

Inverting for Near Shore Bathymetry from Surface Wave Properties

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Many coastal processes are affected by bathymetry

Bathymetry
Inversion
from Waves

Introduction

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Discussion



Bathymetry is submarine topography

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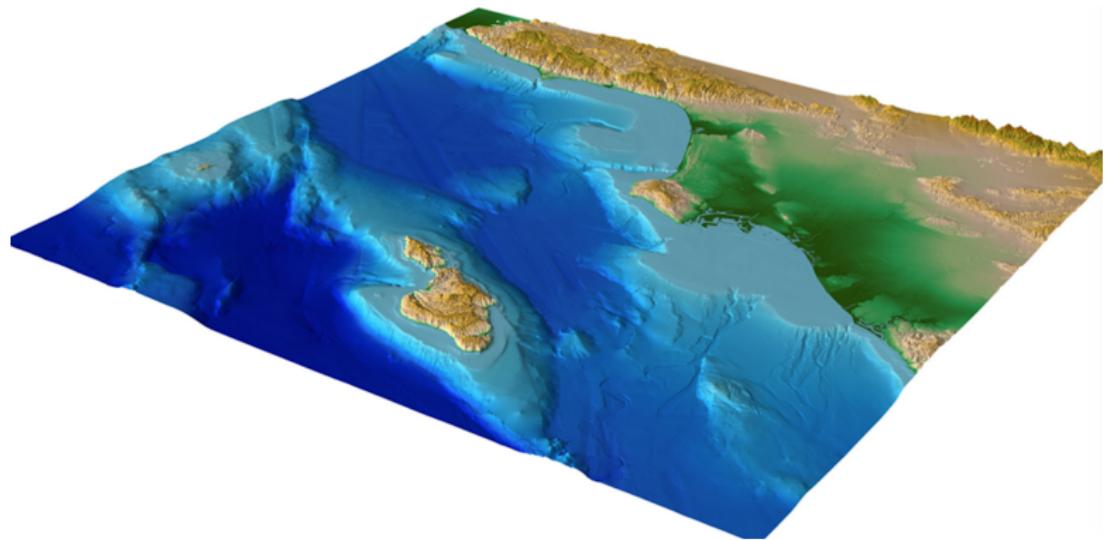
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Direct measurements are expensive and challenging

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LARC



CRAB

Inverse models estimate depth using data & physics

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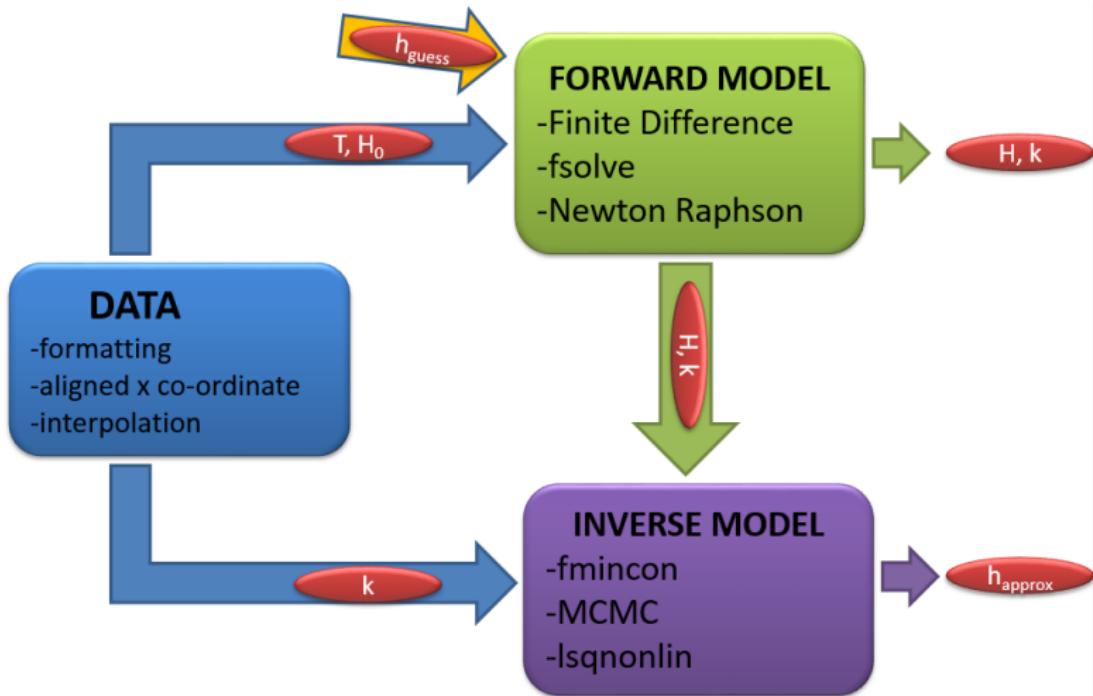
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Bathymetry is related to surface wave properties

