AUTOMAATIOPÄIVÄT24

13.–14.4.2021 | Virtual event

Rejection of Unknown Harmonic Disturbances in a Boundary Controlled Heat Equation

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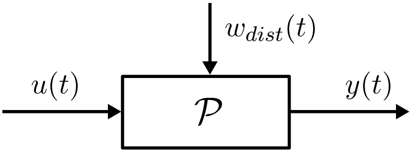
In this presentation

1. An overview of controlled Partial Differential Equations ("PDE Control"), and its applications.
2. Disturbance rejection for a controlled heat equation model.

Part I: PDE Control and Applications – An Overview

Control of Partial Differential Equations



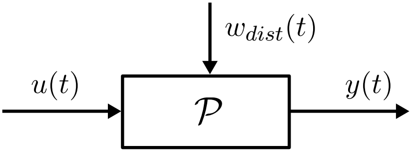
In most control problems (linear or nonlinear), the plant $\mathcal{P}$ is an Ordinary Differential Equation with state $x(t)$ depending on time.

- RLC circuits, mechanical systems
- Different types of robotic models, etc

In **PDE Control**, $\mathcal{P}$ is a **Partial Differential Equation**, whose state depends on several variables, typically,

$$x(t,z) \qquad \text{where} \qquad t = \text{time}, \quad z = \text{position}.$$

Control of Partial Differential Equations



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$$x(t,z) \qquad \text{where} \qquad t = \text{time}, \quad z = \text{position}.$$

Such models describe the dynamics of, for example,

- temperature distributions ("**heat equation**")
- flexible and vibrating structures ("**beam, plate, wave eqns**")
- fluid flows ("**convection and transport equations**")

Example: Robots with Flexible Components

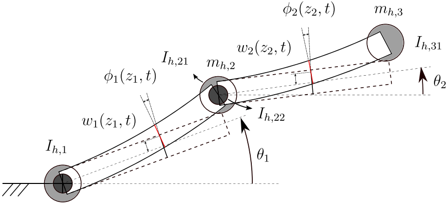


Fig. 2. Flexible two links manipulator.

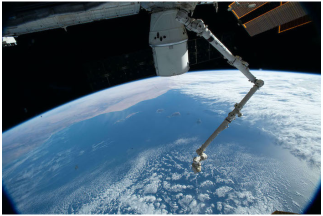


Figure: Mattioni et. al. 2020 (left), www.nasa.gov (right).

Application: Control design for robotic arms with flexible links.

- The dynamics of flexible links modelled with **beam equations**
- Especially lightweight structures cause flexibility (space, nano)

Example: Control of Traffic Flows

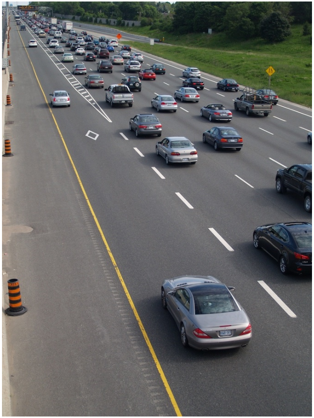


Figure: Wikipedia.

- Dynamics of large configurations of vehicles can be modelled with **transport equations**
- Allows control design to reduce traffic jams
- Speed limits and lights can be used as control actuators
- PDE models can sometimes be used in control design even for small formations

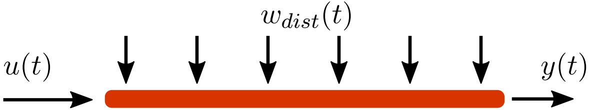
Other Examples

- Control of temperature profiles:
  - Control of room temperature models, heat exchange processes
  - Involves **convection-diffusion equations**
- Control of flows:
  - Mixing processes, pipe networks
  - Depending on the problem, may involve **convection** or **transport equations**
- Vibration control:
  - Vibrations in drilling equipment, large-scale buildings, wind turbine towers
  - Involves **wave, plate and beam equations**

Figure: Wikipedia.

Part II: Rejection of Harmonic Disturbances

Main Case: A Controlled Heat Equation



Consider the controlled PDE: for $x \in [0,1]$

$$
\begin{aligned}
\frac{\partial v}{\partial t}(x,t) &= \frac{\partial}{\partial x}\left(c(x)\frac{\partial v}{\partial x}(x,t)\right) + B_d(x) w_{\textsf{\textbf{dist}}}(t) \\
\frac{\partial v}{\partial x}(0,t) &= u(t), \qquad \frac{\partial v}{\partial x}(1,t) = 0, \\
y(t) &= v(1,t).
\end{aligned}
$$

$v(x,t)$ is the temperature of a metal bar of length $\ell = 1$, heat conductivity $c(x)$, heating control input at $x = 0$, isolated at $x = 1$. Temperature measurement at $x = 1$.

Problem (Output Tracking and Disturbance Rejection)

Choose a control law in such a way that

- *The output $y(t)$ converges to a given reference level $y_{\mathit{ref}} \in \mathbb{R}$ asymptotically, i.e.*

$$|y(t) - y_{\mathit{ref}}| \to 0 \qquad \textit{as } t \to \infty$$

despite the external disturbance $w_{\mathit{dist}}(t)$.

We assume the disturbance has the form

$$w_{\mathit{dist}}(t) = a_0 + \sum_{k=1}^{q} a_k \cos(\omega_k t + \varphi_k),$$

unknown frequencies $\{\omega_k\}_k$, amplitudes $\{a_k\}_k$ and phases $\{\varphi_k\}_k$.

