SLIIT ACADEMY

BSc (IT)

Year 2, Semester 1

Design and Analysis of Algorithms

Introduction to Asymptotic notations

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Lecture Overview

- Asymptotic Notations

  O - Notation

  Θ - Notation

  Ω - Notation

- Selection Sort Algorithm

- Bubble Sort Algorithm

Asymptotic Notations

What is Asymptotic Notation?

Asymptotic notations are the **mathematical notations** used to describe the running time of an algorithm. It used when the input tends towards a particular value or a limiting value.

Why we need of Asymptotic Notation?

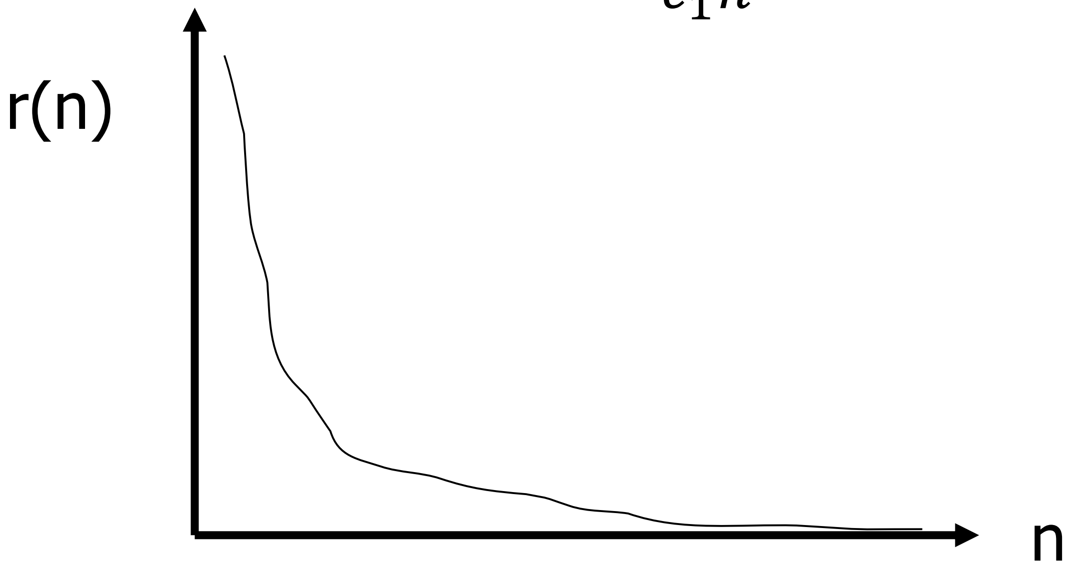
- Ignore machine dependent constants.
- RAM Model have some problems.
- Exact analysis is very complicated
- Sufficiently large size of n.
- Growth of T(n) as n --> ∞

Asymptotic Notations(Contd.)

- Step count is determined to be

  $\mathbf{c_1 n^2 + c_2 n + c_3}$, $c_1 > 0$

Let's take the ratio $r(n) = \frac{c_2 n + c_3}{c_1 n^2}$



When n is large r(n) tends to zero.

Asymptotic Notations(Contd.)