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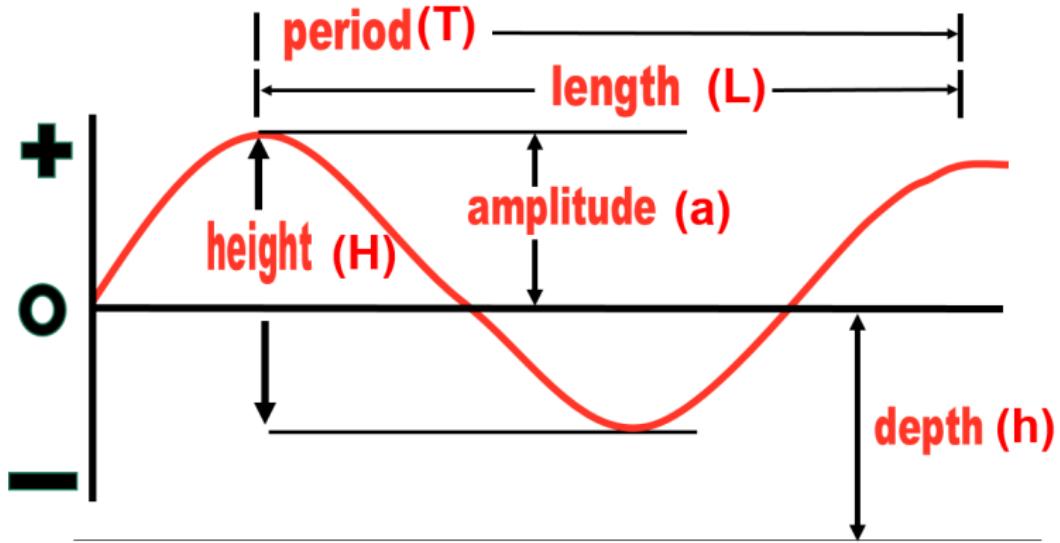
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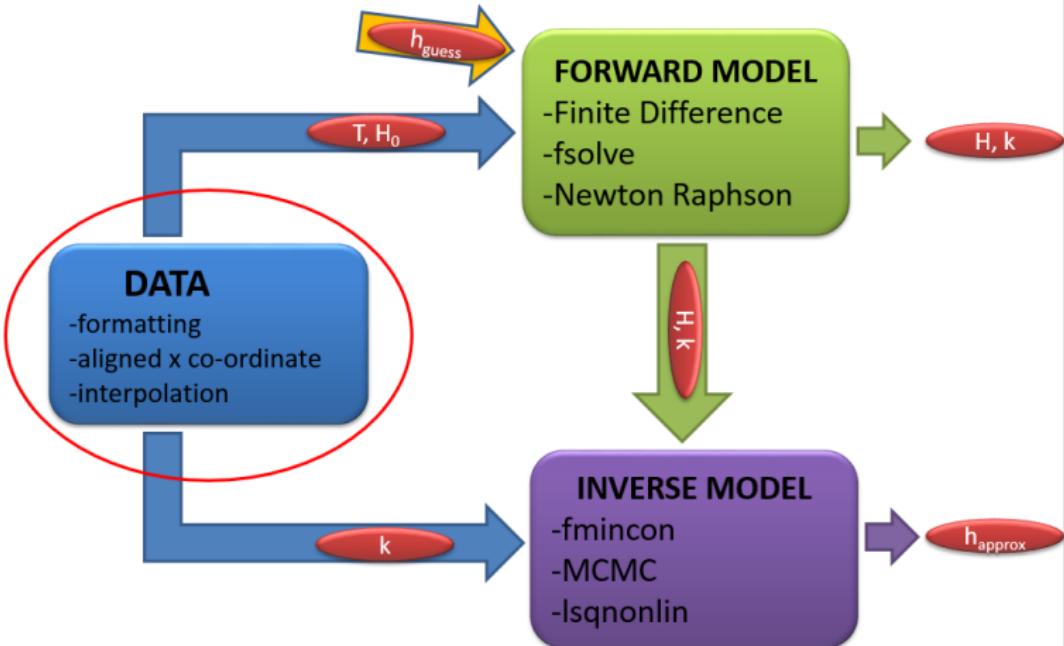
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$$\text{wave number: } k = \frac{2\pi}{L}$$

