 0.7H$.

- E: Wave Energy (ρ, g, H)
- C_g: Group celerity (σ, k, h)

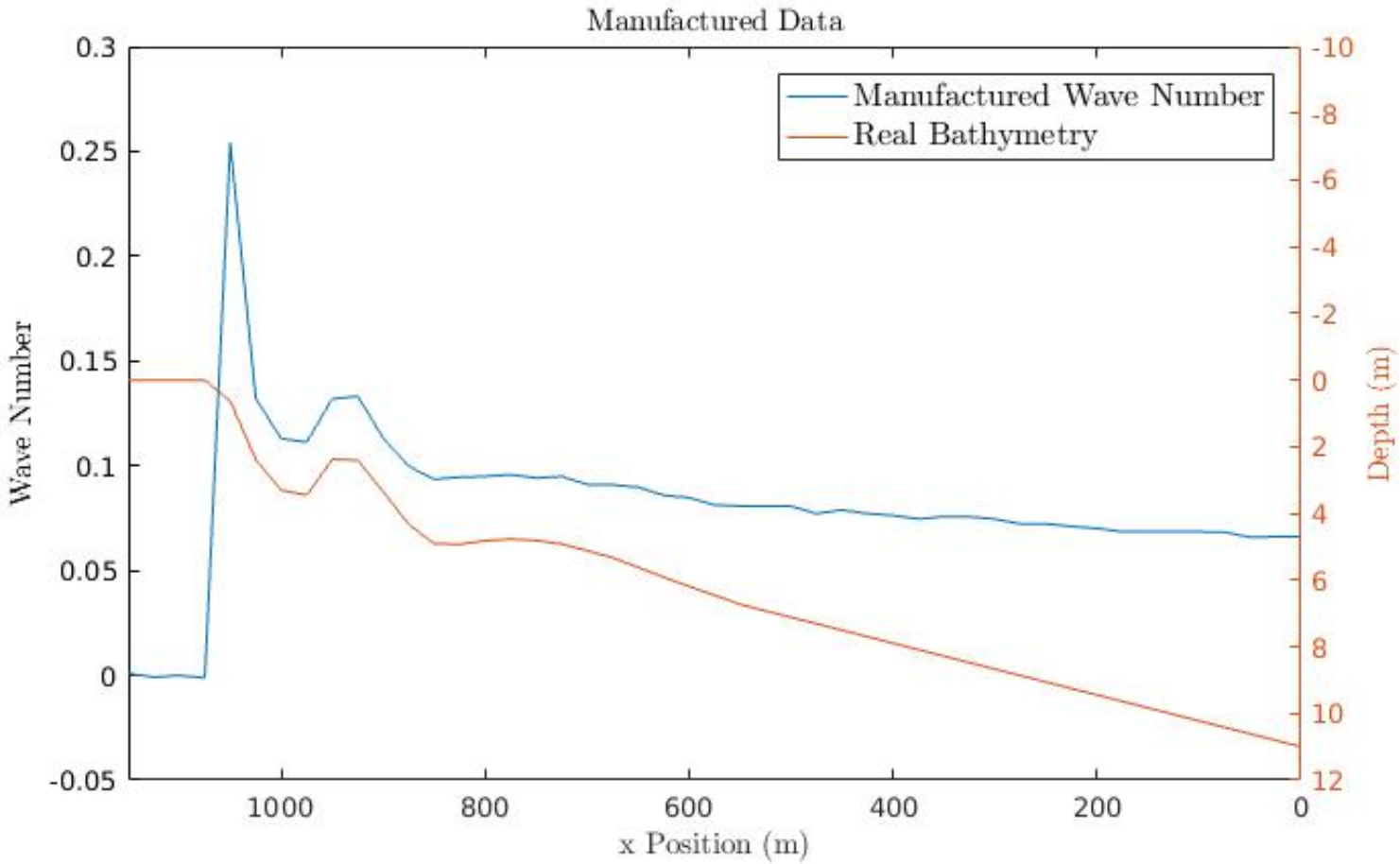
Invert for bathymetry given surface data & physics



