The Type System of VCL Structural and Assertion Diagrams

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Version	Date	Description
0.1	22/04/2011	1st release with type system of structural and assertion diagrams
0.2	23/03/2012	Updated type system of structural and assertion diagrams
0.3	12/06/2012	Updated with section and proofs of information preserving isomorphism between metamodels and grammars.
0.4	15/02/2012	Updated diagrams to meet new concrete syntax. Added Quantifiers.

Table 1: Document Revision History

Contents

Chapter 1

Introduction

This document present a type system for the Visual Contract Language (VCL) [AK10, AKMG10], covering structural and assertion diagrams. This formalises a typed object-oriented system with subtyping. This type system has been implemented in the VCL tool, the *Visual Contract Builder*[1] [AGK11]. The following gives some background on VCL and an outline of the overall document.

1.1 Background: The Visual Contract Language (VCL)

VCL [AK10, AKMG10, AGK11] is a formal language for the abstract modelling of software designs. Its modelling paradigms are set theory, object-orientation and design-by-contract (pre- and post-conditions). VCL's distinguishing features are its capacity to describe predicates visually and its approach to behavioural modelling based on design by contract.

VCL's semantics is based on set theory. Its semantics definition takes a translational approach. Currently, VCL has a Z semantics: VCL diagrams are mapped to ZOO [APS05, Amá07], a semantic domain of object orientation for the language Z [Spi92, ISO02].

1.1.1 VCL Diagrams

A VCL model is made up of diagrams of different kinds. VCL's diagram suite comprises: *package*, *structural*, *behaviour*, *assertion* and *contract* diagrams. Package diagrams (PDs) define VCL packages, coarse-grained modules, and their dependencies with other packages. Structural diagrams (SDs) define state structures and their relations that together make the state space of a package (e.g. Fig. 2.1a). Behaviour diagrams (BDs) provide a map over the behaviour units of a package. ADs define predicates over a single state, which are used to define invariants and query operations (e.g. Figs. 2.1b to 2.1i). Finally, contract diagrams (CDs) describe operations that change state through a contract (a pre- and a post-condition). The type system presented here cover SDs and ADs only.

1.1.2 VCL Syntax and Semantics

VCL's semantic domain is detailed in [APS05]. Briefly, syntax and semantics of SDs and ADs are as follows:

[1] `http://vcl.gforge.uni.lu/`

- All rounded contours in Fig. 2.1a are *sets* (or *blobs*). *Objects* are represented as labelled rectangles; they are atoms, members of a set of possible objects.

- In a SD, a set can either be *value* or *class*. In Fig. 2.1a, `Customer` and `Account` are classes, and all others are value sets. Value sets represent values; class sets (like OO classes) represent objects with identity.

- Property edges are represented as directed arrows. In a SD, property edges define properties shared by all objects of a set (e.g. `custNo`, `accNo` and `balance` in Fig. 2.1a)). In ADs, property edges are used to state predicates that relate the source set or object with some target expression.

- Relation edges are labelled directed lines; direction is indicated by arrow symbol above the line (e.g. `Holds` in Fig. 2.1a).

- SDs define state spaces. ADs describe assertions (conditions or predicates) on a state space. A global (or package) state is a collection of object states, together with states of relation edges. Object states are functions that map object identifiers to their states; there is such a function for each class set. Semantically, a relation edge is a binary relation; it denotes a set of tuples.

1.2 Outline

The remainder of this document is as follows:

- Chapter 2 presents the running example that is used to illustrate the type system presented here.

- Chapter 3 presents the syntactic descriptions of VCL structural and assertion diagrams, from which the type system is defined.

- Chapter 4 discusses the mapping from metamodels to grammars for the purpose of defining the type system, showing that this mapping is sound.

- Chapter 5 presents the actual type system of VCL structural and assertion diagrams.

- Appendix A presents the auxiliary definitions that are used to describe VCL's type system presented here.

- Appendix B presents the VCL metamodels describe using the Alloy formal modelling language.

- Appendix C presents the Z3 encondings for the graphs of metamodels and grammars together with the results of the isomorphism proofs.

Chapter 2

Running Example

This paper's running example is the *Simple Bank* case study [AK10][1]. Figure 2.1 give several diagrams of this case study's VCL model. The SD (Fig. 2.1a) is as follows:

- The two class sets, `Customer` and `Account`, represent, respectively, bank customers and bank accounts.
- Value sets `CustId`, `Name` and `Address` represent, respectively, sets of identifiers, names and addresses of bank customers. `CustType` defines the possible types of customers (a definitional set, symbol ◯): constant objects `corporate` and `personal`. `AccID` represents set of account identifiers. `Int` (a primitive set) represents the integers. `AccType` (a definitional set) represents the possible kinds of accounts: constant objects `savings` and `current`.
- Relation-edge `Holds` relates customers and their accounts. Assertions (elongated hexagons) identify invariants, which can either be *local* (linked to a set) or *global* (not linked).

Local `Account` invariant `SavingsArePositive` (Fig. 2.1b) says, using an implication formula, that savings accounts must be positive. This AD results in the Z predicate: $aType = savings \Rightarrow balance \geq 0$. The same invariant is described globally using a set formula in Fig. 2.1c; this says that the set of negative savings accounts (inner set) must be empty (shading). This results in the Z predicate:

$$\{o : sAccount \mid (stAccount\ o).aType = savings \wedge (stAccount\ o).balance < 0\} = \varnothing$$

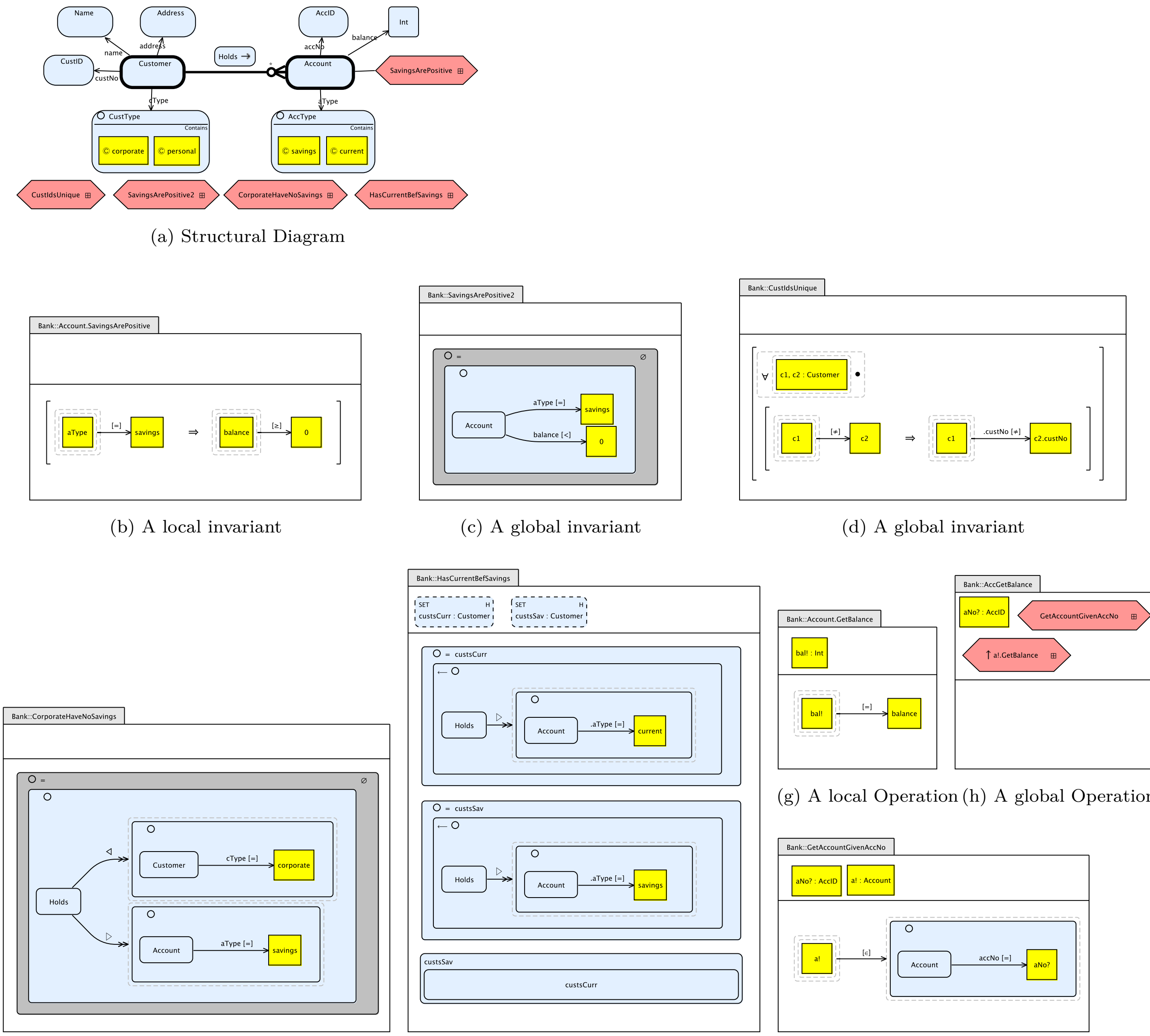
sAccount is set of all existing account objects; *stAccount* is a function mapping account objects to their states.

Global invariant `CustIdsUnique` (Fig. 2.1d) says that customer identifiers are unique. The AD says this using a quantifier formula: for all pairs of distinct customer objects, their customer numbers must also be distinct. This results in the Z predicate:

$$\forall c1, c2 : sCustomer \bullet c1 \neq c2 \Rightarrow (stCustomer\ c1).custNo \neq (stCustomer\ c2).custNo$$

Global invariant `CorporateHaveNoSavings` (Fig. 2.1e) says that corporate customers cannot have savings accounts. The AD builds a set by restricting relation `Holds` using property edge modifiers (edges with double-arrow): the domain is restricted to the set of corporate customers

[1]A tutorial using this case study is available at `http://vcl.gforge.uni.lu/SBDemo`.



(a) Structural Diagram

(b) A local invariant

(c) A global invariant

(d) A global invariant

(e) A global invariant

(f) A global invariant

(g) A local Operation

(h) A global Operation

(i) A global Operation

Figure 2.1: Sample assertion diagrams of the *simple bank* VCL model

(symbol $\lhd$); the range to the set of savings accounts (symbol $\rhd$). The outer set is shaded to say that this constructed set must be empty. The resulting Z is:

$$(\{o : sCustomer \mid (stCustomer\ o).cType = corporate\} \\ \lhd rHolds) \rhd \{o : sAccount \mid (stAccount\ o).aType = savings\} = \varnothing$$

Here, $\lhd$ and $\rhd$ are domain and range restriction relation operators.

Global invariant `HasCurrentBefSavings` (Fig. 2.1f) says that customers must have a current account before opening a savings account using a subset formula. This involves building two auxiliary sets (respectively): (a) set of customers with current accounts (local or hidden variable `custsCurr`) and (b) set of customers with savings accounts (local or hidden variable `custsSav`). Both sets are built similarly by taking the domain (symbol $\leftarrow$) of `Holds` restricted on the range. AD of Fig. 2.1f imports the auxiliary ADs (represented as assertions) and says that `custsSav` is a subset of `custsCurr`; as `custsSav` and `custsCurr` are not declared in Fig. 2.1f, they are internal variables hidden to the outside world. The Z resulting from Fig. 2.1f is:

$$\begin{array}{|l}
\hline
\textit{BankHasCurrentBefSavings0} \\
\hline
\textit{BankGblSt} \\
\textit{custsCurr} : \mathbb{P}\mathbb{O}\ \textit{CustomerCl} \\
\textit{custsSav} : \mathbb{P}\mathbb{O}\ \textit{CustomerCl} \\
\hline
\textit{custsCurr} = \operatorname{dom}(\textit{rHolds} \rhd \{o : \textit{sAccount} \mid (\textit{stAccount}\ o).\textit{aType} = \textit{current}\}) \\
\textit{custsSav} = \operatorname{dom}(\textit{rHolds} \rhd \{o : \textit{sAccount} \mid (\textit{stAccount}\ o).\textit{aType} = \textit{savings}\}) \\
\textit{custsSav} \subseteq \textit{custsCurr} \\
\hline
\end{array}$$

$$\textit{BankHasCurrentBefSavings} == \textit{BankHasCurrentBefSavings0} \setminus (\textit{custsCurr}, \textit{custsSav})$$

Above, variables `custsSav` and `custsCurr` are hidden using the $\setminus$ Z operator.

In VCL, queries are defined using ADs, which can be local or global. Often, global operations are built from local ones. Local operation `Account.GetBalance` (AD of Fig. 2.1g) retrieves the balance of some account object and stores it in output variable `bal!`. Global operation `AccGet-Balance` (Fig. 2.1h) retrieves some account balance given some account number (input `aNo?`); this involves obtaining the object account ($a!$) associated with `aNo?` through operation `GetAccount-GivenAccNo` (Fig, 2.1i) and then retrieving the account's balance using `Account.GetBalance`. The $\uparrow$ symbol says that variables, and not only the predicate, are imported; this means that output $bal!$ is defined also in `AccGetBalance`.

This running example highlights the utility of typing. For instance, in AD of Fig. 2.1e it would be useful to check that `Holds`, `Customer` and `Account` are sets defined in SD of Fig. 2.1a, that the operators $\lhd$ and $\rhd$ are applied correctly, and that the properties `cType` and `aType` exist and are applied correctly with respect to the operator =. Similarly for the remaining ADs.

Chapter 3

Syntax

This chapter presents the syntax of VCL structural and assertion diagrams in terms grammars and class metamodels. The metamodels are the primary representation; metamodels are the basis for constructing the graphical parsers of VCL's tool. The grammars are used to describe the type system; in the implementation the grammar representation is used for type-checking and translation to Z.

3.1 Metamodels

The metamodels of the VCL notations presented here have been defined in the Alloy specification language [Jac06]. They are given in appendix B. Here, we present these metamodels using UML class diagrams, which partially describe what is described in Alloy: the Alloy describes constraints that are not describable using class diagrams.

The Alloy metamodels of VCL package, structural and assertion diagrams comprises the following modules: *common* (section B.1), *structural diagrams* (section B.3) and *assertion diagrams* (section B.4). The following class diagrams describe each of these modules.

3.1.1 Common

The metamodel of the part that is common to both SDs and ADs (Fig. 3.1), corresponding to the Alloy module of section B.1, is as follows:

- Several constructions have a name attribute; the metaclass (`Name`, bottom-left) denotes all names of a VCL model. Several constructions use the type designator (`TypeDesignator`, bottom-left). A type designator can either denote the set of natural numbers (`TypeDesignatorNat`), the set of integers (`TypeDesignatorInt`), or some set defined by a blob or relation edge and denoted by their identifier (`TypeDesignatorId`).

- A property edge (`PropEdge`) can either be of type *predicate* (`PropEdgePred`) or *modifier* (`PropEdgeMod`). `PropEdgePred`s comprise a unary and binary operator (`uop` and `bop` association-ends), an instance of `EdgeOperatorUn` and `EdgeOperatorBin`, respectivelly, a target `Expression` (`target` association-end) and an optional designator (attribute `designator`) to refer to some property of a blob. A `PropEdgeMod` comprises a modifier operator (`mop` association-end) an instance of `EdgeOperatorMod`.

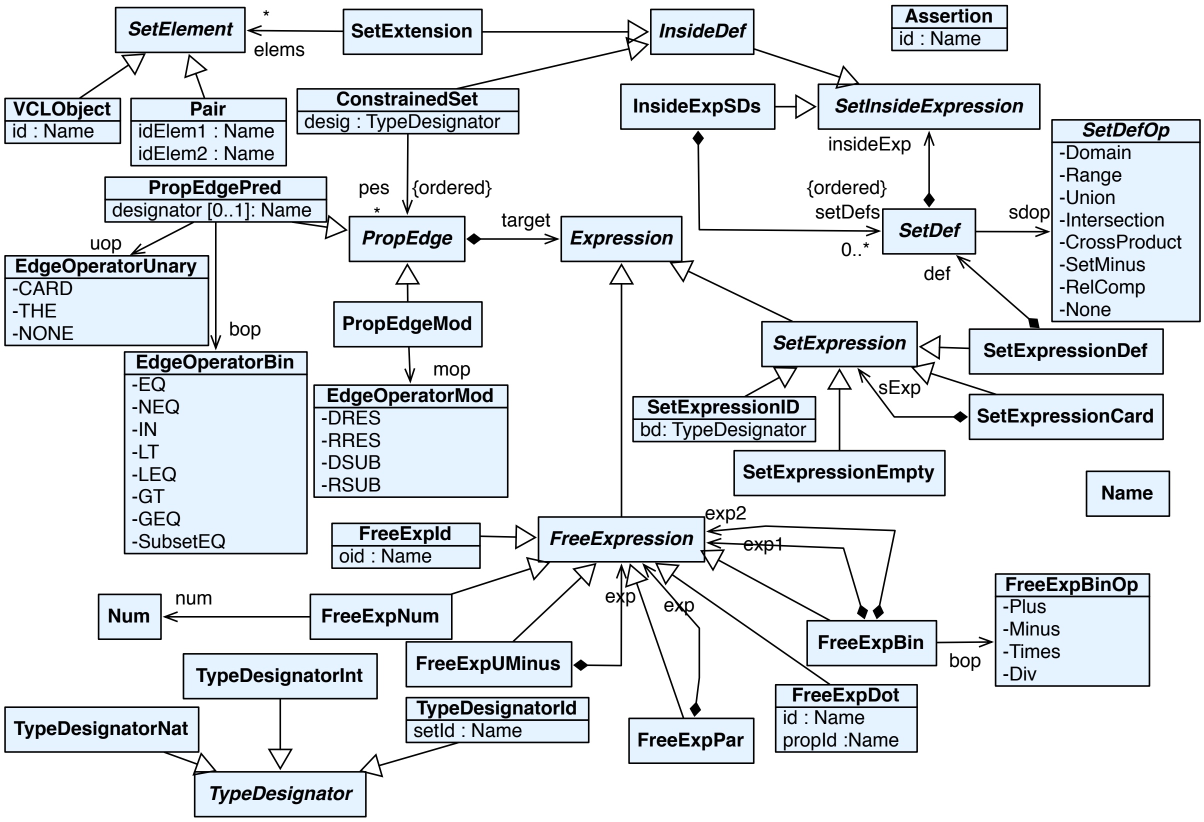


Figure 3.1: The common metamodel

- A modifier edge operator (`EdgeOperatorMod`) is an enumeration comprising the operators: domain restriction (`DRES`, $\lhd$), range restriction (`RRES`, $\rhd$), domain subtraction (`DSUB`, ⩤) and range subtraction (`RSUB`, ⩥). A predicate edge operator is enumeration comprising the operators: equality (`EQ`, $=$), non-equality (`NEQ`, $\neq$), set membership (`IN`, $\in$), less then (`LT`, $<$), less or equal then (`LEQ`, $\leq$), greater then (`GT`, $>$), greater or equal then (`GEQ`, $\geq$), and subsetting (`SubsetEQ`, $\subseteq$).

- There are two kinds of expressions: *object* (`ObjExpression`), represented as objects (rectangles), and *set* (`SetExpression`), represented as blobs (rectangles with rounded corners). An object expression can either be: an identifier (`ObExpId`); a number (`ObjExpNum`); a unary minus expression (`ObjExpUMinus`), comprising another expression (`exp` association-end); a binary object expression, comprising two expressions (association-ends `exp1` and `exp2`) and an infix operator (`bop` association-end); or a parenthesised expression, comprising another expression (`exp` association-end). A binary object-expression operator (`ObjExpBinOp`) is an enumeration comprising the operators: `Plus` ($+$), `Minus` ($-$), `Times` ($*$), and `Div` (div).

- A `SetExpression` can either refer to some existing set (`SetExpressionId`), denote the empty set (a blob that is shaded), be a cardinality operator applied to another set expression `SetExpressionCard`, or be a set definition (`SetExpressionDef`). A `SetExpressionId` comprises a designator of the set being referred (attribute `desig`). A `SetExpressionCard`

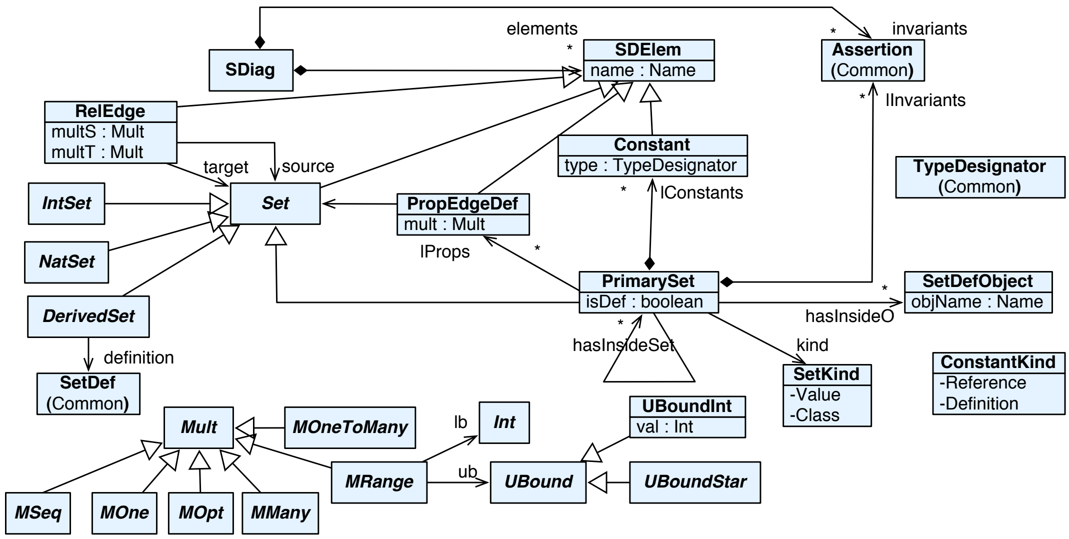


Figure 3.2: The metamodel of VCL Structural diagrams

is the cardinality operator applied to another set expression (`sExp` association-end). A `SetExpressionDef` comprises a set definition (association-end `def`), an instance of `SetDef`.

- Set definitions (`SetDef`) are defined by the things they have inside. They comprise an inside expression (`insideExp` association-end), representing the expression placed inside the blob, and by a set definition operator (`sdop` association-end). A set definition operator (`SetDefOp`) is an enumeration defining the operators `Domain` (symbol $\leftarrow$), `Range` (symbol $\rightarrow$), `Union` (symbol $\cup$), `Intersection` (symbol $\cap$), `CrossProduct` (symbol $\times$), `SetMinus` (symbol $\backslash$) or `None` (no operator).

- A set inside expression (`SetInsideExpression` is either an inside definition (`InsideDef`) or a sequence of set definitions (`InsideExpSDs`). A `InsideExpSDs` comprises a sequence of set definitions (`setDefs` association-end). An `InsideDef` is an abstract class, which comprises either a `SetExtension` or a `ConstrainedSet`. A `ConstrainedSet` represents a set constrained with an ordered collection of property edges (association-end `pes`). A set extension (`SetExtension`) represents a set defined extensionally by a set of elements (association-end `elems`), which are instances of `SetElem`.

- A `SetElem` is represented visually as a rectangles; it can either be a `VCLObject` (a member of set) or a `Pair` (a member of a relation). A `VCLObject` comprises a name (the name of the object); a `Pair` comprises a pair of names.

3.1.2 Structural Diagrams

The metamodel of VCL structural diagrams (SDs) (Fig. 3.2), corresponding to the Alloy module of section B.3, is as follows:

- A SD (`SDDiag`) is made of structural `elements` (`SDElem`) and `invariants` (`Assertion`). A `SDElem` can be a relation edge (`RelEdge`), constant (`Constant`) or set (`Set`).

- In a SD, an `Assertion` represents an invariant. If they belong to the overall SD (association-end `invariants`) they represent global invariants; if they are connected to a set (association-end `lInvariants`), the invariant is local to the set.
- A relation edge (`RelEdge`), or association, represents an edge between two sets: the `source` and the `target`. It holds two attributes to record the multiplicities attached to source and target (`multS` and `multT`).
- Like invariants, constants (`Constant`) are global if they are not connected to any set and local otherwise (association-end `lConstants`).
- A set can be primary (`PrimarySet`), derived (`DerivedSet`) or one of the sets corresponding to primitive types: integers (`IntSet`) or natural numbers (`NatSet`).
- A derived set has a name (attribute `id`) and is associated with a set definition (`SetDef`, defined in common metamodel).
- A primary set has a kind (`SetKind`), indicating whether the set is `Class` or `Value`. A primary set comprises a set of local constants (association-end `lConstants`), a set of local invariants (association-end `lInvariants`), and a set of property edge definitions (association-end `lProps`). A primary set may have other primary sets and objects inside (association-ends `hasInsideSet` and `hasInsideO`).
- A property edge definition (`PropEdgeDef`) has a set has the edge's target (association-end `peTarget`) indicating the type of the property, and a multiplicity constraint (attribute `mult`).
- Multiplicities (`Mult`) are attached to relation edges and property edge definitions. A multiplicity can either be single (`MOne`), optional (`MOpt`), sequence (`MSeq`), multiple with 0 or more (`Many`), multiple with at least one (`MOneToMany`), or be defined as a range (`MRange`) comprising a lower and an upper bound (association-ends `ub` and `lb`).

3.1.3 Assertion Diagrams

The metamodel of VCL assertion diagrams (Fig. 3.3), corresponding to the Alloy module of section B.4, is as follows:

- An assertion diagram (`ADiag`) comprises a name (`aName`), a set of declarations corresponding to the declarations compartment (`declarations` association-end), and a set of formulas corresponding to the predicate compartment (`predicate` association-end).
- A declaration (`Decl`) can either be a typed declaration (`TypedDecl`) or a declaration formula (`DeclFormula`). A typed declaration has a name (`dName`) and a type (`dTy`), and it can either be a declaration of an object (`DeclObj`) or the declaration of a set (`DeclSet`). The `sequence` attribute of `DeclSet` indicates whether the set is a normal set (value `false`) or a sequence (value `true`). The `optional` attribute of `DeclObj` indicates whether the object is optional or not.
- A formula (`Formula`) can either be a negation formula (`FormulaNot`), a binary formula (`FormulaBin`), an arrows formula (`ArrowsFormula`) or a set formula (`SetFormula`).

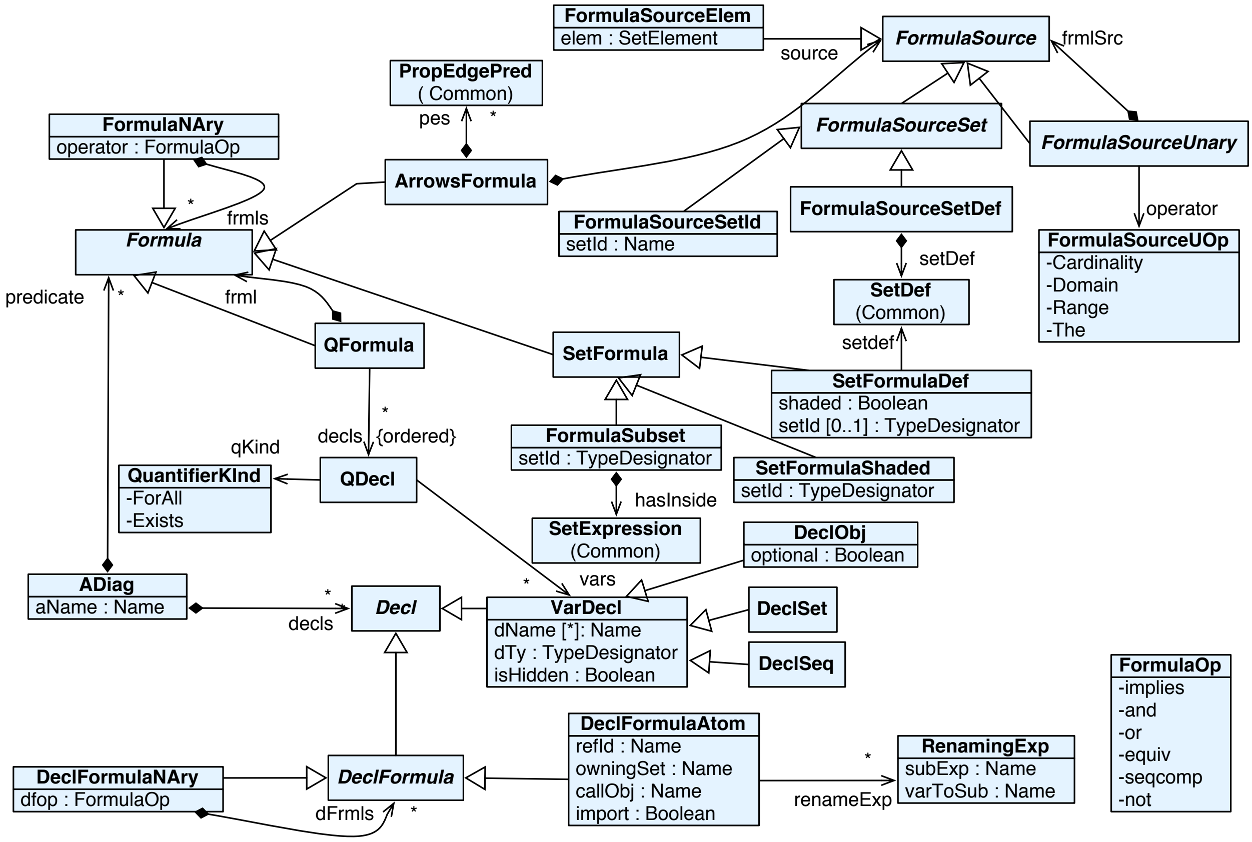


Figure 3.3: The metamodel of VCL assertion diagrams

- A negation formula (`FormulaNot`) comprises another formula corresponding to the formula being negated (`frml` association-end). A binary formula (`FormulaBin`) comprises two formulas corresponding to the formulas being combined (`frml1` and `frml2` association-ends), and an operator (`bop` attribute). A binary formula operator (`FormulaBinOp`) can either be an implication (`implies`), a conjunction (`and`), a disjunction (`or`), an equivalence (`equiv`) or be a sequential composition (`seqcomp`).

- An arrows formula (`ArrowsFormula`) comprises a set of predicate property edges (`pes` association-end).

- A set formula (`SetFormula`) can either be a subset formula (`FormulaSubset`), a shaded blob formula (`SetFormulaShaded`) or a set definition formula (`SetFormulaDef`). A subset formula (`FormulaSubset`) corresponds to the situation where one set is placed inside another to denote the subset relationship; it has a set identifier (attribute `setId`) and a set expression to denote the inside set (`hasInside` association-end). A shaded set formula corresponds to the situation where some set is shaded; it comprises a set identifier (attribute `setId`). A definition set formula (`SetFormulaDef`) comprises a `SetDef` (association-end `setdef`) from the common metamodel (Fig. 3.1); it can be `shaded` or have an identifier (either one or the other).

- A declarations formula (`DeclFormula`) can either be a declarations formula atom (`DeclFormulaAtom`), which comprises a declaration reference, a negated declaration formula (`DeclFormulaNot`), which comprises the declarations formula being negated, or a binary declaration formula (`DeclFormulaBin`), which comprises an operator (`DeclFormulaBinOp`) and two declarations formulas.

- `FormulaSource` represents the source of a predicate formula. This source can either be a set element (`FormulaSourceElem`), which comprises a `SetElement` (defined in Common, Fig. 3.1), a set (`FormulaSourceSet`) or a be some unary operator applied to a formula source `FormulaSourceUnary`.

3.2 Grammars

The following presents the grammars of VCL structural and assertion diagrams; they are equivalent to the visual metamodels presented above.

The grammars use the following operators:

- $\overline{x}$ for zero or more repetitions of x;
- $\overline{x}^1$ for one or more repetitions of x;
- $x \mid y$ for a choice of x or y;
- $[x]$ for an optional x.

In addition,

- $\overline{xc}$ for some character symbol c means zero or more occurrences of x separated with c;
- $\overline{xc}^1$ for some character symbol c means one or more occurrences of x separated with c;

Symbols are set in bold type when they are to be interpreted as terminals to avoid confusion with grammar symbols. We introduce two syntactic sets, representing terminals: the set of identifiers *Id*, and the set of numeric constants (*Num*).

$$
\begin{array}{rcl}
\mathsf{TD} & ::= & \textbf{Int} \mid \textbf{Nat} \mid \mathsf{Id} \\
\mathsf{O} & ::= & \textbf{object}\ \mathsf{Id} \\
\mathsf{P} & ::= & \textbf{pair}\ \textbf{(}\mathsf{Id},\ \mathsf{Id}\textbf{)} \\
\mathsf{SE} & ::= & \mathsf{O} \mid \mathsf{P} \\
\mathsf{A} & ::= & \textbf{assertion}\ \ \mathsf{Id} \\
\mathsf{PE} & ::= & (\mathsf{PEP} \mid \mathsf{PEM})\ \mathsf{TExp} \\
\mathsf{PEP} & ::= & \textbf{[}UEOp\textbf{]}\ [.\mathsf{Id}]\ \mathsf{BEOp} \\
\mathsf{UEOp} & ::= & \# \mid \odot \mid \bot \\
\mathsf{BEOp} & ::= & \textbf{=} \mid\ \neq \mid\ \in\ \mid\ <\ \mid\ \leq \\
 & & \mid > \mid \geq \mid \subseteq \\
\mathsf{PEM} & ::= & \textbf{[}\ \mathsf{MOp}\ \textbf{]} \rightrightarrows \\
\mathsf{MOp} & ::= & \lhd\ \mid \rhd \mid \text{⌫} \mid\ \text{⌦} \\
\mathsf{TExp} & ::= & \mathsf{FExp} \mid \mathsf{SExp} \\
\mathsf{FExp} & ::= & \mathsf{Id} \mid \mathsf{Id.Id} \mid \mathsf{Num} \mid -\mathsf{FExp} \\
 & & \mid \mathsf{FExp}\ \mathsf{FEOP}\ \mathsf{FExp} \\
 & & \mid (\mathsf{FExp}) \\
\mathsf{FEOp} & ::= & + \mid - \mid * \mid \textbf{div} \\
\mathsf{SExp} & ::= & \textbf{set}\ \mathsf{TD} \mid \mathsf{SDef} \\
 & & \mid \textbf{set shaded} \\
 & & \mid \#\ \mathsf{SExp} \\
\mathsf{SDef} & ::= & \textbf{set}\ \bigcirc\ \mathsf{SOp}\ \textbf{hasIn}\ \{\mathsf{IExp}\} \\
\mathsf{SOp} & ::= & \leftarrow \mid \rightarrow \mid \cap \mid \cup \mid \times \mid \setminus \mid \boxdot \mid \bot \\
\mathsf{IExp} & ::= & \mathsf{IDef} \mid \overline{SDef;} \\
\mathsf{IDef} & ::= & \textbf{set}\ \mathsf{TD}\ \textbf{\{}\ \overline{PE}^{1}\ \textbf{\}} \mid \overline{SE}^{1}
\end{array}
$$

(a) Common Syntax

$$
\begin{array}{rcl}
\mathsf{SD} & ::= & \textbf{SD STRUCTURES: } \overline{SDE}\ \textbf{INVARIANTS: } \overline{A} \\
\mathsf{SDE} & ::= & \mathsf{C} \mid \mathsf{RE} \mid \mathsf{Set} \\
\mathsf{C} & ::= & \textbf{const}\ \mathsf{Id}\ \ \textbf{:}\ \mathsf{TD} \\
\mathsf{M} & ::= & \textbf{opt} \mid \textbf{one} \mid \textbf{some} \mid \textbf{many} \\
 & & \mid \textbf{seq} \mid \mathsf{Num}\ \textbf{..}\ (\mathsf{Num} \mid \textbf{*}) \\
\mathsf{RE} & ::= & \textbf{relEdge}\ \mathsf{Id}\ \textbf{(}\mathsf{M}\ \mathsf{TD},\ \ \mathsf{M}\ \mathsf{TD}\textbf{)} \\
\mathsf{SK} & ::= & \textbf{value} \mid \textbf{class} \\
\mathsf{Set} & ::= & \mathsf{PSet} \mid \mathsf{Id} \leftrightarrow \mathsf{SDef} \\
\mathsf{PSet} & ::= & \textbf{set}\ \mathsf{Id}\ \mathsf{SK}\ [\bigcirc]\ \textbf{\{}\ \overline{C}\ \overline{PED}\ \overline{A}\ \textbf{\}} \\
 & & \quad [\textbf{hasIn}\ \textbf{\{}\overline{(O\ \mid\ PSet)}\textbf{\}}] \\
\mathsf{PED} & ::= & \mathsf{Id} \rightarrow \mathsf{M}\ \mathsf{TD}
\end{array}
$$

(b) Structural Diagrams

$$
\begin{array}{rcl}
\mathsf{AD} & ::= & \textbf{AD}\ \mathsf{Id}\ [:\mathsf{Id}]\ \textbf{DECLARATIONS: } \overline{D}\ \textbf{PREDICATE: } \overline{F} \\
\mathsf{D} & ::= & \mathsf{VD} \mid \mathsf{DF} \\
\mathsf{VD} & ::= & [\textbf{hidden}]\ \mathsf{DV}\ \overline{Id,}\ \textbf{:}\ \mathsf{TD} \\
\mathsf{DV} & ::= & [\textbf{opt}]\ \textbf{object} \mid \textbf{set} \mid \textbf{seq} \\
\mathsf{R} & ::= & \mathsf{Id}\ \textbf{/}\ \mathsf{Id} \\
\mathsf{DFA} & ::= & [\uparrow]\ \textbf{assertion}\ [\mathsf{Id} \rightarrow]\ [\mathsf{Id}\ \textbf{.}]\ \mathsf{Id}\ \textbf{[}\overline{R}\textbf{]} \\
\mathsf{DF} & ::= & \mathsf{DFA} \mid \mathsf{FOp}\textbf{[}\overline{DF}\textbf{]} \\
\mathsf{FOp} & ::= & \Rightarrow \mid \Leftrightarrow \mid \wedge \mid \vee \mid \neg\ \mid \boxdot \\
\mathsf{F} & ::= & \mathsf{AF} \mid \mathsf{SF} \mid \mathsf{FOp}\ \textbf{[}\overline{F}\textbf{]} \mid \mathsf{QF} \\
\mathsf{AFS} & ::= & \mathsf{SE} \mid \mathsf{AFSS} \mid \mathsf{FSOp}\ \mathsf{AFS} \\
\mathsf{AFSS} & ::= & \textbf{set}\ \mathsf{Id} \mid \mathsf{SDef} \\
\mathsf{FSOp} & ::= & \# \mid \leftarrow \mid \rightarrow \mid \odot \\
\mathsf{AF} & ::= & \mathsf{AFS}\ \textbf{\{}\ \overline{PEP}\ \textbf{\}} \\
\mathsf{SF} & ::= & [\textbf{shaded}]\ [\mathsf{Id}]\ \mathsf{SDef} \mid \textbf{set shaded}\ \mathsf{TD} \\
 & & \mid \textbf{set}\ \mathsf{TD}\ \textbf{hasIn}\ \{\mathsf{SExp}\} \\
\mathsf{QF} & ::= & \overline{QD,}\ \bullet\ \mathsf{F} \\
\mathsf{QD} & ::= & \textbf{(}\forall \mid \exists\ \textbf{)}\ \overline{VD;}
\end{array}
$$

(c) Assertion Diagrams

Figure 3.4: Syntax of VCL Structural and Assertion diagrams

Chapter 4

From Metamodels to Grammars and Back

This chapter demonstrates that metamodel and grammar representations of chapter 3 are equivalent, which means that it is straightforward to go from one representation to the other. This is important because the type system presented in the next chapter is defined on the grammar, but the graphical editors of VCL's tool are based on metamodels. This chapter shows that this approach based on these two representations is sound.

