3.1

a

Ace of Hearts on the first draw, King of Hearts on the second.
$1/52 * 1/51 = 1/2652 = 0.00038$

b

Two aces.
$4/52 * 3/51 = 1/221 = 0.004$

c

No ace on either draw
$48/52 * 47/51 = 188/221 = 0.85$

d

The probability of picking the ace of hearts on the first draw is 1/52 and 1/51 on the second.
The other option includes picking one of the other aces, and another card that is hearts, which gives 3/52 and 12/51 respectively.
Which gives the final probability of:
$(1/52 * 1/51) + (3/52 * 12/51) = 0.014$

3.2

Suppose that on any given day the probability of rain (R) is 0.25, that of clouds (C) is 0.4, and that of clouds given rain is 0.9. You observe a cloudy sky. What are the chances of rain?

$P(C|R) = 0.9,\ P(C) = 0.4\ ,P(R) = 0.25$
We apply Bayes theorem that says:
$P(R|C) = \frac{P(R)*P(C|R)}{P(C)}$
$P(R|C) = 0.25 * 0.9/0.4 = 0.5625$

3.3

Calculate the posterior probability that the coin is biased given that you flip the coin ten times and observe eight tails followed by two heads.

Your prior belief that the coin is biased is P(B) = 0.01. You also believe that if the coin is biased, then it will fall Heads with a probability of 0.75.

$P(B) = 0.01$, $P(H|B) = 0.75$.
The probability of two heads given biased coin is $P(2H|B) = 0.75^2 * (0.25)^8 = 9/1048576$
The probability of two heads given fair coin is $P(2H|\neg B) = 0.5^{10} = 1/1024$
$P(B|2H) = \frac{P(B)*P(2H|B)}{P(B)*P(2H|B)+(1-P(B))*P(2H|\neg B)} = \frac{0.01*9/1048576}{(0.01*9/1048576)+(0.99*1/1024)} = 1/11265 = 0.00008877$

3.4

Give a Dutch book argument for the probability axiom $P(A) + P(\neg A) = 1$

If you have a subjective degree of belief in $P(A) = 0.55$ and $P(\neg A) = 0.52$ then $P(A) + P(\neg A) \neq 1$.
Given a bet on A you'd be willing to pay 0.55 and 0.52 respectively for a payoff of 1.
So you pay 1.07, and thereby always lose 0.07.
Clearly irrational.