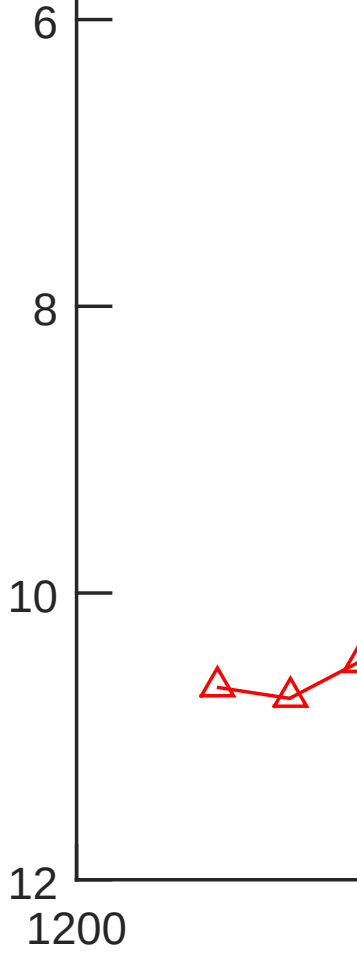
Solutions are computed using 3 inversion methods

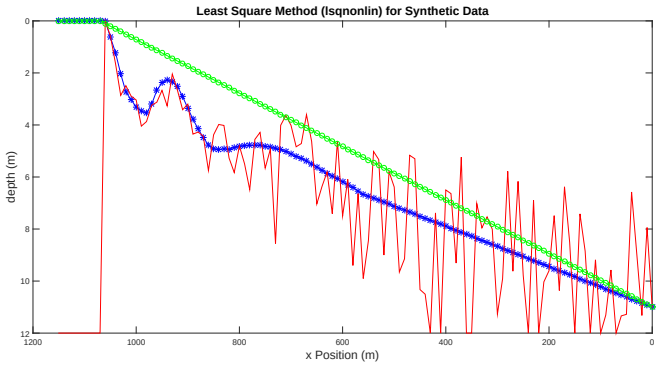
1. Nonlinear Least Squares (lsqnonlin)
   - Logical place to start
2. Bayesian MCMC (Metropolis)
   - Gives a distribution of depth estimates
3. Tikhonov Regularization (fmincon)
   - Bounded-constraint multivariate problem

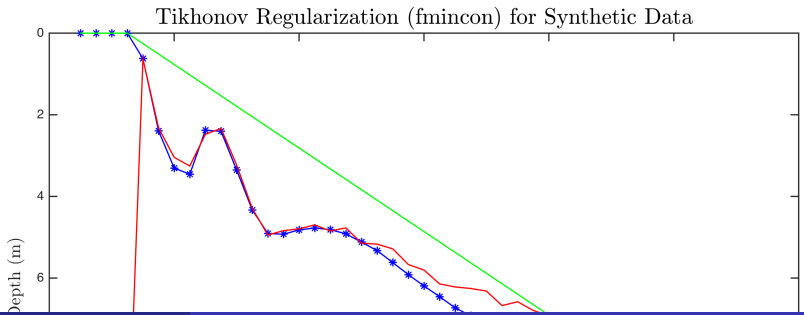
Manufactured "data" is used to test our algorithms