4.1 Overall setting

In [EEPT06], a graph is defined as a tuple $G = \langle V, E, s, t \rangle$, where V is a set of nodes, E is a set of edges, and s, $t : E \to V$ are the source and target functions, respectively, assigning to each edge a source and a target node. A metamodel is actually a typed graph [EEPT06], but this does not concern us here. We are interested in going from the metamodel to the grammar.

A grammar (in our context, a context-free grammar) is defined as the tuple $Gr = \langle V, \Sigma, S, P \rangle$, where V is a set of non-terminals, Σ a set of terminals, S is the starting symbol (it is a member of V) and P is a set of grammar rules or productions. The abstract syntax induced by a grammar can be represented as a graph (a special kind of graph, a tree), where the nodes are the terminals and non-terminals of the grammar, the root node of the tree is the starting symbol, and the edges represent the dependencies between terminal and non-terminals of the grammar as defined by the grammar's productions.

The approach presented here requires the construction of a *graph-isomorphism* between graphs of metamodel and abstract syntax tree. This ensures that we can go from the metamodel to the grammar in a way that preserves the information of the metamodel and back. Given graphs $G_i = (V_i, E_i, s_i, t_i)$, a *graph-morphism* is defined as (from [EEPT06]), $f : G_1 \to G2$, where $f = (f_V, f_E)$ consists of two functions $f_V : V_1 \to V_2$ and $f_E : E_1 \to E_2$ that preserve the source and target functions (that is, $f_V \circ s_1 = t_2 \circ f_E$). f is called isomorphic if both functions f_V and f_E are bijections (both injective and surjective).

4.2 VCL Syntactic Isomorphisms

To show that that metamodel and grammar representations are equivalent, we need to show that there is an information-preserving isomorphism between the metamodels of common (Fig. 3.1), SDs (Fig. 3.2) and ADs (Fig. 3.3) and the corresponding grammars of common (Fig. 3.4a), SDs (Fig. 3.4b) and ADs (Fig. 3.4c), respectively. These proofs are performed using the Z3 theorem prover [dMB08][1]; this involved encoding in Z3 the graphs of metamodel and grammar and all required theorems to prove. Z3 proves automatically all required theorems. The Z3 enconding of graphs and required theorems is given in appendix C.

4.2.1 Isomorphism Theorems and their Proofs

For each pair metamodel and grammar, several theorems need to be proved to demonstrate the existence of the information-preserving isomorphism as defined above. Let, G_{MM} and G_{Gr} be the graphs of metamodel and grammar respectively, such that: $G_{MM} = (V_{MM}, E_{MM}, s_{MM}, t_{MM})$ and $G_{Gr} = (V_{Gr}, E_{Gr}, s_{Gr}, t_{Gr})$. The two mapping functions of the isomorphism are: $f_V : V_{MM} \to V_{Gr}$ and $f_E : E_{MM} \to E_{Gr}$.

In the Z3 prover, the following theorems are proved. The source and target functions of both graphs must be total:

$$\begin{array}{l}
\forall\, emm : E_{MM} \bullet \exists\, vmm : V_{MM} \bullet s_{MM}(emm) = vmm \\
\forall\, emm : E_{MM} \bullet \exists\, vmm : V_{MM} \bullet t_{MM}(emm) = vmm \\
\forall\, egr : E_{Gr} \bullet \exists\, vgr : V_{Gr} \bullet s_{Gr}(egr) = vgr \\
\forall\, egr : E_{Gr} \bullet \exists\, vgr : V_{Gr} \bullet t_{Gr}(egr) = vgr
\end{array}$$

The mapping functions must be total[2]:

$$\begin{array}{l}
\forall\, vmm : V_{MM} \bullet \exists\, vgr : V_{Gr} \bullet f_V(vmm) = vgr \\
\forall\, emm : E_{MM} \bullet \exists\, egr : E_{Gr} \bullet f_E(emm) = egr
\end{array}$$

The mapping functions must be injective:

$$\begin{array}{l}
\forall\, vmm_1, vmm_2 : V_{MM} \bullet f_V(vmm_1) = f_V(vmm_2) \Rightarrow vmm_1 = vmm_2 \\
\forall\, emm_1, emm_2 : E_{MM} \bullet f_E(emm_1) = f_E(emm_2) \Rightarrow emm_1 = emm_2
\end{array}$$

The mapping functions must be surjective:

$$\begin{array}{l}
\forall\, vgr : V_{Gr} \bullet \exists\, vmm : V_{MM} \bullet f_V(vmm) = vgr \\
\forall\, egr : E_{Gr} \bullet \exists\, emm : E_{MM} \bullet f_E(emm) = egr
\end{array}$$

All required Z3 encondings of graphs and theorems are given in appendix C.

Proofs for the common part

There is a direct isomorphism from the metamodel of common (Fig. 3.1) to the corresponding grammar (Fig. 3.4a). Further details of the Z3 proof are given in section C.1.1.

Proofs for the SD part

The SD metamodel of Fig. 3.2 requires a transformation into a another metamodel so that it is then possible to obtain a direct isomorphism. The transformed metamodel of SD that is isomorphic to the grammar of Fig. 3.4b is given in Fig. 4.1. Further details of the Z3 proof for the transformed metamodel are given in section C.2.

[1] `http://research.microsoft.com/en-us/um/redmond/projects/z3/`

[2] In Z3, all functions definitions are total. To prove totality for the mapping functions in Z3, we resorted to a trick based on a special node called `Null`. The mapping functions are defined using Z3's `ite` (`if-then-else`) construct with the ultimate else being an assignment to the special `Null` node. A function is total provided there is one assignment to `Null`.

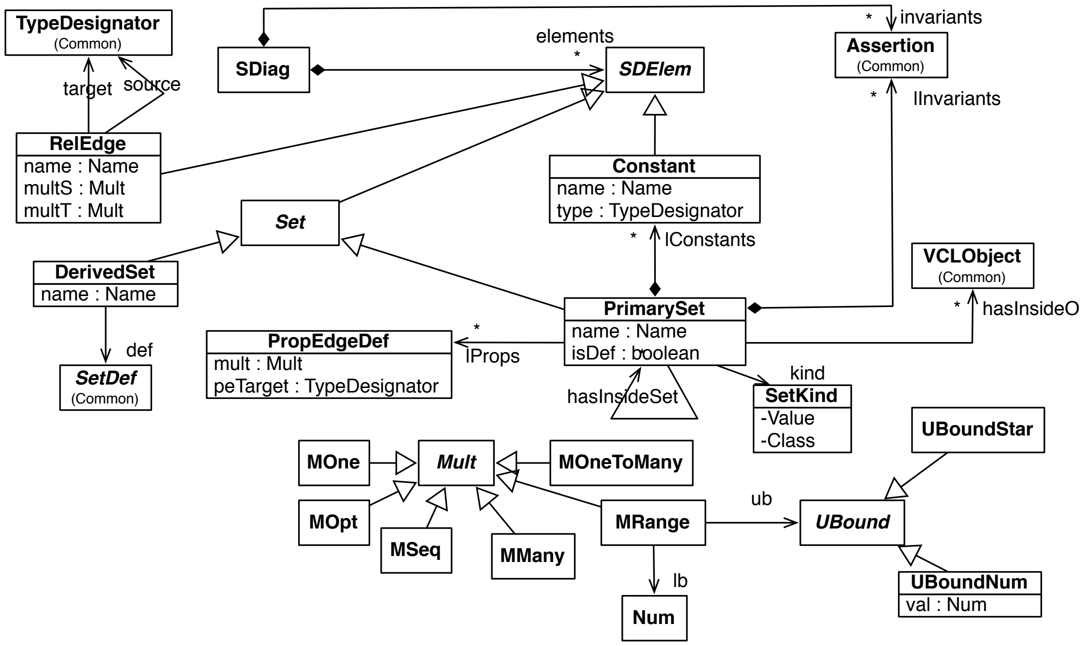


Figure 4.1: The transformed metamodel of SDs that is isomorphic to the grammar

Proofs for the AD part

There is a direct isomorphism from the metamodel of ADs (Fig. 3.3) to the corresponding grammar (Fig. 3.4c). Further details of the Z3 proof are given in section C.3.

Chapter 5

Type System

This chapter presents the type system of VCL structural and assertion diagrams. It starts by defining VCL's types and typing environments (section 5.1).

5.1 Types and Environments

A variable environment (VE) denotes a set of bindings, mapping identifiers to their types:

$$VE == Id \nrightarrow T$$

VCL's types (set T) are as follows:

$$\begin{array}{lll} \mathsf{T} & ::= & \textbf{Int} \mid \textbf{Nat} \mid \textbf{Null} \mid \textbf{Pow}\ \mathsf{T} \mid \textbf{Seq}\ \mathsf{T} \mid \textbf{Opt}\ \mathsf{T} \mid \textbf{Top} \mid \textbf{Obj} \mid \textbf{Set}\ \mathsf{Id} \mid \textbf{Pair}\ \textbf{(}\mathsf{T},\ \mathsf{T}\textbf{)} \\ & & \mid \textbf{Assertion}\ \textbf{[}VE_v,\ VE_h\textbf{]} \end{array}$$

Here, (a) `Int` represents the integers, (b) `Nat` the natural numbers; (c) `Null` represents erroneous results (implementation only); (d) **Pow** `T` represents a powerset of some set; (e) `Seq T` represents a sequence of some type; (f) `Opt T` represents an optional (either it exists or is empty); (g) `Top` is a maximal type (type of all well-formed terms); (h) `Obj` is the maximal type of all well-formed objects; (i) `Set` represents primary sets; (j) `Pair` represents a cartesian product of two types; (k) `Assertion` represents assertions (variable environments indicate assertion's variables, which are either visible, VE_v, or hidden, VE_h).

VCL's type rules use and manipulate environments (set E below), which are made of three components: (a) variable, (b) set and (c) subtyping. Variable environments give the type bindings of some scope. Set environments (SE) map identifiers to a triple made up of the set's kind (value or class), definitional status (DK) and local variable environment. Subtyping environments (set $SubE$) are the subtyping relations between types:

$$\begin{array}{l} SK ::= \textbf{value} \mid \textbf{class} \\ DK ::= \textbf{def} \mid \textbf{notDef} \\ SE == Id \nrightarrow SK \times DK \times VE \\ SubE == T \leftrightarrow T \\ E == VE \times SE \times SubE \end{array}$$

We introduce the following conventions:

Table 5.1 Judgements associated with the base rules of VCL's type system

$E \vdash T$	T is well-formed type in E
$E \vdash T_1 <: T_2$	T_1 is a subtype of T_2 in E
$E \vdash Id : T$	Id is well-formed identifier of type T in E
$E \vdash Id_s.Id_l : T$	Id_l is well-formed local identifier of set Id_s with type T in E

Table 5.2 Basic VCL typing rules

(*Ty Id*)
$$\frac{E.VE(Id) = T}{E \vdash Id : T}$$

(Type)
$$\frac{T = \textbf{Set}\ Id \Rightarrow Id \in \mathrm{dom}\, E.SE}{E \vdash T}$$

(Ty LId)
$$\frac{E \vdash \textbf{Set}\ Id_s \qquad Id_l \in \mathrm{dom}(E.SE\,(Id_s)).VE \qquad E \oplus (E.SE\,(Id_s)).VE \vdash Id_l : T}{E \vdash Id_s.Id_l : T}$$

- $\overline{X}$ and $\widehat{X}$ denote, respectivelly, a sequence and a set of some set X.
- $E_\varnothing$ is an empty environment. $E.VE$, $E.PE$, $E.SE$ and $E.SubE$ denote the different components of E.
- $Id : T$ and $Id \overset{se}{\mapsto} (SK, DK, Id, VE)$ are type (VE) and set (SE) bindings. $T_1 <: T_2$ says that T_1 is a subtype of T_2.
- Disjoint environments are combined using $E_1,\ E_1$; similarly for other types of environments. Bindings are added to an environment using $E,\ Id : T$; similarly for other types of bindings. $E \oplus VE$ means that the environment E is overridden with the set of type bindings VE; similarly for other types of bindings. These operators are defined precisely in appendix A.1.

5.2 Base Rules

The base type rules of VCL's type system manipulate environments and define subtype relations. The judgements are listed in table 5.1. The first judgement asserts that the type T is well-formed in the environment E. The second judgement asserts that the type T_1 is a subtype of T_2 in the environment E. The third judgement says that Id is a well-formed identifier with type T in E. The fourth judgement asserts the well-formedness of a set-property access; it says that property Id_l of set named Id_s has type T in E.

Table 5.2 lists basic rules concerning types. Rule `Ty Id` says that some identifier yields type T provided the variable binding is defined in the variable environment ($E.VE$). Rule `Type` describes the conditions for some type to be valid in some environment E: set types are valid provided their identifiers are defined in the set environment; all remaining types are valid. Rule `Ty LId` yields the type associated with some local identifier Id_l of some set Id_s; the rule checks that the set type is defined and then retrieves the type of the local identifier from the set's variable environment.

Table 5.3 lists basic subtyping rules. Rule `Sub Ty` checks whether some type is a subtype of another; this amounts to check that both types are defined and that the subtyping tuple belongs to the environment's set of subtypes ($E.SubE$). Rules `Sub Refl` and `Sub Trans` says that the subtyping relation is both reflexive and transitive. Rule `Subsumption` is the subsumption rule that says that if some variable has type T_A, and if T_A is subtype of T_B then the variable also

Table 5.3 Basic VCL sub-typing rules

$$\text{(Sub Ty)}\ \frac{E \vdash T_1 \quad E \vdash T_2 \quad (T_1, T_2) \in E.SubE}{E \vdash T_1 <: T_2} \qquad (\mathit{Sub\ Refl})\ \frac{E \vdash T}{E \vdash T <: T} \qquad \text{(Sub Trans)}\ \frac{E \vdash T_A <: T_B \quad E \vdash T_B <: T_C}{E \vdash T_A <: T_C}$$

$$\text{(Subsumption)}\ \frac{E \vdash I : T_A \quad E \vdash T_A <: T_B}{E \vdash I : T_B} \qquad (\mathit{Sub\ Top})\ \frac{E \vdash T}{E \vdash T <: \mathit{Top}}$$

$$(\mathit{Sub\ Obj})\ \frac{E \vdash \mathbf{Set}\ \mathit{Id}_s}{E \vdash \mathbf{Set}\ \mathit{Id}_s <: \mathit{Obj}} \qquad (\mathit{Sub\ NatInt})\ \frac{}{E \vdash \mathit{Nat} <: \mathit{Int}} \qquad (\mathit{Sub\ Pow})\ \frac{E \vdash T_A <: T_B}{E \vdash \mathbf{Pow}\ T_A <: \mathbf{Pow}\ T_B} \qquad (\mathit{Sub\ Seq})\ \frac{E \vdash T_A <: T_B}{E \vdash \mathbf{Seq}\, T_A <: \mathbf{Seq}\, T_B}$$

$$(\mathit{Sub\ Opt})\ \frac{E \vdash T_A <: T_B}{E \vdash \mathbf{Opt}\ T_A <: \mathbf{Opt}\ T_B} \qquad (\mathit{Sub\ Opt\ PSet})\ \frac{E \vdash T_A <: T_B}{E \vdash \mathbf{Opt}\ T_A <: \mathbf{Pow}\ T_B} \qquad (\mathit{Sub\ Pair})\ \frac{E \vdash T_{A1} <: T_{A2} \quad E \vdash T_{B1} <: T_{B2}}{E \vdash \mathbf{Pair}\ (T_{A1}, T_{B1}) <: \mathbf{Pair}\ (T_{A2}, T_{B2})}$$

Table 5.4 Judgements of syntactic constructions common to VCL ADs and SDs

$E \vdash^{td} TD : T$	TD is well-formed type designator with type T in E
$E \vdash^{ta} A \therefore AId : T$	A is well-formed assertion with identifier AId and type T in E
$E \vdash^{se} SE : T$	SE is well-formed set element with type T in E
$E \vdash^{sdef} SDef : T$	Set definition $SDef$ yields type T in E
$E \vdash^{id} IDef : T$	Inside definition $IDef$ yields type T in E
$E \vdash^{so} SOP(\overline{T}) : T$	Application of operator SOp to sequence of types $\overline{T}$ yields type T
$E; T \vdash^{pe} PE$	Property edge PE is well-formed in E
$E; T \vdash^{pep} PEP$	Predicate property edge PEP is well-formed in E
$E; T \vdash^{pem} PEM$	Modifier property edge PEM is well-formed in E
$E \vdash^{te} TExp : T$	Target expression $TExp$ yields type T in E
$E \vdash^{ueo} UEOp(T_1) : T_2$	Application of unary edge operator $UEOp$ to type T_1 yields type T_2
$E \vdash^{eo} BEOP(T_1\ T_2)$	Application of predicate edge operator $BEOP$ to types T_1, T_2 is well-typed
$E; \vdash^{mo} MOP(T_1\ T_2)$	Application of modifier edge operator MOP to types T_1, T_2 is well-typed

has type T_B. The remaining rules say how different types are subtyping related. Rule `Sub Top` says that any valid type is a subtype of `Top`. Rule `Sub Obj` says that any set type is a subtype of `Obj`. Rule `Sub NatInt` says that type of natural numbers is a subtype of the integers. Rules `SubPow`, `Sub Seq` and `Sub Opt` say, respectively, that two powerset, sequence or optional types are subtypes of each other provided their enclosed types (T_A and T_B) are also. Rule `Sub Opt PSet` says that optional types are a subtype of powerset types provided their enclosed types are also. Finally, rule `Sub Pair` says that two pair types are subtypes of each other if their corresponding components are also subtypes of each other.

5.3 Common Rules

The judgements for the syntactic constructions that are common to SDs and ADs (grammar or Fig. 3.4a) are given in table 5.4.

Type designator rules (Table 5.5) derive a type from a designator, yielding a primitive type (`Int` or `Nat`) or some type that is associated with an identifier (rule `TD Id`).

Table 5.6 presents rules for checking the well-formedness of assertions (`T Assertion`), VCL objects (`T SE  Obj`) and pairs (`T SE  Pair`. These rules merely extracts the types associated

Table 5.5 Type rules for type designators (`TD` non-terminal)

$$\frac{}{E \vdash^{td} \mathbf{Nat} \,:\, Nat}\;(TD\ Nat) \qquad \frac{}{E \vdash^{td} \mathbf{Int} \,:\, Int}\;(TD\ Int) \qquad \frac{E \vdash Id \,:\, T}{E \vdash^{td} Id \,:\, T}\;(TD\ Id)$$

Table 5.6 Type rules for assertions and set elements

$$\frac{E \vdash Id : T \quad T = \mathbf{Assertion}[VE_v, VE_h]}{E \vdash^{ta} \mathbf{assertion}\, Id \,\therefore\, Id \,:\, T}\;(\mathrm{T\ \ Assertion}) \qquad \frac{E \vdash Id : T \qquad T <: \mathbf{Obj}}{E \vdash^{se} \mathbf{object}\, Id \,\therefore\, Id \,:\, T}\;(T\ SE\ Obj)$$

$$\frac{E \vdash Id_1 \,:\, T_1 \qquad E \vdash Id_2 \,:\, T_2}{E \vdash^{se} \mathbf{pair}(Id_1, Id_2) : \mathbf{Pair}\ (T_1, T_2)}\;(T\ SE\ Pair)$$

with identifiers from the environment. The assertion rule assumes that the AD associated with the assertion being checked has already been type-checked and its information can, therefore, be retrieved from the environment. Pair rule builds a pair type from the types of its constituent identifiers.

A *set definition* (`SDef` nonterminal, Fig. 3.4a) is a syntactic construct to build sets. The type rules for set definitions (table 5.7) consider two cases, depending on whether the inside expression comprises one inside definition (rule `T SDef IDef`) or a sequence of set definitions (rule `T SDef` $\overline{SDef}$). The rule essentially derive a sequence of types from inside definition (*IDef*) or sequence of set definitions ($\overline{SDef}$) and then apply the rule for the set definition's operator (*SOp*) to retrieve the types yielded by the rules. An inside definition (`IDef` nonterminal, Fig. 3.4a) is a construction associated with set definitions. An inside definition can either be a constrained set or a set expression. The type rules for inside definitions (table 5.7) consider these two cases. The constrained set rule (`IDef CntSet`) derives a type from the given type designator (*TD*) and then checks the sequence of property edges in the context of this derived type (T); the rule says that the set of property edges must either be of only one kind: either predicate or modifier (disjunction). The type rules for set extensions (`IDef SE` and `IDef SE *`) process the sequence of set elements inductively; retrieving the greatest type of all the elements in the sequence, which must be subtypes of each other.

The rules for set definition operators (`SOp` non-terminal) apply to a sequence of types in the context of an environment and a set definition operator; they are given in table 5.8. The rules are as follows:

- Rule `SOp None` considers the case where there is no operator. The rule requires that the sequence of types is made of a single element, and yields the type given in the sequence.
- Rules for domain and range operators (`SOp Dom` and `SOp Ran`) require that there is a single type given in the sequence and that this type is a powerset of a pair (it is a binary relation). Rule `SOp Dom` returns a type formed as the powerset of the first type of the pair (the domain). Rule `SOp Ran` returns a type formed as the powerset of the second type (the range).
- The cross product rules (`SOp Cross` and `SOp * Cross`) consider two cases depending on whether the sequence is made of a pair of types or more than a pair. The pair rule takes

Table 5.7 Type rules for set definitions and associated inside definitions

$$(T\ SDef\ IDef)\quad \frac{E \vdash^{id} IDef : T_1 \qquad E \vdash^{so} SOp(T_1) : T}{E \vdash^{sdef} \mathbf{set} \bigcirc SOp\ \mathbf{hasIn}\ \{IDef\} : T}$$

$$(T\ SDef\ \overline{SDef})\quad \frac{E \vdash^{sdef} \overline{SDef;} : \overline{T_{sd}} \qquad E \vdash^{so} SOp(\overline{T_{sd}}) : T_f}{E \vdash^{sdef} \mathbf{set} \bigcirc SOp\ \mathbf{hasIn}\ \{\overline{SDef;}\} : T_f}$$

$$(\text{IDef CntSet})\quad \frac{E \vdash^{td} TD : T \qquad E; T \vdash^{pe} \overline{PE} \qquad (IsPEP(\overline{PE}) \vee IsPEM(\overline{PE}))}{E \vdash^{id} \mathbf{set}\ TD\ \{\overline{PE}\} : \mathbf{Pow}\, T}$$

$$(IDef\ SE)\quad \frac{E \vdash^{se} SE : T}{E \vdash^{id} SE : \mathbf{Pow}\, T}$$

$$(IDef\ \overline{SE}\,*)\quad \frac{E \vdash^{se} SE : T_1 \qquad E \vdash^{id} \overline{SE} : T_2 \qquad (T_r = T_1 \wedge T_2 <: T_1) \vee (T_r = T_2 \wedge T_1 <: T_2)}{E \vdash^{id} SE\ \overline{SE} : \mathbf{Pow}\, T_r}$$

a pair of powerset types and yields a powerset of a pair type. Rule `SOp * Cross` takes a powerset type and a sequence of types and returns a powerset of a pair type formed with the derived type.

- The intersection (`SOp Pair Intersection` and `SOp * Intersection`) and union rules (`SOp Pair Union` and `SOp * Union`) take a sequence of at least two powerset types and return a powerset of the greatest type in the sequence, according to the subtyping relation (function *getGType*, appendix A). All given types must be subtypes of each other. The set subtraction rule (`SOp Pair SetMinus`) does the same for a pair of powerset types.

Table 5.8 Type rules for set def operators

$(\mathit{SOp\ None})$
$$\frac{}{E \vdash^{so} \bot\,(T) : T}$$

$(\mathit{SOp\ Dom})$
$$\frac{}{E \vdash^{so} \leftarrow (\mathbf{Pow\,Pair}\ (T_d, T_r)) : \mathbf{Pow}\ T_d}$$

$(\mathit{SOp\ Ran})$
$$\frac{}{E \vdash^{so} \rightarrow (\mathbf{Pow\,Pair}\ (T_d, T_r)) : \mathbf{Pow}\ T_r}$$

$(\mathit{SOp\ RelComp})$
$$\frac{}{E \vdash^{so} \boxdot(\mathbf{Pow\,Pair}\ (T_1, T_2)\ \mathbf{Pow\,Pair}\ (T_2, T_3)) : \mathbf{Pow\,Pair}\ (T_1, T_3)}$$

$(\mathit{SOp\ Cross})$
$$\frac{}{E \vdash^{so} \times(\mathbf{Pow}\ T_1\ \mathbf{Pow}\ T_2) : \mathbf{Pow\,Pair}\ (T_1, T_2)}$$

$(\mathit{SOp\ Pair\ Intersection})$
$$\frac{T = \mathit{getGType}(E, T_1, T_2)}{E \vdash^{so} \cap(\mathbf{Pow}\ T_1\ \mathbf{Pow}\ T_2) : \mathbf{Pow}\ T}$$

$(\mathit{SOp} * \mathit{Cross})$
$$\frac{E \vdash^{so} \times(\overline{T}) : \mathbf{Pow}\ T_2}{E \vdash^{so} \times(\mathbf{Pow}\ T_1\ \overline{T}) : \mathbf{Pow\,Pair}\ (T_1, T_2)}$$

$(\mathit{SOp} * \mathit{RelComp})$
$$\frac{E \vdash^{so} \boxdot(\overline{T}) : \mathbf{Pow\,Pair}\ (T_2, T_3)}{E \vdash^{so} \boxdot(\mathbf{Pow\,Pair}\ (T_1, T2)\ \overline{T}) : \mathbf{Pow\,Pair}\ (T_1, T_3)}$$

$(\mathit{SOp} * \mathit{Intersection})$
$$\frac{E \vdash^{so} \cap(\overline{T}^1) : \mathbf{Pow}\ T_2 \qquad T = \mathit{getGType}(E, T_1, T_2)}{E \vdash^{so} \cap(\mathbf{Pow}\ T_1\ \overline{T}^1) : \mathbf{Pow}\ T}$$

$(\mathit{SOp\ Pair\ Union})$
$$\frac{T = \mathit{getGType}(E, T_1, T_2)}{E \vdash^{so} \cup(\mathbf{Pow}\ T_1\ \mathbf{Pow}\ T_2) : \mathbf{Pow}\ T}$$

$(\mathit{SOp} * \mathit{Union})$
$$\frac{E \vdash^{so} \cup(\overline{T}^1) : \mathbf{Pow}\ T_2 \qquad T = \mathit{getGType}(E, T_1, T_2)}{E \vdash^{so} \cup(\mathbf{Pow}\ T_1\ \overline{T}^1) : \mathbf{Pow}\ T}$$

$(\mathit{SOp\ Pair\ SetMinus})$
$$\frac{T = \mathit{getGType}(E, T_1, T_2)}{E \vdash^{so} \setminus(\mathbf{Pow}\ T_1\ \mathbf{Pow}\ T_2) : \mathbf{Pow}\ T}$$

Table 5.9 Type rules for property edges

(PE PEP)
$$\frac{E \vdash^{te} \mathit{TExp} : T_2 \qquad E; (T_1, T_2) \vdash^{pep} \mathit{PEP}}{E; T_1 \vdash^{pe} \mathit{PEP\ TExp}}$$

(PE PEM)
$$\frac{E \vdash^{te} \mathit{TExp} : T_2 \qquad E; (T_1, T_2) \vdash^{pem} \mathit{PEM}}{E; T_1 \vdash^{pe} \mathit{PEM\ TExp}}$$

(PE *)
$$\frac{E; T \vdash^{pe} \mathit{PE} \qquad E; T \vdash^{pe} \overline{\mathit{PE}}^1}{E; T \vdash^{pe} \mathit{PE}\ \overline{\mathit{PE}}^1}$$

(PEP)
$$\frac{E; T_1 \vdash^{peps} [\mathit{Id}] : T_s \qquad E \vdash^{ueo} \mathit{UEOp}(T_s) : T_{sf} \qquad E \vdash^{eo} \mathit{BEOP}(T_{sf}\ T_2)}{E; (T_1, T_2) \vdash^{pep} [\mathit{UEOp}]\,[\mathit{Id}] \rightarrow [\mathit{BEOP}]}$$

$(\mathit{PEPS}\ \epsilon)$
$$\frac{}{E; T \vdash^{peps} \epsilon : T}$$

$(\mathit{PEPS\ PrId})$
$$\frac{T = \mathbf{Set}\ \mathit{Id}_s \qquad E \vdash \mathit{Id}_s.\mathit{Id}_{pr} : T_p}{E; T \vdash^{peps} \mathit{Id}_{pr} : T_p}$$

(PEPM)
$$\frac{E \vdash^{mo} \mathit{MOp}(T_1\ T_2)}{E; (T_1, T_2) \vdash^{pem} [\mathit{MOp}] \rightrightarrows}$$

$(\mathit{UEOp\ No})$
$$\frac{}{E \vdash^{ueo} \bot\,(T) : T}$$

$(\mathit{UEOp\ Card})$
$$\frac{}{E \vdash^{ueo} \#\,(\mathbf{Pow}\ T) : \mathbf{Int}}$$

$(\mathit{UEOp\ The})$
$$\frac{}{E \vdash^{ueo} \odot(\mathbf{Opt}\ T) : T}$$

Table 5.10 Type rules for binary predicate edge operators (BEOp)

$(\mathit{BEOP\ EQNEQ})$
$$\frac{(E \vdash T_1 <: T_2 \vee E \vdash T_2 <: T_1) \qquad \mathit{BEOP} \in \{\neq, =\}}{E \vdash^{eo} \mathit{BEOP}(T_1\ T_2)}$$

$(\mathit{BEOP\ IN})$
$$\frac{E \vdash T_1 <: T_2}{E \vdash^{eo} \in(T_1\ \mathbf{Pow}\ T_2)}$$

$(\mathit{BEOP\ INEQ})$
$$\frac{E \vdash T_1 <: \mathit{Int} \qquad E \vdash T_2 <: \mathit{Int} \qquad \mathit{BEOP} \in \{<, \leq, >, \geq\}}{E \vdash^{eo} \mathit{BEOP}(T_1\ T_2)}$$

$(\mathit{BEOP\ SUBSETEQ})$
$$\frac{E \vdash T_1 <: T_2}{E \vdash^{eo} \subseteq(\mathbf{Pow}\ T_1\ \mathbf{Pow}\ T_2)}$$

Table 5.11 Type rules for modifier edge operators (EOM)

$(\mathit{MOp\ DRES})$
$$\frac{E \vdash T :< T_d}{E \vdash^{mo} \lhd (\mathbf{Pow\,Pair}\ (T_d, T_r), \mathbf{Pow}\, T)}$$

$(\mathit{MOp\ RRES})$
$$\frac{E \vdash T :< T_r}{E \vdash^{mo} \rhd (\mathbf{Pow\,Pair}\ (T_d, T_r), \mathbf{Pow}\, T)}$$

$(\mathit{MOp\ DSUB})$
$$\frac{E \vdash T :< T_d}{E \vdash^{mo} ⩤ (\mathbf{Pow\,Pair}\ (T_d, T_r), \mathbf{Pow}\, T)}$$

$(\mathit{MOp\ RSUB})$
$$\frac{E \vdash T :< T_r}{E \vdash^{mo} ⩥ (\mathbf{Pow\,Pair}\ (T_d, T_r), \mathbf{Pow}\, T)}$$

Table 5.12 Type rules for set expressions

(SExp TD)
$$\frac{E \vdash^{td} TD : T}{E \vdash^{te} \mathbf{set}\, TD : \mathbf{Pow}\ T}$$

(SExp SDef)
$$\frac{E \vdash^{sdef} SDef : T}{E \vdash^{te} SDef : T}$$

(SExp Empty)
$$\frac{}{E \vdash^{te} \mathbf{set\,shaded} : \mathbf{Pow\ Top}}$$

(SExp Card)
$$\frac{E \vdash^{te} SExp : \mathbf{Pow}\, T}{E \vdash^{te} \#\, SExp : \mathbf{Int}}$$

Table 5.13 Type rules for free expressions

$(\mathit{FExp\ ID})$
$$\frac{E \vdash Id : T}{E \vdash^{te} Id : T}$$

(FExp Dot)
$$\frac{E \vdash Id_o : \mathbf{Set}\ Id_s \quad E \vdash Id_s.Id_{pr} : T}{E \vdash^{te} Id_o.Id_{pr} : T}$$

$(\mathit{FExp\ Num})$
$$\frac{}{E \vdash^{te} Num : Nat}$$

$(\mathit{FExp\ Uminus})$
$$\frac{E \vdash^{te} FE : Int}{E \vdash^{te} -FE : Int}$$

(FExp FEOP)
$$\frac{E \vdash^{te} FE_1 : Int \quad E \vdash^{te} FE_2 : Int}{E \vdash^{te} FE_1\ FEOP\ FE_2 : Int}$$

Table 5.14 Judgements for type system of VCL Structural Diagrams

$E; \overline{AD} \vdash^{sd} SD \therefore E'$	SD yields environment E'
$E; \overline{AD} \vdash^{sde} \overline{SDE} \therefore E'$	Sequence of SD elements $\overline{SDE}$ yields environment E'
$E; \overline{AD}; Id_{s\perp} \vdash^{as} \overline{A} \therefore VE$	Sequence of assertions $\overline{A}$ yields variable environment VE
$E \vdash^{cn} \overline{C} \therefore VE$	Sequence of constants $\overline{C}$ yields variable environment VE
$E; \overline{AD}; T \vdash^{pset} PSet \therefore E'$	Primary Set $PSet$ yields environment E'
$E \vdash^{ped} \overline{PED} \therefore VE$	Sequence of edge definitions $\overline{PED}$ yields variable environment VE
$E; M \vdash^{mtd} TD : T$	Designator TD with multiplicity M yields type T
$E; \overline{AD}; T \vdash^{hi} HI \therefore E'$	HI ($HasIn$) yields environment E'
$E; T \vdash^{io} \overline{O} \therefore VE$	Sequence of inside objects $\overline{O}$ yields variable environment VE
$E; T \vdash^{is} \overline{PSet} \therefore E'$	Sequence of inside primary sets $\overline{PSet}$ yields environment E'
$E; \overline{AD}; SId_{\perp} \vdash^{aok} A \therefore AId : T$	A has a well-formed assertion diagram with identifier AId type T in E
$E; \overline{AD}; Id_{s\perp} \vdash^{adok} \overbrace{AD} \therefore VE$	$\overbrace{AD}$ is set of ADs yielding variable environment VE

5.4 Rules for Structural Diagrams

Table 5.14 presents the judgements for structural diagrams (SDs). The first judgement says that a SD is well-formed in the environment E with environment E'. The remaining judgements assert well-formedness for the different components of a SD; namely, sequences of structural diagram element (judgement labelled $\vdash^{sde}$), sequences of assertions denoting invariants ($\vdash^{as}$), sequences of constants ($\vdash^{cn}$), primary sets ($\vdash^{pset}$), sequence of property edge definitions ($\vdash^{ped}$), designators with a multiplicity constraint ($\vdash^{mtd}$), has inside declarations of primary sets ($\vdash^{hi}$), sequence of inside objects ($\vdash^{io}$), sequences of inside primary sets ($\vdash^{is}$), assertion whose AD has not been checked ($\vdash^{aok}$) and set of ADs ($\vdash^{adok}$).