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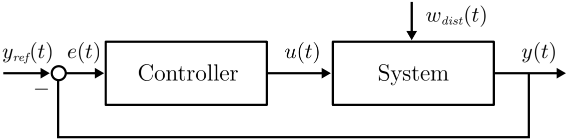

Solution

$$w_{\mathit{dist}}(t) = a_0 + \sum_{k=1}^{q} a_k \cos(\omega_k t + \varphi_k),$$

Background:

- If the frequencies $\{\omega_k\}_k$ were **known**, the problem could be solved using **internal model based control** (developed for PDEs by the TUT/TAU group and many others).

Solution:

- Combine a **frequency estimator** to approximate ω_k with $\hat{\omega}_k(t)$, and an **internal model based** controller.
- $\rightsquigarrow$ Closed-loop system is a time-dependent PDE-ODE system, creates (interesting!) mathematical challenges, methods developed in [Afshar-Paunonen 2019, 2020].

Simulation Results

Parameters: $c(x) \equiv 1$, disturbance profile $B_d(x) = \sin(3.5x)$,

$$w_{dist}(t) = \cos(1.5t - 0.4) + 5\cos(5t + 0.2) \qquad (\omega_1 = 1.5,\ \omega_2 = 5)$$

$$y_{ref} = 56.$$

Choice of the frequency estimator:

- An adaptive estimator by Carnevale & Astolfi, [2008 ACC].
- Multi-frequency estimation
- Input-to-State Stable (required in control design)

Simulation Results

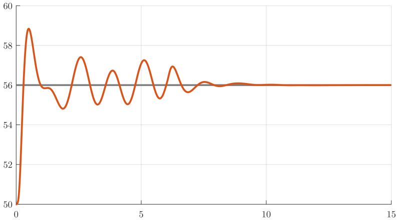

Figure: Controlled output.

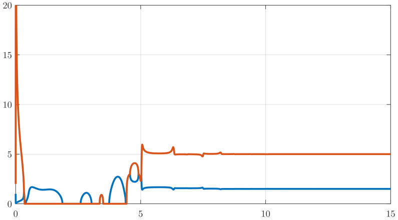

Figure: Frequency estimates.

Simulation Results

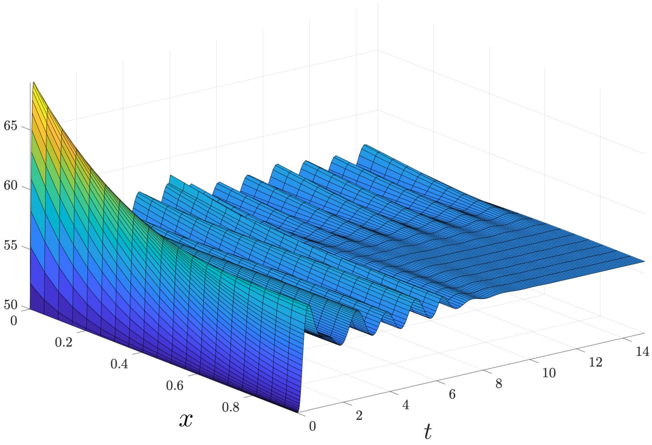

Figure: The controlled heat profile.

Robustness Analysis

Question

Can the control design handle uncertainty in the parameters of the heat equation?

	IM controller	Frequency estimator	Combination
Robustness:	**OK**	**??**	**??**

Robustness Analysis

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Can the control design handle uncertainty in the parameters of the heat equation?

	IM controller	Frequency estimator	Combination
Robustness:	**OK**	**X**	**X**

Short answer: **Very poorly**.

Simulation Results – Perturbed System

Perturbation: An unmodelled reaction term, magnitude $\approx 2 \cdot 10^{-5}$.

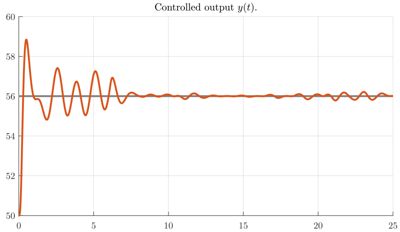


Figure: Controlled output.

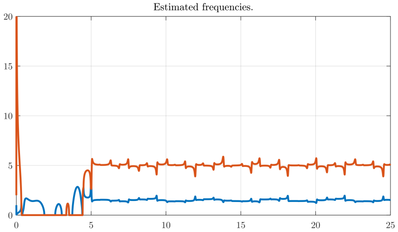
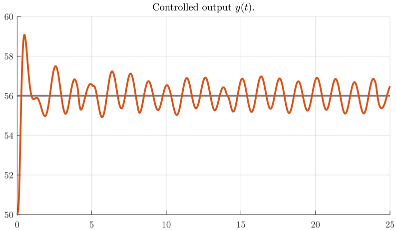


Figure: Frequency estimates.

Simulation Results – Perturbed System

Perturbation: An unmodelled reaction term, magnitude $\approx 10^{-3}$.



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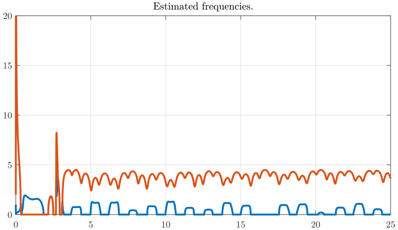

Figure: Frequency estimates.

Robustness Analysis

	IM controller	Frequency estimator	Combination
Robustness:	OK	X	X

Comments:

- The chosen frequency estimator is very sensitive to **persistent** noise.
- In this control design, perturbations cause noise due to "observer model mismatch".
- Potential remedies: Better choices of frequency estimators, changes in the controller design structure, ...?

Summary

In this presentation:

- A brief overview of fundamentals of PDE Control and some of its applications.
- Control design for disturbance rejection in a controlled heat equation.
- Simulation results and analysis of robustness limitations.

Thank You!

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