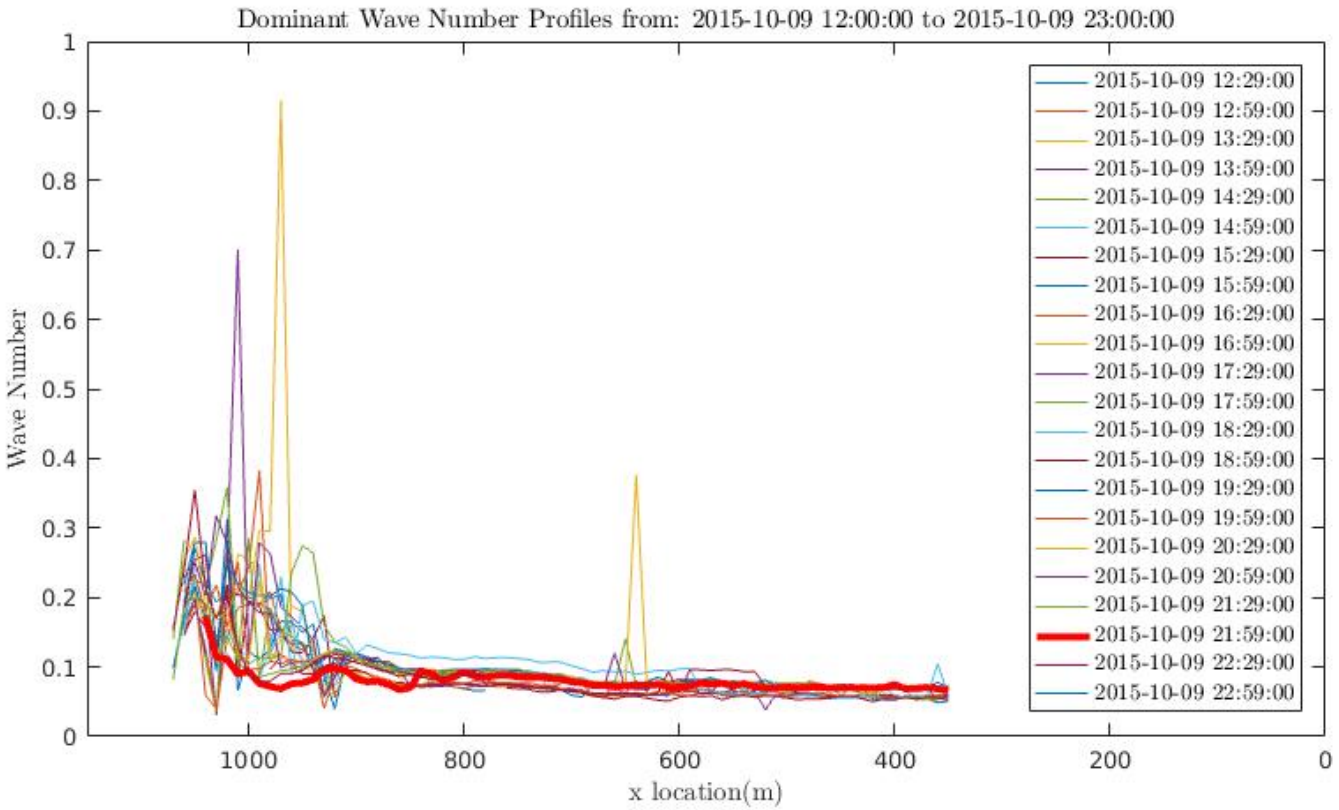
All methods capture the sa



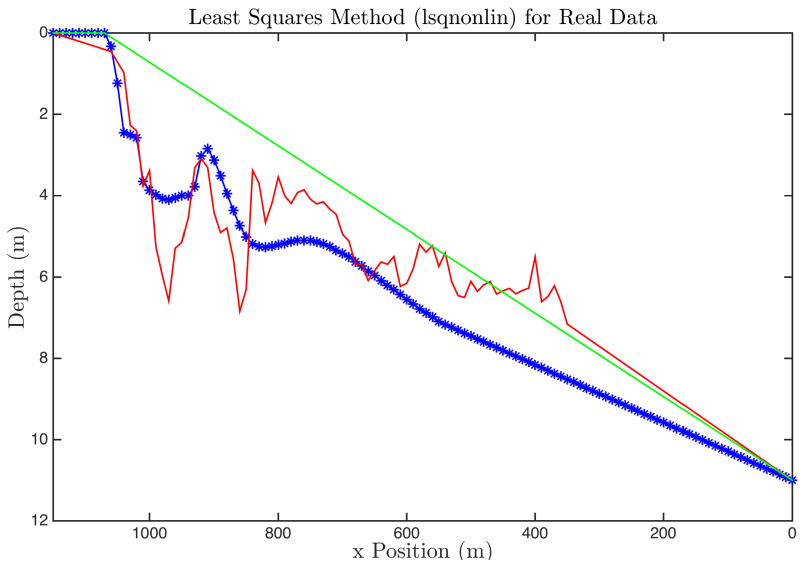


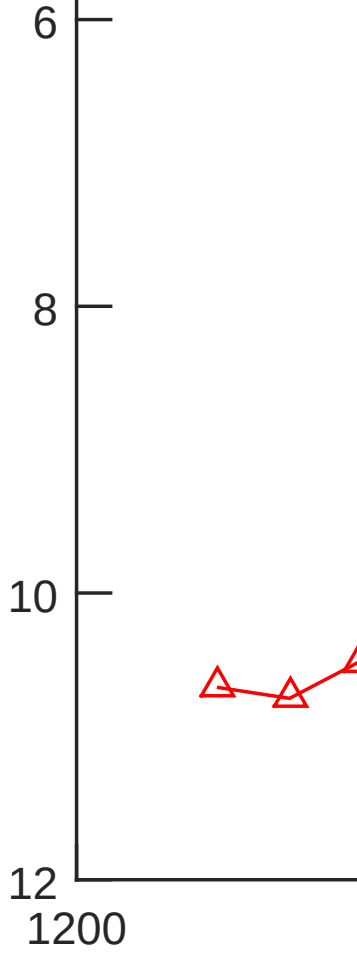


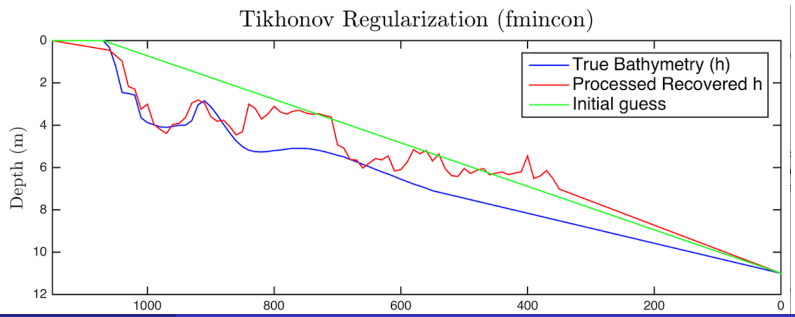
Real k data is selected for a period with low noise