Table 5.15 Type rules for structural diagrams and sequences of diagram elements

(Ok SD)
$$\frac{E; \overline{AD} \vdash^{sde} \overline{SDE} \therefore E' \quad Acyclic\ E'.SE \quad E, E'; \overline{AD}; \perp \vdash^{as} \overline{A} \therefore VE}{E; \overline{AD} \vdash^{sd} \overline{SDE}\, \overline{A} \therefore E, E', VE}$$

($\overline{SDE}$ $*$)
$$\frac{E; \overline{AD} \vdash^{sde} SDE \therefore E' \quad E,\ E'; \overline{AD} \vdash^{sde} \overline{SDE} \therefore E''}{E; \overline{AD} \vdash^{sde} SDE\, \overline{SDE} \therefore E, E', E''}$$

($\overline{SDE}$ ϵ)
$$\frac{}{E; \overline{AD} \vdash^{sde} \epsilon \therefore E_{\varnothing}}$$

Table 5.16 Type rules for constants, relation edges and sets

$$(SDE\ Const)\ \frac{E \vdash^{cn} C \therefore VE_c}{E; \overline{AD} \vdash^{sde} C \therefore E_\varnothing, VE_c} \qquad (SDE\ RelEdge)\ \frac{E \vdash^{td} TD_1 : T_1 \quad E \vdash^{td} TD_2 : T_2 \quad M_1 \neq \mathbf{seq} \quad M_2 \neq \mathbf{seq}}{E; \overline{AD} \vdash^{sde} \mathbf{relEdge}\ Id_{RE}(M_1\ TD_1, M_2\ TD_2) \therefore E_\varnothing, Id_{RE} : \mathbf{Pow\,Pair}\ (T_1, T_2)}$$

$$(Const)\ \frac{E \vdash^{td} TD : T}{E \vdash^{cn} \mathbf{const}\ Id_{Cn} : TD \therefore \{Id_{Cn} : T\}} \qquad (\overline{Const}\,\epsilon)\ \frac{}{E \vdash^{cn} \epsilon \therefore \{\}} \qquad (\overline{Const}\,*)\ \frac{E \vdash^{cn} C \therefore VE_1 \quad E \vdash^{cn} \overline{C} \therefore VE_2 \quad \mathrm{dom}\ VE_1 \cap \mathrm{dom}\ VE_2 = \{\}}{E \vdash^{cn} C\ \overline{C} \therefore VE_1, VE_2}$$

$$(SDE\ PSet)\ \frac{E; \overline{AD}; \mathbf{Obj} \vdash^{pset} PSet \therefore E_b}{E; \overline{AD} \vdash^{sde} PSet \therefore E_b} \qquad (SDE\ Derived)\ \frac{E \vdash^{sdef} SDef : T}{E; \overline{AD} \vdash^{sde} Id_s \leftrightarrow SDef \therefore E_\varnothing, Id_s : T}$$

Table 5.17 Type rules for primary sets

$$(Primary\ Set)\ \frac{\begin{array}{c} Id_s \notin E.VE \\ E \vdash^{cn} \overline{C} \therefore VE_c \quad E \vdash^{ped} \overline{PED} \therefore VE_{pe} \quad E; \overline{AD}; Id_s \vdash^{as} \overline{A} \therefore VE_a \quad VE_i = getVE(E, T) \\ T_s = \mathbf{Set}\ Id_s \quad DK = getDK([\bigcirc]) \quad SI = (SK, DK, (VE_c, VE_{pe}, VE_a, VE_i)) \\ E,\ Id_s : \mathbf{Pow}\ T_s, Id_s \overset{se}{\mapsto} SI; \overline{AD}; T_s \vdash^{hi} [\ \mathbf{hasIn}\ \{\ \overline{(O \mid PSet)}\}\ ] \therefore (E_{hi}) \end{array}}{E; \overline{AD}; T \vdash^{pset} \mathbf{set}\ Id_s\ SK\ [\bigcirc]\ \{\overline{C}\ \overline{PED}\ \overline{A}\}\ [\ \mathbf{hasIn}\ \{\ \overline{(O \mid PSet)}\}\ ] \therefore (E_\varnothing, Id_s : \mathbf{Pow}\,T_s,\ Id_s \overset{se}{\mapsto} SI,\ T_s <: T, E_{hi})}$$

Table 5.18 Type rules for property edge definitions

$$(\overline{PED}\,\epsilon)\ \frac{}{E \vdash^{ped} \epsilon \therefore VE_\varnothing} \qquad (\overline{PED}\,*)\ \frac{E; M \vdash^{mtd} TD : T \quad E \vdash^{ped} \overline{PED} \therefore VE_2}{E \vdash^{ped} M\ Id_{Pe} \to TD\ \overline{PED} \therefore \{Id_{Pe} : T\}, VE_2}$$

$$(MTD\ One)\ \frac{E \vdash^{td} TD \therefore T}{E; \mathbf{one} \vdash^{mtd} TD : T} \qquad (MTD\ Pow)\ \frac{E \vdash^{td} TD \therefore T \quad M = \mathbf{some} \lor M = \mathbf{many} \lor M = Num\,..\,(Num|\mathbf{*})}{E; M \vdash^{mtd} TD : \mathbf{Pow}\ T}$$

$$(MTD\ Opt)\ \frac{E \vdash^{td} TD \therefore T}{E; \mathbf{opt} \vdash^{mtd} TD : \mathbf{Opt}\ T} \qquad (MTD\ Seq)\ \frac{E \vdash^{td} TD \therefore T}{E; \mathbf{seq} \vdash^{mtd} TD : \mathbf{Seq}\ T}$$

Table 5.19 Type rules for sequences of invariants

$$(\overline{A}\,\epsilon)\ \frac{}{E; \overline{AD}; Id_{s\perp} \vdash^{as} \epsilon \therefore \{\}} \qquad (\overline{A}\,*)\ \frac{E; \overline{AD}; Id_{s\perp} \vdash^{aok} A \therefore VE_1 \quad E; \overline{AD}; Id_{s\perp} \vdash^{as} \overline{A} \therefore VE_2}{E; \overline{AD}; Id_{s\perp} \vdash^{as} A\,\overline{A} \therefore VE_1,\ VE_2}$$

Table 5.20 Type rules for has inside declarations

$$(\mathit{HasInside}\,\epsilon)\quad \frac{}{E; T \vdash^{hi} \epsilon \therefore E_\varnothing}$$

$$(\mathit{HasInside}\,*)\quad \frac{E; T \vdash^{io} \overline{O} \therefore VE \qquad E; T \vdash^{is} \overline{PSet} \therefore E'}{E; T \vdash^{hi} \textbf{hasIn}\, \{\overline{O}\; \overline{PSet}\} \therefore E', VE}$$

$$(\mathit{HasInObjs}\,\epsilon)\quad \frac{}{E; T \vdash^{io} \epsilon \therefore \{\}}$$

$$(\mathit{HasInObjs}\,*)\quad \frac{E; T \vdash^{io} \overline{O} \therefore VE}{E; T \vdash^{io} \textbf{object}\; Id_o\; \overline{O} \therefore VE, \{Id_o : T\}}$$

$$(\mathit{HasInSet}\,\epsilon)\quad \frac{}{E; T \vdash^{is} \epsilon \therefore E_\varnothing}$$

$$(\mathit{HasInSets}\,*)\quad \frac{\begin{array}{c} E; T \vdash^{pset} PSet \therefore E' \\ E, E'; T \vdash^{is} \overline{PSet} \therefore E'' \end{array}}{E; T \vdash^{is} PSet\; \overline{PSet} \therefore E', E''}$$

Table 5.21 Type rules for checking assertions

$$(\mathit{AssertionOk})\quad \frac{Id_A \notin \operatorname{dom} E.VE \qquad AD = \mathit{findAD}(\overline{AD}, Id_A, Id_{s\perp}) \qquad E; \overline{AD}; Id_{s\perp} \vdash^{adok} AD \therefore VE}{E; \overline{AD}; Id_{s\perp} \vdash^{aok} \textbf{assertion}\; Id_A \therefore VE}$$

$$(\mathit{AD\ Ok})\quad \frac{\begin{array}{c} \widehat{AD} = \mathit{getDepsOfAD}\,(AD, \overline{AD}, Id_{s\perp}) \\ E; \overline{AD}; Id_{s\perp} \vdash^{adok} \widehat{AD} \therefore VE \qquad E, VE; \overline{AD} \vdash^{ad} AD \therefore Id_A : T \end{array}}{E; \overline{AD}; Id_{s\perp} \vdash^{adok} AD \therefore VE, Id_A : T}$$

$$(\mathit{ADs\ Ok}\,\epsilon)\quad \frac{}{E; \overline{AD}; Id_{s\perp} \vdash^{adok} \{\} \therefore \{\}}$$

$$(\mathit{ADs\ Ok}\,*)\quad \frac{E; \overline{AD}; Id_{s\perp} \vdash^{adok} AD \therefore VE \qquad E; \overline{AD}; Id_{s\perp} \vdash^{ads} \widehat{AD} \therefore VE'}{E; \overline{AD}; Id_{s\perp} \vdash^{adok} \{AD\} \cup \widehat{AD} \therefore VE, VE'}$$

Table 5.21 presents the rules for checking ADs associated with some assertion. These rules are used when the AD type information is to be loaded into the environment. The rules are as follows:

- Rule `Assertion Ok` derives the name of the assertion diagram through function `getFAId`, which considers the special case of assertions associated with constants, and then looks for the AD using function `findAD` (both functions defined in appendix A, section A.3.5). The retrieved AD is then checked (rule associated with judgement $\vdash^{adok}$) to yield variable environment VE.
- Rule `AD Ok` processes a single AD. It retrieves all the ADs that are included in the given AD through function `getDepsOfAD` (appendix A, section A.3.4) to yield set $\widehat{AD}$ and then checks them using the rules associated with judgement $\vdash^{adok}$ to derive variable environment VE. The current AD is also checked using the rule for assertion diagrams to yield a variable binding. The rule yields a variable environment formed by adding the retrieved variable binding to the variable environment VE.
- Rules $\mathit{AD\ Ok}\,\epsilon$ and $\mathit{AD\ Ok}\,*$ process a set of ADs inductively. Rule $\mathit{AD\ Ok}\,\epsilon$ considers the case where the set is empty, yielding an empty set of variable bindings. Rule $\mathit{AD\ Ok}\,*$ considers the case where the set has at least one element; it builds a variable environment

by joining the variable environment derived from the current single AD and the variable environment derived from the remaining set of ADs.

Table 5.22 Judgements for typing of assertion diagrams

$E \vdash^{ad} AD \therefore I : T$	AD yields binding $I : T$ in E
$E \vdash^{vd} \overline{VD} \therefore (VE_v, VE_h)$	Variable declarations block VD yields binding sets (VE_v, VE_h)
$E \vdash^{d} \overline{D} \therefore (VE_v, VE_h)$	Sequence of declarations $\overline{D}$ yields binding sets (VE_v, VE_h)
$E \vdash^{df} DF \therefore (VE_v, VE_h)$	Declarations formula atom DF yields binding sets (VE_v, VE_h)
$E \vdash^{f} \overline{F}$	Sequence of formulas $\overline{F}$ is well-formed in E
$E \vdash^{afs} AFS : T$	Arrows formula source AFS yields type T

Table 5.23 Type rules for assertion diagrams

$(AD\ GBL)$

$$\frac{E \vdash^{d} \overline{D} \therefore (VE_v; VE_h) \qquad E \oplus (VE_v, VE_h) \vdash^{f} \overline{F}}{E \vdash^{ad} \mathbf{AD}\ Id_A\ \mathbf{decls}\ \{\overline{D}\}\ \mathbf{pred}\ \{\overline{F}\} \therefore Id_A : \mathbf{Assertion}[VE_v, VE_h]}$$

$(AD\ LOCAL)$

$$\frac{\begin{array}{c} E \vdash Id_s : \mathbf{Pow\,Set}\ Id_s \\ E.SE(Id_s) = (SK,\ DK,\ VE_s) \qquad E \oplus VE_s \vdash^{d} \overline{D} \therefore (VE_v; VE_h) \qquad E \oplus (VE_s, VE_v, VE_h) \vdash^{f} \overline{F} \end{array}}{E \vdash^{ad} \mathbf{AD}\ Id_A : Id_s\ \mathbf{decls}\ \{\overline{D}\}\ \mathbf{pred}\ \{\overline{F}\} \therefore Id_A : \mathbf{Assertion}\ [VE_v,\ VE_h]}$$

5.5 Rules for Assertion Diagrams

The judgements for ADs are listed in Table 5.22. In the judgements's contexts, E is an environment; the AD rules assume that all relevant ADs have been checked and its information can be found in the environment. The judgements are as follows. The first judgement ($\vdash^{ad}$) asserts the well-formedness of some AD, yielding a binding made up of the AD's identifier and type. The remaining judgements concern either the declarations or predicate compartment of ADs. The declarations judgements include: judgement $\vdash^{d}$, which says that a sequence of declarations ($\overline{D}$) is well-formed and $\vdash^{df}$, which says that a particular declaration formula (DF) is well-formed. The predicate compartment includes judgements for formulas ($\vdash^{f}$) and arrows formula source ($\vdash^{afs}$).

The typing rules for ADs (table 5.23) consider two cases, corresponding to global (`AD GBL`) and local ADs (`AD LOCAL`). The rules are similar: the typing of declarations is followed by the typing of the predicate. The local rule requires the local variable environment, which it retrieves from the set environment component of the environment ($E.SE$). The processing of the declaration yields two variable environments: the visible (VE_v) and the hidden (VE_h) variables. The visible variables are visible in the assertions predicate and to the outside world; the hidden variables are only visible within the assertion.

The type rules for the declarations (table 5.24) build the visible and hidden variable environments. They are follows:

- Rules $\overline{D}\,\epsilon$ and $\overline{D}\,*$ handle a sequence of declaration inductively. Rule $\overline{D}\,\epsilon$ yields the empty variable environments ($\{\}$) for both visible and hidden: there are no declarations to process. Rule $\overline{D}\,*$ retrieves the variable environments from the current declaration ($VE_v,\ VE_h$) and from the remaining declarations ($VE_{vs},\ VE_{hs}$); the variables environments to be yielded by the rule are then merged (operator $\bowtie$), which requires that identifiers in common in the variable environments being combined must be bound to the same type; furthermore, all variables from the visible list (VE_{vf}) are removed in the hidden list (operator $\boxslash$).
- Rules `D Obj` and `D Set` consider the cases where there is a declaration of a scalar (object)

Table 5.24 Type rules for declarations

$$(\overline{D}\,\epsilon)\quad \frac{}{E \vdash^{d} \epsilon \therefore (\{\};\{\})}$$

$$(\overline{D}\,*)\quad \frac{E \vdash^{d} D \therefore (VE_v; VE_h) \qquad E, VE_v, VE_h \vdash^{d} \overline{D} \therefore (VE_{vs}; VE_{hs}) \\ VE_{vf} = VE_v \bowtie VE_{vs} \qquad VE_{hf} = (VE_h \bowtie VE_{hs}) \boxbslash VE_{vf}}{E \vdash^{d} D\,\overline{D} \therefore (VE_{vf}; VE_{hf})}$$

$$(\text{VD Obj})\quad \frac{\begin{array}{c} E \vdash^{td} TD : T \\ (OQ = \mathbf{opt} \wedge T_f = \mathbf{Opt}\ T \vee OQ = \epsilon \wedge T_f = T) \\ VE = \{\overline{Id_O,} : T_f\} \\ (HQ = \epsilon \wedge VE_v = VE \wedge VE_h = \{\} \\ \vee\ HQ = \mathbf{hidden} \wedge VE_v = \{\} \wedge VE_h = VE) \end{array}}{E \vdash^{vd} HQ\,\mathbf{object}\ OQ\ \overline{Id_O,}{:}TD \therefore (VE_v, VE_h)}$$

$$(\text{VD Set})\quad \frac{\begin{array}{c} E \vdash^{td} TD : T \\ VE = \{\overline{Id_s,} : \mathbf{Pow}\,T\} \\ (HQ = \epsilon \wedge VE_v = VE \wedge VE_h = \{\} \\ \vee\ HQ = \mathbf{hidden} \wedge VE_v = \{\} \wedge VE_h = VE) \end{array}}{E \vdash^{vd} HQ\,\mathbf{set}\ \overline{Id_s,}{:}TD \therefore (VE_v, VE_h)}$$

$$(\text{VD Seq})\quad \frac{\begin{array}{c} E \vdash^{td} TD : T \qquad VE = \{\overline{Id_s,} : \mathbf{Seq}\ T\} \\ (HQ = \epsilon \wedge VE_v = VE \wedge VE_h = \{\} \\ \vee\ HQ = \mathbf{hidden} \wedge VE_v = \{\} \wedge VE_h = VE) \end{array}}{E \vdash^{vd} HQ\,\mathbf{seq}\ \overline{Id_s,}{:}TD \therefore (VE_v, VE_h)}$$

$$(\text{VD }*)\quad \frac{\begin{array}{c} E \vdash^{vd} VD \therefore (VE_{v1}, VE_{h1}) \\ E \vdash^{vd} \overline{VD} \therefore (VE_{v2}, VE_{h2}) \\ VE_{v1} \cap VE_{v2} \cap VE_{h1} \cap VE_{h2} = \{\} \end{array}}{E \vdash^{vd} VD\ \overline{VD}{:}TD \therefore (VE_{v1} \cup VE_{v2}, VE_{h1} \cup VE_{h2})}$$

$$(D\ VD)\quad \frac{E \vdash^{vd} VD \therefore (VE_v, VE_h)}{E \vdash^{d} VD \therefore (VE_v, VE_h)} \qquad (D\ DF)\quad \frac{E \vdash^{df} DF \therefore (VE_v, VE_h)}{E \vdash^{d} DF \therefore (VE_v, VE_h)}$$

or set. Both rules retrieves a type from the declaration's type designator (TD) and then yield a visible binding made of the variable's identifier and appropriate type. Rule `D Obj` considers whether there is an optional qualifier ; type to yield is optional if there is a qualifier (**Opt**T) or the type derived from the type designator otherwise (T). Rule `D Set` also considers whether there is a sequence qualifier; type to yield is sequence of there is a qualifier ($\mathbf{Seq}\,T$) or a powerset otherwise ($\mathbf{Pow}\,T$).

- Rule `D DF` considers the case where the declaration comprises a declarations formula. In this case, the type rule for declaration formulas is called.

Table 5.25 presents the type rules for declaration formulas. The rules are as follows:

- Rules `DFA Assertion`, `DFA OCall` and `DFA ClCall` deal with declaration formula atoms (`DFA` non-terminal, Fig. 3.4c). Rule `DFA Assertion` considers the case where the construction refers to a normal assertion defined in the same scope (either local or global); rule `DFA OCall` considers the case where there is a local assertion being called on some object; and rule `DFA ClCall` considers the case where a class assertion is called.

- Rules `DFA Assertion`, `DFA OCall` and `DFA ClCall` assume that the AD associated with the assertion being checked has already been type-checked: the assertion's type can be retrieved from the environment. These rules retrieve the appropriate assertion type from the environment to obtain the assertion's visible and hidden bindings (VE_v and VE_h). From the assertion's visible bindings (VE_v), the rule then builds the visible and hidden bindings for the declaration using function $conVEs$, which takes into account the presence of symbol $\uparrow$, and from these constructed bindings the rule makes the required substitutions

Table 5.25 Type rules for declaration formulas

(DFA Assertion)

$$\frac{E \vdash^{ta} A \therefore Id_A : \mathbf{Assertion}[VE_v; VE_h] \quad (VE_{cv}, VE_{ch}) = consVEs(VE_v, [\uparrow]) \quad (VE_{fv}, VE_{fh}) = doSubs(VE_{cv}, VE_{ch}, [\overline{R},])}{E \vdash^{df} [\uparrow] A[\overline{R},] \therefore (VE_{fv}; VE_{fh})}$$

(DFA OCall)

$$\frac{E \vdash Id_O : T_s \quad Id_s = getSIdFrTy(T_s) \quad E \vdash Id_s\textbf{.}Id_A : \mathbf{Assertion}[VE_v, VE_h] \quad (VE_{cv}, VE_{ch}) = consVEs(VE_v, [\uparrow]) \quad (VE_{fv}, VE_{fh}) = doSubs(VE_{cv}, VE_{ch}, [\overline{R},])}{E \vdash^{df} [\uparrow]\mathbf{assertion}\ Id_O\textbf{.}Id_A[\overline{R},] \therefore (VE_{fv}; VE_{fh})}$$

(*DFA ClCall*)

$$\frac{E \vdash Id_s\textbf{.}Id_A : \mathbf{Assertion}[VE_v, VE_h] \quad (VE_{cv}, VE_{ch}) = consVEs(VE_v, [\uparrow]) \quad (VE_{fv}, VE_{fh}) = doSubs(VE_{cv}, VE_{ch}, [\overline{R},])}{E \vdash^{df} [\uparrow]\mathbf{assertion}\ Id_s \rightarrow Id_A[\overline{R},] \therefore (VE_{fv}; VE_{fh})}$$

(*DF Neg*)

$$\frac{E \vdash^{df} DF \therefore (VE_v; VE_h)}{E \vdash^{df} \neg\ [DF] \therefore (VE_v; VE_h)}$$

(DF Bin)

$$\frac{E \vdash^{df} DF_1 \therefore (VE_{v1}; VE_{h1}) \quad E \vdash^{df} DF_2 \therefore (VE_{v2}; VE_{h2}) \quad FOp \in \{\Rightarrow, equiv\}}{E \vdash^{df} FOp[DF_1\ DF_2] \therefore (VE_{v1} \bowtie VE_{v2}; VE_{h1} \bowtie VE_{h2})}$$

(*DF NAry* 2∗)

$$\frac{E \vdash^{df} \overline{DF} \therefore (VE_v; VE_h) \quad FOp \in \{\vee, \wedge, \boxdot\} \quad \#\,\overline{DF} \geq 2}{E \vdash^{df} FOp[\overline{DF}] \therefore (VE_v; VE_h)}$$

(*DF NAry* ϵ)

$$\frac{}{E \vdash^{df} \epsilon \therefore (\{\}; \{\})}$$

(*DF NAry* ∗)

$$\frac{E \vdash^{df} DF \therefore (VE_{v1}; VE_{h1}) \quad E \vdash^{df} \overline{DF} \therefore (VE_{v2}; VE_{h2})}{E \vdash^{df} DF\ \overline{DF} \therefore (VE_{v1} \bowtie VE_{v2}; VE_{h1} \bowtie VE_{h2})}$$

according to what is defined in the sequence of renamings ($\overline{R}$) using function *applySubs*. All it varies in the rules is the way the assertion type is obtained; rule `DFA Assertion` obtains the assertion type directly from the environment; rule `DFA OCall` obtains the assertion type from the object's set; and rule `DFA ClCall` obtains the assertion type from the given set identifier.

- Rule `DF Neg` obtains the visible and hidden variables of a negated declarations formula from the enclosed declarations formula.
- Rule `DF Bin` handles a binary declarations formula combined using a binary operator. The rules obtains the visible and hidden bindings from the two declarations formulas being combined and then merges them using the operator *mergeves*.

Table 5.26 Type rules for Formulas (F)

$$(\overline{F}\,\epsilon)\quad \frac{}{E \vdash^{f} \epsilon}$$

$$(\overline{F}\,*)\quad \frac{\begin{array}{c} E \vdash^{f} F \\ E \vdash^{f} \overline{F} \end{array}}{E \vdash^{f} F\,\overline{F}}$$

$$(\textit{F Not})\quad \frac{E \vdash^{f} F}{E \vdash^{f} \neg\,[F]}$$

$$(\text{F Bin})\quad \frac{\begin{array}{c} E \vdash^{f} F_1 \qquad E \vdash^{f} F_2 \\ FOp \in \{\Rightarrow, \Leftrightarrow\} \end{array}}{E \vdash^{f} FOp[F_1 F_2]}$$

$$(\text{F NAry})\quad \frac{\begin{array}{c} E \vdash^{f} \overline{F} \qquad \#\,\overline{F} \geq 2 \\ FOp \in \{\wedge, \vee\} \end{array}}{E \vdash^{f} FOp\,[\overline{F}]}$$

$$(\text{F AF})\quad \frac{\begin{array}{c} E \vdash^{afs} AFS : T \\ E; T \vdash^{peps} \overline{PEP}^{1} \end{array}}{E \vdash^{f} AFS\,\{\overline{PEP}^{1}\}}$$

$$(\textit{F SF Shaded})\quad \frac{E \vdash^{sdef}\ SDef : T}{E \vdash^{f} \textbf{shaded}\ SDef}$$

$$(\text{F SF Id})\quad \frac{\begin{array}{c} E \vdash Id_s : T_1 \\ E \vdash^{sdef}\ SDef : T_2 \\ (E \vdash T_2 <: T_1 \\ \vee\ E \vdash T_2 <: T_1) \end{array}}{E \vdash^{f} [\textbf{shaded}]\ Id_s\ SDef}$$

$$(\text{F SF Inside})\quad \frac{\begin{array}{c} E \vdash^{td}\ TD : T_1 \\ E \vdash^{te}\ SExp : T_2 \\ E \vdash\ T_1 <: T_2 \end{array}}{E \vdash^{f} \textbf{set}\ TD\ \textbf{hasIn}\ \{SExp\}}$$

$$(\textit{AFS SE})\quad \frac{E \vdash^{td}\ TD : \textbf{Pow}\,T_2}{E \vdash^{f} \textbf{set shaded}\ TD}$$

$$(\text{QF})\quad \frac{\begin{array}{c} E \vdash^{gd} \overline{QD}, \therefore VE \\ E \oplus VE \vdash^{f} F \end{array}}{E \vdash^{f} \overline{QD}, \bullet\ F}$$

$$(\text{QD})\quad \frac{\begin{array}{c} E \vdash^{vd} \overline{VD} \therefore (VE_v, VE_h) \\ VE_h = \{\} \end{array}}{E \vdash^{gd} Q\ \overline{VD}; \therefore VE_v}$$

$$(\text{QD }*)\quad \frac{\begin{array}{c} E \vdash^{vd} \overline{VD} \therefore (VE_v, VE_h) \\ VE_h = \{\} \qquad E \vdash^{gd} \overline{QD}, \therefore VE \end{array}}{E \vdash^{gd} Q\ \overline{VD}; \overline{QD}, \therefore VE_v \oplus VE}$$

Table 5.27 Type rules for Arrows Formula Source (production *AFS*)

$$(\textit{AFS SE})\quad \frac{E \vdash^{se} SE : T}{E \vdash^{afs}\ SE : T}$$

$$(\textit{AFS SetId})\quad \frac{E \vdash Id_s : T}{E \vdash^{afs} \textbf{set}\ Id_s : T}$$

$$(\textit{AFS SDef})\quad \frac{E \vdash^{sdef} SDef : T}{E \vdash^{afs} SDef : T}$$

$$(\textit{AFSB Un Card})\quad \frac{E \vdash^{afs} AFS : \textbf{Pow}\ T}{E \vdash^{afs} \#\ AFS : \textbf{Int}}$$

$$(\textit{AFS Un Dom})\quad \frac{E \vdash^{afs} AFS : \textbf{Pow Pair}\ (T_1, T_2)}{E \vdash^{afs} \leftarrow\ AFS : \textbf{Pow}\ T_1}$$

$$(\textit{AFS Un Ran})\quad \frac{E \vdash^{afs} AFS : \textbf{Pow Pair}\ (T_1, T_2)}{E \vdash^{afs} \rightarrow\ AFS : \textbf{Pow}\ T_2}$$

$$(\textit{AFSB Un The})\quad \frac{E \vdash^{afs} AFS : \textbf{Opt}\ T}{E \vdash^{afs} \odot\ AFS : T}$$

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Appendix A

Auxiliary Definitions

This appendix presents the auxiliary definitions that are used to describe the VCL type system presented in chapter 5.

A.1 Environment Operators

Several operators manipulate environments. E_1, E_2 means that two disjoint environments are combined into one. This is defined as set union for each component of the environments being combined:

$$E_1, E_2 = (VE_1 \cup VE_2, SE_1 \cup SE_2, SubE_1 \cup SubE_2)$$
$$\text{where, } E_1 = (VE_1, SE_1, SubE_1) \wedge E_2 = (VE_2, SE_2, SubE_2)$$

VE_1, VE_2 means that two disjoint variable environments are combined into one. This is defined as set union:

$$VE_1, VE_2 = VE_1 \cup VE_2 \Leftrightarrow \text{dom } VE_1 \cap \text{dom } VE_2 = \varnothing$$

Another operation on variable environments is $\bowtie$, which merges two variable environments. This requires that if there are identifiers in common in both variable environments, then they must be bound to the same type. This operator is defined as a partial function:

$$\_ \bowtie \_ : VE \times VE \nrightarrow VE$$

This is defined inductively by the following equations:

$$\{\} \bowtie VE = VE$$
$$(\{id : T\} \cup VE_1) \bowtie VE_2 = VE_1 \bowtie (VE_2 \cup \{Id : T\}) \Leftrightarrow id \notin \text{dom } VE_2 \vee VE_2(Id) = T$$

We define an operator for performing subtractions on variable environments that require that identifiers in common in both variable environments are bound to the same type. This operator is defined as a partial function:

$$\_ \boxslash \_ : VE \times VE \nrightarrow VE$$

This is defined by the following equation:

$$VE_1 \boxbslash VE_2 = VE_1 \setminus VE_2 \Leftrightarrow (\forall\, Id \in (\mathrm{dom}\, VE_1 \cap \mathrm{dom}\, VE_2) \bullet VE_1(Id) = VE_2(Id))$$

$E, Id : T$ means that a variable binding is added to an environment. This is defined as:

$$E, Id : T = \begin{cases} (VE \cup \{Id \mapsto T\}, SE, SubE) & \text{If } E = (VE, SE, SubE) \wedge \neg\, Id \in \mathrm{dom}\, E.VE \\ \mathit{undefined} & \text{otherwise} \end{cases}$$

$E, T_1 <: T_2$ means that a subtyping tuple is added to an environment. This is defined as:

$$E, T_1 <: T_2 = (VE, SE, SubE \cup \{T_1 \mapsto T_2\}) \quad \text{where, } E = (VE, SE, SubE)$$

$E, Id \overset{se}{\mapsto} (SK, DK, Id, VE)$ means that a set environment binding is added to an environment. This is defined as:

$$E, Id \overset{se}{\mapsto} (SK, DK, Id, VE) = \\ \begin{cases} (VE, PE, SE \cup \{(SK, DK, Id, VE)\}, SubE) & \text{If } E = (VE, SE, SubE) \wedge \neg\, Id \in \mathrm{dom}\, E.SE \\ \mathit{undefined} & \text{otherwise} \end{cases}$$

$E \oplus VE$ means that an environment is overridden with a set of variable bindings. This is defined as:

$$E \oplus VE_2 = (VE_1 \oplus VE_2, PE, SE, SubE) \quad \text{where, } E = (VE_1, PE, SE, SubE)$$

A.2 Predicates

$$\mathit{Acyclic}\,(R) \Leftrightarrow R \in \{rel : X \leftrightarrow X \mid rel^{+} \cap \mathrm{id}\, X = \varnothing\}$$

$$\mathit{IsPEP}(PE\,\overline{PEP}) \Leftrightarrow PE = PEP \wedge (\overline{PE} = \epsilon \vee \mathit{IsPEP}(\overline{PE}))$$

$$\mathit{IsPEM}(PE\,\overline{PEP}) \Leftrightarrow PE = PEM \wedge (\overline{PE} = \epsilon \vee \mathit{IsPEM}(\overline{PE}))$$

A.3 Auxiliary Functions

A.3.1 Function *getGType*

The function *getGType* gets the greatest type between two types ordered by the subtyping relation:

$$\mathbf{getGType} : E \times \mathit{Type} \times \mathit{Type} \nrightarrow \mathit{Type}$$

$$\mathit{getGType}\,(E, T_1, T_2) = \begin{cases} T_1 & \text{If } E \vdash T_2 <: T1 \\ T_2 & \text{If } E \vdash T_1 <: T2 \\ \mathit{undefined} & \text{otherwise} \end{cases}$$

A.3.2 Functions producing variable environments (VEs)

The function *getVE* extracts variable environments from set types:

$$\begin{aligned}
&\mathbf{getVE} : T \times E \rightarrow VE\\
&\quad getVE\,(T, E) \;=\; \begin{cases} VE & \text{If } T \;=\; \mathbf{Set}\, Id_s \;\wedge\; E.SE(Id_s) = (SK,\; DK,\, VE) \\ \{\} & \text{otherwise} \end{cases}
\end{aligned}$$

The function *consVEs* constructs a pair of variable environments given an optional imports qualifier and a variable environment (VE). This function simply makes the given VE the first component of the pair if there is an imports qualifier and makes it the second component of the pair otherwise:

$$\begin{aligned}
&\mathbf{consVEs} \;:\; VE \times \uparrow_\perp \rightarrow VE \times VE\\
&\quad consVEs\,(VE, \uparrow_\perp) = \begin{cases} (VE, \{\}) & \text{if } \uparrow_\perp = \uparrow) \\ (\{\}, VE) & \text{if } \uparrow_\perp = \perp) \end{cases}
\end{aligned}$$

A.3.3 Function *getDK*

The function *getDK* extracts the definitional kind:

$$\begin{aligned}
&\mathbf{getDK} : [\bigcirc] \rightarrow DK\\
&\quad getDK\,(\bigcirc) \;= \mathbf{def}\\
&\quad getDK\,(\epsilon) \;= \mathbf{notDef}
\end{aligned}$$

A.3.4 Functions to extract information from ADs

The following functions extract the AD identifier, set identifier and declarations from ADs:

$$\begin{aligned}
&getIdOfAD : AD \rightarrow Id\\
&\quad getIdOfAD(\mathbf{AD}\; Id_A\, [: Id_s]\, \mathbf{decls}\, \{\overline{D}\}\, \mathbf{pred}\, \{\overline{F}\}) = Id_A\\
&getSIdOfAD : AD \nrightarrow Id_\perp\\
&\quad getSIdOfAD(\mathbf{AD}\; Id_A : Id_s\, \mathbf{decls}\, \{\overline{D}\}\, \mathbf{pred}\, \{\overline{F}\}) = Id_s\\
&\quad getSIdOfAD(\mathbf{AD}\; Id_A\, \mathbf{decls}\, \{\overline{D}\}\, \mathbf{pred}\, \{\overline{F}\}) = \perp\\
&getDeclsOfAD : AD \rightarrow \overline{D}\\
&\quad getDeclsOfAD(\mathbf{AD}\; Id_A\, [: Id_s]\, \mathbf{decls}\, \{\overline{D}\}\, \mathbf{pred}\, \{\overline{F}\}) = \overline{D}
\end{aligned}$$

The following functions get the set of ADs that are included in some AD:

$\mathbf{getDepsOfAD} : AD \times \overline{AD} \times Id_\perp \to \widehat{AD}$
$\quad getDepsOfAD\ (AD, \overline{AD}, Id_{s\perp}) = getADsOfDecls\ (getDeclsOfAD\ (AD), \overline{AD}, Id_{s\perp})$

$getADsOfDecls : \overline{D} \times \overline{AD} \times Id_\perp \to \widehat{AD}$
$\quad getADsOfDecls\ (\epsilon, \overline{AD}, Id_{s\perp}) = \{\}$
$\quad getADsOfDecls\ (D\,\overline{D}, \overline{AD}, Id_{s\perp}) =$
$\qquad getADsOfDecl(D, \overline{AD}, Id_{s\perp}) \cup getADsOfDecls\ (\overline{D}, \overline{AD}, Id_{s\perp})$

$getADsOfDecl : Decl \times \overline{AD} \times Id_\perp \to \widehat{AD}$
$\quad getADsOfDecl\ (DV\ Id\ \mathbf{:}\ TD, \overline{AD}, Id_{s\perp}) = \{\}$
$\quad getADsOfDecl\ (DF, \overline{AD}, Id_{s\perp}) = getADsOfDF\ (DF, \overline{AD}, Id_{s\perp})$

$\mathbf{getADsOfDF} : DF \times \overline{AD} \times Id_\perp \to \widehat{AD}$
$\quad getADsOfDF\ ([\uparrow]\mathbf{assertion}\ Id_A\ [\overline{R},], \overline{AD}, Id_{s\perp}) = \{findAD(\overline{AD}, Id_{s\perp}, Id_A)\}$
$\quad getADsOfDF\ ([\uparrow]\mathbf{assertion}\ Id_o\mathbf{.}Id_A\ [\overline{R},], \overline{AD}, Id_{s\perp}) = findLADsWithName(\overline{AD}, Id_A)$
$\quad getADsOfDF\ ([\uparrow]\mathbf{assertion}\ Id_s \to Id_A\ [\overline{R},], \overline{AD}, Id_{s\perp}) = \{findAD(\overline{AD}, Id_s, Id_A)\}$
$\quad getADsOfDF\ (\neg\ \mathbf{(}DF\mathbf{)}, \overline{AD}, Id_{s\perp}) = getADsOfDF(DF, \overline{AD}, Id_{s\perp})$
$\quad getADsOfDF\ (\mathbf{(}DF_1\ FOp\ DF_2\mathbf{)}, \overline{AD}, Id_{s\perp}) = getADsOfDF(DF_1, \overline{AD}, Id_{s\perp})$
$\qquad \cup\ getADsOfDF(DF_2, \overline{AD}, Id_{s\perp})$

$\mathbf{getMatchingAD} : AD \times Id \to \widehat{AD}$

$$getMatchingAD(AD, Id_A) = \begin{cases} \{AD\} & \text{If } getSIdOfAD(AD) \neq \perp \wedge\ getIdOfAD(AD) = Id_A \\ \{\} & \text{otherwise} \end{cases}$$

$\mathbf{findLADsWithName} : \overline{AD} \times Id \nrightarrow \widehat{AD}$
$\quad findLADsWithName(\epsilon, Id_A) = \{\}$
$\quad findLADsWithName(AD\,\overline{AD}, Id_A) = getMatchingAD(AD, Id_A) \cup findLADsWithName(\overline{AD}, Id_A)$

A.3.5 Functions for AD lookup

The following functions look for some AD in a sequence of ADs:

$\mathbf{findAD} : \overline{AD} \times Id_{s\perp} \times Id_A \nrightarrow AD$
$\quad findAD\ (\overline{AD}, \perp, Id_A) = findGblAD(\overline{AD}, Id_A)$
$\quad findAD\ (\overline{AD}, Id_s, Id_A) = findLAD(\overline{AD}, Id_s, Id_A)$
$\mathbf{findGblAD} : \overline{AD} \times Id_A \to AD$
$\quad findGblAD\ (AD, Id_A) = AD \Leftrightarrow getIdOfAD(AD) = Id_A$
$\quad findGblAD\ (AD\,\overline{AD}, Id_A) = AD \Leftrightarrow getIdOfAD(AD) = Id_A$
$\quad findGblAD\ (AD\,\overline{AD}, Id_A) = findGblAD\ (\overline{AD}, Id_A) \Leftrightarrow getIdOfAD(AD) \neq Id_A$
$\mathbf{findLAD} : \overline{AD} \times Id_s \times Id_A \nrightarrow AD$
$\quad findLAD\ (AD, Id_s, Id_A) = AD \Leftrightarrow getSIdOfAD(AD) = Id_s \wedge getIdOfAD(AD) = Id_A$
$\quad findLAD\ (AD\,\overline{AD}, Id_s, Id_A) = AD \Leftrightarrow getSIdOfAD(AD) = Id_S \wedge getIdOfAD(AD) = Id_A$
$\quad findLAD\ (AD\,\overline{AD}, Id_s, Id_A) = findLAD(\overline{AD}, Id_s, Id_A)$
$\qquad \Leftrightarrow getIdOfAD(AD) \neq Id_A \vee getSIdOfAD(AD) \neq Id_s$