- Since the term $c_2n + c_3$ is not significant ,the run time is approximately

  $$\mathbf{c_1n^2}$$

  Let $n_1$and n_2 be two large values of n. Therefore

  $$\frac{t(n_1)}{t(n_2)} \rightarrow \frac{n_1^{\ 2}}{n_2^{\ 2}}$$

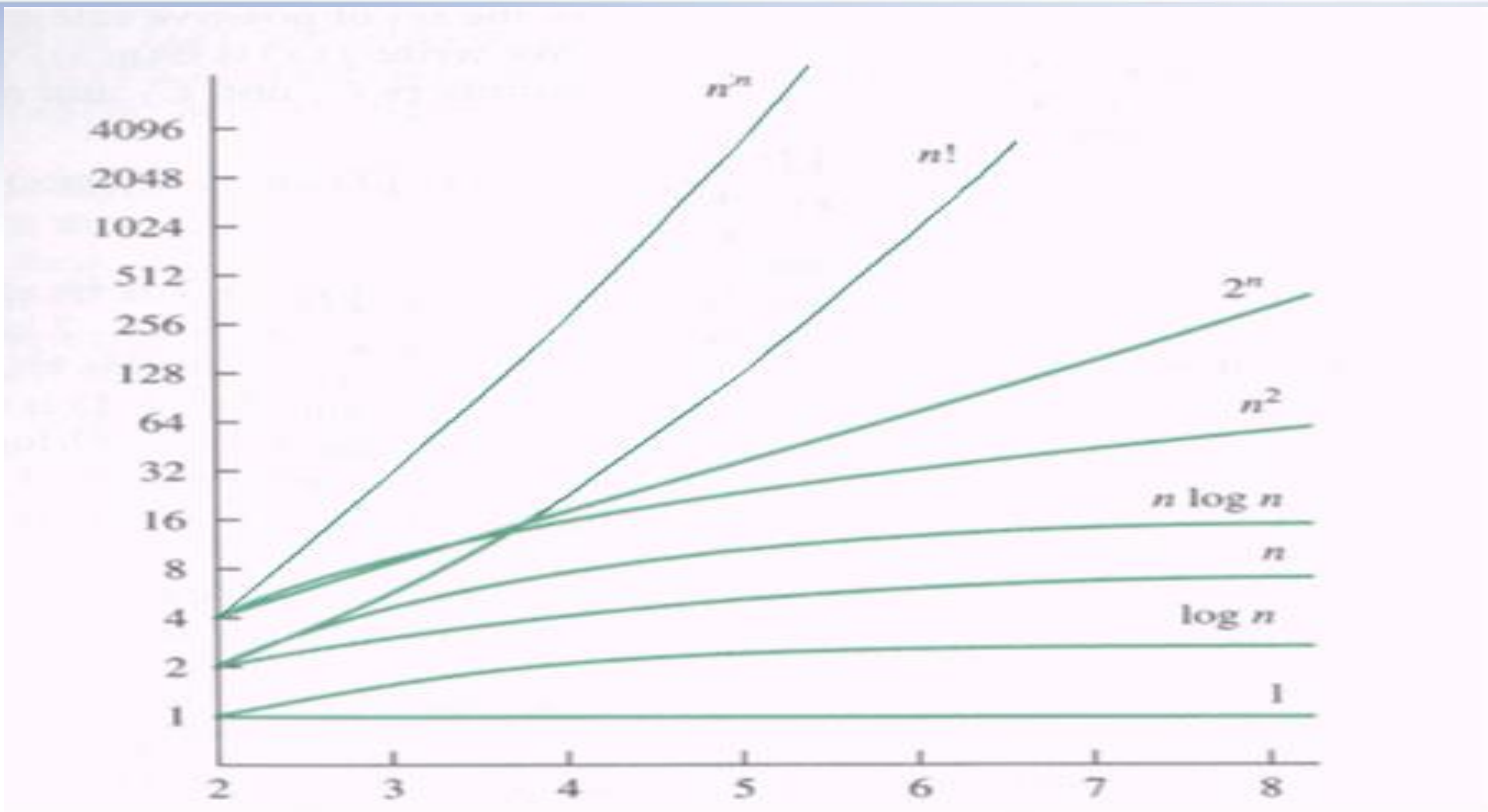
Therefore the run time is expected to increase by a factor of 4 when the instance size is double(2).

Asymptotic Notations(Contd.)

Suppose that programs A and B perform the same task. Assume that one person has determined the step counts of these programs to be $t_A(n)=2n^2+3n$ and $t_B(n)=13n$.

- Which program is the faster one ?
- What is the answer ,if the step count of the program B is 2^n+n^2 ?

Graphs of functions



Asymptotic Notations(Contd.)

There are three notations.

O - Notation

Θ - Notation

Ω - Notation

Asymptotic Notations (Contd.)

- Focus on what's important by abstracting away low-order terms and constant factors.
- How we indicate running times of algorithms.
- A way to compare "sizes" of functions:
  - $O \approx \leq$ -- Consider the **Upper Bound**
  - $\Omega \approx \geq$ -- Consider the **Lower Bound**
  - $\Theta \approx =$ -- **Consider the Both(Average)**

Big O - Notation

- Introduced by Paul Bechman in 1892.
- We use Big O-notation to give an upper bound on a function.

Definition:

$$O(g(n)) = \{ f(n) : \text{there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \le f(n) \le cg(n) \text{ for all } n \ge n_0 \} .$$

Eg: What is the big O value of f(n)=2n + 6 ?

g(n)=n therefore f(n)=O(n)

a_n x^n + … + a_1 x + a_0 is **O(x^n)** for any real numbers **a_n , …, a_0** and any nonnegative number **n .**

Big O – Notation(Contd.)

Find the Big Oh value for following fragment of code.

for i $\leftarrow$ 1 to n

  for j $\leftarrow$ 1 to i $O(n^2)$

    Print j

Big O – Notation(Contd.)

Assignment ($s \leftarrow 1$)

Addition (s+1)

Multiplication (s*2)

Comparison (S<10)

O(1)

Big O – Notation(Contd.)

Find the Big O value for the following functions.