Before we invert we need data



Data was collected by the USACE in Duck, NC

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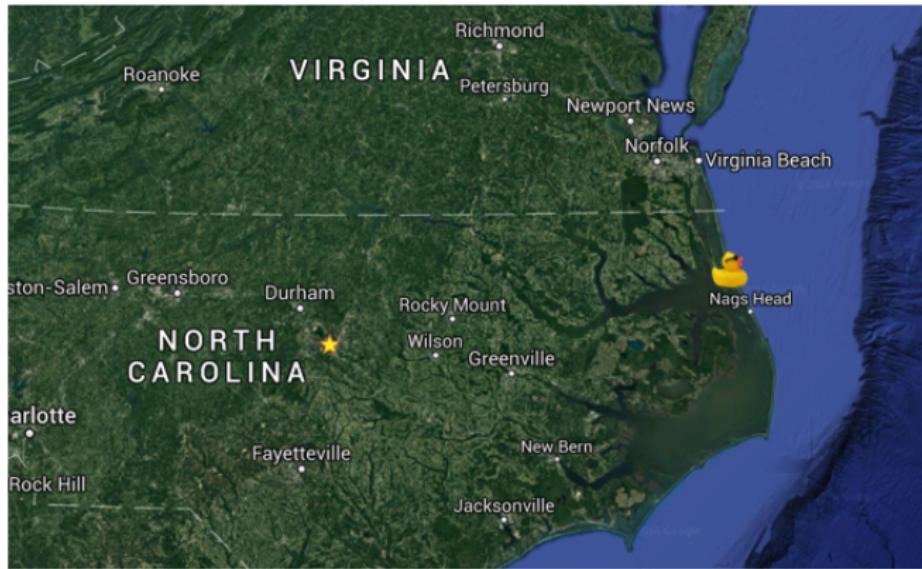
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Model coordinate system has $x = 0$ m offshore

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Remote sensing of surface properties are possible

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Data includes T , H at offshore boundary; 1D k

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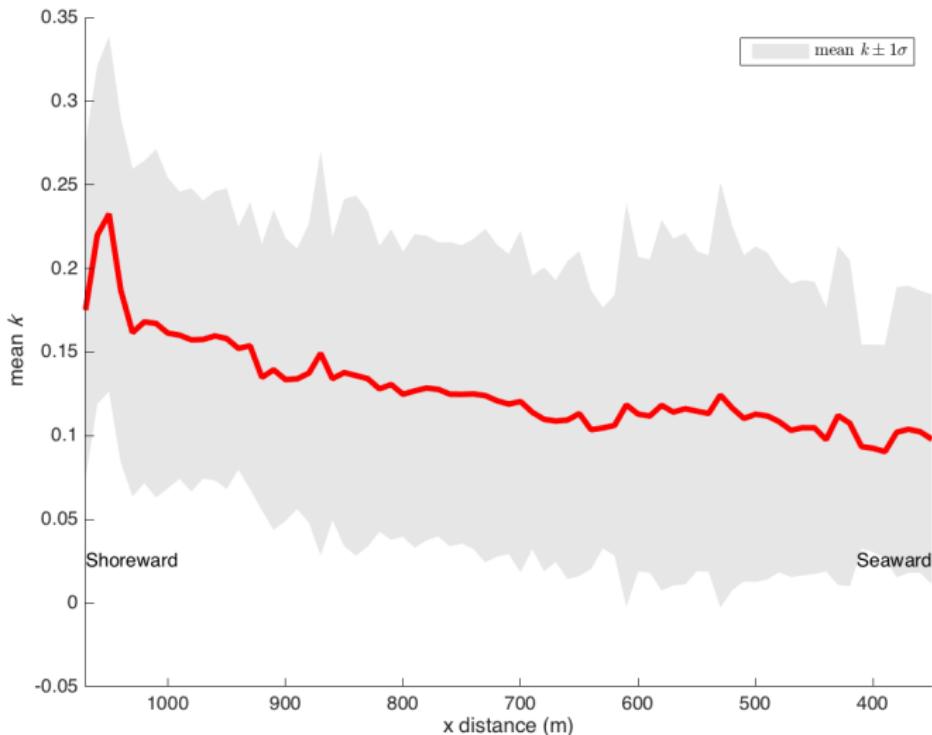
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Known bathymetry is used for testing our results