A.3.6 Functions for substitutions

The following functions deal with substitutions in variable environments:

$\mathbf{doSubs} : VE \times VE \times \overline{R} \nrightarrow VE \times VE$
$\quad doSubs\ (VE_v, VE_h, \overline{R}) = (applySubs(VE_v, \overline{R}), applySubs(VE_h, \overline{R}))$

$$\textbf{applySubs} : VE \times \overline{R} \nrightarrow VE$$
$$substitute\,(VE, idn/ido) = \begin{cases} (VE \setminus \{(ido,\, VE(ido))\}) \cup \{(idn,\, VE(ido))\} & \text{If } ido \in \text{dom } VE \wedge idn \notin \text{dom } VE \\ undefined & \text{otherwise} \end{cases}$$
$$applySubs\,(VE, \epsilon) = VE$$
$$applySubs\,(VE, R\,\overline{R}) = applySubs(substitute(VE, R), \overline{R})$$

A.3.7 Function *getSIdFrScalarOrCollection*

The following function retrieves a set identifier from types involving set types, which may either denote a scalar or a collection:

$$\textbf{getSIdFrTy} : Type \nrightarrow Id$$
$$getSIdFrTyn\,(\textbf{Set}\, Id_s) = Id_s$$
$$getSIdFrTy\,(\textbf{Pow}\,\textbf{Set}\, Id_s) = Id_s$$
$$getSIdFrTy\,(\textbf{Seq}\,\textbf{Set}\, Id_s) = Id_s$$
$$getSIdFrTy\,(\textbf{Opt}\,\textbf{Set}\, Id_s) = Id_s$$

Appendix B

Alloy Metamodels

B.1 VCL Common

```
--=========================================================================
-- Name: 'VCL_Common'
-- Description:
--    + Common entities of VCL ADs and SDs
--=========================================================================

module VCL_Common

--=========================================================================
-- Name: 'Name'
-- Description:
--    + Introduces set of labels to be attached to nodes and edges
--=========================================================================
sig Name {}

--=========================================================================
-- Name: 'SetElement'
-- Description:
--    + Defines a set element
--    + Either a single object or a pair
--=========================================================================
abstract sig SetElement {
}

--=========================================================================
-- Name: 'VCLObject'
-- Description:
--    + A named VCL object
--    + Elements that can be inside a set (either primitive or derived)
--=========================================================================
sig VCLObject extends SetElement {
   id : Name
```

```
}

--==========================================================================
-- Name: 'Pair'
-- Description:
--    + Represents a pair made of two named objects
--==========================================================================
sig Pair extends SetElement {
   idElem1 : Name,
   idElem2 : Name
}

--===========================================================================
-- Name: 'Assertion'
-- Description:
--    + Defines assertions whose symbol is the elongated hexagon.
--===========================================================================
sig Assertion {
   idAssertion  : Name
}

--============================================================================
-- Name: 'TypeDesignator'
--
-- Description:
--    + Defines a designator for types.
--============================================================================

abstract sig TypeDesignator {
}

--============================================================================
-- Name: 'TypeDesignator', ' TypeDesignatorNat'
--
-- Description:
--    + Defines a type designator naturals and integers.
--============================================================================
sig TypeDesignatorInt, TypeDesignatorNat extends TypeDesignator {
}

--============================================================================
-- Name: 'TypeDesignatorId'
--
-- Description:
--    + Defines a designator of sets with an identifier.
--============================================================================
sig TypeDesignatorId extends TypeDesignator {
   setId : Name
}
```

```
--==========================================================================
-- Name: 'PropEdge'
--
-- Description:
--    + Defines property edges with a source and a target.
--==========================================================================
abstract sig PropEdge {
   op : EdgeOperator,
   target : Expression,
}

--==========================================================================
-- Name: 'PropEdgePred'
--
-- Description:
--    + Defines property edges attached to predicate elements.
--==========================================================================
sig PropEdgePred extends PropEdge {
   unop : lone EdgeOperatorUnary,
   designator : lone Name
}{
   -- 'op' must be a 'EdgeOperatorPred'
   op in EdgeOperatorBin
}

--==========================================================================
-- Name: 'PropEdgeMod'
--
-- Description:
--    + Defines the property edge modifier that applies some operation to
--      the source.
--==========================================================================
sig PropEdgeMod extends PropEdge {
}{
   -- 'op' must be a 'EdgeOperatorMod'
   op in EdgeOperatorMod
}

--==========================================================================
-- Name: 'EdgeOperator'
--
-- Description:
--    + Defines edge operarator used in edges.
--==========================================================================
abstract sig EdgeOperator {
}

--==========================================================================
```

```
-- Name: 'EdgeOperatorBin'
--
-- Description:
--    + Defines edge operarator used in predicate edges.
--==========================================================================
abstract sig EdgeOperatorBin extends EdgeOperator{
}

--==========================================================================
-- Name: 'EdgeOperatorMod'
--
-- Description:
--    + Defines edge operarator used in modifer edges.
--==========================================================================
abstract sig EdgeOperatorMod extends EdgeOperator{
}

--==========================================================================
-- Name: 'EdgeOperatorUnary'
--
-- Description:
--    + Defines edge operarator used in modifer edges.
--==========================================================================
abstract sig EdgeOperatorUnary extends EdgeOperator{
}

--============================================================================
-- Name: 'EdgeOperatorEq',  'EdgeOperatorIn', 'EdgeOperatorSubsetEQ'
--'EdgeOperatorLT', 'EdgeOperatorLEQ', 'EdgeOperatorGT', 'EdgeOperatorGEQ'
--
-- Description:
--    + Defines different kinds of edge operators.
--    + Eq (=), Neq ( ), In ( ), LT, (<), LEQ ( ), GT (>), GEQ ( )
--    + SubsetEQ ( )
--=============================================================================
one sig EdgeOperatorEq,
EdgeOperatorNEq,
EdgeOperatorIn,
EdgeOperatorLT,
EdgeOperatorLEQ,
EdgeOperatorGT,
EdgeOperatorGEQ,
EdgeOperatorSubsetEQ
  extends EdgeOperatorBin {
}

--============================================================================
-- Name: 'EdgeOperatorDRES', 'EdgeOperatorRRES'
--
```

```
-- Description:
--    + Edge Operators used in property edge modifiers.
--    + DRES ( , domain restriction), and RRES ( , range restriction)
--    + DSUB ( , domain subtraction) and RSUB ( , range subtraction)
--==============================================================================

one sig EdgeOperatorDRES,
EdgeOperatorRRES,
   EdgeOperatorDSUB,
EdgeOperatorRSUB
extends EdgeOperatorMod {
}

--=============================================================================
-- Name: 'EdgeOperatorCARD', 'EdgeOperatorTHE'
-- Description:
--    + Unary edge operator used in predicate property edges
--    + CARD (#, cardinality)
--    + THE ( , the)
--==============================================================================
one sig EdgeOperatorCARD, EdgeOperatorTHE
   extends EdgeOperatorUnary {
}

--===========================================================================
-- Name: 'Num'
--
-- Description:
--    + String representing natural numbers.
--===========================================================================
sig Num {}

--===========================================================================
-- Name: 'Expression'
--
-- Description:
--    + Defines expressions associated with property edges.
--===========================================================================
abstract sig Expression {
}

--===========================================================================
-- Name: 'FreeExpression'
--
-- Description:
--    + Defines a free (editable) expression.
--===========================================================================
abstract sig FreeExpression extends Expression {
}
```

```
--===========================================================================
-- Name: 'FreeExpId'
--
-- Description:
--    + Defines object expressions comprising an identifier (a name).
--===========================================================================

sig FreeExpId extends FreeExpression {
   oid : Name,
 pkgId : lone Name,
}

--===========================================================================
-- Name: 'FreeExpDot'
--
-- Description:
--    + Defines expressions that access the state of objects.
--===========================================================================
abstract sig FreeExpDot extends FreeExpression {
   oid : Name, -- Identifier of the object
   propId : Name -- Identifier of the property
}

--===========================================================================
-- Name: 'FreeExpNum'
--
-- Description:
--    + Defines expressions comprising a number.
--===========================================================================

sig FreeExpNum extends  FreeExpression {
   num : Num
}

--===========================================================================
-- Name: 'FreeExpUMinus'
--
-- Description:
--    + Defines unary minus expression (-e).
--===========================================================================
sig FreeExpUMinus extends FreeExpression {
   e : FreeExpression
}

--===========================================================================
-- Name: 'FreeExpBin'
--
```

```
-- Description:
--    + Defines expressions that can be combined with binary operators.
--===========================================================================
abstract sig FreeExpBin extends FreeExpression {
   e1, e2 : FreeExpression,
   op :  FreeExpBinOp
}{
   e1 != e2
}

--===========================================================================
-- Name: 'FreeExpPar'
--
-- Description:
--    + Defines expressions that can be placed within parenthesis.
--===========================================================================
abstract sig FreeExpPar extends FreeExpression {
   e : FreeExpression
}

--===========================================================================
-- Name: 'FreeExpBinOp'
--
-- Description:
--    + Infix operators for sum (+), subtraction (-), product (*), div (/).
--===========================================================================
abstract sig FreeExpBinOp {}

one sig FreeExpBinOpPlus,
   FreeExpBinOpMinus,
   FreeExpBinOpTimes,
   FreeExpBinOpDiv extends FreeExpBinOp {}

--===========================================================================
-- Name: 'SetExpression'
--
-- Description:
--    + Defines a set expression.
--===========================================================================
abstract sig SetExpression extends Expression {
}

--===========================================================================
-- Name: 'SetExpressionID'
--
-- Description:
--    + Defines a set expression defined using a type designator.
--===========================================================================
sig SetExpressionID extends SetExpression {
```

```
   bd : TypeDesignator
}

--===========================================================================
-- Name: 'SetExpressionEmpty'
--
-- Description:
--    + Defines a set that is shaded to represent the empty set.
--===========================================================================
sig SetExpressionEmpty extends SetExpression {
}

--===========================================================================
-- Name: 'SetExpressionCard'
--
-- Description:
--    + Defines a set with a cardinality unary operator attached.
--===========================================================================
sig SetExpressionCard extends SetExpression {
   setExp : SetExpression
}

--===========================================================================
-- Name: 'SetDef'
--
-- Description:
--    + Defines a set definition (symbol  ).
--===========================================================================
sig SetDef {
   bdop      : SetDefOp, -- optional blob def operator
   insideExp : SetInsideExpression
}


--===========================================================================
-- Name: 'SetDefOp'
--
-- Description:
--    + Defines set def operators
--    + Domain operator is represented as symbol
--    + Range operator is represented as symbol
--    + None represents no symbol
--    + Union operator is represented as symbol
--    + Intersection operator is represented as symbol
--    + Cross product operator is represented as symbol
--    + Relation composition operator is represented by symbol
--    + Set difference operator is represented as symbol
--===========================================================================
abstract sig SetDefOp {
```

```
}

one sig SetDefOpDomain,
   SetDefOpRange,
   SetDefOPNone,
   SetDefOpUnion,
   SetDefOpIntersection,
   SetDefOpCrossProduct,
   SetDefOpSetMinus,
 SetDefOpRelComp
 extends SetDefOp {
}

--============================================================================
-- Name: 'SetExpressionDef'
--
-- Description:
--    + Defines a set expression defined using a set definition.
--============================================================================

sig SetExpressionDef extends SetExpression {
  def : SetDef
}

--============================================================================
-- Name: 'SetInsideExpression'
--
-- Description:
--    + Expression inside the set definition
--============================================================================
abstract sig SetInsideExpression {
}


--============================================================================
-- Name: 'InsideExpBlDs'
--
-- Description:
--    + Expression inside the set def
--============================================================================
sig InsideExpBlDs extends SetInsideExpression {
  blobDefs   : seq SetDef
}


--============================================================================
-- Name: 'InsideDef'
--
-- Description:
```

```
--    + Definition of the blob def
--    + Either a constrained blob or a a set extension
--==========================================================================
abstract sig InsideDef extends SetInsideExpression {
}

--==========================================================================
-- Name: 'ConstrainedSet'
--
-- Description:
--    + Defines a set with restrictions (constraints).
--==========================================================================

sig ConstrainedSet extends InsideDef {
    bd  : TypeDesignator,
    pes : seq PropEdge -- 0 or more predicate property edges
}

fact PropEdgesOfConstrainedSetAreOfSomeKind {
     all be :ConstrainedSet |
       all disj pe1, pe2 : univ.(be.pes) |
     pe1+pe2 in PropEdgePred || pe1+pe2 in PropEdgeMod
}

--==========================================================================
-- Name: 'SetExtension'
--
-- Description:
--    + Defines a set extensionally by listing its members.
--==========================================================================

sig SetExtension extends SetInsideExpression {
    elems : some SetElement
}
```

B.2 Bool Module

```
--==========================================================================
-- Name: 'Bool'
--
-- Description:
--    + Signature of booleans: 'True' or 'False'.
--==========================================================================
abstract sig Bool {}

one sig True, False extends Bool {}
```

B.3 VCL Structural Diagrams

```
--=========================================================================
-- Name: 'VCL_SD'
-- Description:
--    + Defines meta-model of VCL structural diagrams (SDs).
--===============================================================================
module VCL_SD

open VCL_Common as c
open Bool

--========================================================================
-- Name: 'Mult' (Multiplicity)
--
-- Description:
--    + Defines what a multiplicity is.
--    + Multiplicities are attached to ends of edges.
-- Details:
--    + There are the folowing kinds of multiplicity: one, optional (0..1),
--    many (0..*), one or many (1..*), range (n1..n2) and sequence.
--    + Multiplicities of kind range have a lower and an upper bound.
--========================================================================
abstract sig Mult {}

one sig MOne, MOpt, MMany, MOneOrMany, MSeq extends Mult {}

one sig MStar {}

sig MRange extends Mult {
   -- lower and upper bounds for 'range' multiplicities.
   lb : Int,
 ub : (Int+MStar)
}{
   -- lower and upper bounds must be greater or equal than 0
   -- and 'ub' greater or equal than 'lb'.
   lb >= 0 && (ub = MStar || ub >= lb)
}

--==========================================================================
-- Name: 'SDElem'
--
-- Description:
--    + Introduces the labelled structural diagram element.
--    + To be extended by 'Set', 'Object', 'Edge'.
--==========================================================================
abstract sig SDElem {
   name  : Name -- a modelling element has a name (a label).
}
```

```
--=========================================================================
-- Name: 'Constant'
--
-- Description:
--    + Represents constants. A constant has a type (field 'type).
--    + Constants can be 'local' or 'global'.
--    + A constant definition has a type
--========================================================================

sig Constant extends SDElem {
 type       : lone Name
}

--===============================================================
-- Name: 'RelEdge' (Relational Edge)
--
-- Description:
--    + Set relational edges are binary edges connecting sets.
--    + They have multiplicities at each end of edge.
--===============================================================

sig RelEdge extends SDElem {
 source, target : Set,
   sourceMult, targetMult : Mult,
}{
  -- Relation edges cannot have multiplicities of type sequence
not (sourceMult+targetMult) in MSeq
}


--===========================================================================
-- Name: 'Set' (Set Definitions)
--
-- Description:
--    + Defines a global set definition.
--    + It's characterised by inside property.
--
--===========================================================================

abstract sig Set extends SDElem {
}


--===========================================================================
-- Name: 'IntSet' (Integer Set)
--
-- Description:
--    + Defines a set representing the integers
```

```
--=========================================================================
one sig IntBlob extends Set {}

--=========================================================================
-- Name: 'NatSet' (Natural numbers Set)
--
-- Description:
--    + Defines a set representing the natural numbers
--=========================================================================
one sig NatSet extends Set {}

abstract sig SetKind {}

--=========================================================================
-- Name: 'Value', 'Class
--
-- Description:
--    + Defines two set kinds: 'value' and 'class'.
--=========================================================================

one sig Value, Class extends SetKind {}

--============================================================================
-- Name: 'SetDefObject'
--
-- Description:
--    + An object that can be inside a primitive set.
--============================================================================

sig SetDefObject {
objName : Name
}

--=========================================================================
-- Name: 'PrimarySet'
-- Description:
--    + Defines a primary set
--    + A Primary set can have sets ad objects inside.
--=========================================================================
sig PrimarySet extends Set {
   kind         : SetKind,
   lProps       : set PropEdgeDef,
 hasInsideSet : set PrimarySet,
   isDefSet     : Bool, -- (symbol   if 'True')
 hasInsideO   : set SetDefObject,
 lInvariants  : set Assertion,
   lConstants   : set Constant,
}
```

```
--
-- The following defines what it means for VCL structures to be well-formed
-- regarding the 'inside' property

--
-- The graph representing the 'inside' relation should be acyclic.
fact acyclicInside {
   no ^(hasInsideSet) & iden
}

--
-- An object should be in at most one set (the inverse of the relation is a partial function)
fact setInAtMostOneBlob {
all s : PrimarySet | lone s.~hasInsideSet
}

--
-- An object should be in at most one set (the inverse of the relation is a partial function)
fact objInAtMostOneBlob {
all n : SetDefObject | lone n.~hasInsideO
}

--
-- Each 'Set' has its own set of local invariants.
-- Or local invariants are not shared.
fact LInvariantsNotShared {
   all c : Assertion | (some lInvariants.c)
      =>  one lInvariants.c
}

--
-- Each 'Set' has its own set of local constants
-- Or local constants are not shared.
fact LConstantsNotShared {
   all c : Constant | (some lConstants.c)
      =>  one lConstants.c
}

-- Definitional sets must have things inside.
fact DefSetsHasThingsInside {
    all b : isDefSet.True | #b.hasInsideO > 0 ||  #b.hasInsideSet > 0
}

--
-- Each class set can contain other classes obly
-- and they can be inside of class sets only.
fact ClSetHasClSetsInside {
   all b : PrimarySet | b.kind = Class
      => (b.hasInsideSet) in kind.Class && hasInsideSet.b in kind.Class
```

```
}

--=========================================================================
-- Name: 'PropEdgeDef' (Property Edge Definition)
-- Description:
--    + Defines properties of sets.
--    + Relates one blob (having property) to another (type of property).
--    + A property edge has a 'Set' as target.
--    + A property edge may have a multiplicity.
--
--   ----------          0..*-------
--   |PropEdge|------------->|Set   |
--   ----------       target -------
--=========================================================================
sig PropEdgeDef extends SDElem {
   peTarget : Set,
   mult     : Mult
}
{
   -- a PropEdgeDef cannot have its blob or his inside blobs as target
   not (peTarget in ((this.~lProps) + (this.~lProps).^(hasInsideSet)))
}

--
-- Each 'Set' has its own set of property edge definitions
-- Or property edges are not shared. All property edges belong to some set
fact propEdgesNotSharedAndBelongToSomeSet {
   all pe : PropEdgeDef | one lProps.pe
}

fun nameOf (elem : SDElem + Assertion) : Name {
elem in SDElem implies elem.name else elem.idAssertion
}

--
-- Local Names in the scope of a 'Set'must be unique
--
fact LocalNamesAreUnique {
all s : Set |
all e1, e2 : (s.lConstants+s.lInvariants+s.lProps+(s.hasInsideO))
| nameOf [e1] = nameOf [e2]
       =>  e1 = e2
}

--
-- All global names must be unique
fact GblNamesAreUnique {
   all e1, e2 :
   (Set+(Assertion-(PrimarySet.lInvariants))+
```

```
      RelEdge+(Constant-(PrimarySet.lConstants)))
 | nameOf[e1] = nameOf[e2] implies e1 = e2
}

--===========================================================================
-- Name: 'DerivedSet'
--
-- Description:
--    + Defines a derived set
--    + Derived sets make use of symbol ' '
--===========================================================================
sig DerivedSet extends Set {
   definition : SetDef
}

--===========================================================================
-- Name: 'SDiag'
--
-- Description:
--    + Defines a structural diagram
--===========================================================================
sig SDDiag {
sdelems : set SDElem,
  invs : set Assertion
}
```

B.4 VCL Assertion Diagrams

```
--=========================================================================
-- Name: 'VCL_AD'
--
-- Description:
--    + Module defining the meta-model of VCL assertion diagrams.
--================================================================================

open VCL_Common as c
open Bool

--=================================================================================
-- Name: 'Decl'
--
-- Description:
--    + Defines a declaration of an assertion diagram.
--=================================================================================
abstract sig Decl {
}
```

```
--==================================================================================
-- Name: 'VarDecl'
--
-- Description:
--    + Defines a typed variable declaration of AD or CD.
--    + A typed variable declaration has a name, type and hidden status
--   + In EMF metamodel 'dNames' is just a string (to be parsed by type-checker)
--==================================================================================
abstract sig VarDecl extends Decl {
   dNames : set Name, -- set of declaration names separated by commas
   dTy : TypeDesignator, // Type of declaration
   isHidden : Bool, // Indicates whether the variable is hidden or not
}

--============================================================================
-- Name: 'DeclObj'
--
-- Description:
--    + Defines declarations of objects.
--    + Declarations of objects are represented as objects (rectangles).
--    + field optional indicates whether declaration is optional or not
--    + If optional is true, then '?' precedes the object's type.
--============================================================================

sig DeclObj extends VarDecl{
   optional : Bool
}

--============================================================================
-- Name: 'DeclSet'
--
-- Description:
--    + Defines declarations of sets.
--    + Sets are represented as blobs (ovals); they include the word "SET" on
--    top-left corner
--============================================================================

sig DeclSet extends VarDecl {
}

--============================================================================
-- Name: 'DeclSeq'
--
-- Description:
--    + Defines declarations of sequence.
--    + Sequences are represented as blobs (ovals); they include the word
--    "SEQUENCE" on top-left corner
--============================================================================
```

```
sig DeclSeq extends VarDecl {
}

--==========================================================================
-- Name: 'DeclFormula'
--
-- Description:
--    + Defines a declaration reference formula.
--    + This enables declaration references (either assertions or contracts)
--    to be combined using logical operators.
--==========================================================================

abstract sig DeclFormula extends Decl {
}

--==========================================================================
-- Name: 'RenamingExp'
--
-- Description:
--    + Defines a renaming expression, denoted in logic as [u/y]
--      where expression u denoted as the susbtition for variable y.
--==========================================================================
sig RenamingExp {
subExp : Name, -- Substituting expression
  varToSub : Name -- Variable to substitute
}

--==========================================================================
-- Name: 'DeclFormulaAtom'
--
-- Description:
--   + A declarations formula atom holds represents references to assertions or contracts
--    + The import is represented by the symbol '↑'
--    + Optional 'callObj' indicates a call a local operation on an object
--       represented as "a.op".
--   + Optional field 'origin' indicates origin of the operation (blob or package).
--    + Optional owning set indicates set of local contract or assertion
--    + Renaming expressions represented as '[t/x,u/y]'. In Ecore,
--    'RenamingExp' is just a String.
--==========================================================================

abstract sig DeclFormulaAtom extends DeclFormula {
   refId     : Name, -- Name of assertion or contract
   owningSet : lone Name, -- Id of set that owns local assertion or contract
  callObj   : lone Name, -- Id of obj on which local assertion or contract is called
 origin    : lone Name, -- optional originating package
   import    : Bool,      -- Whether import symbol is present or not
   renameExp : set RenamingExp -- a set of renaming expressions
}
```

```
--=========================================================================
-- Name: 'DeclAssertion'
--
-- Description:
--   + Represents an assertion reference of a declarations formula
--========================================================================
sig DeclAssertion extends DeclFormulaAtom {
}

--=========================================================================
-- Name: 'FormulaOp'
--
-- Description:
--    + Defines a formula operator.
--=========================================================================
abstract sig FormulaOp {
}

--=========================================================================
-- Name: FImplies, FAnd, FOr, FEquiv
--
-- Description:
--    + Defines formula operators for implication ( ), conjunction ( ),
--       disjunction ( ), equivalence ( ), negation (¬),
--    + and sequential composition ( )
--=========================================================================
one sig FImplies, FAnd, FOr, FEquiv, FNot,  FSComp extends FormulaOp {
}


--=========================================================================
-- Name: 'DeclFormulaNAry'
--
-- Description:
--    + Defines a declaration binary formula
--    + This supports the logical operators  ,
--=========================================================================
sig DeclFormulaNAr extends DeclFormula {
  dfrmls  : DeclFormula,
  dfop    : FormulaOp
}{
 all df1, df2 : dfrmls | df1 != df2
   dfop != FSComp
   dfop in FAnd+FOr implies #dfrmls >= 2
   dfop in FNot implies #dfrmls = 1
   dfop in FImplies+FEquiv implies #dfrmls = 2
}
```

```
--=========================================================================
-- Name: 'FormulaSource'
--
-- Description:
--    + Defines the source of an arrows formula
--    + It cain either be: obj, blob or pair
--=========================================================================
abstract sig FormulaSource {
}

--=========================================================================
-- Name: 'FormulaSourceElement'
--
-- Description:
--    + Defines source formula of type object
--    + 'elem' indicates the 'SetElement' either object or pair
--=========================================================================
sig FormulaSourceElem extends FormulaSource {
 elem : SetElement
}

--=========================================================================
-- Name: 'FormulaSourceSet'
--
-- Description:
--    + Defines source formula of type set
--=========================================================================
abstract sig FormulaSourceSet extends FormulaSource {
}

--=========================================================================
-- Name: 'FormulaSourceSetId'
--
-- Description:
--    + Defines source formula of type blob identifier
--    + 'bId' indicates identifier of the set
--=========================================================================
sig FormulaSourceSetId extends FormulaSourceSet {
   bId : Name
}

--=========================================================================
-- Name: 'FormulaSourceSetDef'
--
-- Description:
--    + Defines source formula of type set definition
--    + 'blDef' holds set definition
--=========================================================================
sig FormulaSourceSetDef extends FormulaSourceSet {
```

```
   blDef : SetDef
}

--=========================================================================
-- Name: 'FormulaSourceUOp'
--
-- Description:
--    + Defines a unary Formula operator for a formula source.
--=========================================================================
abstract sig FormulaSourceUOp {
}

--=========================================================================
-- Name: FSBCardinality, FSBDom, FSBRan
--
-- Description:
--    + Symbol of Formula source operator cardinality is #
--    + Symbol of Formula source operator domain is ' '
--    + Symbol of Formula source operator range is ' '
--    + Symbol of Formula source operator the is ' '
--========================================================================
one sig FSBCardinality, FSBDom, FSBRan ,  FSBThe
extends FormulaSourceUOp  {
}

--=========================================================================
-- Name: 'FormulaSourceUnary'
--
-- Description:
--    + Defines source formula with unary operator
--    + Let 'O' be a blob, this construction is expressed as # [O]
--=========================================================================
sig FormulaSourceUnary extends FormulaSource {
   operator : FormulaSourceUOp,
   frmlSrc  : FormulaSource
}


--=========================================================================
-- Name: 'AD'
--
-- Description:
--    + Defines what an assertion diagram is.
--=========================================================================

abstract sig AD {
   aName        : Name,
   declarations : set Decl,
   predicate    : set Formula
```

```
}

--==========================================================================
-- Name: 'Formula'
--
-- Description:
--    + Defines a Formula.
--==========================================================================
abstract sig Formula {
}


--==========================================================================
-- Name: 'FormulaNAry'
--
-- Description:
--    + Defines an n-ary Formula
--==========================================================================
sig FormulaNAry extends Formula {
   frmls : set Formula,
   operator : FormulaOp
}{
   all f1, f2 : frmls | f1 != f2
   operator != FSComp
   operator in FAnd+FOr implies #frmls >= 2
   operator in FNot implies #frmls = 1
   operator in FImplies+FEquiv implies #frmls = 2
}


--==========================================================================
-- Name: 'QFormula' (Quantified formula)
--
-- Description:
--    + Defines a quantified Formula.
--    + Includes a set of variable declarations and one formula
--==========================================================================
sig QFormula extends Formula {
   decls : seq QDecl,
   frml  : Formula
}{
   -- all elements in the sequence must be distinct
  not decls.hasDups
}

--==========================================================================
-- Name: 'QDecl' (Quantified Declaration)
--
-- Description:
```

```
--    + Defines a quantified declaration.
--    + Includes a set of variable declarations and one variable kind
--===========================================================================
sig QDecl {
   qkind : QuantifierKind, -- quantifier kind
   vars  : set VarDecl  -- variable declarations
}{
   -- Vars are distinct
   all v1, v2 : vars | v1 != v2
   -- hidden variables not allowed in quantified formulas
   all v : vars | v.isHidden = False
}

--===========================================================================
-- Name: 'QuantifierKind'
--
-- Description:
--    + Defines the kind of quantifier: forall or exists.
--===========================================================================
abstract sig QuantifierKind {
}

--===========================================================================
-- Name: 'QForAll'
--
-- Description:
--    + Defines the 'forall' (or universal) quantifier kind
--    + Represented in terms of concrete syntax by symbol classic symbol ' '
--===========================================================================
one sig QForAll extends QuantifierKind {
}

--===========================================================================
-- Name: 'QExists'
--
-- Description:
--    + Defines the 'exists' quantifier kind
--    + Represented in terms of concrete syntax by symbol classic symbol ' '
--===========================================================================
one sig QExists extends QuantifierKind {
}


--===========================================================================
-- Name: 'ArrowsFormula'
--
-- Description:
--    + Defines an arrows formula
--    + Made of predicate property edges
```

```
--    + With a source, which can either be: obj, blob or pair
--=========================================================================
sig ArrowsFormula extends Formula {
   source : FormulaSource,
   pes : some PropEdgePred
}

--=========================================================================
-- Name: 'SetFormula'
--
-- Description:
--    + Defines a 'Set' formula.
--=========================================================================
abstract sig SetFormula extends Formula {
}

--=========================================================================
-- Name: 'SetFormulaDef'
--
-- Description:
--    + Defines a 'Set' formula using a set definition (symbol  )
--=========================================================================
sig SetFormulaDef extends SetFormula {
   shaded : Bool, -- set may be shaded to mean empty set
   bid  : lone TypeDesignator, -- optional set designator
   bdef : SetDef -- set definition
}

--=========================================================================
-- Name: 'SetFormulaSubset'
--
-- Description:
--    + Defines a 'Set' formula defined using a subset definition.
--=========================================================================
sig SetFormulaSubset extends SetFormula {
   bid : TypeDesignator,
   hasInside : SetExpression
}

--=========================================================================
-- Name: 'SetFormulaShaded'
--
-- Description:
--    + Defines a 'Set' formula defined using shading.
--=========================================================================
sig SetFormulaShaded extends SetFormula {
   bid : TypeDesignator
}
```