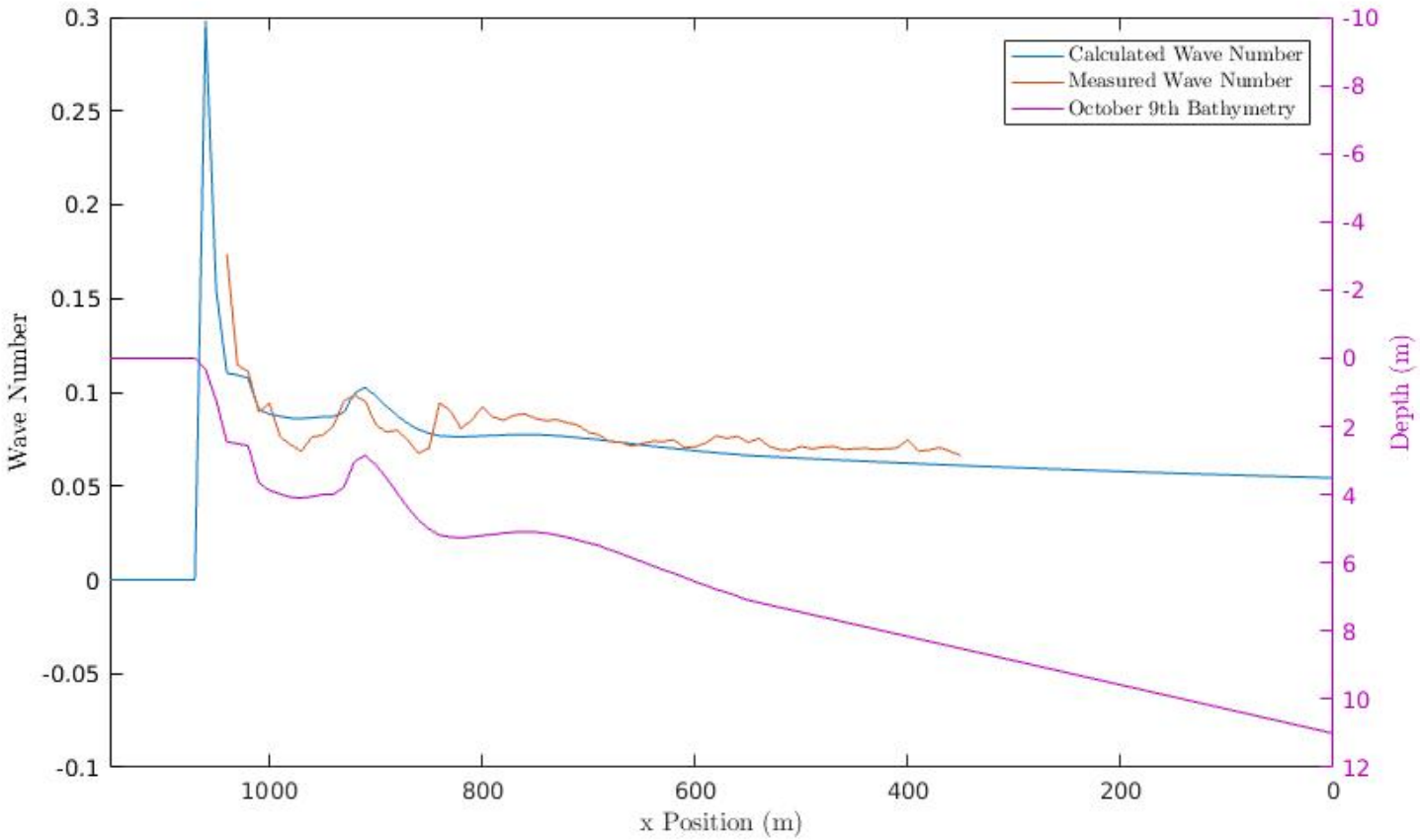
Bathymetry estimates perf



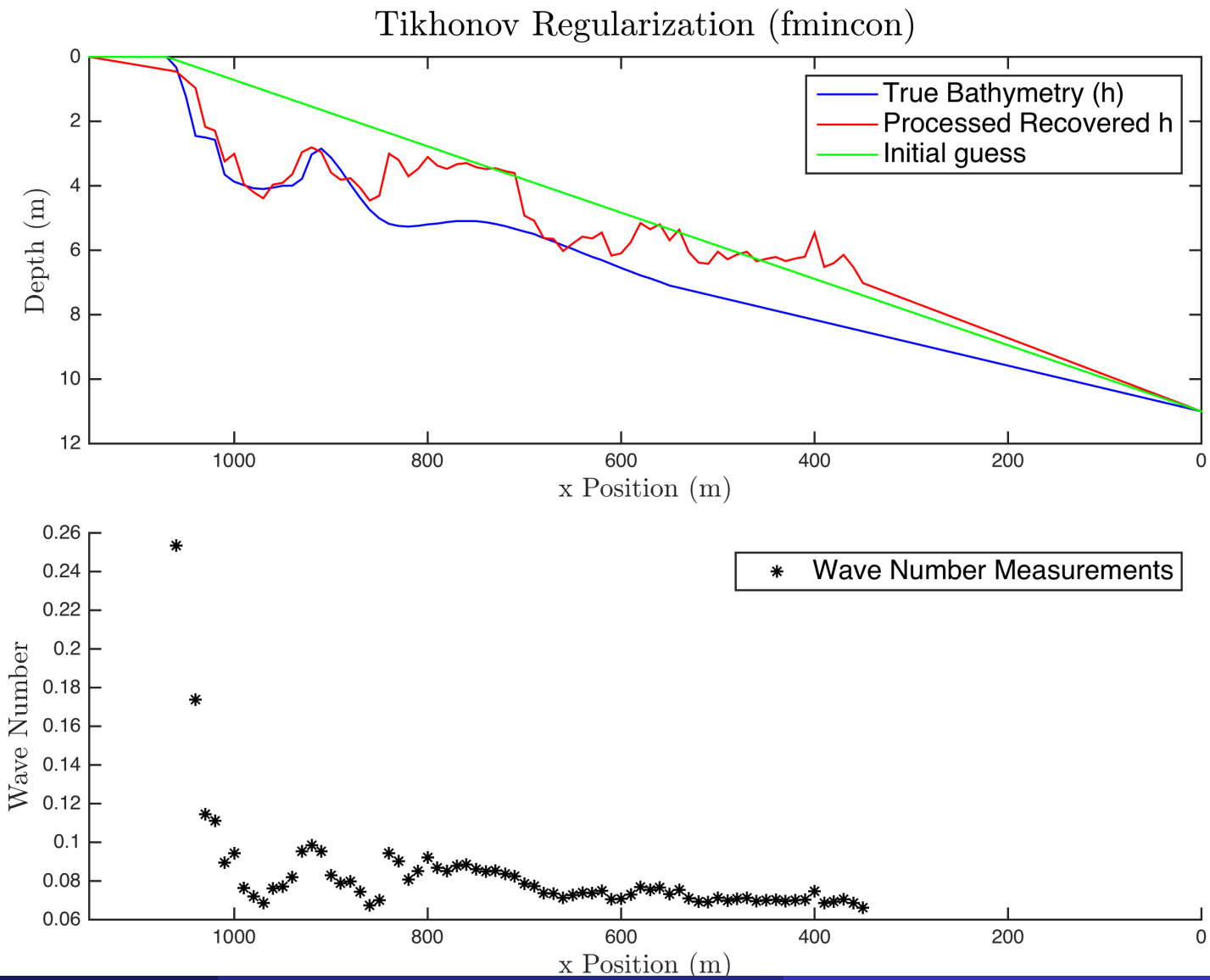




Bathymetry estimates are limited by noisy k data



Missing data & deep water cause poor h offshore



Future Directions

- Non-linear wave theory
  - Linear wave theory is just a starting point!
- Inclusion of observed wave height, H, or other measured variables, along the profile
  - this method will allow for assimilation of other measured variables
- Application of further regularization methods
  - we heuristically "tuned" our regulariztion
  - mayhaps incorporate prior knowledge?
- Incorporate uncertainty in measurements
- Expansion to 2D wave physics

Thank you!
Questions?

Acknowledgements:
NSF
USACE
SAMSI
Anusha Madushani
Kimberly Kaufeld