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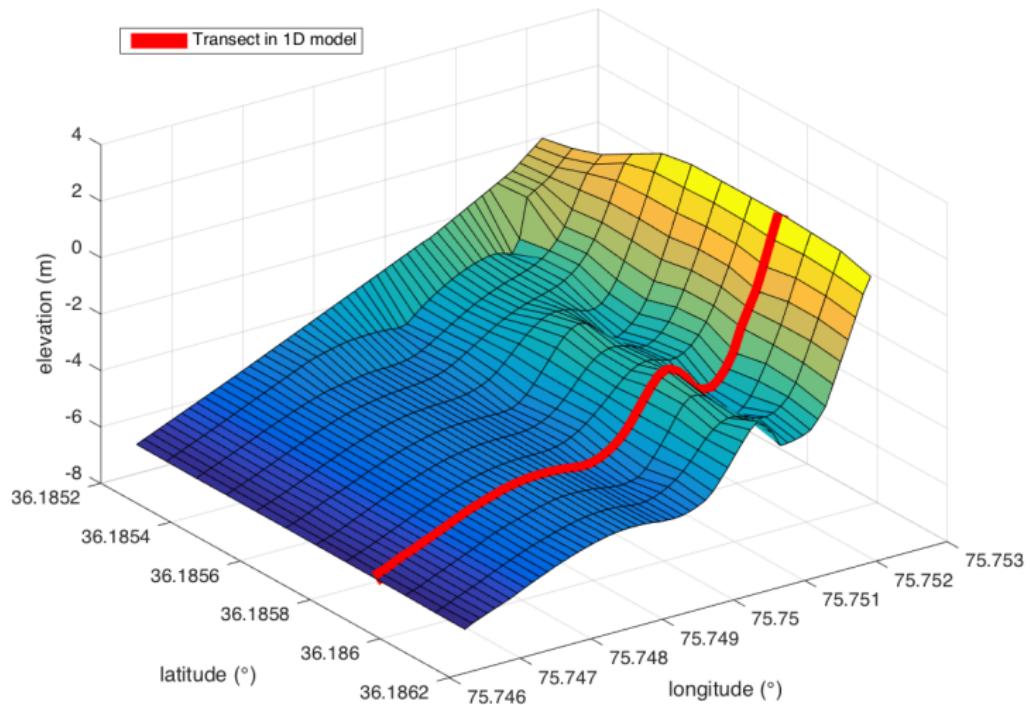
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Forward model computes k assuming h_{guess} & BC