Appendix C

Z3 Proofs

C.1 Common

C.1.1 Z3 Encoding

```
(set-option :mbqi true)
(set-option :macro-finder true)
(set-option :pull-nested-quantifiers true)
(set-option :produce-unsat-cores true)
(set-option :produce-models true)

(declare-sort V_MM)
(declare-sort E_MM)

(declare-sort V_G)
(declare-sort E_G)

(declare-const MM_Name              V_MM)
(declare-const MM_Num               V_MM)
(declare-const MM_Assertion         V_MM)
(declare-const MM_VCLObj            V_MM)
(declare-const MM_Pair              V_MM)
(declare-const MM_SetElement        V_MM)
(declare-const MM_InsideDef         V_MM)
(declare-const MM_SetExtension      V_MM)
(declare-const MM_ConstrainedSet    V_MM)
(declare-const MM_SetInsideExpression V_MM)
(declare-const MM_InsideExpSDs      V_MM)
(declare-const MM_SetDef            V_MM)
;; SetDefOp
(declare-const MM_SetDefOp          V_MM)
(declare-const MM_SOp_Domain        V_MM)
(declare-const MM_SOp_Range         V_MM)
(declare-const MM_SOp_Union         V_MM)
(declare-const MM_SOp_Intersection  V_MM)
```

```
(declare-const MM_SOp_CrossProduct   V_MM)
(declare-const MM_SOp_SetMinus       V_MM)
(declare-const MM_SOp_RelComp        V_MM)
(declare-const MM_SOp_None           V_MM)
;; Type Designator
(declare-const MM_TypeDesignator     V_MM)
(declare-const MM_TypeDesignatorNat  V_MM)
(declare-const MM_TypeDesignatorInt  V_MM)
(declare-const MM_TypeDesignatorId   V_MM)
;; FreeExpression
(declare-const MM_FreeExpression     V_MM)
(declare-const MM_FreeExpId          V_MM)
(declare-const MM_FreeExpNum         V_MM)
(declare-const MM_FreeExpUMinus      V_MM)
(declare-const MM_FreeExpPar         V_MM)
(declare-const MM_FreeExpDot         V_MM)
(declare-const MM_FreeExpBin         V_MM)
;; FreeExpBinOp
(declare-const MM_FreeExpBinOp       V_MM)
(declare-const MM_FreeExpBinOp_Plus  V_MM)
(declare-const MM_FreeExpBinOp_Minus V_MM)
(declare-const MM_FreeExpBinOp_Times V_MM)
(declare-const MM_FreeExpBinOp_Div   V_MM)
;; SetExpression
(declare-const MM_SetExpression      V_MM)
(declare-const MM_SetExpressionCard  V_MM)
(declare-const MM_SetExpressionId    V_MM)
(declare-const MM_SetExpressionEmpty V_MM)
(declare-const MM_SetExpressionDef   V_MM)
;; Expression
(declare-const MM_Expression         V_MM)
;; PropEdge
(declare-const MM_PropEdge           V_MM)
(declare-const MM_PropEdgePred       V_MM)
(declare-const MM_PropEdgeMod        V_MM)
(declare-const MM_EdgeOperatorBin    V_MM)
(declare-const MM_EdgeOperatorMod    V_MM)
(declare-const MM_EdgeOperatorUn     V_MM)
;; EdgeOperatorUnary
(declare-const MM_UOpCard            V_MM)
(declare-const MM_UOpThe             V_MM)
(declare-const MM_UOpNone            V_MM)
;; EdgeOperatorBin
(declare-const MM_BOpEQ              V_MM)
(declare-const MM_BOpNEQ             V_MM)
(declare-const MM_BOpIN              V_MM)
(declare-const MM_BOpLT              V_MM)
(declare-const MM_BOpLEQ             V_MM)
(declare-const MM_BOpGT              V_MM)
```

```
(declare-const MM_BOpGEQ             V_MM)
(declare-const MM_BOpSubsetEQ        V_MM)
;; EdgeOperatorMod
(declare-const MM_MOpDRES            V_MM)
(declare-const MM_MOpRRES            V_MM)
(declare-const MM_MOpDSUB            V_MM)
(declare-const MM_MOpRSUB            V_MM)
;; Special 'Null' constant to check totality
(declare-const MM_Null               V_MM)

(declare-const G_Num                 V_G)
(declare-const G_Id                  V_G)
;; TD
(declare-const G_TD                  V_G)
(declare-const G_TD_Int              V_G)
(declare-const G_TD_Nat              V_G)
(declare-const G_TD_Id               V_G)
;; A, O, P, SE
(declare-const G_A                   V_G)
(declare-const G_O                   V_G)
(declare-const G_P                   V_G)
(declare-const G_SE                  V_G)
;; PE
(declare-const G_PE                  V_G)
(declare-const G_PEP                 V_G)
(declare-const G_PEM                 V_G)
;; UEOp
(declare-const G_UEOp                V_G)
(declare-const G_UEOp_Card           V_G)
(declare-const G_UEOp_The            V_G)
(declare-const G_UEOp_None           V_G)
;; BEOp
(declare-const G_BEOp                V_G)
(declare-const G_BEOp_EQ             V_G)
(declare-const G_BEOp_NEQ            V_G)
(declare-const G_BEOp_IN             V_G)
(declare-const G_BEOp_LT             V_G)
(declare-const G_BEOp_LEQ            V_G)
(declare-const G_BEOp_GT             V_G)
(declare-const G_BEOp_GEQ            V_G)
(declare-const G_BEOp_SubsetEQ       V_G)
;; MOp
(declare-const G_MOp                 V_G)
(declare-const G_MOp_DRES            V_G)
(declare-const G_MOp_RRES            V_G)
(declare-const G_MOp_DSUB            V_G)
(declare-const G_MOp_RSUB            V_G)
;; TExp
(declare-const G_TExp                V_G)
```

```
;; FExp
(declare-const G_FExp               V_G)
(declare-const G_FExpId             V_G)
(declare-const G_FExpDot            V_G)
(declare-const G_FExpNum            V_G)
(declare-const G_FExpUMinus         V_G)
(declare-const G_FExpPar            V_G)
(declare-const G_FExpBin            V_G)
;; FEOp
(declare-const G_FEOp               V_G)
(declare-const G_FEOp_Plus          V_G)
(declare-const G_FEOp_Minus         V_G)
(declare-const G_FEOp_Times         V_G)
(declare-const G_FEOp_Div           V_G)
;; SExp
(declare-const G_SExp               V_G)
(declare-const G_SExpTD             V_G)
(declare-const G_SExpSDef           V_G)
(declare-const G_SExpEmpty          V_G)
(declare-const G_SExpCard           V_G)
;; IDef
(declare-const G_IDef               V_G)
(declare-const G_IDef_SExt          V_G)
(declare-const G_IDef_CntSet        V_G)
;; IExp
(declare-const G_IExp               V_G)
(declare-const G_IExp_SDs           V_G)
(declare-const G_IExp_IDef          V_G)
(declare-const G_SDef               V_G)
;; SOp
(declare-const G_SOp                V_G)
(declare-const G_SOp_Domain         V_G)
(declare-const G_SOp_Range          V_G)
(declare-const G_SOp_Union          V_G)
(declare-const G_SOp_Intersection   V_G)
(declare-const G_SOp_CrossProduct   V_G)
(declare-const G_SOp_SetMinus       V_G)
(declare-const G_SOp_RelComp        V_G)
(declare-const G_SOp_None           V_G)
;; Special 'Null' constant to check totality
(declare-const G_Null               V_G)

; Assertion, VCLObj, Pair, SetElement
(declare-const MM_EAssertion_Id           E_MM)
(declare-const MM_EVCLObj_Id              E_MM)
(declare-const MM_EIVCLObj                E_MM)
(declare-const MM_EPair_Id1               E_MM)
(declare-const MM_EPair_Id2               E_MM)
(declare-const MM_EIPair                  E_MM)
```

```
(declare-const MM_ESetExtension_Elems      E_MM)
(declare-const MM_EConstrainedSet_Desig    E_MM)
(declare-const MM_EConstrainedSet_PropEdge E_MM)
(declare-const MM_EISetExtension           E_MM)
(declare-const MM_EIConstrainedSet         E_MM)
(declare-const MM_EIInsideDef              E_MM)
(declare-const MM_EIInsideExpSDs           E_MM)
(declare-const MM_ESetDef_insideExp        E_MM)
(declare-const MM_ESetDef_sdop             E_MM)
(declare-const MM_EInsideExpSDs_setDefs    E_MM)
;; SetDefOp
(declare-const MM_EISOp_Domain             E_MM)
(declare-const MM_EISOp_Range              E_MM)
(declare-const MM_EISOp_Union              E_MM)
(declare-const MM_EISOp_Intersection       E_MM)
(declare-const MM_EISOp_CrossProduct       E_MM)
(declare-const MM_EISOp_SetMinus           E_MM)
(declare-const MM_EISOp_RelComp            E_MM)
(declare-const MM_EISOp_None               E_MM)
;; Type Designator
(declare-const MM_ETypeDesignatorId        E_MM)
(declare-const MM_EITypeDesignatorId       E_MM)
(declare-const MM_EITypeDesignatorNat      E_MM)
(declare-const MM_EITypeDesignatorInt      E_MM)
;; FreeExpression
(declare-const MM_EFreeExpNum              E_MM)
(declare-const MM_EIFreeExpNum             E_MM)
(declare-const MM_EFreeExpId               E_MM)
(declare-const MM_EIFreeExpId              E_MM)
(declare-const MM_EFreeExpUMinus           E_MM)
(declare-const MM_EIFreeExpUMinus          E_MM)
(declare-const MM_EFreeExpPar              E_MM)
(declare-const MM_EIFreeExpPar             E_MM)
(declare-const MM_EFreeExpDotId            E_MM)
(declare-const MM_EFreeExpDotPropId        E_MM)
(declare-const MM_EIFreeExpDot             E_MM)
(declare-const MM_EFreeExpBinExp1          E_MM)
(declare-const MM_EFreeExpBinExp2          E_MM)
(declare-const MM_EFreeExpBinOp            E_MM)
(declare-const MM_EIFreeExpBinExp          E_MM)
;; FreeExpBinOp
(declare-const MM_EIFreeExpBinOp_Plus      E_MM)
(declare-const MM_EIFreeExpBinOp_Minus     E_MM)
(declare-const MM_EIFreeExpBinOp_Times     E_MM)
(declare-const MM_EIFreeExpBinOp_Div       E_MM)
;; SetExpression
(declare-const MM_ESetExpressionCard       E_MM)
(declare-const MM_EISetExpressionCard      E_MM)
(declare-const MM_ESetExpressionId         E_MM)
```

```
(declare-const MM_EISetExpressionId        E_MM)
(declare-const MM_EISetExpressionEmpty     E_MM)
(declare-const MM_ESetExpressionDef        E_MM)
(declare-const MM_EISetExpressionDef       E_MM)
;; Expression
(declare-const MM_EISetExpression          E_MM)
(declare-const MM_EIFreeExp                E_MM)
;; PropEdge
(declare-const MM_EPropEdgeTarget          E_MM)
(declare-const MM_EPropEdgePredBOp         E_MM)
(declare-const MM_EPropEdgePredName        E_MM)
(declare-const MM_EPropEdgePredUOp         E_MM)
(declare-const MM_EPropEdgeModMOp          E_MM)
(declare-const MM_EIPropEdgeMod            E_MM)
(declare-const MM_EIPropEdgePred           E_MM)
;; EdgeOperatorUnary
(declare-const MM_EIUOp_Card               E_MM)
(declare-const MM_EIUOp_The                E_MM)
(declare-const MM_EIUOp_None               E_MM)
;; EdgeOperatorBin
(declare-const MM_EIBOp_EQ                 E_MM)
(declare-const MM_EIBOp_NEQ                E_MM)
(declare-const MM_EIBOp_In                 E_MM)
(declare-const MM_EIBOp_LT                 E_MM)
(declare-const MM_EIBOp_LEQ                E_MM)
(declare-const MM_EIBOp_GT                 E_MM)
(declare-const MM_EIBOp_GEQ                E_MM)
(declare-const MM_EIBOp_SubsetEQ           E_MM)
;; EdgeOperatorMod
(declare-const MM_EIMOp_DRES               E_MM)
(declare-const MM_EIMOp_RRES               E_MM)
(declare-const MM_EIMOp_DSUB               E_MM)
(declare-const MM_EIMOp_RSUB               E_MM)
;; Special 'Null' constant to check totality
(declare-const MM_ENull                    E_MM)

;; TD
(declare-const G_E_TD_Id                   E_G)
(declare-const G_E_TD_Def_Id               E_G)
(declare-const G_E_TD_Def_Nat              E_G)
(declare-const G_E_TD_Def_Int              E_G)
;; A, O, P, SE
(declare-const G_E_A_Id                    E_G)
(declare-const G_E_SE_Def_O                E_G)
(declare-const G_E_SE_Def_P                E_G)
(declare-const G_E_O_Id                    E_G)
(declare-const G_E_P_Id_1                  E_G)
(declare-const G_E_P_Id_2                  E_G)
;; PE
```

```
(declare-const G_E_PE_TExp                E_G)
(declare-const G_E_PE_PEP                 E_G)
(declare-const G_E_PE_PEM                 E_G)
(declare-const G_E_PEP_UEOp               E_G)
(declare-const G_E_PEP_Id                 E_G)
(declare-const G_E_PEP_BEOp               E_G)
(declare-const G_E_PEM_MOp                E_G)
;; UEOp
(declare-const G_E_UEOp_Def_Card          E_G)
(declare-const G_E_UEOp_Def_The           E_G)
(declare-const G_E_UEOp_Def_None          E_G)
;; BEOp
(declare-const G_E_BEOp_Def_Eq           E_G)
(declare-const G_E_BEOp_Def_Neq          E_G)
(declare-const G_E_BEOp_Def_In           E_G)
(declare-const G_E_BEOp_Def_LT           E_G)
(declare-const G_E_BEOp_Def_LEQ          E_G)
(declare-const G_E_BEOp_Def_GT           E_G)
(declare-const G_E_BEOp_Def_GEQ          E_G)
(declare-const G_E_BEOp_Def_SUBSETEQ     E_G)
;; MOp
(declare-const G_E_MOp_Def_DRES          E_G)
(declare-const G_E_MOp_Def_RRES          E_G)
(declare-const G_E_MOp_Def_DSUB          E_G)
(declare-const G_E_MOp_Def_RSUB          E_G)
;; TExp
(declare-const G_E_TExp_Def_SExp         E_G)
(declare-const G_E_TExp_Def_FExp         E_G)
;; FExp
(declare-const G_E_FExpId                E_G)
(declare-const G_E_FExpNum               E_G)
(declare-const G_E_FExpUMinus            E_G)
(declare-const G_E_FExpPar               E_G)
(declare-const G_E_FExpDot_Id            E_G)
(declare-const G_E_FExpDot_PropId        E_G)
(declare-const G_E_FExpBinExp1           E_G)
(declare-const G_E_FExpBinExp2           E_G)
(declare-const G_E_FExpBinOp             E_G)
(declare-const G_E_FExp_Def_Id           E_G)
(declare-const G_E_FExp_Def_Num          E_G)
(declare-const G_E_FExp_Def_UMinus       E_G)
(declare-const G_E_FExp_Def_Par          E_G)
(declare-const G_E_FExp_Def_Dot          E_G)
(declare-const G_E_FExp_Def_Bin          E_G)
;; FEOp
(declare-const G_E_FEOp_Def_Plus          E_G)
(declare-const G_E_FEOp_Def_Minus         E_G)
(declare-const G_E_FEOp_Def_Times         E_G)
(declare-const G_E_FEOp_Def_Div           E_G)
```

```
;; SExp
(declare-const G_E_SExpTD               E_G)
(declare-const G_E_SExpSDef             E_G)
(declare-const G_E_SExpCard             E_G)
(declare-const G_E_SExp_Def_TD          E_G)
(declare-const G_E_SExp_Def_SetDef      E_G)
(declare-const G_E_SExp_Def_Empty       E_G)
(declare-const G_E_SExp_Def_Card        E_G)
;; IDef
(declare-const G_E_IDef_SExt_SEs          E_G)
(declare-const G_E_IDef_CntSet_TD         E_G)
(declare-const G_E_IDef_CntSet_PEs        E_G)
(declare-const G_E_IDef_Def_SExt          E_G)
(declare-const G_E_IDef_Def_CntSet        E_G)
;; IExp
(declare-const G_E_IExp_Def_IDef          E_G)
(declare-const G_E_IExp_Def_SDs           E_G)
(declare-const G_E_IExpSDs_SDef           E_G)
;; SDef
(declare-const G_E_SDef_IExp              E_G)
(declare-const G_E_SDef_SOp               E_G)
;; SOp
(declare-const G_E_SOp_Def_Domain         E_G)
(declare-const G_E_SOp_Def_Range          E_G)
(declare-const G_E_SOp_Def_Union          E_G)
(declare-const G_E_SOp_Def_Intersection   E_G)
(declare-const G_E_SOp_Def_CrossProduct   E_G)
(declare-const G_E_SOp_Def_SetMinus       E_G)
(declare-const G_E_SOp_Def_RelComp        E_G)
(declare-const G_E_SOp_Def_None           E_G)
;; Special 'Null' constant to check totality
(declare-const G_E_Null                   E_G)

(assert (distinct
   MM_Null
   MM_Num
   MM_Name
   MM_TypeDesignator
   MM_TypeDesignatorNat
   MM_TypeDesignatorInt
   MM_TypeDesignatorId
   MM_FreeExpression
   MM_FreeExpId
   MM_FreeExpNum
   MM_FreeExpUMinus
   MM_FreeExpPar
   MM_FreeExpDot
   MM_FreeExpBin
   MM_FreeExpBinOp
```

```
MM_FreeExpBinOp_Plus
MM_FreeExpBinOp_Minus
MM_FreeExpBinOp_Times
MM_FreeExpBinOp_Div
MM_SetExpression
MM_SetExpressionId
MM_SetExpressionDef
MM_SetExpressionEmpty
MM_PropEdge
MM_PropEdgePred
MM_PropEdgeMod
MM_EdgeOperatorUn
MM_EdgeOperatorBin
MM_EdgeOperatorMod
MM_UOpCard
MM_UOpThe
MM_UOpNone
MM_BOpEQ
MM_BOpNEQ
MM_BOpIN
MM_BOpLT
MM_BOpLEQ
MM_BOpGT
MM_BOpGEQ
MM_BOpSubsetEQ
MM_MOpDRES
MM_MOpRRES
MM_MOpDSUB
MM_MOpRSUB
MM_Assertion
MM_VCLObj
MM_Pair
MM_SetElement
MM_InsideDef
MM_SetExtension
MM_ConstrainedSet
MM_SetInsideExpression
MM_InsideExpSDs
MM_SetDef
MM_SetDefOp
MM_SOp_Domain
MM_SOp_Range
MM_SOp_Union
MM_SOp_Intersection
MM_SOp_CrossProduct
MM_SOp_SetMinus
MM_SOp_RelComp
MM_SOp_None))
```

```
(assert (distinct
   G_Null
   G_Id
   G_PE
   G_PEP
   G_PEM
   G_BEOp
   G_UEOp
   G_MOp
   G_UEOp_Card
   G_UEOp_The
   G_UEOp_None
   G_BEOp_EQ
   G_BEOp_NEQ
   G_BEOp_IN
   G_BEOp_LT
   G_BEOp_LEQ
   G_BEOp_GT
   G_BEOp_GEQ
   G_BEOp_SubsetEQ
   G_MOp_DRES
   G_MOp_RRES
   G_MOp_DSUB
   G_MOp_RSUB
   G_TD
   G_TD_Nat
   G_TD_Int
   G_TD_Id
   G_FExp
   G_FExpId
   G_FExpNum
   G_FExpUMinus
   G_FExpPar
   G_FExpDot
   G_FExpBin
   G_FEOp
   G_FEOp_Plus
   G_FEOp_Minus
   G_FEOp_Times
   G_FEOp_Div
   G_SExp
   G_SExpTD
   G_SExpSDef
   G_SExpEmpty
   G_A
   G_O
   G_P
   G_SE
   G_IDef
```

```
   G_IDef_SExt
   G_IDef_CntSet
   G_IExp
   G_IExp_SDs
   G_IExp_IDef
   G_SDef
   G_SOp
   G_SOp_Domain
   G_SOp_Range
   G_SOp_Union
   G_SOp_Intersection
   G_SOp_CrossProduct
   G_SOp_SetMinus
   G_SOp_RelComp
   G_SOp_None))

(assert (distinct
   MM_ENull
   MM_ETypeDesignatorId
   MM_EITypeDesignatorId
   MM_EITypeDesignatorNat
   MM_EITypeDesignatorInt
   MM_EISetExpression
   MM_EIFreeExp
   MM_EFreeExpNum
   MM_EIFreeExpNum
   MM_EFreeExpId
   MM_EIFreeExpId
   MM_EFreeExpUMinus
   MM_EIFreeExpUMinus
   MM_EFreeExpPar
   MM_EIFreeExpPar
   MM_EFreeExpDotId
   MM_EFreeExpDotPropId
   MM_EIFreeExpDot
   MM_EFreeExpBinExp1
   MM_EFreeExpBinExp2
   MM_EFreeExpBinOp
   MM_EIFreeExpBinExp
   MM_EIFreeExpBinOp_Plus
   MM_EIFreeExpBinOp_Minus
   MM_EIFreeExpBinOp_Times
   MM_EIFreeExpBinOp_Div
   MM_ESetExpressionId
   MM_EISetExpressionId
   MM_ESetExpressionCard
   MM_EISetExpressionCard
   MM_ESetExpressionDef
   MM_EISetExpressionDef
```

```
  MM_EISetExpressionEmpty
  MM_EPropEdgeTarget
  MM_EPropEdgePredBOp
  MM_EPropEdgePredName
  MM_EPropEdgeModMOp
  MM_EIPropEdgeMod
  MM_EIPropEdgePred
  MM_EPropEdgePredUOp
  MM_EIUOp_Card
  MM_EIUOp_The
  MM_EIUOp_None
  MM_EIBOp_EQ
  MM_EIBOp_NEQ
  MM_EIBOp_In
  MM_EIBOp_LT
  MM_EIBOp_LEQ
  MM_EIBOp_GT
  MM_EIBOp_GEQ
  MM_EIMOp_DRES
  MM_EIMOp_RRES
  MM_EIMOp_DSUB
  MM_EIMOp_RSUB
  MM_EIBOp_SubsetEQ
  MM_EAssertion_Id
  MM_EVCLObj_Id
  MM_EPair_Id1
  MM_EPair_Id2
  MM_EIVCLObj
  MM_EIPair
  MM_ESetExtension_Elems
  MM_EConstrainedSet_Desig
  MM_EConstrainedSet_PropEdge
  MM_EISetExtension
  MM_EIConstrainedSet
  MM_EIInsideDef
  MM_EIInsideExpSDs
  MM_ESetDef_insideExp
  MM_ESetDef_sdop
  MM_EInsideExpSDs_setDefs
  MM_EISOp_Domain
  MM_EISOp_Range
  MM_EISOp_Union
  MM_EISOp_Intersection
  MM_EISOp_CrossProduct
  MM_EISOp_SetMinus
  MM_EISOp_RelComp
  MM_EISOp_None))

(assert (distinct
```

```
G_E_Null
G_E_TD_Id
G_E_TD_Def_Id
G_E_TD_Def_Nat
G_E_TD_Def_Int
G_E_TExp_Def_SExp
G_E_TExp_Def_FExp
G_E_FExpId
G_E_FExp_Def_Id
G_E_FExpNum
G_E_FExp_Def_Num
G_E_FExpUMinus
G_E_FExp_Def_UMinus
G_E_FExpPar
G_E_FExp_Def_Par
G_E_FExpBinExp1
G_E_FExpBinExp2
G_E_FExpBinOp
G_E_FExp_Def_Bin
G_E_FExpDot_Id
G_E_FExpDot_PropId
G_E_FExp_Def_Dot
G_E_FEOp_Def_Plus
G_E_FEOp_Def_Minus
G_E_FEOp_Def_Times
G_E_FEOp_Def_Div
G_E_SExpTD
G_E_SExp_Def_TD
G_E_SExpSDef
G_E_SExp_Def_SetDef
G_E_SExpCard
G_E_SExp_Def_Card
G_E_SExp_Def_Empty
G_E_PE_TExp
G_E_PE_PEP
G_E_PE_PEM
G_E_PEP_UEOp
G_E_PEP_Id
G_E_PEP_BEOp
G_E_PEM_MOp
G_E_UEOp_Def_Card
G_E_UEOp_Def_The
G_E_UEOp_Def_None
G_E_BEOp_Def_Eq
G_E_BEOp_Def_Neq
G_E_BEOp_Def_In
G_E_BEOp_Def_LT
G_E_BEOp_Def_LEQ
G_E_BEOp_Def_GT
```

```
   G_E_BEOp_Def_GEQ
   G_E_BEOp_Def_SUBSETEQ
   G_E_MOp_Def_DRES
   G_E_MOp_Def_RRES
   G_E_MOp_Def_DSUB
   G_E_MOp_Def_RSUB
   G_E_A_Id
   G_E_SE_Def_O
   G_E_SE_Def_P
   G_E_O_Id
   G_E_P_Id_1
   G_E_P_Id_2
   G_E_IDef_SExt_SEs
   G_E_IDef_CntSet_TD
   G_E_IDef_CntSet_PEs
   G_E_IDef_Def_SExt
   G_E_IDef_Def_CntSet
   G_E_IExp_Def_IDef
   G_E_IExp_Def_SDs
   G_E_SDef_IExp
   G_E_SDef_SOp
   G_E_IExpSDs_SDef
   G_E_SOp_Def_Domain
   G_E_SOp_Def_Range
   G_E_SOp_Def_Union
   G_E_SOp_Def_Intersection
   G_E_SOp_Def_CrossProduct
   G_E_SOp_Def_SetMinus
   G_E_SOp_Def_RelComp
   G_E_SOp_Def_None))

(define-fun Map_V ((v V_MM)) V_G
   (ite (= v MM_Num)                  G_Num
   (ite (= v MM_Name)                 G_Id
   (ite (= v MM_TypeDesignator)       G_TD
   (ite (= v MM_TypeDesignatorNat)    G_TD_Nat
   (ite (= v MM_TypeDesignatorInt)    G_TD_Int
   (ite (= v MM_TypeDesignatorId)     G_TD_Id
   (ite (= v MM_FreeExpression)       G_FExp
   (ite (= v MM_FreeExpId)            G_FExpId
   (ite (= v MM_FreeExpNum)           G_FExpNum
   (ite (= v MM_FreeExpUMinus)        G_FExpUMinus
   (ite (= v MM_FreeExpPar)           G_FExpPar
   (ite (= v MM_FreeExpDot)           G_FExpDot
   (ite (= v MM_FreeExpBin)           G_FExpBin
   (ite (= v MM_FreeExpBinOp)         G_FEOp
   (ite (= v MM_FreeExpBinOp_Plus)    G_FEOp_Plus
   (ite (= v MM_FreeExpBinOp_Minus)   G_FEOp_Minus
   (ite (= v MM_FreeExpBinOp_Times)   G_FEOp_Times
```

```
(ite (= v MM_FreeExpBinOp_Div)    G_FEOp_Div
(ite (= v MM_TypeDesignator)      G_TD
(ite (= v MM_SetExpression)       G_SExp
(ite (= v MM_SetExpressionId)     G_SExpTD
(ite (= v MM_SetExpressionDef)    G_SExpSDef
(ite (= v MM_SetExpressionEmpty)  G_SExpEmpty
(ite (= v MM_PropEdge)            G_PE
(ite (= v MM_PropEdgePred)        G_PEP
(ite (= v MM_PropEdgeMod)         G_PEM
(ite (= v MM_EdgeOperatorUn)      G_UEOp
(ite (= v MM_EdgeOperatorBin)     G_BEOp
(ite (= v MM_EdgeOperatorMod)     G_MOp
(ite (= v MM_UOpCard)             G_UEOp_Card
(ite (= v MM_UOpThe)              G_UEOp_The
(ite (= v MM_UOpNone)             G_UEOp_None
(ite (= v MM_BOpEQ)               G_BEOp_EQ
(ite (= v MM_BOpNEQ)              G_BEOp_NEQ
(ite (= v MM_BOpIN)               G_BEOp_IN
(ite (= v MM_BOpLT)               G_BEOp_LT
(ite (= v MM_BOpLEQ)              G_BEOp_LEQ
(ite (= v MM_BOpGT)               G_BEOp_GT
(ite (= v MM_BOpGEQ)              G_BEOp_GEQ
(ite (= v MM_BOpSubsetEQ)         G_BEOp_SubsetEQ
(ite (= v MM_MOpDRES)             G_MOp_DRES
(ite (= v MM_MOpRRES)             G_MOp_RRES
(ite (= v MM_MOpDSUB)             G_MOp_DSUB
(ite (= v MM_MOpRSUB)             G_MOp_RSUB
(ite (= v MM_PropEdge)            G_PE
(ite (= v MM_Assertion)           G_A
(ite (= v MM_VCLObj)              G_O
(ite (= v MM_Pair)                G_P
(ite (= v MM_SetElement)          G_SE
(ite (= v MM_InsideDef)           G_IDef
(ite (= v MM_SetExtension)        G_IDef_SExt
(ite (= v MM_ConstrainedSet)      G_IDef_CntSet
(ite (= v MM_SetInsideExpression) G_IExp
(ite (= v MM_InsideExpSDs)        G_IExp_SDs
(ite (= v MM_SetDef)              G_SDef
(ite (= v MM_SetDefOp)            G_SOp
(ite (= v MM_SOp_Domain)          G_SOp_Domain
(ite (= v MM_SOp_Range)           G_SOp_Range
(ite (= v MM_SOp_Union)           G_SOp_Union
(ite (= v MM_SOp_Intersection)    G_SOp_Intersection
(ite (= v MM_SOp_CrossProduct)    G_SOp_CrossProduct
(ite (= v MM_SOp_SetMinus)        G_SOp_SetMinus
(ite (= v MM_SOp_RelComp)         G_SOp_RelComp
(ite (= v MM_SOp_None)            G_SOp_None
G_Null)))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))
```

```
(define-fun Map_E ((e E_MM)) E_G
   (ite (= e MM_ETypeDesignatorId)        G_E_TD_Id
   (ite (= e MM_EITypeDesignatorId)       G_E_TD_Def_Id
   (ite (= e MM_EITypeDesignatorNat)      G_E_TD_Def_Nat
   (ite (= e MM_EITypeDesignatorInt)      G_E_TD_Def_Int
   (ite (= e MM_EFreeExpNum)              G_E_FExpNum
   (ite (= e MM_EIFreeExpNum)             G_E_FExp_Def_Num
   (ite (= e MM_EFreeExpId)               G_E_FExpId
   (ite (= e MM_EIFreeExpId)              G_E_FExp_Def_Id
   (ite (= e MM_EFreeExpUMinus)           G_E_FExpUMinus
   (ite (= e MM_EIFreeExpUMinus)          G_E_FExp_Def_UMinus
   (ite (= e MM_EFreeExpPar)              G_E_FExpPar
   (ite (= e MM_EIFreeExpPar)             G_E_FExp_Def_Par
   (ite (= e MM_EFreeExpDotId)            G_E_FExpDot_Id
   (ite (= e MM_EFreeExpDotPropId)        G_E_FExpDot_PropId
   (ite (= e MM_EIFreeExpDot)             G_E_FExp_Def_Dot
   (ite (= e MM_EFreeExpBinExp1)          G_E_FExpBinExp1
   (ite (= e MM_EFreeExpBinExp2)          G_E_FExpBinExp2
   (ite (= e MM_EFreeExpBinOp)            G_E_FExpBinOp
   (ite (= e MM_EIFreeExpBinExp)          G_E_FExp_Def_Bin
   (ite (= e MM_EIFreeExpBinOp_Plus)      G_E_FEOp_Def_Plus
   (ite (= e MM_EIFreeExpBinOp_Minus)     G_E_FEOp_Def_Minus
   (ite (= e MM_EIFreeExpBinOp_Times)     G_E_FEOp_Def_Times
   (ite (= e MM_EIFreeExpBinOp_Div)       G_E_FEOp_Def_Div
   (ite (= e MM_ESetExpressionCard)       G_E_SExpCard
   (ite (= e MM_EISetExpressionCard)      G_E_SExp_Def_Card
   (ite (= e MM_ESetExpressionId)         G_E_SExpTD
   (ite (= e MM_EISetExpressionId)        G_E_SExp_Def_TD
   (ite (= e MM_EISetExpressionEmpty)     G_E_SExp_Def_Empty
   (ite (= e MM_ESetExpressionDef)        G_E_SExpSDef
   (ite (= e MM_EISetExpressionDef)       G_E_SExp_Def_SetDef
   (ite (= e MM_EISetExpression)          G_E_TExp_Def_SExp
   (ite (= e MM_EIFreeExp)                G_E_TExp_Def_FExp
   (ite (= e MM_EAssertion_Id)            G_E_A_Id
   (ite (= e MM_EPropEdgeTarget)          G_E_PE_TExp
   (ite (= e MM_EPropEdgePredBOp)         G_E_PEP_BEOp
   (ite (= e MM_EPropEdgePredName)        G_E_PEP_Id
   (ite (= e MM_EPropEdgeModMOp)          G_E_PEM_MOp
   (ite (= e MM_EPropEdgePredUOp)         G_E_PEP_UEOp
   (ite (= e MM_EIPropEdgePred)           G_E_PE_PEP
   (ite (= e MM_EIPropEdgeMod)            G_E_PE_PEM
   (ite (= e MM_EIUOp_Card)               G_E_UEOp_Def_Card
   (ite (= e MM_EIUOp_The)                G_E_UEOp_Def_The
   (ite (= e MM_EIUOp_None)               G_E_UEOp_Def_None
   (ite (= e MM_EIBOp_EQ)                 G_E_BEOp_Def_Eq
   (ite (= e MM_EIBOp_NEQ)                G_E_BEOp_Def_Neq
   (ite (= e MM_EIBOp_In)                 G_E_BEOp_Def_In
   (ite (= e MM_EIBOp_LT)                 G_E_BEOp_Def_LT
   (ite (= e MM_EIBOp_LEQ)                G_E_BEOp_Def_LEQ
```

```
   (ite (= e MM_EIBOp_GT)                 G_E_BEOp_Def_GT
   (ite (= e MM_EIBOp_GEQ)                G_E_BEOp_Def_GEQ
   (ite (= e MM_EIBOp_SubsetEQ)           G_E_BEOp_Def_SUBSETEQ
   (ite (= e MM_EIMOp_DRES)               G_E_MOp_Def_DRES
   (ite (= e MM_EIMOp_RRES)               G_E_MOp_Def_RRES
   (ite (= e MM_EIMOp_DSUB)               G_E_MOp_Def_DSUB
   (ite (= e MM_EIMOp_RSUB)               G_E_MOp_Def_RSUB
   (ite (= e MM_EIVCLObj)                 G_E_SE_Def_O
   (ite (= e MM_EIPair)                   G_E_SE_Def_P
   (ite (= e MM_EVCLObj_Id)               G_E_O_Id
   (ite (= e MM_EPair_Id1)                G_E_P_Id_1
   (ite (= e MM_EPair_Id2)                G_E_P_Id_2
   (ite (= e MM_ESetExtension_Elems)      G_E_IDef_SExt_SEs
   (ite (= e MM_EConstrainedSet_Desig)    G_E_IDef_CntSet_TD
   (ite (= e MM_EConstrainedSet_PropEdge) G_E_IDef_CntSet_PEs
   (ite (= e MM_EISetExtension)           G_E_IDef_Def_SExt
   (ite (= e MM_EIConstrainedSet)         G_E_IDef_Def_CntSet
   (ite (= e MM_EIInsideDef)              G_E_IExp_Def_IDef
   (ite (= e MM_EIInsideExpSDs)           G_E_IExp_Def_SDs
   (ite (= e MM_ESetDef_insideExp)        G_E_SDef_IExp
   (ite (= e MM_ESetDef_sdop)             G_E_SDef_SOp
   (ite (= e MM_EInsideExpSDs_setDefs)    G_E_IExpSDs_SDef
   (ite (= e MM_EISOp_Domain)             G_E_SOp_Def_Domain
   (ite (= e MM_EISOp_Range)              G_E_SOp_Def_Range
   (ite (= e MM_EISOp_Union)              G_E_SOp_Def_Union
   (ite (= e MM_EISOp_Intersection)       G_E_SOp_Def_Intersection
   (ite (= e MM_EISOp_CrossProduct)       G_E_SOp_Def_CrossProduct
   (ite (= e MM_EISOp_SetMinus)           G_E_SOp_Def_SetMinus
   (ite (= e MM_EISOp_RelComp)            G_E_SOp_Def_RelComp
   (ite (= e MM_EISOp_None)               G_E_SOp_Def_None
  G_E_Null))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))

(push)
(echo "Testing function 'Map_V' (1) --> sat")
(assert (= (Map_V MM_PropEdge) G_PE))
(check-sat)
(pop)

(push)
(echo "Testing function 'Map_V' (2) --> sat")
(assert (= (Map_V MM_VCLObj) G_O))
(check-sat)
(pop)

(push)
(echo "Testing function 'Map_V' (3) --> unsat")
(assert (= (Map_V MM_SetElement) G_P))
(check-sat)
(pop)
```

```
(push)
(echo "Checking Totality of 'Map_V' --> sat")
(assert (forall ((vmm V_MM))
   (=> (= (Map_V vmm) G_Null) (= vmm MM_Null))))
(check-sat)
(pop)

(push)
(echo "Checking injectiveness of 'Map_V' --> sat")
(assert (forall ((vmm1 V_MM) (vmm2 V_MM))
   (=> (= (Map_V vmm1) (Map_V vmm2)) (= vmm1 vmm2))))
(check-sat)
(pop)

(push)
(echo "Checking Surjectiveness of 'Map_V' --> sat")
(assert (forall ((vg V_G))
   (exists ((vmm V_MM))
      (= (Map_V vmm) vg))))
(check-sat)
(pop)

;(push)
;(echo "Checking Surjectiveness of 'Map_V' (2)--> sat")
;(declare-fun svmm (V_G) V_MM)
;(assert (forall ((vg V_G))
;      (= (Map_V (svmm vg)) vg)))
;(check-sat)
;(pop)


(push)
(echo "Testing function 'Map_E' (1) --> sat")
(assert (= (Map_E MM_EAssertion_Id) G_E_A_Id))
(check-sat)
(pop)

(push)
(echo "Testing function 'Map_E' (2) --> sat")
(assert (= (Map_E MM_ESetExtension_Elems) G_E_IDef_SExt_SEs))
(check-sat)
(pop)

(push)
(echo "Testing function 'Map_E' (3) --> unsat")
(assert (= (Map_E MM_EAssertion_Id) G_E_SOp_Def_None))
(check-sat)
(pop)
```

```
(push)
(echo "Checking Totality of 'Map_E' --> sat")
(assert (forall ((emm E_MM))
   (=> (= (Map_E emm) G_E_Null) (= emm MM_ENull))))
(check-sat)
(pop)

(push)
(echo "Checking injectiveness of 'Map_E' --> sat")
(assert (forall ((emm1 E_MM) (emm2 E_MM))
   (=> (= (Map_E emm1) (Map_E emm2)) (= emm1 emm2))))
(check-sat)
(pop)

(push)
(echo "Checking Surjectiveness of 'Map_E' --> sat")
(assert (forall ((eg E_G))
   (exists ((emm E_MM))
      (= (Map_E emm) eg))))
(check-sat)
(pop)

;(push)
;(echo "Checking surjectiveness of 'Map_E' (2)--> sat")
;(declare-fun semm (E_G) E_MM)
;(assert (forall ((eg E_G))
;   (= (Map_E (semm eg)) eg)))
;(check-sat)
;(pop)

(define-fun Target_MM ((e E_MM)) V_MM
   (ite (= e MM_ETypeDesignatorId)        MM_Name
   (ite (= e MM_EITypeDesignatorId)       MM_TypeDesignator
   (ite (= e MM_EITypeDesignatorNat)      MM_TypeDesignator
   (ite (= e MM_EITypeDesignatorInt)      MM_TypeDesignator
   (ite (= e MM_EISetExpression)          MM_Expression
   (ite (= e MM_EIFreeExp)                MM_Expression
   (ite (= e MM_EFreeExpNum)              MM_Num
   (ite (= e MM_EIFreeExpNum)             MM_FreeExpression
   (ite (= e MM_EFreeExpId)               MM_Name
   (ite (= e MM_EIFreeExpId)              MM_FreeExpression
   (ite (= e MM_EFreeExpUMinus)           MM_FreeExpression
   (ite (= e MM_EIFreeExpUMinus)          MM_FreeExpression
   (ite (= e MM_EFreeExpPar)              MM_FreeExpression
   (ite (= e MM_EIFreeExpPar)             MM_FreeExpression
   (ite (= e MM_EFreeExpDotId)            MM_Name
   (ite (= e MM_EFreeExpDotPropId)        MM_Name
   (ite (= e MM_EIFreeExpDot)             MM_FreeExpression
```

```
(ite (= e MM_EFreeExpBinExp1)          MM_FreeExpression
(ite (= e MM_EFreeExpBinExp2)          MM_FreeExpression
(ite (= e MM_EFreeExpBinOp)            MM_FreeExpBinOp
(ite (= e MM_EIFreeExpBinExp)          MM_FreeExpression
(ite (= e MM_EIFreeExpBinOp_Plus)      MM_FreeExpBinOp
(ite (= e MM_EIFreeExpBinOp_Minus)     MM_FreeExpBinOp
(ite (= e MM_EIFreeExpBinOp_Times)     MM_FreeExpBinOp
(ite (= e MM_EIFreeExpBinOp_Div)       MM_FreeExpBinOp
(ite (= e MM_ESetExpressionId)         MM_TypeDesignator
(ite (= e MM_ESetExpressionDef)        MM_SetDef
(ite (= e MM_ESetExpressionCard)       MM_SetExpression
(ite (= e MM_EISetExpressionId)        MM_SetExpression
(ite (= e MM_EISetExpressionDef)       MM_SetExpression
(ite (= e MM_EISetExpressionCard)      MM_SetExpression
(ite (= e MM_EISetExpressionEmpty)     MM_SetExpression
(ite (= e MM_EPropEdgeTarget)          MM_Expression
(ite (= e MM_EPropEdgePredBOp)         MM_EdgeOperatorBin
(ite (= e MM_EPropEdgePredName)        MM_Name
(ite (= e MM_EPropEdgeModMOp)          MM_EdgeOperatorMod
(ite (= e MM_EIPropEdgeMod)            MM_PropEdge
(ite (= e MM_EIPropEdgePred)           MM_PropEdge
(ite (= e MM_EPropEdgePredUOp)         MM_EdgeOperatorUn
(ite (= e MM_EIUOp_Card)               MM_EdgeOperatorUn
(ite (= e MM_EIUOp_The)                MM_EdgeOperatorUn
(ite (= e MM_EIUOp_None)               MM_EdgeOperatorUn
(ite (= e MM_EIBOp_EQ)                 MM_EdgeOperatorBin
(ite (= e MM_EIBOp_NEQ)                MM_EdgeOperatorBin
(ite (= e MM_EIBOp_In)                 MM_EdgeOperatorBin
(ite (= e MM_EIBOp_LT)                 MM_EdgeOperatorBin
(ite (= e MM_EIBOp_LEQ)                MM_EdgeOperatorBin
(ite (= e MM_EIBOp_GT)                 MM_EdgeOperatorBin
(ite (= e MM_EIBOp_GEQ)                MM_EdgeOperatorBin
(ite (= e MM_EIBOp_SubsetEQ)           MM_EdgeOperatorBin
(ite (= e MM_EIMOp_DRES)               MM_EdgeOperatorMod
(ite (= e MM_EIMOp_RRES)               MM_EdgeOperatorMod
(ite (= e MM_EIMOp_DSUB)               MM_EdgeOperatorMod
(ite (= e MM_EIMOp_RSUB)               MM_EdgeOperatorMod
(ite (= e MM_EAssertion_Id)            MM_Name
(ite (= e MM_EIVCLObj)                 MM_SetElement
(ite (= e MM_EIPair)                   MM_SetElement
(ite (= e MM_EVCLObj_Id)               MM_Name
(ite (= e MM_EPair_Id1)                MM_Name
(ite (= e MM_EPair_Id2)                MM_Name
(ite (= e MM_ESetExtension_Elems)      MM_SetElement
(ite (= e MM_EConstrainedSet_Desig)    MM_TypeDesignator
(ite (= e MM_EConstrainedSet_PropEdge) MM_PropEdge
(ite (= e MM_EISetExtension)           MM_InsideDef
(ite (= e MM_EIConstrainedSet)         MM_InsideDef
(ite (= e MM_EIInsideDef)              MM_SetInsideExpression
```

```
   (ite (= e MM_EIInsideExpSDs)          MM_SetInsideExpression
   (ite (= e MM_ESetDef_insideExp)       MM_SetInsideExpression
   (ite (= e MM_ESetDef_sdop)            MM_SetDefOp
   (ite (= e MM_EInsideExpSDs_setDefs)   MM_SetDef
   (ite (= e MM_EISOp_Domain)            MM_SetDefOp
   (ite (= e MM_EISOp_Range)             MM_SetDefOp
   (ite (= e MM_EISOp_Union)             MM_SetDefOp
   (ite (= e MM_EISOp_Intersection)      MM_SetDefOp
   (ite (= e MM_EISOp_CrossProduct)      MM_SetDefOp
   (ite (= e MM_EISOp_SetMinus)          MM_SetDefOp
   (ite (= e MM_EISOp_RelComp)           MM_SetDefOp
   (ite (= e MM_EISOp_None)              MM_SetDefOp
  MM_Null)))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))

(define-fun Source_MM ((e E_MM)) V_MM
   (ite (= e MM_ETypeDesignatorId)       MM_TypeDesignatorId
   (ite (= e MM_EITypeDesignatorId)      MM_TypeDesignatorId
   (ite (= e MM_EITypeDesignatorNat)     MM_TypeDesignatorNat
   (ite (= e MM_EITypeDesignatorInt)     MM_TypeDesignatorInt
   (ite (= e MM_EISetExpression)         MM_SetExpression
   (ite (= e MM_EIFreeExp)               MM_FreeExpression
   (ite (= e MM_EFreeExpNum)             MM_FreeExpNum
   (ite (= e MM_EIFreeExpNum)            MM_FreeExpNum
   (ite (= e MM_EFreeExpId)              MM_FreeExpId
   (ite (= e MM_EIFreeExpId)             MM_FreeExpId
   (ite (= e MM_EFreeExpUMinus)          MM_FreeExpUMinus
   (ite (= e MM_EIFreeExpUMinus)         MM_FreeExpUMinus
   (ite (= e MM_EFreeExpPar)             MM_FreeExpPar
   (ite (= e MM_EIFreeExpPar)            MM_FreeExpPar
   (ite (= e MM_EFreeExpDotId)           MM_FreeExpDot
   (ite (= e MM_EFreeExpDotPropId)       MM_FreeExpDot
   (ite (= e MM_EIFreeExpDot)            MM_FreeExpDot
   (ite (= e MM_EFreeExpBinExp1)         MM_FreeExpBin
   (ite (= e MM_EFreeExpBinExp2)         MM_FreeExpBin
   (ite (= e MM_EFreeExpBinOp)           MM_FreeExpBin
   (ite (= e MM_EIFreeExpBinExp)         MM_FreeExpBin
   (ite (= e MM_EIFreeExpBinOp_Plus)     MM_FreeExpBinOp_Plus
   (ite (= e MM_EIFreeExpBinOp_Minus)    MM_FreeExpBinOp_Minus
   (ite (= e MM_EIFreeExpBinOp_Times)    MM_FreeExpBinOp_Times
   (ite (= e MM_EIFreeExpBinOp_Div)      MM_FreeExpBinOp_Div
   (ite (= e MM_ESetExpressionId)        MM_SetExpressionId
   (ite (= e MM_ESetExpressionDef)       MM_SetExpressionDef
   (ite (= e MM_ESetExpressionCard)      MM_SetExpressionCard
   (ite (= e MM_EISetExpressionId)       MM_SetExpressionCard
   (ite (= e MM_EISetExpressionDef)      MM_SetExpressionDef
   (ite (= e MM_EISetExpressionCard)     MM_SetExpressionCard
   (ite (= e MM_EISetExpressionEmpty)    MM_SetExpressionEmpty
   (ite (= e MM_EPropEdgeTarget)         MM_PropEdge
   (ite (= e MM_EPropEdgePredBOp)        MM_PropEdgePred
```

