

3.1

a

$$1/52 * 1/51 = 1/2652 = 0.00038$$

b

$$4/52 * 3/51 = 1/221 = 0.004$$

c

$$48/52 * 47/51 = 188/221 = 0.85$$

d

The probability of picking the ace of hearts on the first draw is $1/52$ and $1/51$ on the second.

The other option includes picking one of the other aces, and another card that is hearts, which gives $3/52$ and $12/51$ respectively.

Which gives the final probability of:

$$(1/52 * 1/51) + (3/52 * 12/51) = 0.014$$

3.2

$$P(C|R) = 0.9, P(C) = 0.4, P(R) = 0.25$$

We apply Bayes theorem that says:

$$P(R|C) = P(R) * P(C|R) / P(C)$$

$$P(R|C) = 0.25 * 0.9 / 0.4 = 0.5625$$

3.3

$$P(B) = 0.01, P(H|B) = 0.75.$$

The probability of two heads given biased coin is $P(2H|B) = \binom{10}{2} * 0.75^2 * (0.25)^8 = 405/1048576$

The probability of two heads given fair coin is $P(2H|\neg B) = \binom{10}{2} * 0.5^{10} = 45/1024$

$$P(B|2H) = \frac{P(B) * P(2H|B)}{P(B) * P(2H|B) + (1 - P(B)) * P(2H|\neg B)} = \frac{0.01 * 405 / 1048576}{(0.01 * 405 / 1048576) + (0.99 * 45 / 1024)} = 1/11265 = 0.00008877$$