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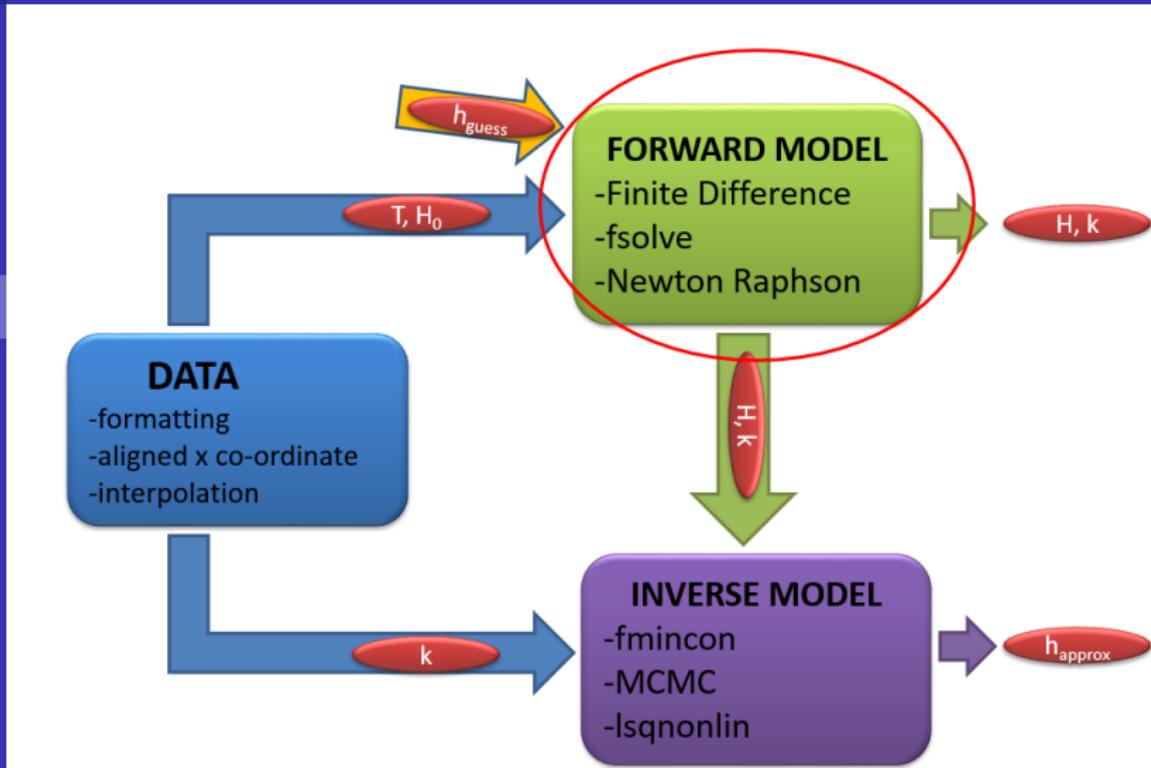
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Wave dispersion relationship relates k to h

Dispersion Relation:

$$\sigma^2 = gk \tanh(kh) \iff \left(\frac{2\pi}{T}\right)^2 = g \left(\frac{2\pi}{L}\right) \tanh\left(\frac{2\pi h}{L}\right)$$

- Relates wave number (k) and Period (T)
- Wave length (L) varies with depth (h)
- Period (T) remains constant

1D forward model relates H and h

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Energy Flux Method:

$$\frac{d}{dx} (EC_g) = -\delta$$

$$\delta = \frac{1}{4h} B \rho g f H_{rms}^3 \left[(R^3 + \frac{3}{2}R)e^{-R^2} + \frac{3}{4}\sqrt{\pi}(1 - erf(R)) \right]$$

(Janssen and Battjes, 2007)

where $R = \frac{H_b}{H_{rms}}$, $H_b = 0.78h$, $H_{rms} = 0.7H$.

- E : Wave Energy (ρ, g, H)
- C_g : Group celerity (σ, k, h)

Invert for bathymetry given surface data & physics

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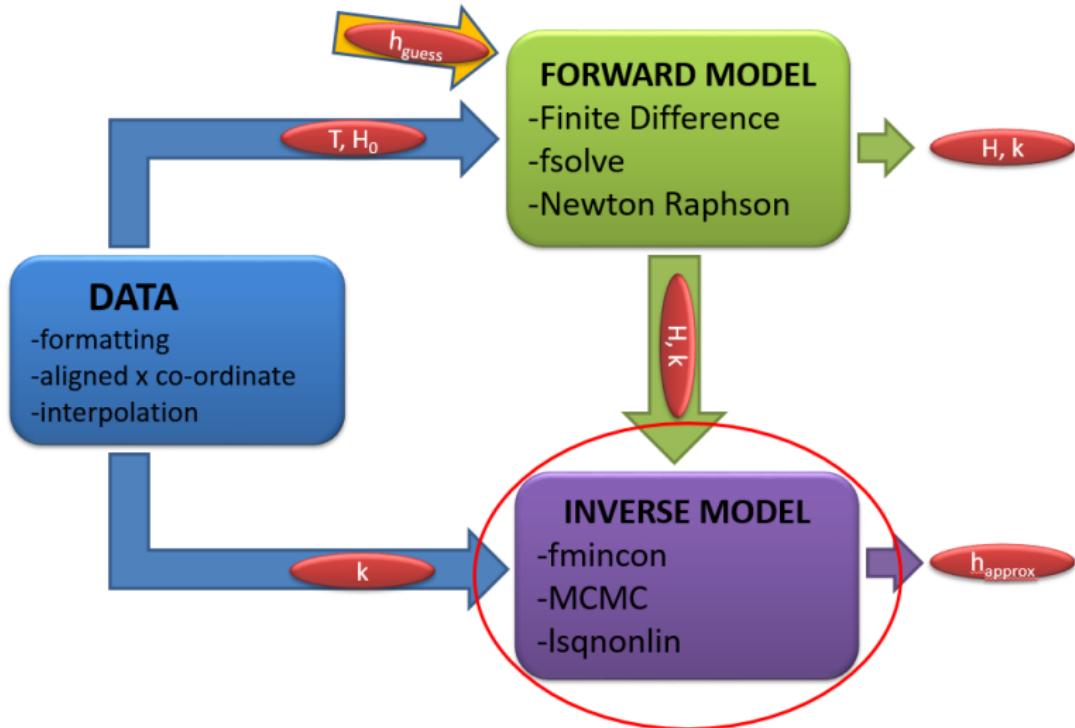
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Solutions are computed using 3 inversion methods