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   (ite (= e MM_EPropEdgePredName)        MM_PropEdgePred
   (ite (= e MM_EPropEdgeModMOp)          MM_PropEdgeMod
   (ite (= e MM_EIPropEdgeMod)            MM_PropEdgeMod
   (ite (= e MM_EIPropEdgePred)           MM_PropEdgePred
   (ite (= e MM_EPropEdgePredUOp)         MM_PropEdgePred
   (ite (= e MM_EIUOp_Card)               MM_UOpCard
   (ite (= e MM_EIUOp_The)                MM_UOpThe
   (ite (= e MM_EIUOp_None)               MM_UOpNone
   (ite (= e MM_EIBOp_EQ)                 MM_BOpEQ
   (ite (= e MM_EIBOp_NEQ)                MM_BOpNEQ
   (ite (= e MM_EIBOp_In)                 MM_BOpIN
   (ite (= e MM_EIBOp_LT)                 MM_BOpLT
   (ite (= e MM_EIBOp_LEQ)                MM_BOpLEQ
   (ite (= e MM_EIBOp_GT)                 MM_BOpGT
   (ite (= e MM_EIBOp_GEQ)                MM_BOpGEQ
   (ite (= e MM_EIBOp_SubsetEQ)           MM_BOpSubsetEQ
   (ite (= e MM_EIMOp_DRES)               MM_MOpDRES
   (ite (= e MM_EIMOp_RRES)               MM_MOpRRES
   (ite (= e MM_EIMOp_DSUB)               MM_MOpDSUB
   (ite (= e MM_EIMOp_RSUB)               MM_MOpRSUB
   (ite (= e MM_EAssertion_Id)            MM_Assertion
   (ite (= e MM_EIVCLObj)                 MM_VCLObj
   (ite (= e MM_EIPair)                   MM_Pair
   (ite (= e MM_EVCLObj_Id)               MM_VCLObj
   (ite (= e MM_EPair_Id1)                MM_Pair
   (ite (= e MM_EPair_Id2)                MM_Pair
   (ite (= e MM_ESetExtension_Elems)      MM_SetExtension
   (ite (= e MM_EConstrainedSet_Desig)    MM_ConstrainedSet
   (ite (= e MM_EConstrainedSet_PropEdge) MM_ConstrainedSet
   (ite (= e MM_EISetExtension)           MM_SetExtension
   (ite (= e MM_EIConstrainedSet)         MM_ConstrainedSet
   (ite (= e MM_EIInsideDef)              MM_InsideDef
   (ite (= e MM_EIInsideExpSDs)           MM_InsideExpSDs
   (ite (= e MM_ESetDef_insideExp)        MM_SetDef
   (ite (= e MM_ESetDef_sdop)             MM_SetDef
   (ite (= e MM_EInsideExpSDs_setDefs)    MM_InsideExpSDs
   (ite (= e MM_EISOp_Domain)             MM_SOp_Domain
   (ite (= e MM_EISOp_Range)              MM_SOp_Range
   (ite (= e MM_EISOp_Union)              MM_SOp_Union
   (ite (= e MM_EISOp_Intersection)       MM_SOp_Intersection
   (ite (= e MM_EISOp_CrossProduct)       MM_SOp_CrossProduct
   (ite (= e MM_EISOp_SetMinus)           MM_SOp_SetMinus
   (ite (= e MM_EISOp_RelComp)            MM_SOp_RelComp
   (ite (= e MM_EISOp_None)               MM_SOp_None
  MM_Null)))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))

(define-fun Target_G ((e E_G)) V_G
   (ite (= e G_E_TD_Id)                   G_Id
   (ite (= e G_E_TD_Def_Id)               G_TD
```

```
(ite (= e G_E_TD_Def_Nat)              G_TD
(ite (= e G_E_TD_Def_Int)              G_TD
(ite (= e G_E_TExp_Def_SExp)           G_TExp
(ite (= e G_E_TExp_Def_FExp)           G_TExp
(ite (= e G_E_FExpNum)                 G_Num
(ite (= e G_E_FExpId)                  G_Id
(ite (= e G_E_FExpUMinus)              G_FExp
(ite (= e G_E_FExpPar)                 G_FExp
(ite (= e G_E_FExpBinExp1)             G_FExp
(ite (= e G_E_FExpBinExp2)             G_FExp
(ite (= e G_E_FExpBinOp)               G_FEOp
(ite (= e G_E_FExp_Def_Id)             G_FExp
(ite (= e G_E_FExp_Def_Num)            G_FExp
(ite (= e G_E_FExp_Def_UMinus)         G_FExp
(ite (= e G_E_FExp_Def_Par)            G_FExp
(ite (= e G_E_FExp_Def_Bin)            G_FExp
(ite (= e G_E_FExpDot_Id)              G_Id
(ite (= e G_E_FExpDot_PropId)          G_Id
(ite (= e G_E_FExp_Def_Dot)            G_FExp
(ite (= e G_E_FEOp_Def_Plus)           G_FEOp
(ite (= e G_E_FEOp_Def_Minus)          G_FEOp
(ite (= e G_E_FEOp_Def_Times)          G_FEOp
(ite (= e G_E_FEOp_Def_Div)            G_FEOp
(ite (= e G_E_SExpTD)                  G_TD
(ite (= e G_E_SExpSDef)                G_SDef
(ite (= e G_E_SExpCard)                G_SExp
(ite (= e G_E_SExp_Def_TD)             G_SExp
(ite (= e G_E_SExp_Def_SetDef)         G_SExp
(ite (= e G_E_SExp_Def_Card)           G_SExp
(ite (= e G_E_SExp_Def_Empty)          G_SExp
(ite (= e G_E_PE_TExp)                 G_TExp
(ite (= e G_E_PE_PEP)                  G_PE
(ite (= e G_E_PE_PEM)                  G_PE
(ite (= e G_E_PEP_UEOp)                G_UEOp
(ite (= e G_E_PEP_Id)                  G_Id
(ite (= e G_E_PEP_BEOp)                G_BEOp
(ite (= e G_E_PEM_MOp)                 G_MOp
(ite (= e G_E_UEOp_Def_Card)           G_UEOp
(ite (= e G_E_UEOp_Def_The)            G_UEOp
(ite (= e G_E_UEOp_Def_None)           G_UEOp
(ite (= e G_E_BEOp_Def_Eq)             G_BEOp
(ite (= e G_E_BEOp_Def_Neq)            G_BEOp
(ite (= e G_E_BEOp_Def_In)             G_BEOp
(ite (= e G_E_BEOp_Def_LT)             G_BEOp
(ite (= e G_E_BEOp_Def_LEQ)            G_BEOp
(ite (= e G_E_BEOp_Def_GT)             G_BEOp
(ite (= e G_E_BEOp_Def_GEQ)            G_BEOp
(ite (= e G_E_BEOp_Def_SUBSETEQ)       G_BEOp
(ite (= e G_E_MOp_Def_DRES)            G_MOp
```

```
    (ite (= e G_E_MOp_Def_RRES)            G_MOp
    (ite (= e G_E_MOp_Def_DSUB)            G_MOp
    (ite (= e G_E_MOp_Def_RSUB)            G_MOp
    (ite (= e G_E_A_Id)                    G_Id
    (ite (= e G_E_O_Id)                    G_Id
    (ite (= e G_E_P_Id_1)                  G_Id
    (ite (= e G_E_P_Id_2)                  G_Id
    (ite (= e G_E_SE_Def_O)                G_SE
    (ite (= e G_E_SE_Def_P)                G_SE
    (ite (= e G_E_IDef_SExt_SEs)           G_SE
    (ite (= e G_E_IDef_CntSet_TD)          G_TD
    (ite (= e G_E_IDef_CntSet_PEs)         G_PE
    (ite (= e G_E_IDef_Def_SExt)           G_IDef
    (ite (= e G_E_IDef_Def_CntSet)         G_IDef
    (ite (= e G_E_IExp_Def_IDef)           G_IExp
    (ite (= e G_E_IExp_Def_SDs)            G_IExp
    (ite (= e G_E_SDef_IExp)               G_IExp
    (ite (= e G_E_SDef_SOp)                G_SOp
    (ite (= e G_E_IExpSDs_SDef)            G_SDef
    (ite (= e G_E_SOp_Def_Domain)          G_SOp
    (ite (= e G_E_SOp_Def_Range)           G_SOp
    (ite (= e G_E_SOp_Def_Union)           G_SOp
    (ite (= e G_E_SOp_Def_Intersection)    G_SOp
    (ite (= e G_E_SOp_Def_CrossProduct)    G_SOp
    (ite (= e G_E_SOp_Def_SetMinus)        G_SOp
    (ite (= e G_E_SOp_Def_RelComp)         G_SOp
    (ite (= e G_E_SOp_Def_None)            G_SOp
  G_Null)))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))

(define-fun Source_G ((e E_G)) V_G
    (ite (= e G_E_TD_Id)                   G_TD_Id
    (ite (= e G_E_TD_Def_Id)               G_TD_Id
    (ite (= e G_E_TD_Def_Nat)              G_TD_Nat
    (ite (= e G_E_TD_Def_Int)              G_TD_Int
    (ite (= e G_E_TExp_Def_SExp)           G_SExp
    (ite (= e G_E_TExp_Def_FExp)           G_FExp
    (ite (= e G_E_FExpNum)                 G_FExpNum
    (ite (= e G_E_FExpId)                  G_FExpId
    (ite (= e G_E_FExpUMinus)              G_FExpUMinus
    (ite (= e G_E_FExpPar)                 G_FExpPar
    (ite (= e G_E_FExpBinExp1)             G_FExpBin
    (ite (= e G_E_FExpBinExp2)             G_FExpBin
    (ite (= e G_E_FExpBinOp)               G_FExpBin
    (ite (= e G_E_FExp_Def_Id)             G_FExpId
    (ite (= e G_E_FExp_Def_Num)            G_FExpNum
    (ite (= e G_E_FExp_Def_UMinus)         G_FExpUMinus
    (ite (= e G_E_FExp_Def_Par)            G_FExpPar
    (ite (= e G_E_FExp_Def_Bin)            G_FExpBin
    (ite (= e G_E_FExpDot_Id)              G_FExpDot
```

```
(ite (= e G_E_FExpDot_PropId)          G_FExpDot
(ite (= e G_E_FExp_Def_Dot)            G_FExpDot
(ite (= e G_E_FEOp_Def_Plus)           G_FEOp_Plus
(ite (= e G_E_FEOp_Def_Minus)          G_FEOp_Minus
(ite (= e G_E_FEOp_Def_Times)          G_FEOp_Times
(ite (= e G_E_FEOp_Def_Div)            G_FEOp_Div
(ite (= e G_E_SExpTD)                  G_SExpTD
(ite (= e G_E_SExpSDef)                G_SExpSDef
(ite (= e G_E_SExpCard)                G_SExpCard
(ite (= e G_E_SExp_Def_TD)             G_SExpTD
(ite (= e G_E_SExp_Def_SetDef)         G_SExpSDef
(ite (= e G_E_SExp_Def_Card)           G_SExpCard
(ite (= e G_E_SExp_Def_Empty)          G_SExpEmpty
(ite (= e G_E_PE_TExp)                 G_PE
(ite (= e G_E_PE_PEP)                  G_PEP
(ite (= e G_E_PE_PEM)                  G_PEM
(ite (= e G_E_PEP_UEOp)                G_PEP
(ite (= e G_E_PEP_Id)                  G_PEP
(ite (= e G_E_PEP_BEOp)                G_PEP
(ite (= e G_E_PEM_MOp)                 G_PEM
(ite (= e G_E_UEOp_Def_Card)           G_UEOp_Card
(ite (= e G_E_UEOp_Def_The)            G_UEOp_The
(ite (= e G_E_UEOp_Def_None)           G_UEOp_None
(ite (= e G_E_BEOp_Def_Eq)             G_BEOp_EQ
(ite (= e G_E_BEOp_Def_Neq)            G_BEOp_NEQ
(ite (= e G_E_BEOp_Def_In)             G_BEOp_IN
(ite (= e G_E_BEOp_Def_LT)             G_BEOp_LT
(ite (= e G_E_BEOp_Def_LEQ)            G_BEOp_LEQ
(ite (= e G_E_BEOp_Def_GT)             G_BEOp_GT
(ite (= e G_E_BEOp_Def_GEQ)            G_BEOp_GEQ
(ite (= e G_E_BEOp_Def_SUBSETEQ)       G_BEOp_SubsetEQ
(ite (= e G_E_MOp_Def_DRES)            G_MOp_DRES
(ite (= e G_E_MOp_Def_RRES)            G_MOp_RRES
(ite (= e G_E_MOp_Def_DSUB)            G_MOp_DSUB
(ite (= e G_E_MOp_Def_RSUB)            G_MOp_RSUB
(ite (= e G_E_A_Id)                    G_A
(ite (= e G_E_O_Id)                    G_O
(ite (= e G_E_P_Id_1)                  G_P
(ite (= e G_E_P_Id_2)                  G_P
(ite (= e G_E_SE_Def_O)                G_O
(ite (= e G_E_SE_Def_P)                G_P
(ite (= e G_E_IDef_SExt_SEs)           G_IDef_SExt
(ite (= e G_E_IDef_CntSet_TD)          G_IDef_CntSet
(ite (= e G_E_IDef_CntSet_PEs)         G_IDef_CntSet
(ite (= e G_E_IDef_Def_SExt)           G_IDef_SExt
(ite (= e G_E_IDef_Def_CntSet)         G_IDef_CntSet
(ite (= e G_E_IExp_Def_IDef)           G_IDef
(ite (= e G_E_IExp_Def_SDs)            G_IExp_SDs
(ite (= e G_E_SDef_IExp)               G_SDef
```

```
   (ite (= e G_E_SDef_SOp)                G_SDef
   (ite (= e G_E_IExpSDs_SDef)            G_IExp_SDs
   (ite (= e G_E_SOp_Def_Domain)          G_SOp_Domain
   (ite (= e G_E_SOp_Def_Range)           G_SOp_Range
   (ite (= e G_E_SOp_Def_Union)           G_SOp_Union
   (ite (= e G_E_SOp_Def_Intersection)    G_SOp_Intersection
   (ite (= e G_E_SOp_Def_CrossProduct)    G_SOp_CrossProduct
   (ite (= e G_E_SOp_Def_SetMinus)        G_SOp_SetMinus
   (ite (= e G_E_SOp_Def_RelComp)         G_SOp_RelComp
   (ite (= e G_E_SOp_Def_None)            G_SOp_None
  G_Null)))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))

(push)
(echo "Testing the 'Target_MM' function (1) --> sat")
(assert (= (Target_MM MM_EAssertion_Id) MM_Name))
(check-sat)
(pop)

(push)
(echo "Testing the target 'Target_MM' function (2) --> sat")
(assert (= (Target_MM MM_EISOp_Range) MM_SetDefOp))
(check-sat)
(pop)

(push)
(echo "Testing the 'Target_MM' function (3) --> unsat")
(assert (= (Target_MM MM_EISetExtension) MM_PropEdge))
(check-sat)
(pop)

(push)
(echo "Checking totality of 'Target_MM' --> sat")
(assert (forall ((emm E_MM))
   (=> (= (Target_MM emm) MM_Null) (= emm MM_ENull))))
(check-sat)
(pop)

(push)
(echo "Testing the 'Source_MM' function (1) --> sat")
(assert (= (Source_MM MM_EAssertion_Id) MM_Assertion))
(check-sat)
(pop)

(push)
(echo "Testing the 'Source_MM' function (2) --> sat")
(assert (= (Source_MM MM_EISOp_Range) MM_SOp_Range))
(check-sat)
(pop)
```

```
(push)
(echo "Testing the 'Source_MM' function (3) --> unsat")
(assert (= (Source_MM MM_EISetExtension) MM_SetDef))
(check-sat)
(pop)

(push)
(echo "Checking Totality of 'Source_MM' --> sat")
(assert (forall ((emm E_MM))
   (=> (= (Source_MM emm) MM_Null) (= emm MM_ENull))))
(check-sat)
(pop)

(push)
(echo "Checking totality of 'Target_G' ->sat")
(assert (forall ((eg E_G))
   (=> (= (Target_G eg) G_Null) (= eg G_E_Null))))
(check-sat)
(pop)

(push)
(echo "Checking that the target function 'Target_MM' is preserved -> sat")
(assert (forall ((emm1 E_MM))
   (= (Map_V (Target_MM emm1)) (Target_G (Map_E emm1)))))
(check-sat)
(pop)

(push)
(echo "Testing the 'Source_G' function (1)-> sat")
(assert (= (Source_G G_E_IDef_Def_SExt) G_IDef_SExt))
(check-sat)
(pop)

(push)
(echo "Testing the 'Source_G' function (2) -> sat")
(assert (= (Source_G G_E_SDef_SOp) G_SDef))
(check-sat)
(pop)

(push)
(echo "Testing the 'Source_G' function (3) -> unsat")
(assert (= (Source_G  G_E_IDef_CntSet_TD) G_TD))
(check-sat)
(pop)

(push)
(echo "Checking Totality of 'Source_G' ->sat")
(assert (forall ((eg E_G))
   (=> (= (Source_G eg) G_Null) (= eg G_E_Null))))
```

```
(check-sat)
(pop)

(push)
(echo "Checking that the source function 'Source_MM' is preserved -> sat")
(assert (forall ((emm1 E_MM))
   (= (Map_V (Source_MM emm1)) (Source_G (Map_E emm1)))))
(check-sat)
(pop)
```

C.1.2 Z3 Proof Output

```
Testing function 'Map_V' (1) --> sat
sat
Testing function 'Map_V' (2) --> sat
sat
Testing function 'Map_V' (3) --> unsat
unsat
Checking Totality of 'Map_V' --> sat
sat
Checking injectiveness of 'Map_V' --> sat
sat
Checking Surjectiveness of 'Map_V' --> sat
sat
Testing function 'Map_E' (1) --> sat
sat
Testing function 'Map_E' (2) --> sat
sat
Testing function 'Map_E' (3) --> unsat
unsat
Checking Totality of 'Map_E' --> sat
sat
Checking injectiveness of 'Map_E' --> sat
sat
Checking Surjectiveness of 'Map_E' --> sat
sat
Testing the 'Target_MM' function (1) --> sat
sat
Testing the target 'Target_MM' function (2) --> sat
sat
Testing the 'Target_MM' function (3) --> unsat
unsat
Checking totality of 'Target_MM' --> sat
sat
Testing the 'Source_MM' function (1) --> sat
sat
Testing the 'Source_MM' function (2) --> sat
sat
Testing the 'Source_MM' function (3) --> unsat
```

```
unsat
Checking Totality of 'Source_MM' --> sat
sat
Checking totality of 'Target_G' ->sat
sat
Checking that the target function 'Target_MM' is preserved -> sat
sat
Testing the 'Source_G' function (1)-> sat
sat
Testing the 'Source_G' function (2) -> sat
sat
Testing the 'Source_G' function (3) -> unsat
unsat
Checking Totality of 'Source_G' ->sat
sat
Checking that the source function 'Source_MM' is preserved -> sat
sat
```

C.2 Structural diagrams

```
(set-option :mbqi true)
(set-option :macro-finder true)
(set-option :pull-nested-quantifiers true)
(set-option :produce-unsat-cores true)
(set-option :produce-models true)

(declare-sort V_MM)
(declare-sort E_MM)

(declare-sort V_G)
(declare-sort E_G)

(declare-const MM_Name            V_MM)
(declare-const MM_Num             V_MM)
(declare-const MM_Bool            V_MM)
(declare-const MM_Assertion       V_MM)
(declare-const MM_TypeDesignator  V_MM)
(declare-const MM_SetDef          V_MM)
;; 'Mult'
(declare-const MM_Mult            V_MM)
(declare-const MM_MSeq            V_MM)
(declare-const MM_MOne            V_MM)
(declare-const MM_MOpt            V_MM)
(declare-const MM_MMany           V_MM)
(declare-const MM_MRange          V_MM)
(declare-const MM_MOneToMany      V_MM)
(declare-const MM_UBound          V_MM)
(declare-const MM_UBoundNum       V_MM)
```

```
(declare-const MM_UBoundStar      V_MM)
;; 'SetKind'
(declare-const MM_SetKind         V_MM)
(declare-const MM_SetKind_Value   V_MM)
(declare-const MM_SetKind_Class   V_MM)
;; 'SDElem'
(declare-const MM_SDElem          V_MM)
;; 'Constant'
(declare-const MM_Constant        V_MM)
;; 'Relation Edge'
(declare-const MM_RelEdge         V_MM)
;; 'PropEdgeDef'
(declare-const MM_PropEdgeDef     V_MM)
;; 'Set'
(declare-const MM_Set             V_MM)
(declare-const MM_PrimarySet      V_MM)
(declare-const MM_DerivedSet      V_MM)
;; 'SetDefObject'
(declare-const MM_SetDefObject    V_MM)
;; 'SDiag'
(declare-const MM_SDiag           V_MM)
;; Special 'Null' constant to check totality
(declare-const MM_Null            V_MM)

;; Mult
(declare-const MM_E_I_MSeq           E_MM)
(declare-const MM_E_I_MOne           E_MM)
(declare-const MM_E_I_MOpt           E_MM)
(declare-const MM_E_I_MMany          E_MM)
(declare-const MM_E_I_MOneToMany     E_MM)
(declare-const MM_E_I_MRange         E_MM)
(declare-const MM_E_MRange_lb        E_MM)
(declare-const MM_E_MRange_ub        E_MM)
(declare-const MM_E_I_UBoundNum      E_MM)
(declare-const MM_E_I_UBoundStar     E_MM)
;; 'SetKind'
(declare-const MM_E_I_SetKind_Value  E_MM)
(declare-const MM_E_I_SetKind_Class  E_MM)
;; 'Constant'
(declare-const MM_E_I_Constant       E_MM)
(declare-const MM_E_Constant_name    E_MM)
(declare-const MM_E_Constant_TD      E_MM)
;; 'PropEdgeDef'
(declare-const MM_E_PropEdgeDef_mult E_MM)
(declare-const MM_E_PropEdgeDef_tgt  E_MM)
(declare-const MM_E_PropEdgeDef_id   E_MM)
;; 'RelEdge'
(declare-const MM_E_I_RelEdge        E_MM)
(declare-const MM_E_RelEdge_name     E_MM)
```

```
(declare-const MM_E_RelEdge_Src      E_MM)
(declare-const MM_E_RelEdge_MultS    E_MM)
(declare-const MM_E_RelEdge_Tgt      E_MM)
(declare-const MM_E_RelEdge_MultT    E_MM)
;; 'Set'
(declare-const MM_E_I_Set            E_MM)
(declare-const MM_E_I_PrimarySet     E_MM)
(declare-const MM_E_PrimarySet_name  E_MM)
(declare-const MM_E_PrimarySet_isDef E_MM)
(declare-const MM_E_PrimarySet_lcs   E_MM)
(declare-const MM_E_PrimarySet_lis   E_MM)
(declare-const MM_E_PrimarySet_hio   E_MM)
(declare-const MM_E_PrimarySet_his   E_MM)
(declare-const MM_E_PrimarySet_kind  E_MM)
(declare-const MM_E_PrimarySet_lps   E_MM)
(declare-const MM_E_I_DerivedSet     E_MM)
(declare-const MM_E_DerivedSet_name  E_MM)
(declare-const MM_E_DerivedSet_def   E_MM)
;; 'SetDefObject'
(declare-const MM_E_SetDefObject_objName E_MM)
;; 'SDiag'
(declare-const MM_E_SDiag_elements   E_MM)
(declare-const MM_E_SDiag_invariants E_MM)
;; Special 'Null' constant to check totality
(declare-const MM_ENull              E_MM)

(declare-const G_Num           V_G)
(declare-const G_Id            V_G)
(declare-const G_Bool          V_G)
(declare-const G_A             V_G)
(declare-const G_O             V_G)
(declare-const G_TD            V_G)
(declare-const G_SDef          V_G)
;; M
(declare-const G_M             V_G)
(declare-const G_M_One         V_G)
(declare-const G_M_Opt         V_G)
(declare-const G_M_Some        V_G)
(declare-const G_M_Many        V_G)
(declare-const G_M_Seq         V_G)
(declare-const G_M_Range       V_G)
(declare-const G_UBound        V_G)
(declare-const G_UBound_Num    V_G)
(declare-const G_UBound_Star   V_G)
;; 'SK'
(declare-const G_SK            V_G)
(declare-const G_SK_Value      V_G)
(declare-const G_SK_Class      V_G)
;; 'SDE'
```

```
(declare-const G_SDE           V_G)
;; 'C'
(declare-const G_C             V_G)
;; 'RE'
(declare-const G_RE            V_G)
;; 'PED'
(declare-const G_PED           V_G)
;; 'Set'
(declare-const G_Set           V_G)
(declare-const G_DSet          V_G)
(declare-const G_PSet          V_G)
;; 'SD'
(declare-const G_SD            V_G)
;; Special 'Null' constant to check totality
(declare-const G_Null          V_G)


;; O
(declare-const G_E_O_Id            E_G)
;; Mult
(declare-const G_E_M_Def_opt        E_G)
(declare-const G_E_M_Def_one        E_G)
(declare-const G_E_M_Def_some       E_G)
(declare-const G_E_M_Def_many       E_G)
(declare-const G_E_M_Def_seq        E_G)
(declare-const G_E_M_Def_range      E_G)
(declare-const G_E_MRange_lb        E_G)
(declare-const G_E_MRange_ub        E_G)
(declare-const G_E_UBound_Def_Num   E_G)
(declare-const G_E_UBound_Def_Star  E_G)
;; 'SK'
(declare-const G_E_SK_Def_Value     E_G)
(declare-const G_E_SK_Def_Class     E_G)
;; 'SDE'
(declare-const G_E_SDE_Def_C        E_G)
(declare-const G_E_SDE_Def_RE       E_G)
(declare-const G_E_SDE_Def_Set      E_G)
;; 'C'
(declare-const G_E_C_TD             E_G)
(declare-const G_E_C_Id             E_G)
;; 'RE'
(declare-const G_E_RE_Id            E_G)
(declare-const G_E_RE_Src_TD        E_G)
(declare-const G_E_RE_Src_M         E_G)
(declare-const G_E_RE_Tgt_TD        E_G)
(declare-const G_E_RE_Tgt_M         E_G)
;; 'PED'
(declare-const G_E_PED_M            E_G)
(declare-const G_E_PED_TD           E_G)
```

```
(declare-const G_E_PED_Id           E_G)
;; 'Set'
(declare-const G_E_Set_Def_PSet     E_G)
(declare-const G_E_Set_Def_DSet     E_G)
(declare-const G_E_PSet_Id          E_G)
(declare-const G_E_PSet_SK          E_G)
(declare-const G_E_PSet_isDef       E_G)
(declare-const G_E_PSet_Cs          E_G)
(declare-const G_E_PSet_PEDs        E_G)
(declare-const G_E_PSet_As          E_G)
(declare-const G_E_PSet_hiOs        E_G)
(declare-const G_E_PSet_hiPSs       E_G)
(declare-const G_E_DSet_SDef        E_G)
(declare-const G_E_DSet_Id          E_G)
;; 'SD'
(declare-const G_E_SD_SDEs          E_G)
(declare-const G_E_SD_As            E_G)
;; Special 'Null' constant to check totality
(declare-const G_E_Null             E_G)

 (assert (distinct
   MM_Null
   MM_Num
   MM_Name
   MM_Bool
   MM_TypeDesignator
   MM_Assertion
   MM_SetDef
   MM_Mult
   MM_MSeq
   MM_MOne
   MM_MOpt
   MM_MMany
   MM_MRange
   MM_MOneToMany
   MM_UBound
   MM_UBoundNum
   MM_UBoundStar
   MM_SetKind
   MM_SetKind_Class
   MM_SetKind_Value
   MM_SDElem
   MM_Constant
   MM_RelEdge
   MM_PropEdgeDef
   MM_Set
   MM_PrimarySet
   MM_DerivedSet
   MM_SetDefObject
```

```
   MM_SDiag))

(assert (distinct
   G_Null
   G_Id
   G_Num
   G_Bool
   G_A
   G_O
   G_TD
   G_SDef
   G_M
   G_M_One
   G_M_Opt
   G_M_Some
   G_M_Many
   G_M_Seq
   G_M_Range
   G_UBound
   G_UBound_Num
   G_UBound_Star
   G_SK
   G_SK_Value
   G_SK_Class
   G_SDE
   G_C
   G_RE
   G_PED
   G_Set
   G_DSet
   G_PSet
   G_SD))

(assert (distinct
   MM_ENull
   MM_E_I_MSeq
   MM_E_I_MOne
   MM_E_I_MOpt
   MM_E_I_MMany
   MM_E_I_MOneToMany
   MM_E_I_MRange
   MM_E_MRange_lb
   MM_E_MRange_ub
   MM_E_I_UBoundNum
   MM_E_I_UBoundStar
   MM_E_I_SetKind_Value
   MM_E_I_SetKind_Class
   MM_E_I_Constant
   MM_E_Constant_TD
```

```
   MM_E_Constant_name
   MM_E_I_RelEdge
   MM_E_RelEdge_name
   MM_E_RelEdge_Src
   MM_E_RelEdge_Tgt
   MM_E_RelEdge_MultS
   MM_E_RelEdge_MultT
   MM_E_PropEdgeDef_mult
   MM_E_PropEdgeDef_tgt
   MM_E_PropEdgeDef_id
   MM_E_I_Set
   MM_E_I_PrimarySet
   MM_E_PrimarySet_name
   MM_E_PrimarySet_isDef
   MM_E_PrimarySet_lcs
   MM_E_PrimarySet_lis
   MM_E_PrimarySet_hio
   MM_E_PrimarySet_his
   MM_E_PrimarySet_kind
   MM_E_PrimarySet_lps
   MM_E_I_DerivedSet
   MM_E_DerivedSet_name
   MM_E_DerivedSet_def
   MM_E_SetDefObject_objName
   MM_E_SDiag_elements
   MM_E_SDiag_invariants))

(assert (distinct
   G_E_Null
   G_E_O_Id
   G_E_M_Def_opt
   G_E_M_Def_one
   G_E_M_Def_some
   G_E_M_Def_many
   G_E_M_Def_seq
   G_E_M_Def_range
   G_E_MRange_lb
   G_E_MRange_ub
   G_E_UBound_Def_Num
   G_E_UBound_Def_Star
   G_E_SK_Def_Value
   G_E_SK_Def_Class
   G_E_SDE_Def_C
   G_E_C_Id
   G_E_C_TD
   G_E_SDE_Def_RE
   G_E_RE_Id
   G_E_RE_Src_TD
   G_E_RE_Tgt_TD
```

```
   G_E_RE_Src_M
   G_E_RE_Tgt_M
   G_E_PED_M
   G_E_PED_TD
   G_E_PED_Id
   G_E_SDE_Def_Set
   G_E_Set_Def_PSet
   G_E_Set_Def_DSet
   G_E_PSet_Id
   G_E_PSet_SK
   G_E_PSet_isDef
   G_E_PSet_Cs
   G_E_PSet_PEDs
   G_E_PSet_As
   G_E_PSet_hiOs
   G_E_PSet_hiPSs
   G_E_DSet_Id
   G_E_DSet_SDef
   G_E_SD_SDEs
   G_E_SD_As))

(define-fun Map_V ((v V_MM)) V_G
   (ite (= v MM_Num)                G_Num
   (ite (= v MM_Name)               G_Id
   (ite (= v MM_Bool)               G_Bool
   (ite (= v MM_TypeDesignator)     G_TD
   (ite (= v MM_Assertion)          G_A
   (ite (= v MM_SetDef)             G_SDef
   (ite (= v MM_Mult)               G_M
   (ite (= v MM_MSeq)               G_M_Seq
   (ite (= v MM_MOne)               G_M_One
   (ite (= v MM_MOpt)               G_M_Opt
   (ite (= v MM_MMany)              G_M_Many
   (ite (= v MM_MRange)             G_M_Range
   (ite (= v MM_MOneToMany)         G_M_Some
   (ite (= v MM_UBound)             G_UBound
   (ite (= v MM_UBoundNum)          G_UBound_Num
   (ite (= v MM_UBoundStar)         G_UBound_Star
   (ite (= v MM_SetKind)            G_SK
   (ite (= v MM_SetKind_Value)      G_SK_Value
   (ite (= v MM_SetKind_Class)      G_SK_Class
   (ite (= v MM_SDElem)             G_SDE
   (ite (= v MM_Constant)           G_C
   (ite (= v MM_RelEdge)            G_RE
   (ite (= v MM_PropEdgeDef)        G_PED
   (ite (= v MM_Set)                G_Set
   (ite (= v MM_PrimarySet)         G_PSet
   (ite (= v MM_DerivedSet)         G_DSet
   (ite (= v MM_SetDefObject)       G_O
```

```
   (ite (= v MM_SDiag)              G_SD
   G_Null))))))))))))))))))))))))))))