(i) $T(n)= 3 +5n + 3n^2$

(ii) $f(n)= 2^n + n^2 +8n +7$

(iii) $T(n)= n + \log n +6$

Answers:

(i) $O(n^2)$

(ii) $O(2^n)$

(iii) $O(n)$

Back to the example

Alternative calculation:

	cost	times
$sum \leftarrow 0$	c_1	1
for $i \leftarrow 1$ to n	c_2	n+1
$\quad sum \leftarrow sum + A[i]$	c_3	n

$T(n) = c_1 + c_2\,(n+1) + c_3\,n = (c_1 + c_2) + (c_2 + c_3)\,n = c_4 + \mathbf{c_5}\,\boldsymbol{n}$

$\rightarrow$ O (n)

Proof: $c_4 + c_5\,n \leq c\,n$ $\rightarrow$ TRUE for $n \geq 1$ and $c \geq c_4 + c_5$

Ω - Notation

Which will provide the **lower bound** of the function.

Definition:

$\Omega(g(n))$ = { f(n) : there exist positive constants c and n_0 such that $0 \leq cg(n) \leq f(n)$ for all $n \geq n_o$}

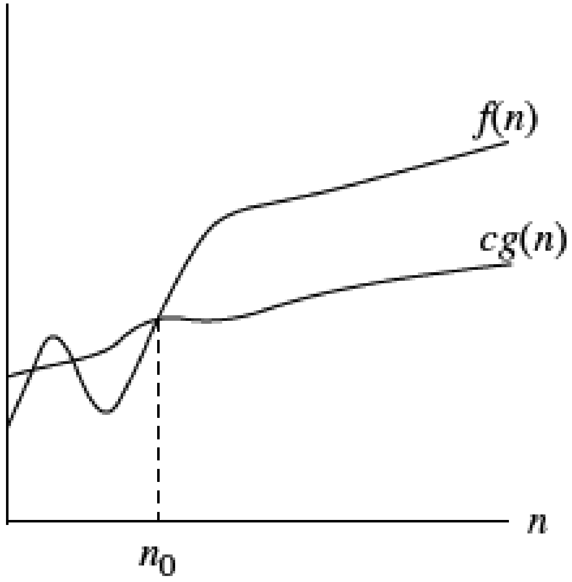
ex:Find the Ω value of the of functions.

(i) $f(n)=6 * 2^n + n^2$

(ii) $f(n)=3n + 2$

Ω-notation

$$\Omega(g(n)) = \{f(n) : \text{ there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \le cg(n) \le f(n) \text{ for all } n \ge n_0\} .$$



$g(n)$ is an ***asymptotic lower bound*** for $f(n)$.

Θ - Notation

This is used when the function *f* can be bounded both from above and below by the same function *g*.

Definition:

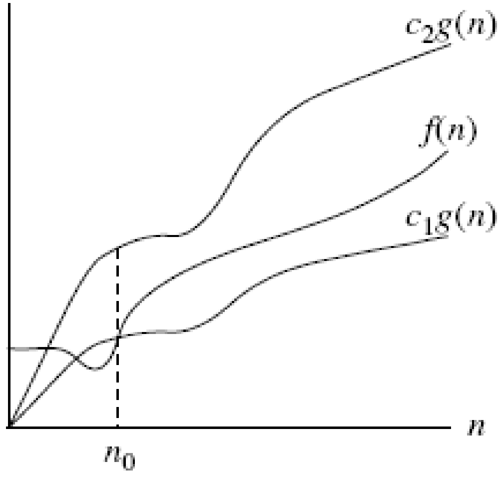
$\Theta(g(n))$ ={ f(n): there exist positive constant c_1, c_2, and n_0 such that $0 \le c_1\, g(n) \le f(n) \le c_2 g(n)$ for all $n \ge n_0$ }

Θ - Notation

Θ-notation

$$\Theta(g(n)) = \{f(n) : \text{there exist positive constants } c_1, c_2, \text{ and } n_0 \text{ such that } 0 \le c_1 g(n) \le f(n) \le c_2 g(n) \text{ for all } n \ge n_0\}.$$

Lecture Notes for Chapter 3: Growth of Functions



$g(n)$ is an ***asymptotically tight bound*** for $f(n)$.

Analysis of Selection Sort Algorithm

- This is an another efficient algorithm for *sorting small number of elements.*
- *Selection Sort Algorithm consist of 5 main steps.*
  1. *Initialize the "min" as leftmost element*
  2. *Search the minimum value in the list*
  3. *Swap with leftmost value and minimum value*
  4. *leftmost "min" incremented by 1, to go for next occurance*
  5. *Repeat the process until the numbers are sorted*

Pseudocode for Selection Sort

```
Selection-SORT(A)
for i = 1 to n - 1
    min = i

      for j = i+1 to n
          if A[j] < A[min] then
              min = j;
         end if
      end for

         swap A[min] and A[i]
   end for
```

Pseudocode for Bubble Sort

```
Bubble-SORT(A)
for i = 1 to n - 1
        for j = 1 to n-i
            if A[i] > A[i+1] then
                swap A[i] and A[i+1]
          end if
        end for
end for
```

Activity

- Convert this number set into Acsending Order using,
  - Selection Sort
  - Bubble Sort

1.

3	9	7	4	1	5

2.

5	3	1	4	7	6

Questions ???

Thank You

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