- ① Nonlinear Least Squares (lsqnonlin)
 - Logical place to start
- ② Bayesian MCMC (Metropolis)
 - Gives a distribution of depth estimates
- ③ Tikhonov Regularization (fmincon)
 - Bounded-constraint multivariate problem

Manufactured “data” is used to test our algorithms

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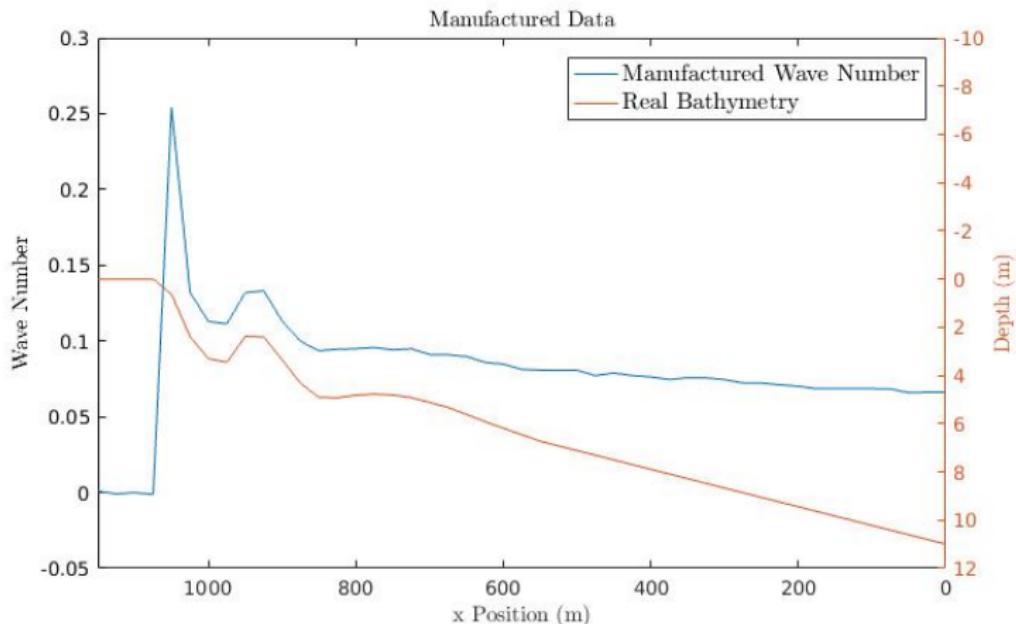
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All methods capture the sandbar well

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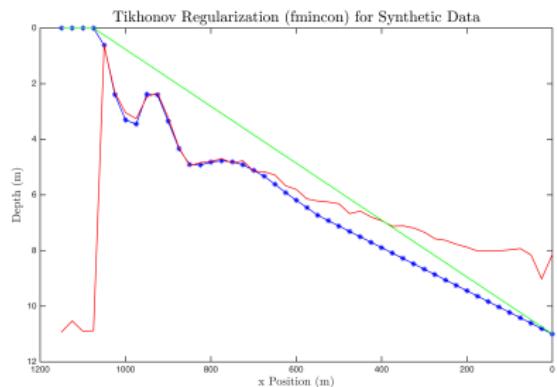
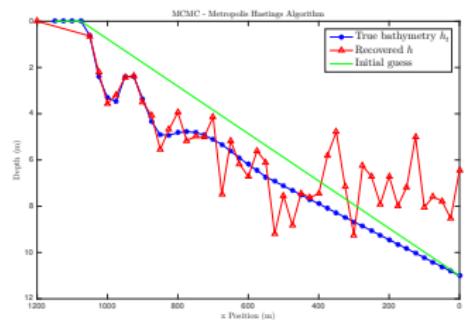
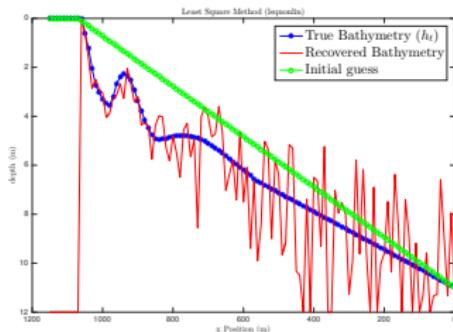
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Real k data is selected for a period with low noise