(define-fun Map_E ((e E_MM)) E_G
   (ite (= e MM_E_I_MSeq)           G_E_M_Def_seq
   (ite (= e MM_E_I_MOne)           G_E_M_Def_one
   (ite (= e MM_E_I_MOpt)           G_E_M_Def_opt
   (ite (= e MM_E_I_MMany)          G_E_M_Def_many
   (ite (= e MM_E_I_MOneToMany)     G_E_M_Def_some
   (ite (= e MM_E_I_MRange)         G_E_M_Def_range
   (ite (= e MM_E_MRange_lb)        G_E_MRange_lb
   (ite (= e MM_E_MRange_ub)        G_E_MRange_ub
   (ite (= e MM_E_I_UBoundNum)      G_E_UBound_Def_Num
   (ite (= e MM_E_I_UBoundStar)     G_E_UBound_Def_Star
   (ite (= e MM_E_I_SetKind_Value)  G_E_SK_Def_Value
   (ite (= e MM_E_I_SetKind_Class)  G_E_SK_Def_Class
   (ite (= e MM_E_I_Constant)       G_E_SDE_Def_C
   (ite (= e MM_E_Constant_name)    G_E_C_Id
   (ite (= e MM_E_Constant_TD)      G_E_C_TD
   (ite (= e MM_E_I_RelEdge)        G_E_SDE_Def_RE
   (ite (= e MM_E_RelEdge_name)     G_E_RE_Id
   (ite (= e MM_E_RelEdge_Src)      G_E_RE_Src_TD
   (ite (= e MM_E_RelEdge_Tgt)      G_E_RE_Tgt_TD
   (ite (= e MM_E_RelEdge_MultS)    G_E_RE_Src_M
   (ite (= e MM_E_RelEdge_MultT)    G_E_RE_Tgt_M
   (ite (= e MM_E_PropEdgeDef_mult) G_E_PED_M
   (ite (= e MM_E_PropEdgeDef_tgt)  G_E_PED_TD
   (ite (= e MM_E_PropEdgeDef_id)   G_E_PED_Id
   (ite (= e MM_E_I_Set)            G_E_SDE_Def_Set
   (ite (= e MM_E_I_PrimarySet)     G_E_Set_Def_PSet
   (ite (= e MM_E_PrimarySet_name)  G_E_PSet_Id
   (ite (= e MM_E_PrimarySet_isDef) G_E_PSet_isDef
   (ite (= e MM_E_PrimarySet_lcs)   G_E_PSet_Cs
   (ite (= e MM_E_PrimarySet_lis)   G_E_PSet_As
   (ite (= e MM_E_PrimarySet_hio)   G_E_PSet_hiOs
   (ite (= e MM_E_PrimarySet_his)   G_E_PSet_hiPSs
   (ite (= e MM_E_PrimarySet_kind)  G_E_PSet_SK
   (ite (= e MM_E_PrimarySet_lps)   G_E_PSet_PEDs
   (ite (= e MM_E_I_DerivedSet)     G_E_Set_Def_DSet
   (ite (= e MM_E_DerivedSet_name)  G_E_DSet_Id
   (ite (= e MM_E_DerivedSet_def)   G_E_DSet_SDef
   (ite (= e MM_E_SetDefObject_objName) G_E_O_Id
   (ite (= e MM_E_SDiag_elements)   G_E_SD_SDEs
   (ite (= e MM_E_SDiag_invariants) G_E_SD_As
   G_E_Null))))))))))))))))))))))))))))))))))))))))))

(push)
(echo "Testing function 'Map_V' (1) --> sat")
(assert (= (Map_V MM_SDElem) G_SDE))
```

```
(check-sat)
(pop)

(push)
(echo "Testing function 'Map_V' (2) --> sat")
(assert (= (Map_V MM_PropEdgeDef) G_PED))
(check-sat)
(pop)

(push)
(echo "Testing function 'Map_V' (3) --> unsat")
(assert (= (Map_V MM_SetKind_Class) G_SK_Value))
(check-sat)
(pop)

(push)
(echo "Checking Totality of 'Map_V' --> sat")
(assert (forall ((vmm V_MM))
   (=> (= (Map_V vmm) G_Null) (= vmm MM_Null))))
(check-sat)
(pop)

(push)
(echo "Checking injectiveness of 'Map_V' --> sat")
(assert (forall ((vmm1 V_MM) (vmm2 V_MM))
   (=> (= (Map_V vmm1) (Map_V vmm2)) (= vmm1 vmm2))))
(check-sat)
(pop)

(push)
(echo "Checking Surjectiveness of 'Map_V' (1) --> sat")
(assert (forall ((vg V_G))
   (exists ((vmm V_MM))
      (= (Map_V vmm) vg))))
(check-sat)
(pop)

;(push)
;(echo "Checking Surjectiveness of 'Map_V' (2)->sat")
;(declare-fun svmm (V_G) V_MM)
;(assert (forall ((vg V_G))
;      (= (Map_V (svmm vg)) vg)))
;(check-sat)
;(pop)

(push)
(echo "Testing function 'Map_E' (1) --> sat")
(assert (= (Map_E MM_E_I_Constant) G_E_SDE_Def_C))
(check-sat)
```

```
(pop)

(push)
(echo "Testing function 'Map_E' (2) --> sat")
(assert (= (Map_E MM_E_I_RelEdge) G_E_SDE_Def_RE))
(check-sat)
(pop)

(push)
(echo "Testing function 'Map_E' (3) --> unsat")
(assert (= (Map_E MM_E_MRange_lb) G_E_M_Def_opt))
(check-sat)
(pop)

(push)
(echo "Checking Totality of 'Map_E' --> sat")
(assert (forall ((emm E_MM))
   (=> (= (Map_E emm) G_E_Null) (= emm MM_ENull))))
(check-sat)
(pop)

(push)
(echo "Checking injectiveness of 'Map_E' --> sat")
(assert (forall ((emm1 E_MM) (emm2 E_MM))
   (=> (= (Map_E emm1) (Map_E emm2)) (= emm1 emm2))))
(check-sat)
(pop)

(push)
(echo "Checking Surjectiveness of 'Map_E' (1) --> sat")
(assert (forall ((eg E_G))
   (exists ((emm E_MM))
      (= (Map_E emm) eg))))
(check-sat)
(pop)

;(push)
;(echo "Checking surjectiveness of 'Map_E' (2) -> sat")
;(declare-fun semm (E_G) E_MM)
;(assert (forall ((eg E_G))
;   (= (Map_E (semm eg)) eg)))
;(check-sat)
;(pop)

(define-fun Target_MM ((e E_MM)) V_MM
   (ite (= e MM_E_I_MSeq)          MM_Mult
   (ite (= e MM_E_I_MOne)          MM_Mult
   (ite (= e MM_E_I_MOpt)          MM_Mult
   (ite (= e MM_E_I_MMany)         MM_Mult
```

```
   (ite (= e MM_E_I_MOneToMany)     MM_Mult
   (ite (= e MM_E_I_MRange)         MM_Mult
   (ite (= e MM_E_MRange_lb)        MM_Num
   (ite (= e MM_E_MRange_ub)        MM_UBound
   (ite (= e MM_E_I_UBoundNum)      MM_UBound
   (ite (= e MM_E_I_UBoundStar)     MM_UBound
   (ite (= e MM_E_I_SetKind_Value)  MM_SetKind
   (ite (= e MM_E_I_SetKind_Class)  MM_SetKind
   (ite (= e MM_E_I_Constant)       MM_SDElem
   (ite (= e MM_E_Constant_name)    MM_Name
   (ite (= e MM_E_Constant_TD)      MM_TypeDesignator
   (ite (= e MM_E_I_RelEdge)        MM_SDElem
   (ite (= e MM_E_RelEdge_name)     MM_Name
   (ite (= e MM_E_RelEdge_Src)      MM_TypeDesignator
   (ite (= e MM_E_RelEdge_Tgt)      MM_TypeDesignator
   (ite (= e MM_E_RelEdge_MultS)    MM_Mult
   (ite (= e MM_E_RelEdge_MultT)    MM_Mult
   (ite (= e MM_E_PropEdgeDef_mult) MM_Mult
   (ite (= e MM_E_PropEdgeDef_tgt)  MM_TypeDesignator
   (ite (= e MM_E_PropEdgeDef_id)   MM_Name
   (ite (= e MM_E_I_Set)            MM_SDElem
   (ite (= e MM_E_I_PrimarySet)     MM_Set
   (ite (= e MM_E_PrimarySet_name)  MM_Name
   (ite (= e MM_E_PrimarySet_isDef) MM_Bool
   (ite (= e MM_E_PrimarySet_lcs)   MM_Constant
   (ite (= e MM_E_PrimarySet_lis)   MM_Assertion
   (ite (= e MM_E_PrimarySet_hio)   MM_SetDefObject
   (ite (= e MM_E_PrimarySet_his)   MM_PrimarySet
   (ite (= e MM_E_PrimarySet_kind)  MM_SetKind
   (ite (= e MM_E_PrimarySet_lps)   MM_PropEdgeDef
   (ite (= e MM_E_I_DerivedSet)     MM_Set
   (ite (= e MM_E_DerivedSet_name)  MM_Name
   (ite (= e MM_E_DerivedSet_def)   MM_SetDef
   (ite (= e MM_E_SetDefObject_objName) MM_Name
   (ite (= e MM_E_SDiag_elements)   MM_SDElem
   (ite (= e MM_E_SDiag_invariants) MM_Assertion
   MM_Null)))))))))))))))))))))))))))))))))))))))

(define-fun Source_MM ((e E_MM)) V_MM
   (ite (= e MM_E_I_MSeq)           MM_MSeq
   (ite (= e MM_E_I_MOne)           MM_MOne
   (ite (= e MM_E_I_MOpt)           MM_MOpt
   (ite (= e MM_E_I_MMany)          MM_MMany
   (ite (= e MM_E_I_MOneToMany)     MM_MOneToMany
   (ite (= e MM_E_I_MRange)         MM_MRange
   (ite (= e MM_E_MRange_lb)        MM_MRange
   (ite (= e MM_E_MRange_ub)        MM_MRange
   (ite (= e MM_E_I_UBoundNum)      MM_UBoundNum
   (ite (= e MM_E_I_UBoundStar)     MM_UBoundStar
```

```
   (ite (= e MM_E_I_SetKind_Value)  MM_SetKind_Value
   (ite (= e MM_E_I_SetKind_Class)  MM_SetKind_Class
   (ite (= e MM_E_I_Constant)       MM_Constant
   (ite (= e MM_E_Constant_name)    MM_Constant
   (ite (= e MM_E_Constant_TD)      MM_Constant
   (ite (= e MM_E_I_RelEdge)        MM_RelEdge
   (ite (= e MM_E_RelEdge_name)     MM_RelEdge
   (ite (= e MM_E_RelEdge_Src)      MM_RelEdge
   (ite (= e MM_E_RelEdge_Tgt)      MM_RelEdge
   (ite (= e MM_E_RelEdge_MultS)    MM_RelEdge
   (ite (= e MM_E_RelEdge_MultT)    MM_RelEdge
   (ite (= e MM_E_PropEdgeDef_mult) MM_PropEdgeDef
   (ite (= e MM_E_PropEdgeDef_tgt)  MM_PropEdgeDef
   (ite (= e MM_E_PropEdgeDef_id)   MM_PropEdgeDef
   (ite (= e MM_E_I_Set)            MM_Set
   (ite (= e MM_E_I_PrimarySet)     MM_PrimarySet
   (ite (= e MM_E_PrimarySet_name)  MM_PrimarySet
   (ite (= e MM_E_PrimarySet_isDef) MM_PrimarySet
   (ite (= e MM_E_PrimarySet_lcs)   MM_PrimarySet
   (ite (= e MM_E_PrimarySet_lis)   MM_PrimarySet
   (ite (= e MM_E_PrimarySet_hio)   MM_PrimarySet
   (ite (= e MM_E_PrimarySet_his)   MM_PrimarySet
   (ite (= e MM_E_PrimarySet_kind)  MM_PrimarySet
   (ite (= e MM_E_PrimarySet_lps)   MM_PrimarySet
   (ite (= e MM_E_I_DerivedSet)     MM_DerivedSet
   (ite (= e MM_E_DerivedSet_name)  MM_DerivedSet
   (ite (= e MM_E_DerivedSet_def)   MM_DerivedSet
   (ite (= e MM_E_SetDefObject_objName) MM_SetDefObject
   (ite (= e MM_E_SDiag_elements)   MM_SDiag
   (ite (= e MM_E_SDiag_invariants) MM_SDiag
   MM_Null)))))))))))))))))))))))))))))))))))))))

(define-fun Target_G ((e E_G)) V_G
   (ite (= e G_E_O_Id)            G_Id
   (ite (= e G_E_M_Def_opt)       G_M
   (ite (= e G_E_M_Def_one)       G_M
   (ite (= e G_E_M_Def_some)      G_M
   (ite (= e G_E_M_Def_many)      G_M
   (ite (= e G_E_M_Def_seq)       G_M
   (ite (= e G_E_M_Def_range)     G_M
   (ite (= e G_E_MRange_lb)       G_Num
   (ite (= e G_E_MRange_ub)       G_UBound
   (ite (= e G_E_UBound_Def_Num)  G_UBound
   (ite (= e G_E_UBound_Def_Star) G_UBound
   (ite (= e G_E_SK_Def_Value)    G_SK
   (ite (= e G_E_SK_Def_Class)    G_SK
   (ite (= e G_E_SDE_Def_C)       G_SDE
   (ite (= e G_E_C_TD)            G_TD
   (ite (= e G_E_C_Id)            G_Id
```

```
   (ite (= e G_E_SDE_Def_RE)      G_SDE
   (ite (= e G_E_RE_Id)           G_Id
   (ite (= e G_E_RE_Src_TD)       G_TD
   (ite (= e G_E_RE_Tgt_TD)       G_TD
   (ite (= e G_E_RE_Src_M)        G_M
   (ite (= e G_E_RE_Tgt_M)        G_M
   (ite (= e G_E_PED_M)           G_M
   (ite (= e G_E_PED_TD)          G_TD
   (ite (= e G_E_PED_Id)          G_Id
   (ite (= e G_E_SDE_Def_Set)     G_SDE
   (ite (= e G_E_Set_Def_PSet)    G_Set
   (ite (= e G_E_Set_Def_DSet)    G_Set
   (ite (= e G_E_PSet_Id)         G_Id
   (ite (= e G_E_PSet_SK)         G_SK
   (ite (= e G_E_PSet_isDef)      G_Bool
   (ite (= e G_E_PSet_Cs)         G_C
   (ite (= e G_E_PSet_PEDs)       G_PED
   (ite (= e G_E_PSet_As)         G_A
   (ite (= e G_E_PSet_hiOs)       G_O
   (ite (= e G_E_PSet_hiPSs)      G_PSet
   (ite (= e G_E_DSet_Id)         G_Id
   (ite (= e G_E_DSet_SDef)       G_SDef
   (ite (= e G_E_SD_SDEs)         G_SDE
   (ite (= e G_E_SD_As)           G_A
   G_Null)))))))))))))))))))))))))))))))))))))))))

(define-fun Source_G ((e E_G)) V_G
   (ite (= e G_E_O_Id)           G_O
   (ite (= e G_E_M_Def_opt)       G_M_Opt
   (ite (= e G_E_M_Def_one)       G_M_One
   (ite (= e G_E_M_Def_some)      G_M_Some
   (ite (= e G_E_M_Def_many)      G_M_Many
   (ite (= e G_E_M_Def_seq)       G_M_Seq
   (ite (= e G_E_M_Def_range)     G_M_Range
   (ite (= e G_E_MRange_lb)       G_M_Range
   (ite (= e G_E_MRange_ub)       G_M_Range
   (ite (= e G_E_UBound_Def_Num)  G_UBound_Num
   (ite (= e G_E_UBound_Def_Star) G_UBound_Star
   (ite (= e G_E_SK_Def_Value)    G_SK_Value
   (ite (= e G_E_SK_Def_Class)    G_SK_Class
   (ite (= e G_E_SDE_Def_C)       G_C
   (ite (= e G_E_C_TD)            G_C
   (ite (= e G_E_C_Id)            G_C
   (ite (= e G_E_SDE_Def_RE)      G_RE
   (ite (= e G_E_RE_Id)           G_RE
   (ite (= e G_E_RE_Src_TD)       G_RE
   (ite (= e G_E_RE_Tgt_TD)       G_RE
   (ite (= e G_E_RE_Src_M)        G_RE
   (ite (= e G_E_RE_Tgt_M)        G_RE
```

```
   (ite (= e G_E_PED_M)          G_PED
   (ite (= e G_E_PED_TD)         G_PED
   (ite (= e G_E_PED_Id)         G_PED
   (ite (= e G_E_SDE_Def_Set)    G_Set
   (ite (= e G_E_Set_Def_PSet)   G_PSet
   (ite (= e G_E_Set_Def_DSet)   G_DSet
   (ite (= e G_E_PSet_Id)        G_PSet
   (ite (= e G_E_PSet_SK)        G_PSet
   (ite (= e G_E_PSet_isDef)     G_PSet
   (ite (= e G_E_PSet_Cs)        G_PSet
   (ite (= e G_E_PSet_PEDs)      G_PSet
   (ite (= e G_E_PSet_As)        G_PSet
   (ite (= e G_E_PSet_hiOs)      G_PSet
   (ite (= e G_E_PSet_hiPSs)     G_PSet
   (ite (= e G_E_DSet_Id)        G_DSet
   (ite (= e G_E_DSet_SDef)      G_DSet
   (ite (= e G_E_SD_SDEs)        G_SD
   (ite (= e G_E_SD_As)          G_SD
   G_Null)))))))))))))))))))))))))))))))))))))))))

(push)
(echo "Testing function 'Target_MM' (1) --> sat")
(assert (= (Target_MM MM_E_Constant_TD) MM_TypeDesignator))
(check-sat)
(pop)

(push)
(echo "Testing function 'Target_MM' (2)->sat")
(assert (= (Target_MM MM_E_I_RelEdge) MM_SDElem))
(check-sat)
(pop)

(push)
(echo "Testing function 'Target_MM' (3) -> unsat")
(assert (= (Target_MM MM_E_I_SetKind_Value) MM_Num))
(check-sat)
(pop)

(push)
(echo "Checking totality of 'Target_MM' ->sat")
(assert (forall ((emm E_MM))
   (=> (= (Target_MM emm) MM_Null) (= emm MM_ENull))))
(check-sat)
(pop)

(push)
(echo "Checking totality of 'Target_G' ->sat")
(assert (forall ((eg E_G))
   (=> (= (Target_G eg) G_Null) (= eg G_E_Null))))
```

```
(check-sat)
(pop)

(push)
(echo "Checking that the target function 'Target_MM' is preserved -> sat")
(assert (forall ((emm1 E_MM))
   (= (Map_V (Target_MM emm1)) (Target_G (Map_E emm1)))))
(check-sat)
(pop)

(push)
(echo "Testing the 'Source_MM' function (1)-> sat")
(assert (= (Source_MM MM_E_RelEdge_Tgt) MM_RelEdge))
(check-sat)
(pop)

(push)
(echo "Testing the 'Source_MM' function (2)-> sat")
(assert (= (Source_MM MM_E_I_SetKind_Value) MM_SetKind_Value))
(check-sat)
(pop)

(push)
(echo "Testing the 'Source_MM' function (3) -> unsat")
(assert (= (Source_MM MM_E_I_SetKind_Class) MM_SetKind_Value))
(check-sat)
(pop)

(push)
(echo "Testing the 'Source_G' function (1)-> sat")
(assert (= (Source_G G_E_RE_Src_M) G_RE))
(check-sat)
(pop)

(push)
(echo "Testing the 'Source_G' function (2) -> sat")
(assert (= (Source_G G_E_SK_Def_Value) G_SK_Value))
(check-sat)
(pop)

(push)
(echo "Testing the 'Source_G' function (3) -> unsat")
(assert (= (Source_G  G_E_MRange_lb) G_TD))
(check-sat)
(pop)

(push)
(echo "Checking Totality of 'Source_MM' ->sat")
(assert (forall ((emm E_MM))
```

```
   (=> (= (Source_MM emm) MM_Null) (= emm MM_ENull))))
(check-sat)
(pop)

(push)
(echo "Checking Totality of 'Source_G' ->sat")
(assert (forall ((eg E_G))
   (=> (= (Source_G eg) G_Null) (= eg G_E_Null))))
(check-sat)
(pop)

(push)
(echo "Checking that the source function 'Source_MM' is preserved -> sat")
(assert (forall ((emm1 E_MM))
   (= (Map_V (Source_MM emm1)) (Source_G (Map_E emm1)))))
(check-sat)
(pop)
```

C.2.1 Z3 Output

```
Testing function 'Map_V' (1) --> sat
sat
Testing function 'Map_V' (2) --> sat
sat
Testing function 'Map_V' (3) --> unsat
unsat
Checking Totality of 'Map_V' --> sat
sat
Checking injectiveness of 'Map_V' --> sat
sat
Checking Surjectiveness of 'Map_V' (1) --> sat
sat
Testing function 'Map_E' (1) --> sat
sat
Testing function 'Map_E' (2) --> sat
sat
Testing function 'Map_E' (3) --> unsat
unsat
Checking Totality of 'Map_E' --> sat
sat
Checking injectiveness of 'Map_E' --> sat
sat
Checking Surjectiveness of 'Map_E' (1) --> sat
sat
Testing function 'Target_MM' (1) --> sat
sat
Testing function 'Target_MM' (2)->sat
sat
Testing function 'Target_MM' (3) -> unsat
```

```
unsat
Checking totality of 'Target_MM' ->sat
sat
Checking totality of 'Target_G' ->sat
sat
Checking that the target function 'Target_MM' is preserved -> sat
sat
Testing the 'Source_MM' function (1)-> sat
sat
Testing the 'Source_MM' function (2)-> sat
sat
Testing the 'Source_MM' function (3) -> unsat
unsat
Testing the 'Source_G' function (1)-> sat
sat
Testing the 'Source_G' function (2) -> sat
sat
Testing the 'Source_G' function (3) -> unsat
unsat
Checking Totality of 'Source_MM' ->sat
sat
Checking Totality of 'Source_G' ->sat
sat
Checking that the source function 'Source_MM' is preserved -> sat
sat
```

C.3 Assertion diagrams

```
(set-option :mbqi true)
(set-option :macro-finder true)
(set-option :pull-nested-quantifiers true)
(set-option :produce-unsat-cores true)
(set-option :produce-models true)

(declare-sort V_MM)
(declare-sort E_MM)

(declare-sort V_G)
(declare-sort E_G)

(declare-const MM_Name                    V_MM)
(declare-const MM_Bool                    V_MM)
; From 'Common'
(declare-const MM_TypeDesignator          V_MM)
(declare-const MM_SetDef                  V_MM)
(declare-const MM_SetElement              V_MM)
(declare-const MM_PropEdgePred            V_MM)
(declare-const MM_SetExpression           V_MM)
```

```
;; 'FormulaSource'
(declare-const MM_FormulaSource           V_MM)
(declare-const MM_FormulaSourceSet        V_MM)
(declare-const MM_FormulaSourceElem       V_MM)
(declare-const MM_FormulaSourceUnary      V_MM)
(declare-const MM_FormulaSourceSetId      V_MM)
(declare-const MM_FormulaSourceSetDef     V_MM)
(declare-const MM_FormulaSourceUOp        V_MM)
(declare-const MM_FormulaSourceUOp_Card   V_MM)
(declare-const MM_FormulaSourceUOp_Domain V_MM)
(declare-const MM_FormulaSourceUOp_Range  V_MM)
(declare-const MM_FormulaSourceUOp_The    V_MM)
;; 'Formula'
(declare-const MM_Formula                 V_MM)
(declare-const MM_FormulaNAry             V_MM)
(declare-const MM_ArrowsFormula           V_MM)
(declare-const MM_SetFormula              V_MM)
(declare-const MM_FormulaSubset           V_MM)
(declare-const MM_SetFormulaDef           V_MM)
(declare-const MM_SetFormulaShaded        V_MM)
;; 'QFormula'
(declare-const MM_QFormula                V_MM)
(declare-const MM_QDecl                   V_MM)
(declare-const MM_QuantifierKind          V_MM)
(declare-const MM_QuantifierKind_ForAll   V_MM)
(declare-const MM_QuantifierKind_Exists   V_MM)
;; 'FormulaOp'
(declare-const MM_FormulaOp            V_MM)
(declare-const MM_FOp_Implies          V_MM)
(declare-const MM_FOp_And              V_MM)
(declare-const MM_FOp_Or               V_MM)
(declare-const MM_FOp_Equiv            V_MM)
(declare-const MM_FOp_SeqComp          V_MM)
(declare-const MM_FOp_Not              V_MM)
;; 'Decl'
(declare-const MM_Decl                 V_MM)
(declare-const MM_VarDecl              V_MM)
(declare-const MM_DeclObj              V_MM)
(declare-const MM_DeclSet              V_MM)
(declare-const MM_DeclSeq              V_MM)
;; 'DeclFormula'
(declare-const MM_DeclFormula          V_MM)
(declare-const MM_DeclFormulaNAry      V_MM)
(declare-const MM_DeclFormulaAtom      V_MM)
;; 'Renaming'
(declare-const MM_RenamingExp          V_MM)
;; 'ADiag'
(declare-const MM_ADiag                V_MM)
;; Special 'Null' constant to check totality
```

```
(declare-const MM_Null                     V_MM)

(declare-const G_Id            V_G)
(declare-const G_Bool          V_G)
; From 'Common'
(declare-const G_TD            V_G)
(declare-const G_SDef          V_G)
(declare-const G_SE            V_G)
(declare-const G_PEP           V_G)
(declare-const G_SExp          V_G)
;; 'AFS'
(declare-const G_AFS           V_G)
(declare-const G_AFS_SE        V_G)
(declare-const G_AFSS          V_G)
(declare-const G_AFS_FSOp      V_G)
(declare-const G_AFSS_SetId    V_G)
(declare-const G_AFSS_SDef     V_G)
(declare-const G_FSOp          V_G)
(declare-const G_FSOp_Card     V_G)
(declare-const G_FSOp_Domain   V_G)
(declare-const G_FSOp_Range    V_G)
(declare-const G_FSOp_The      V_G)
;; 'F'
(declare-const G_F             V_G)
(declare-const G_AF            V_G)
(declare-const G_F_NAry        V_G)
(declare-const G_SF            V_G)
(declare-const G_SF_SDef       V_G)
(declare-const G_SF_shaded     V_G)
(declare-const G_SF_hasIn      V_G)
;; 'QF'
(declare-const G_QF            V_G)
(declare-const G_QD            V_G)
(declare-const G_QK            V_G)
(declare-const G_QK_All        V_G)
(declare-const G_QK_Exists     V_G)
;; 'FOp'
(declare-const G_FOp           V_G)
(declare-const G_FOp_Implies   V_G)
(declare-const G_FOp_Equiv     V_G)
(declare-const G_FOp_And       V_G)
(declare-const G_FOp_Or        V_G)
(declare-const G_FOp_Not       V_G)
(declare-const G_FOp_SeqComp   V_G)
;; 'D'
(declare-const G_D             V_G)
(declare-const G_VD            V_G)
(declare-const G_VD_O          V_G)
(declare-const G_VD_Set        V_G)
```

```
(declare-const G_VD_Seq        V_G)
;; 'DF'
(declare-const G_DF            V_G)
(declare-const G_DFA           V_G)
(declare-const G_DF_NAry       V_G)
;; 'R'
(declare-const G_R             V_G)
;; 'AD'
(declare-const G_AD            V_G)
;; Special 'Null' constant to check totality
(declare-const G_Null          V_G)

;; 'FormulaSource'
(declare-const MM_E_I_FormulaSourceElem          E_MM)
(declare-const MM_E_I_FormulaSourceSet           E_MM)
(declare-const MM_E_I_FormulaSourceUnary         E_MM)
(declare-const MM_E_FormulaSourceUnary_frmlSrc   E_MM)
(declare-const MM_E_FormulaSourceUnary_operator E_MM)
(declare-const MM_E_I_FormulaSourceSetId         E_MM)
(declare-const MM_E_FormulaSourceSetId_setId     E_MM)
(declare-const MM_E_I_FormulaSourceSetDef        E_MM)
(declare-const MM_E_FormulaSourceSetDef_setDef   E_MM)
; 'Formula'
(declare-const MM_E_I_FormulaNAry                E_MM)
(declare-const MM_E_FormulaNAry_frmls            E_MM)
(declare-const MM_E_FormulaNAry_operator         E_MM)
(declare-const MM_E_I_SetFormula                 E_MM)
(declare-const MM_E_I_ArrowsFormula              E_MM)
(declare-const MM_E_ArrowsFormula_source         E_MM)
(declare-const MM_E_ArrowsFormula_pes            E_MM)
(declare-const MM_E_I_FormulaSubset              E_MM)
(declare-const MM_E_I_SetFormulaShaded           E_MM)
(declare-const MM_E_I_SetFormulaDef              E_MM)
(declare-const MM_E_FormulaSubset_setId          E_MM)
(declare-const MM_E_FormulaSubset_hasIn          E_MM)
(declare-const MM_E_SetFormulaDef_shaded         E_MM)
(declare-const MM_E_SetFormulaDef_setId          E_MM)
(declare-const MM_E_SetFormulaDef_setDef         E_MM)
(declare-const MM_E_SetFormulaShaded_setId       E_MM)
(declare-const MM_E_I_QFormula                   E_MM)
(declare-const MM_E_QFormula_decls               E_MM)
(declare-const MM_E_QFormula_frml                E_MM)
; 'QDecl'
(declare-const MM_E_QDecl_vars                   E_MM)
(declare-const MM_E_QDecl_qkind                  E_MM)
(declare-const MM_E_I_QuantifierKind_ForAll      E_MM)
(declare-const MM_E_I_QuantifierKind_Exists      E_MM)
;; 'FormulaOp'
(declare-const MM_E_I_FOp_Implies                E_MM)
```

```
(declare-const MM_E_I_FOp_And                     E_MM)
(declare-const MM_E_I_FOp_Or                      E_MM)
(declare-const MM_E_I_FOp_Equiv                   E_MM)
(declare-const MM_E_I_FOp_SeqComp                 E_MM)
(declare-const MM_E_I_FOp_Not                     E_MM)
;; 'Decl'
(declare-const MM_E_I_VarDecl                     E_MM)
(declare-const MM_E_I_DeclSet                     E_MM)
(declare-const MM_E_I_DeclSeq                     E_MM)
(declare-const MM_E_I_DeclObj                     E_MM)
(declare-const MM_E_DeclObj_optional              E_MM)
(declare-const MM_E_VarDecl_dName                 E_MM)
(declare-const MM_E_VarDecl_dTy                   E_MM)
(declare-const MM_E_VarDecl_isHidden              E_MM)
;; 'DeclFormula'
(declare-const MM_E_I_DeclFormula                 E_MM)
(declare-const MM_E_I_DeclFormulaNAry             E_MM)
(declare-const MM_E_DeclFormulaNAry_dfop          E_MM)
(declare-const MM_E_DeclFormulaNAry_dFrmls        E_MM)
(declare-const MM_E_I_DeclFormulaAtom             E_MM)
(declare-const MM_E_DeclFormulaAtom_refId         E_MM)
(declare-const MM_E_DeclFormulaAtom_owningSet     E_MM)
(declare-const MM_E_DeclFormulaAtom_callObj       E_MM)
(declare-const MM_E_DeclFormulaAtom_import        E_MM)
(declare-const MM_E_DeclFormulaAtom_renameExp     E_MM)
;; 'Renaming'
(declare-const MM_E_Renaming_subExp               E_MM)
(declare-const MM_E_Renaming_varToSub             E_MM)
;; 'ADiag'
(declare-const MM_E_ADiag_aName                   E_MM)
(declare-const MM_E_ADiag_predicate               E_MM)
(declare-const MM_E_ADiag_decls                   E_MM)
;; Special 'Null' constant to check totality
(declare-const MM_E_Null                          E_MM)

;; 'AFS'
(declare-const G_E_AFS_Def_SE         E_G)
(declare-const G_E_AFS_Def_AFSS       E_G)
(declare-const G_E_AFS_Def_FSOp       E_G)
(declare-const G_E_AFS_FSOp_Op        E_G)
(declare-const G_E_AFS_FSOp_AFS       E_G)
;; 'AFSS'
(declare-const G_E_AFSS_Def_SetId     E_G)
(declare-const G_E_AFSS_Def_SDef      E_G)
(declare-const G_E_AFSS_SetId_Id      E_G)
(declare-const G_E_AFSS_SDef_SDef     E_G)
;; 'F'
(declare-const G_E_F_Def_AF           E_G)
(declare-const G_E_F_Def_SF           E_G)
```

```
(declare-const G_E_F_Def_NAry        E_G)
(declare-const G_E_F_NAry_Fs         E_G)
(declare-const G_E_F_NAry_FOp        E_G)
(declare-const G_E_AF_AFS            E_G)
(declare-const G_E_AF_PEPs           E_G)
(declare-const G_E_SF_Def_SDef       E_G)
(declare-const G_E_SF_Def_shaded     E_G)
(declare-const G_E_SF_Def_hasIn      E_G)
(declare-const G_E_SF_SDef_shaded    E_G)
(declare-const G_E_SF_SDef_Id        E_G)
(declare-const G_E_SF_SDef_SDef      E_G)
(declare-const G_E_SF_shaded_TD      E_G)
(declare-const G_E_SF_hasIn_TD       E_G)
(declare-const G_E_SF_hasIn_SExp     E_G)
;; 'QF'
(declare-const G_E_F_Def_QF          E_G)
(declare-const G_E_QF_QDs            E_G)
(declare-const G_E_QF_F              E_G)
(declare-const G_E_QD_QK             E_G)
(declare-const G_E_QD_VDs            E_G)
(declare-const G_E_QK_Def_ForAll     E_G)
(declare-const G_E_QK_Def_Exists     E_G)
;; 'FOp'
(declare-const G_E_FOp_Def_Implies   E_G)
(declare-const G_E_FOp_Def_Equiv     E_G)
(declare-const G_E_FOp_Def_And       E_G)
(declare-const G_E_FOp_Def_Or        E_G)
(declare-const G_E_FOp_Def_SeqComp   E_G)
(declare-const G_E_FOp_Def_Not       E_G)
;; 'D'
(declare-const G_E_D_Def_VD          E_G)
(declare-const G_E_D_Def_DF          E_G)
(declare-const G_E_DV_Def_O          E_G)
(declare-const G_E_DV_Def_Set        E_G)
(declare-const G_E_DV_Def_Seq        E_G)
(declare-const G_E_D_VD_Id           E_G)
(declare-const G_E_D_VD_TD           E_G)
(declare-const G_E_D_VD_isHidden     E_G)
(declare-const G_E_DV_Def_O_opt      E_G)
;; 'DF'
(declare-const G_E_DF_Def_DFA        E_G)
(declare-const G_E_DF_Def_NAry       E_G)
(declare-const G_E_DF_NAry_DFs       E_G)
(declare-const G_E_DF_NAry_FOp       E_G)
;; 'DFA'
(declare-const G_E_DFA_uparrow       E_G)
(declare-const G_E_DFA_RefId         E_G)
(declare-const G_E_DFA_ObjId         E_G)
(declare-const G_E_DFA_SetId         E_G)
```

```
(declare-const G_E_DFA_Rs           E_G)
;; 'R'
(declare-const G_E_R_Id1            E_G)
(declare-const G_E_R_Id2            E_G)
;; 'AD'
(declare-const G_E_AD_Id            E_G)
(declare-const G_E_AD_Ds            E_G)
(declare-const G_E_AD_Fs            E_G)
;; Special 'Null' constant to check totality
(declare-const G_E_Null             E_G)

(assert (distinct
   MM_Null
   MM_Name
   MM_Bool
   MM_TypeDesignator
   MM_SetDef
   MM_SetElement
   MM_PropEdgePred
   MM_SetExpression
   MM_FormulaSource
   MM_FormulaSourceSet
   MM_FormulaSourceElem
   MM_FormulaSourceUnary
   MM_FormulaSourceSetId
   MM_FormulaSourceSetDef
   MM_FormulaSourceUOp
   MM_FormulaSourceUOp_Card
   MM_FormulaSourceUOp_Domain
   MM_FormulaSourceUOp_Range
   MM_FormulaSourceUOp_The
   MM_Formula
   MM_ArrowsFormula
   MM_SetFormula
   MM_FormulaNAry
   MM_FormulaSubset
   MM_SetFormulaDef
   MM_SetFormulaShaded
   MM_QFormula
   MM_QDecl
   MM_QuantifierKind
   MM_QuantifierKind_ForAll
   MM_QuantifierKind_Exists
   MM_Decl
   MM_VarDecl
   MM_DeclObj
   MM_DeclSet
   MM_DeclSeq
   MM_DeclFormula
```

```
  MM_DeclFormulaNAry
  MM_DeclFormulaAtom
  MM_FormulaOp
  MM_FOp_Implies
  MM_FOp_And
  MM_FOp_Or
  MM_FOp_Equiv
  MM_FOp_SeqComp
  MM_FOp_Not
  MM_RenamingExp
  MM_ADiag))

(assert (distinct
  G_Null
  G_Id
  G_Bool
  G_TD
  G_SDef
  G_SE
  G_PEP
  G_SExp
  G_AFS
  G_AFS_SE
  G_AFSS
  G_AFS_FSOp
  G_AFSS_SetId
  G_AFSS_SDef
  G_FSOp
  G_FSOp_Card
  G_FSOp_Domain
  G_FSOp_Range
  G_FSOp_The
  G_F
  G_AF
  G_SF
  G_F_NAry
  G_QF
  G_QD
  G_QK
  G_QK_All
  G_QK_Exists
  G_D
  G_VD
  G_VD_O
  G_VD_Set
  G_VD_Seq
  G_DF
  G_DFA
  G_DF_NAry
```

```
   G_FOp
   G_FOp_Implies
   G_FOp_Equiv
   G_FOp_And
   G_FOp_Or
   G_FOp_SeqComp
   G_FOp_Not
   G_R
   G_AD))

(assert (distinct
   MM_E_Null
   MM_E_I_FormulaSourceElem
   MM_E_I_FormulaSourceSet
   MM_E_I_FormulaSourceUnary
   MM_E_FormulaSourceUnary_frmlSrc
   MM_E_FormulaSourceUnary_operator
   MM_E_I_FormulaSourceSetId
   MM_E_FormulaSourceSetId_setId
   MM_E_I_FormulaSourceSetDef
   MM_E_FormulaSourceSetDef_setDef
   MM_E_ArrowsFormula_source
   MM_E_ArrowsFormula_pes
   MM_E_I_FormulaNAry
   MM_E_FormulaNAry_frmls
   MM_E_FormulaNAry_operator
   MM_E_I_FormulaSubset
   MM_E_I_SetFormulaShaded
   MM_E_I_SetFormulaDef
   MM_E_FormulaSubset_setId
   MM_E_FormulaSubset_hasIn
   MM_E_SetFormulaDef_shaded
   MM_E_SetFormulaDef_setId
   MM_E_SetFormulaDef_setDef
   MM_E_SetFormulaShaded_setId
   MM_E_I_QFormula
   MM_E_QFormula_decls
   MM_E_QFormula_frml
   MM_E_QDecl_vars
   MM_E_QDecl_qkind
   MM_E_I_QuantifierKind_ForAll
   MM_E_I_QuantifierKind_Exists
   MM_E_I_VarDecl
   MM_E_I_DeclSet
   MM_E_I_DeclSeq
   MM_E_I_DeclObj
   MM_E_I_DeclFormula
   MM_E_I_DeclFormulaNAry
   MM_E_I_DeclFormulaAtom
```