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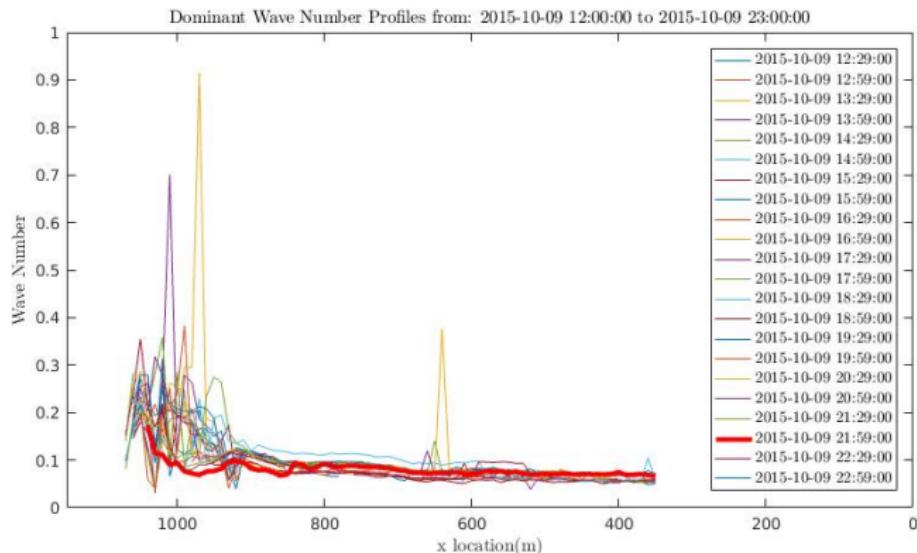
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Bathymetry estimates perform well in shallow water

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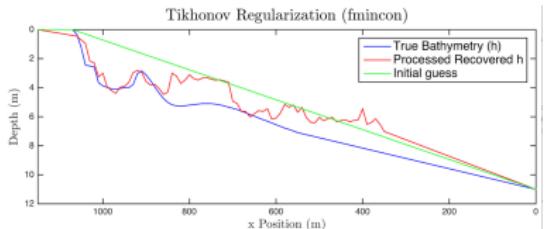
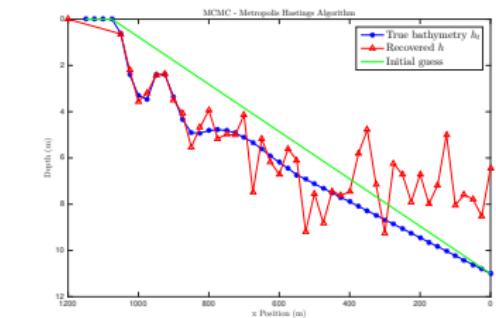
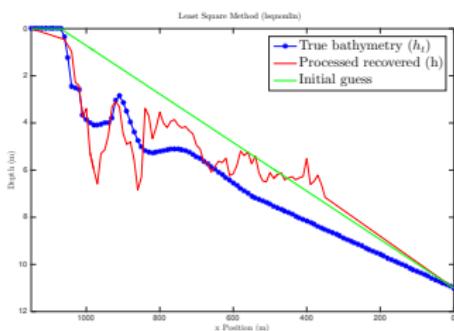
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Bathymetry estimates are limited by noisy k data