```
  MM_E_DeclFormulaNAry_dfop
  MM_E_DeclFormulaNAry_dFrmls
  MM_E_DeclFormulaAtom_refId
  MM_E_DeclFormulaAtom_import
  MM_E_DeclFormulaAtom_renameExp
  MM_E_VarDecl_dName
  MM_E_VarDecl_dTy
  MM_E_VarDecl_isHidden
  MM_E_DeclObj_optional
  MM_E_I_FOp_Implies
  MM_E_I_FOp_And
  MM_E_I_FOp_Or
  MM_E_I_FOp_Equiv
  MM_E_I_FOp_SeqComp
  MM_E_I_FOp_Not
  MM_E_Renaming_subExp
  MM_E_Renaming_varToSub
  MM_E_ADiag_aName
  MM_E_ADiag_predicate
  MM_E_ADiag_decls))

(assert (distinct
  G_E_Null
  G_E_AFS_Def_SE
  G_E_AFS_Def_AFSS
  G_E_AFS_Def_FSOp
  G_E_AFS_FSOp_Op
  G_E_AFS_FSOp_AFS
  G_E_AFSS_Def_SetId
  G_E_AFSS_Def_SDef
  G_E_AFSS_SetId_Id
  G_E_AFSS_SDef_SDef
  G_E_F_Def_AF
  G_E_F_Def_SF
  G_E_F_Def_NAry
  G_E_F_NAry_Fs
  G_E_F_NAry_FOp
  G_E_AF_AFS
  G_E_AF_PEPs
  G_E_SF_Def_SDef
  G_E_SF_Def_shaded
  G_E_SF_Def_hasIn
  G_E_SF_SDef_shaded
  G_E_SF_SDef_Id
  G_E_SF_SDef_SDef
  G_E_SF_shaded_TD
  G_E_SF_hasIn_TD
  G_E_SF_hasIn_SExp
  G_E_F_Def_QF
```

```
  G_E_QF_QDs
  G_E_QF_F
  G_E_QD_QK
  G_E_QD_VDs
  G_E_QK_Def_ForAll
  G_E_QK_Def_Exists
  G_E_D_Def_VD
  G_E_D_Def_DF
  G_E_DV_Def_O
  G_E_DV_Def_Set
  G_E_DV_Def_Seq
  G_E_DF_Def_DFA
  G_E_DF_Def_NAry
  G_E_D_VD_Id
  G_E_D_VD_TD
  G_E_D_VD_isHidden
  G_E_DV_Def_O_opt
  G_E_DF_NAry_DFs
  G_E_DF_NAry_FOp
  G_E_FOp_Def_Implies
  G_E_FOp_Def_Equiv
  G_E_FOp_Def_And
  G_E_FOp_Def_Or
  G_E_FOp_Def_SeqComp
  G_E_FOp_Def_Not
  G_E_DFA_uparrow
  G_E_DFA_RefId
  G_E_DFA_ObjId
  G_E_DFA_SetId
  G_E_DFA_Rs
  G_E_R_Id1
  G_E_R_Id2
  G_E_AD_Id
  G_E_AD_Ds
  G_E_AD_Fs))

(define-fun Map_V ((v V_MM)) V_G
  (ite (= v MM_Name)                    G_Id
  (ite (= v MM_Bool)                    G_Bool
  (ite (= v MM_TypeDesignator)          G_TD
  (ite (= v MM_SetDef)                  G_SDef
  (ite (= v MM_SetElement)              G_SE
  (ite (= v MM_PropEdgePred)            G_PEP
  (ite (= v MM_SetExpression)           G_SExp
  (ite (= v MM_FormulaSource)           G_AFS
  (ite (= v MM_FormulaSourceSet)        G_AFSS
  (ite (= v MM_FormulaSourceElem)       G_AFS_SE
  (ite (= v MM_FormulaSourceUnary)      G_AFS_FSOp
  (ite (= v MM_FormulaSourceSetId)      G_AFSS_SetId
```

```
   (ite (= v MM_FormulaSourceSetDef)     G_AFSS_SDef
   (ite (= v MM_FormulaSourceUOp)        G_FSOp
   (ite (= v MM_FormulaSourceUOp_Card)   G_FSOp_Card
   (ite (= v MM_FormulaSourceUOp_Domain) G_FSOp_Domain
   (ite (= v MM_FormulaSourceUOp_Range)  G_FSOp_Range
   (ite (= v MM_FormulaSourceUOp_The)    G_FSOp_The
   (ite (= v MM_Formula)                 G_F
   (ite (= v MM_FormulaNAry)             G_F_NAry
   (ite (= v MM_SetFormula)              G_SF
   (ite (= v MM_ArrowsFormula)           G_AF
   (ite (= v MM_FormulaNAry)             G_F_NAry
   (ite (= v MM_FormulaSubset)           G_SF_hasIn
   (ite (= v MM_SetFormulaDef)           G_SF_SDef
   (ite (= v MM_SetFormulaShaded)        G_SF_shaded
   (ite (= v MM_QFormula)                G_QF
   (ite (= v MM_QDecl)                   G_QD
   (ite (= v MM_QuantifierKind)          G_QK
   (ite (= v MM_QuantifierKind_ForAll)   G_QK_All
   (ite (= v MM_QuantifierKind_Exists)   G_QK_Exists
   (ite (= v MM_Decl)                    G_D
   (ite (= v MM_VarDecl)                 G_VD
   (ite (= v MM_DeclObj)                 G_VD_O
   (ite (= v MM_DeclSet)                 G_VD_Set
   (ite (= v MM_DeclSeq)                 G_VD_Seq
   (ite (= v MM_DeclFormula)             G_DF
   (ite (= v MM_DeclFormulaNAry)         G_DF_NAry
   (ite (= v MM_DeclFormulaAtom)         G_DFA
   (ite (= v MM_FormulaOp)               G_FOp
   (ite (= v MM_FOp_Implies)             G_FOp_Implies
   (ite (= v MM_FOp_And)                 G_FOp_And
   (ite (= v MM_FOp_Or)                  G_FOp_Or
   (ite (= v MM_FOp_Equiv)               G_FOp_Equiv
   (ite (= v MM_FOp_SeqComp)             G_FOp_SeqComp
   (ite (= v MM_FOp_Not)                 G_FOp_Not
   (ite (= v MM_RenamingExp)             G_R
   (ite (= v MM_ADiag)                   G_AD
   G_Null))))))))))))))))))))))))))))))))))))))))))))))))))))))

(define-fun Map_E ((e E_MM)) E_G
   (ite (= e MM_E_I_FormulaSourceElem)         G_E_AFS_Def_SE
   (ite (= e MM_E_I_FormulaSourceSet)          G_E_AFS_Def_AFSS
   (ite (= e MM_E_I_FormulaSourceUnary)        G_E_AFS_Def_FSOp
   (ite (= e MM_E_FormulaSourceUnary_frmlSrc)  G_E_AFS_FSOp_AFS
   (ite (= e MM_E_FormulaSourceUnary_operator) G_E_AFS_FSOp_Op
   (ite (= e MM_E_I_FormulaSourceSetId)        G_E_AFSS_Def_SetId
   (ite (= e MM_E_FormulaSourceSetId_setId)    G_E_AFSS_SetId_Id
   (ite (= e MM_E_I_FormulaSourceSetDef)       G_E_AFSS_Def_SDef
   (ite (= e MM_E_FormulaSourceSetDef_setDef)  G_E_AFSS_SDef_SDef
   (ite (= e MM_E_I_SetFormula)                G_E_F_Def_SF
```

```
(ite (= e MM_E_I_ArrowsFormula)            G_E_F_Def_AF
(ite (= e MM_E_I_FormulaNAry)              G_E_F_Def_NAry
(ite (= e MM_E_FormulaNAry_frmls)          G_E_F_NAry_Fs
(ite (= e MM_E_FormulaNAry_operator)       G_E_F_NAry_FOp
(ite (= e MM_E_ArrowsFormula_source)       G_E_AF_AFS
(ite (= e MM_E_ArrowsFormula_pes)          G_E_AF_PEPs
(ite (= e MM_E_I_FormulaSubset)            G_E_SF_Def_hasIn
(ite (= e MM_E_I_SetFormulaShaded)         G_E_SF_Def_shaded
(ite (= e MM_E_I_SetFormulaDef)            G_E_SF_Def_SDef
(ite (= e MM_E_FormulaSubset_setId)        G_E_SF_hasIn_TD
(ite (= e MM_E_FormulaSubset_hasIn)        G_E_SF_hasIn_SExp
(ite (= e MM_E_SetFormulaDef_shaded)       G_E_SF_SDef_shaded
(ite (= e MM_E_SetFormulaDef_setId)        G_E_SF_SDef_Id
(ite (= e MM_E_SetFormulaDef_setDef)       G_E_SF_SDef_SDef
(ite (= e MM_E_SetFormulaShaded_setId)     G_E_SF_shaded_TD
(ite (= e MM_E_I_QFormula)                 G_E_F_Def_QF
(ite (= e MM_E_QFormula_decls)             G_E_QF_QDs
(ite (= e MM_E_QFormula_frml)              G_E_QF_F
(ite (= e MM_E_QDecl_vars)                 G_E_QD_VDs
(ite (= e MM_E_QDecl_qkind)                G_E_QD_QK
(ite (= e MM_E_I_QuantifierKind_ForAll)    G_E_QK_Def_ForAll
(ite (= e MM_E_I_QuantifierKind_Exists)    G_E_QK_Def_Exists
(ite (= e MM_E_I_VarDecl)             G_E_D_Def_VD
(ite (= e MM_E_I_DeclSet)             G_E_DV_Def_Set
(ite (= e MM_E_I_DeclSeq)             G_E_DV_Def_Seq
(ite (= e MM_E_I_DeclObj)             G_E_DV_Def_O
(ite (= e MM_E_I_DeclFormula)             G_E_D_Def_DF
(ite (= e MM_E_I_DeclFormulaNAry)          G_E_DF_Def_NAry
(ite (= e MM_E_I_DeclFormulaAtom)          G_E_DF_Def_DFA
(ite (= e MM_E_DeclFormulaNAry_dfop)       G_E_DF_NAry_FOp
(ite (= e MM_E_DeclFormulaNAry_dFrmls)     G_E_DF_NAry_DFs
(ite (= e MM_E_VarDecl_dName)              G_E_D_VD_Id
(ite (= e MM_E_VarDecl_dTy)                G_E_D_VD_TD
(ite (= e MM_E_VarDecl_isHidden)           G_E_D_VD_isHidden
(ite (= e MM_E_DeclObj_optional)           G_E_DV_Def_O_opt
(ite (= e MM_E_I_FOp_Implies)              G_E_FOp_Def_Implies
(ite (= e MM_E_I_FOp_And)                  G_E_FOp_Def_And
(ite (= e MM_E_I_FOp_Or)                   G_E_FOp_Def_Or
(ite (= e MM_E_I_FOp_Equiv)                G_E_FOp_Def_Equiv
(ite (= e MM_E_I_FOp_SeqComp)              G_E_FOp_Def_SeqComp
(ite (= e MM_E_I_FOp_Not)                  G_E_FOp_Def_Not
(ite (= e MM_E_DeclFormulaAtom_refId)      G_E_DFA_RefId
(ite (= e MM_E_DeclFormulaAtom_callObj)    G_E_DFA_ObjId
(ite (= e MM_E_DeclFormulaAtom_owningSet)  G_E_DFA_SetId
(ite (= e MM_E_DeclFormulaAtom_import)     G_E_DFA_uparrow
(ite (= e MM_E_DeclFormulaAtom_renameExp)  G_E_DFA_Rs
(ite (= e MM_E_Renaming_subExp)            G_E_R_Id1
(ite (= e MM_E_Renaming_varToSub)          G_E_R_Id2
(ite (= e MM_E_ADiag_aName)                G_E_AD_Id
```

```
   (ite (= e MM_E_ADiag_predicate)             G_E_AD_Fs
   (ite (= e MM_E_ADiag_decls)                 G_E_AD_Ds
   G_E_Null))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))

(push)
(echo "Testing function 'Map_V' (1) --> sat")
(assert (= (Map_V MM_SetDef) G_SDef))
(check-sat)
(pop)

(push)
(echo "Testing function 'Map_V' (2) --> sat")
(assert (= (Map_V MM_PropEdgePred) G_PEP))
(check-sat)
(pop)

(push)
(echo "Testing function 'Map_V' (3) --> unsat")
(assert (= (Map_V MM_FormulaSourceUOp) G_FSOp_Range))
(check-sat)
(pop)

(push)
(echo "Checking Totality of 'Map_V' --> sat")
(assert (forall ((vmm V_MM))
   (=> (= (Map_V vmm) G_Null) (= vmm MM_Null))))
(check-sat)
(pop)

(push)
(echo "Checking injectiveness of 'Map_V' --> sat")
(assert (forall ((vmm1 V_MM) (vmm2 V_MM))
   (=> (= (Map_V vmm1) (Map_V vmm2)) (= vmm1 vmm2))))
(check-sat)
(pop)

(push)
(echo "Checking Surjectiveness of 'Map_V' -->sat")
(assert (forall ((vg V_G))
   (exists ((vmm V_MM))
      (= (Map_V vmm) vg))))
(check-sat)
(pop)

;(push)
;(echo "Checking Surjectiveness of 'Map_V' (2)->sat")
;(declare-fun svmm (V_G) V_MM)
;(assert (forall ((vg V_G))
;      (= (Map_V (svmm vg)) vg)))
```

```
;(check-sat)
;(pop)

(push)
(echo "Testing the 'Map_E' function (1) --> sat")
(assert (= (Map_E MM_E_I_DeclFormulaNAry) G_E_DF_Def_NAry))
(check-sat)
(pop)

(push)
(echo "Testing the 'Map_E' function (2) --> sat")
(assert (= (Map_E MM_E_I_FormulaNAry) G_E_F_Def_NAry))
(check-sat)
(pop)

(push)
(echo "Testing the 'Map_E' function (3) --> unsat")
(assert (= (Map_E MM_E_I_FormulaSourceSetDef) G_E_Null))
(check-sat)
(pop)

(push)
(echo "Checking Totality of 'Map_E' --> sat")
(assert (forall ((emm E_MM))
   (=> (= (Map_E emm) G_E_Null) (= emm MM_E_Null))))
(check-sat)
(pop)

(push)
(echo "Checking injectiveness of 'Map_E' --> sat")
(assert (forall ((emm1 E_MM) (emm2 E_MM))
   (=> (= (Map_E emm1) (Map_E emm2)) (= emm1 emm2))))
(check-sat)
(pop)

(push)
(echo "Checking Surjectiveness of 'Map_E' (1) --> sat")
(assert (forall ((eg E_G))
   (exists ((emm E_MM))
      (= (Map_E emm) eg))))
(check-sat)
(pop)

;(push)
;(echo "Checking surjectiveness of 'Map_E' (2) --> sat")
;(declare-fun semm (E_G) E_MM)
;(assert (forall ((eg E_G))
;   (= (Map_E (semm eg)) eg)))
;(check-sat)
```

```
;(pop)

(define-fun Target_MM ((e E_MM)) V_MM
   (ite (= e MM_E_I_FormulaSourceElem)          MM_FormulaSource
   (ite (= e MM_E_I_FormulaSourceSet)           MM_FormulaSource
   (ite (= e MM_E_I_FormulaSourceUnary)         MM_FormulaSource
   (ite (= e MM_E_FormulaSourceUnary_frmlSrc)   MM_FormulaSource
   (ite (= e MM_E_FormulaSourceUnary_operator)  MM_FormulaSourceUOp
   (ite (= e MM_E_I_FormulaSourceSetId)         MM_FormulaSourceSet
   (ite (= e MM_E_I_FormulaSourceSetDef)        MM_FormulaSourceSet
   (ite (= e MM_E_FormulaSourceSetId_setId)     MM_Name
   (ite (= e MM_E_FormulaSourceSetDef_setDef)   MM_SetDef
   (ite (= e MM_E_ArrowsFormula_source)         MM_FormulaSource
   (ite (= e MM_E_ArrowsFormula_pes)            MM_PropEdgePred
   (ite (= e MM_E_I_FormulaNAry)                MM_Formula
   (ite (= e MM_E_FormulaNAry_frmls)            MM_Formula
   (ite (= e MM_E_FormulaNAry_operator)         MM_FormulaOp
   (ite (= e MM_E_I_FormulaSubset)              MM_SetFormula
   (ite (= e MM_E_I_SetFormulaShaded)           MM_SetFormula
   (ite (= e MM_E_I_SetFormulaDef)              MM_SetFormula
   (ite (= e MM_E_FormulaSubset_setId)          MM_TypeDesignator
   (ite (= e MM_E_FormulaSubset_hasIn)          MM_SetExpression
   (ite (= e MM_E_SetFormulaDef_shaded)         MM_Bool
   (ite (= e MM_E_SetFormulaDef_setId)          MM_Name
   (ite (= e MM_E_SetFormulaDef_setDef)         MM_SetDef
   (ite (= e MM_E_SetFormulaShaded_setId)       MM_TypeDesignator
   (ite (= e MM_E_I_QFormula)                   MM_Formula
   (ite (= e MM_E_QFormula_decls)               MM_QDecl
   (ite (= e MM_E_QFormula_frml)                MM_Formula
   (ite (= e MM_E_QDecl_vars)                   MM_VarDecl
   (ite (= e MM_E_QDecl_qkind)                  MM_QuantifierKind
   (ite (= e MM_E_I_QuantifierKind_ForAll)      MM_QuantifierKind
   (ite (= e MM_E_I_QuantifierKind_Exists)      MM_QuantifierKind
   (ite (= e MM_E_I_VarDecl)                    MM_Decl
   (ite (= e MM_E_I_DeclSet)                    MM_VarDecl
   (ite (= e MM_E_I_DeclObj)                    MM_VarDecl
   (ite (= e MM_E_I_DeclSeq)                    MM_VarDecl
   (ite (= e MM_E_I_DeclFormula)                MM_Decl
   (ite (= e MM_E_I_DeclFormulaNAry)            MM_DeclFormula
   (ite (= e MM_E_I_DeclFormulaAtom)            MM_DeclFormula
   (ite (= e MM_E_VarDecl_dName)                MM_Name
   (ite (= e MM_E_VarDecl_dTy)                  MM_TypeDesignator
   (ite (= e MM_E_VarDecl_isHidden)             MM_Bool
   (ite (= e MM_E_DeclObj_optional)             MM_Bool
   (ite (= e MM_E_DeclFormulaNAry_dFrmls)       MM_DeclFormula
   (ite (= e MM_E_DeclFormulaNAry_dfop)         MM_FormulaOp
   (ite (= e MM_E_I_FOp_Implies)                MM_FormulaOp
   (ite (= e MM_E_I_FOp_And)                    MM_FormulaOp
   (ite (= e MM_E_I_FOp_Or)                     MM_FormulaOp
```

```
   (ite (= e MM_E_I_FOp_Equiv)                MM_FormulaOp
   (ite (= e MM_E_I_FOp_SeqComp)              MM_FormulaOp
   (ite (= e MM_E_I_FOp_Not)                  MM_FormulaOp
   (ite (= e MM_E_DeclFormulaAtom_refId)      MM_Name
   (ite (= e MM_E_DeclFormulaAtom_import)     MM_Bool
   (ite (= e MM_E_DeclFormulaAtom_callObj)    MM_Name
   (ite (= e MM_E_DeclFormulaAtom_owningSet)  MM_Name
   (ite (= e MM_E_DeclFormulaAtom_renameExp)  MM_RenamingExp
   (ite (= e MM_E_Renaming_subExp)            MM_Name
   (ite (= e MM_E_Renaming_varToSub)          MM_Name
   (ite (= e MM_E_ADiag_aName)                MM_Name
   (ite (= e MM_E_ADiag_predicate)            MM_Formula
   (ite (= e MM_E_ADiag_decls)                MM_Decl
   MM_Null))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))

(define-fun Source_MM ((e E_MM)) V_MM
   (ite (= e MM_E_I_FormulaSourceElem)         MM_FormulaSourceElem
   (ite (= e MM_E_I_FormulaSourceSet)          MM_FormulaSourceSet
   (ite (= e MM_E_I_FormulaSourceUnary)        MM_FormulaSourceUnary
   (ite (= e MM_E_FormulaSourceUnary_frmlSrc)  MM_FormulaSourceUnary
   (ite (= e MM_E_FormulaSourceUnary_operator) MM_FormulaSourceUnary
   (ite (= e MM_E_I_FormulaSourceSetId)        MM_FormulaSourceSetId
   (ite (= e MM_E_I_FormulaSourceSetDef)       MM_FormulaSourceSetDef
   (ite (= e MM_E_FormulaSourceSetId_setId)    MM_FormulaSourceSetId
   (ite (= e MM_E_FormulaSourceSetDef_setDef)  MM_FormulaSourceSetDef
   (ite (= e MM_E_ArrowsFormula_source)        MM_ArrowsFormula
   (ite (= e MM_E_ArrowsFormula_pes)           MM_ArrowsFormula
   (ite (= e MM_E_I_FormulaNAry)               MM_FormulaNAry
   (ite (= e MM_E_FormulaNAry_frmls)           MM_FormulaNAry
   (ite (= e MM_E_FormulaNAry_operator)        MM_FormulaNAry
   (ite (= e MM_E_I_FormulaSubset)             MM_FormulaSubset
   (ite (= e MM_E_I_SetFormulaShaded)          MM_SetFormulaShaded
   (ite (= e MM_E_I_SetFormulaDef)             MM_SetFormulaDef
   (ite (= e MM_E_FormulaSubset_setId)         MM_FormulaSubset
   (ite (= e MM_E_FormulaSubset_hasIn)         MM_FormulaSubset
   (ite (= e MM_E_SetFormulaDef_shaded)        MM_SetFormulaDef
   (ite (= e MM_E_SetFormulaDef_setId)         MM_SetFormulaDef
   (ite (= e MM_E_SetFormulaDef_setDef)        MM_SetFormulaDef
   (ite (= e MM_E_SetFormulaShaded_setId)      MM_SetFormulaShaded
   (ite (= e MM_E_I_QFormula)                  MM_QFormula
   (ite (= e MM_E_QFormula_decls)              MM_QFormula
   (ite (= e MM_E_QFormula_frml)               MM_QFormula
   (ite (= e MM_E_QDecl_vars)                  MM_QDecl
   (ite (= e MM_E_QDecl_qkind)                 MM_QDecl
   (ite (= e MM_E_I_QuantifierKind_ForAll)     MM_QuantifierKind_ForAll
   (ite (= e MM_E_I_QuantifierKind_Exists)     MM_QuantifierKind_Exists
   (ite (= e MM_E_I_VarDecl)                   MM_VarDecl
   (ite (= e MM_E_I_DeclSet)                   MM_DeclSet
   (ite (= e MM_E_I_DeclObj)                   MM_DeclObj
```

```
   (ite (= e MM_E_I_DeclSeq)                    MM_DeclSeq
   (ite (= e MM_E_I_DeclFormula)                MM_DeclFormula
   (ite (= e MM_E_I_DeclFormulaNAry)            MM_DeclFormulaNAry
   (ite (= e MM_E_I_DeclFormulaAtom)            MM_DeclFormulaAtom
   (ite (= e MM_E_VarDecl_dName)                MM_VarDecl
   (ite (= e MM_E_VarDecl_dTy)                  MM_VarDecl
   (ite (= e MM_E_DeclObj_optional)             MM_DeclObj
   (ite (= e MM_E_VarDecl_isHidden)             MM_VarDecl
   (ite (= e MM_E_DeclFormulaNAry_dFrmls)       MM_DeclFormulaNAry
   (ite (= e MM_E_DeclFormulaNAry_dfop)         MM_DeclFormulaNAry
   (ite (= e MM_E_I_FOp_Implies)                MM_FOp_Implies
   (ite (= e MM_E_I_FOp_And)                    MM_FOp_And
   (ite (= e MM_E_I_FOp_Or)                     MM_FOp_Or
   (ite (= e MM_E_I_FOp_Equiv)                  MM_FOp_Equiv
   (ite (= e MM_E_I_FOp_SeqComp)                MM_FOp_SeqComp
   (ite (= e MM_E_I_FOp_Not)                    MM_FOp_Not
   (ite (= e MM_E_DeclFormulaAtom_refId)        MM_DeclFormulaAtom
   (ite (= e MM_E_DeclFormulaAtom_import)       MM_DeclFormulaAtom
   (ite (= e MM_E_DeclFormulaAtom_owningSet)    MM_DeclFormulaAtom
   (ite (= e MM_E_DeclFormulaAtom_callObj)      MM_DeclFormulaAtom
   (ite (= e MM_E_DeclFormulaAtom_renameExp)    MM_DeclFormulaAtom
   (ite (= e MM_E_Renaming_subExp)              MM_RenamingExp
   (ite (= e MM_E_Renaming_varToSub)            MM_RenamingExp
   (ite (= e MM_E_ADiag_aName)                  MM_ADiag
   (ite (= e MM_E_ADiag_predicate)              MM_ADiag
   (ite (= e MM_E_ADiag_decls)                  MM_ADiag
   MM_Null))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))

(define-fun Target_G ((e E_G)) V_G
   (ite (= e G_E_AFS_Def_SE)        G_AFS
   (ite (= e G_E_AFS_Def_AFSS)      G_AFS
   (ite (= e G_E_AFS_Def_FSOp)      G_AFS
   (ite (= e G_E_AFS_FSOp_Op)       G_FSOp
   (ite (= e G_E_AFS_FSOp_AFS)      G_AFS
   (ite (= e G_E_AFSS_Def_SetId)    G_AFSS
   (ite (= e G_E_AFSS_Def_SDef)     G_AFSS
   (ite (= e G_E_AFSS_SetId_Id)     G_Id
   (ite (= e G_E_AFSS_SDef_SDef)    G_SDef
   (ite (= e G_E_AF_AFS)            G_AFS
   (ite (= e G_E_AF_PEPs)           G_PEP
   (ite (= e G_E_F_Def_AF)          G_F
   (ite (= e G_E_F_Def_SF)          G_F
   (ite (= e G_E_F_Def_NAry)        G_F
   (ite (= e G_E_F_NAry_Fs)         G_F
   (ite (= e G_E_F_NAry_FOp)        G_FOp
   (ite (= e G_E_SF_Def_hasIn)      G_SF
   (ite (= e G_E_SF_Def_shaded)     G_SF
   (ite (= e G_E_SF_Def_SDef)       G_SF
   (ite (= e G_E_SF_hasIn_TD)       G_TD
```

```
  (ite (= e G_E_SF_hasIn_SExp)    G_SExp
  (ite (= e G_E_SF_SDef_shaded)   G_Bool
  (ite (= e G_E_SF_SDef_Id)       G_Id
  (ite (= e G_E_SF_SDef_SDef)     G_SDef
  (ite (= e G_E_SF_shaded_TD)     G_TD
  (ite (= e G_E_F_Def_QF)         G_F
  (ite (= e G_E_QF_QDs)           G_QD
  (ite (= e G_E_QF_F)             G_F
  (ite (= e G_E_QD_QK)            G_QK
  (ite (= e G_E_QD_VDs)           G_VD
  (ite (= e G_E_QK_Def_ForAll)    G_QK
  (ite (= e G_E_QK_Def_Exists)    G_QK
  (ite (= e G_E_D_Def_VD)         G_D
  (ite (= e G_E_DV_Def_Set)       G_VD
  (ite (= e G_E_DV_Def_Seq)       G_VD
  (ite (= e G_E_DV_Def_O)         G_VD
  (ite (= e G_E_D_Def_DF)         G_D
  (ite (= e G_E_DF_Def_NAry)      G_DF
  (ite (= e G_E_DF_Def_DFA)       G_DF
  (ite (= e G_E_D_VD_Id)          G_Id
  (ite (= e G_E_D_VD_TD)          G_TD
  (ite (= e G_E_D_VD_isHidden)    G_Bool
  (ite (= e G_E_DV_Def_O_opt)     G_Bool
  (ite (= e G_E_DF_NAry_DFs)      G_DF
  (ite (= e G_E_DF_NAry_FOp)      G_FOp
  (ite (= e G_E_FOp_Def_Implies)  G_FOp
  (ite (= e G_E_FOp_Def_Equiv)    G_FOp
  (ite (= e G_E_FOp_Def_And)      G_FOp
  (ite (= e G_E_FOp_Def_Or)       G_FOp
  (ite (= e G_E_FOp_Def_SeqComp)  G_FOp
  (ite (= e G_E_FOp_Def_Not)      G_FOp
  (ite (= e G_E_DFA_RefId)        G_Id
  (ite (= e G_E_DFA_ObjId)        G_Id
  (ite (= e G_E_DFA_SetId)        G_Id
  (ite (= e G_E_DFA_uparrow)      G_Bool
  (ite (= e G_E_DFA_Rs)           G_R
  (ite (= e G_E_R_Id1)            G_Id
  (ite (= e G_E_R_Id2)            G_Id
  (ite (= e G_E_AD_Id)            G_Id
  (ite (= e G_E_AD_Fs)            G_F
  (ite (= e G_E_AD_Ds)            G_D
  G_Null))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))

(define-fun Source_G ((e E_G)) V_G
  (ite (= e G_E_AFS_Def_SE)       G_AFS_SE
  (ite (= e G_E_AFS_Def_AFSS)     G_AFSS
  (ite (= e G_E_AFS_Def_FSOp)     G_AFS_FSOp
  (ite (= e G_E_AFS_FSOp_Op)      G_AFS_FSOp
  (ite (= e G_E_AFS_FSOp_AFS)     G_AFS_FSOp
```

```
(ite (= e G_E_AFSS_Def_SetId)   G_AFSS_SetId
(ite (= e G_E_AFSS_Def_SDef)    G_AFSS_SDef
(ite (= e G_E_AFSS_SetId_Id)    G_AFSS_SetId
(ite (= e G_E_AFSS_SDef_SDef)   G_AFSS_SDef
(ite (= e G_E_AF_AFS)           G_AF
(ite (= e G_E_AF_PEPs)          G_AF
(ite (= e G_E_F_Def_AF)         G_AF
(ite (= e G_E_F_Def_SF)         G_SF
(ite (= e G_E_F_Def_NAry)       G_F_NAry
(ite (= e G_E_F_NAry_Fs)        G_F_NAry
(ite (= e G_E_F_NAry_FOp)       G_F_NAry
(ite (= e G_E_SF_Def_hasIn)     G_SF_hasIn
(ite (= e G_E_SF_Def_shaded)    G_SF_shaded
(ite (= e G_E_SF_Def_SDef)      G_SDef
(ite (= e G_E_SF_hasIn_TD)      G_SF_hasIn
(ite (= e G_E_SF_hasIn_SExp)    G_SF_hasIn
(ite (= e G_E_SF_SDef_shaded)   G_SF_SDef
(ite (= e G_E_SF_SDef_Id)       G_SF_SDef
(ite (= e G_E_SF_SDef_SDef)     G_SF_SDef
(ite (= e G_E_SF_shaded_TD)     G_SF_shaded
(ite (= e G_E_F_Def_QF)         G_QF
(ite (= e G_E_QF_QDs)           G_QF
(ite (= e G_E_QF_F)             G_QF
(ite (= e G_E_QD_QK)            G_QD
(ite (= e G_E_QD_VDs)           G_QD
(ite (= e G_E_QK_Def_ForAll)    G_QK_All
(ite (= e G_E_QK_Def_Exists)    G_QK_Exists
(ite (= e G_E_D_Def_VD)         G_VD
(ite (= e G_E_DV_Def_Set)       G_VD_Set
(ite (= e G_E_DV_Def_Seq)       G_VD_Seq
(ite (= e G_E_DV_Def_O)         G_VD_O
(ite (= e G_E_D_Def_DF)         G_DF
(ite (= e G_E_DF_Def_NAry)      G_DF_NAry
(ite (= e G_E_DF_Def_DFA)       G_DFA
(ite (= e G_E_D_VD_Id)          G_VD
(ite (= e G_E_D_VD_TD)          G_VD
(ite (= e G_E_D_VD_isHidden)    G_VD
(ite (= e G_E_DV_Def_O_opt)     G_VD_O
(ite (= e G_E_DF_NAry_DFs)      G_DF_NAry
(ite (= e G_E_DF_NAry_FOp)      G_DF_NAry
(ite (= e G_E_FOp_Def_Implies)  G_FOp_Implies
(ite (= e G_E_FOp_Def_Equiv)    G_FOp_Equiv
(ite (= e G_E_FOp_Def_And)      G_FOp_And
(ite (= e G_E_FOp_Def_Or)       G_FOp_Or
(ite (= e G_E_FOp_Def_SeqComp)  G_FOp_SeqComp
(ite (= e G_E_FOp_Def_Not)      G_FOp_Not
(ite (= e G_E_DFA_RefId)        G_DFA
(ite (= e G_E_DFA_ObjId)        G_DFA
(ite (= e G_E_DFA_SetId)        G_DFA
```

```
   (ite (= e G_E_DFA_uparrow)      G_DFA
   (ite (= e G_E_DFA_Rs)           G_DFA
   (ite (= e G_E_R_Id1)            G_R
   (ite (= e G_E_R_Id2)            G_R
   (ite (= e G_E_AD_Id)            G_AD
   (ite (= e G_E_AD_Fs)            G_AD
   (ite (= e G_E_AD_Ds)            G_AD
   G_Null))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))))

(push)
(echo "Testing function 'Target_MM' (1) --> sat")
(assert (= (Target_MM MM_E_I_FormulaSourceSet) MM_FormulaSource))
(check-sat)
(pop)

(push)
(echo "Testing function 'Target_MM'  (2) --> sat")
(assert (= (Target_MM MM_E_FormulaSourceUnary_operator) MM_FormulaSourceUOp))
(check-sat)
(pop)

(push)
(echo "Testing function 'Target_MM' (3) --> unsat")
(assert (= (Target_MM MM_E_FormulaSourceSetDef_setDef) MM_Null))
(check-sat)
(pop)

(push)
(echo "Checking totality of 'Target_MM' --> sat")
(assert (forall ((emm E_MM))
   (=> (= (Target_MM emm) MM_Null) (= emm MM_E_Null))))
(check-sat)
(pop)

(push)
(echo "Checking totality of 'Target_G' --> sat")
(assert (forall ((eg E_G))
   (=> (= (Target_G eg) G_Null) (= eg G_E_Null))))
(check-sat)
(pop)

(push)
(echo "Checking that the target function 'Target_MM' is preserved --> sat")
(assert (forall ((emm1 E_MM))
   (= (Map_V (Target_MM emm1)) (Target_G (Map_E emm1)))))
(check-sat)
(pop)

(push)
```

```
(echo "Testing function 'Source_MM' (1) --> sat")
(assert (= (Source_MM MM_E_I_FormulaSourceSetId) MM_FormulaSourceSetId))
(check-sat)
(pop)

(push)
(echo "Testing function 'Source_MM' (2) --> sat")
(assert (= (Source_MM MM_E_I_FormulaSourceSetDef) MM_FormulaSourceSetDef))
(check-sat)
(pop)

(push)
(echo "Testing function 'Source_MM' (3) --> unsat")
(assert (= (Source_MM MM_E_FormulaSourceUnary_operator) MM_Null))
(check-sat)
(pop)

(push)
(echo "Checking Totality of 'Source_MM' --> sat")
(assert (forall ((emm E_MM))
   (=> (= (Source_MM emm) MM_Null) (= emm MM_E_Null))))
(check-sat)
(pop)

(push)
(echo "Testing function 'Source_G' (1) --> sat")
(assert (= (Source_G G_E_AFS_FSOp_AFS) G_AFS_FSOp))
(check-sat)
(pop)

(push)
(echo "Testing function 'Source_G' (2) --> sat")
(assert (= (Source_G G_E_AFSS_Def_SetId) G_AFSS_SetId))
(check-sat)
(pop)

(push)
(echo "Testing function 'Source_G' (3) --> unsat")
(assert (= (Source_G G_E_AFSS_SDef_SDef) G_Null))
(check-sat)
(pop)

(push)
(echo "Checking Totality of 'Source_G' -->sat")
(assert (forall ((eg E_G))
   (=> (= (Source_G eg) G_Null) (= eg G_E_Null))))
(check-sat)
(pop)
```

```
(push)
(echo "Checking that the source function 'Source_MM' is preserved --> sat")
(assert (forall ((emm1 E_MM))
   (= (Map_V (Source_MM emm1)) (Source_G (Map_E emm1)))))
(check-sat)
(pop)
```

C.3.1 Z3 Output

```
Testing function 'Map_V' (1) --> sat
sat
Testing function 'Map_V' (2) --> sat
sat
Testing function 'Map_V' (3) --> unsat
unsat
Checking Totality of 'Map_V' --> sat
sat
Checking injectiveness of 'Map_V' --> sat
sat
Checking Surjectiveness of 'Map_V' -->sat
sat
Testing the 'Map_E' function (1) --> sat
sat
Testing the 'Map_E' function (2) --> sat
sat
Testing the 'Map_E' function (3) --> unsat
unsat
Checking Totality of 'Map_E' --> sat
sat
Checking injectiveness of 'Map_E' --> sat
sat
Checking Surjectiveness of 'Map_E' (1) --> sat
sat
Testing function 'Target_MM' (1) --> sat
sat
Testing function 'Target_MM'  (2) --> sat
sat
Testing function 'Target_MM' (3) --> unsat
unsat
Checking totality of 'Target_MM' --> sat
sat
Checking totality of 'Target_G' --> sat
sat
Checking that the target function 'Target_MM' is preserved --> sat
sat
Testing function 'Source_MM' (1) --> sat
sat
```

```
Testing function 'Source_MM' (2) --> sat
sat
Testing function 'Source_MM' (3) --> unsat
unsat
Checking Totality of 'Source_MM' --> sat
sat
Testing function 'Source_G' (1) --> sat
sat
Testing function 'Source_G' (2) --> sat
sat
Testing function 'Source_G' (3) --> unsat
unsat
Checking Totality of 'Source_G' -->sat
sat
Checking that the source function 'Source_MM' is preserved --> sat
sat
```