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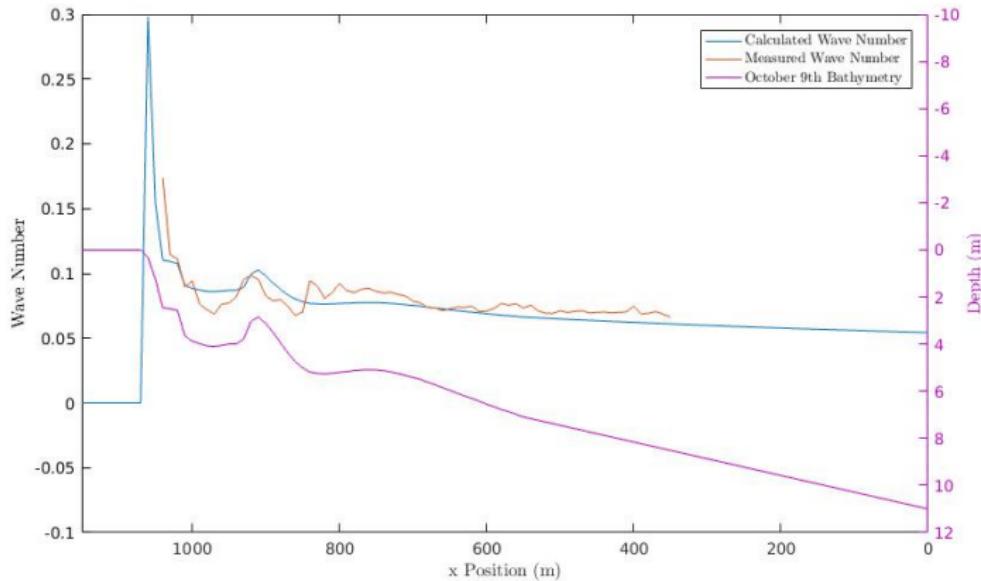
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Missing data & deep water cause poor h offshore

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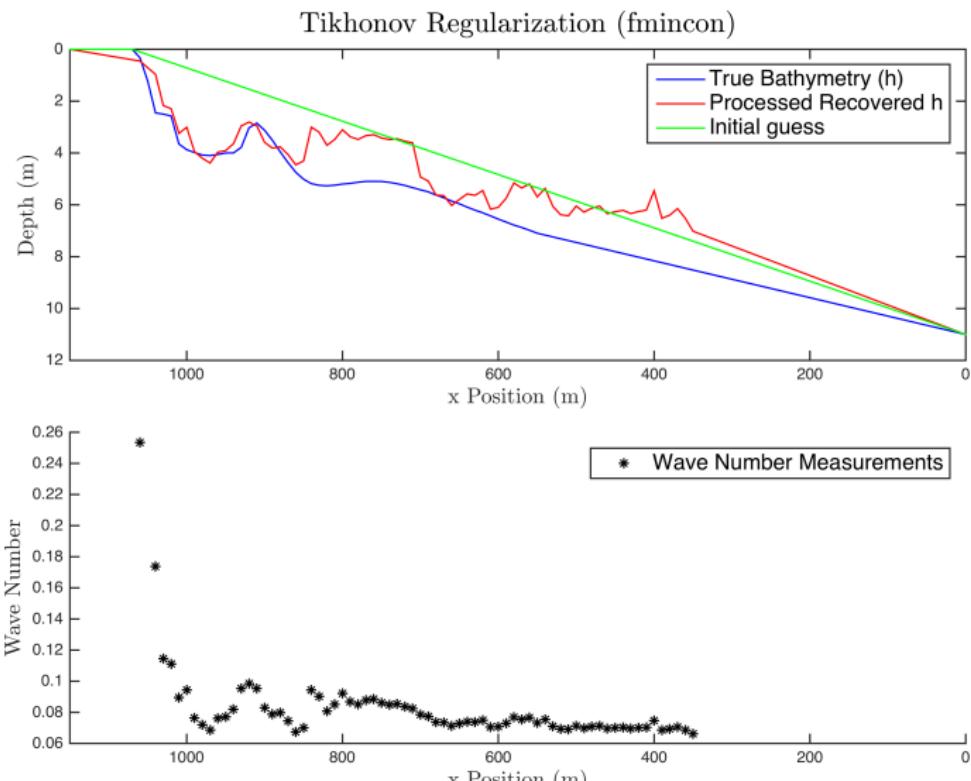
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Future Directions

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- Inclusion of observed wave height, H , along the profile
- Application of further regularization methods
- Expansion to 2D wave physics and higher fidelity models

**Thank you!
Questions?**

Acknowledgements:

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