**CIS 511: Theory of Computation** **Spring 2017**

# Lecture 1: Intro and Some Regular Languages

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**NB**: *These notes are from CIS511 at Penn. The course followed Michael Sipser's Introduction to the Theory of Computation (3ed) text.*

## Introduction

Why theory?

1. Minimal approach to understanding the idea of **computation**.
2. What makes computation tick?
3. Theory anticipates technology.
4. Models of computation are interesting.

## Mathematics!

Should know:

1. Sets
2. Functions
3. Relations
4. Logic
5. Proofs
6. Graphs

**Definition 1.1** *An **alphabet** is a non-empty, finite set of characters.*

***Definition 1.2*** *A **string** $s$ (over an alphabet $\Sigma$) is a finite ordered sequence of elements of $\Sigma$.*

***Definition 1.3*** *The **empty string**, $\epsilon$, is the sequence of no symbols, and is in fact a valid string.*

***Definition 1.4*** *Let $\Sigma^*$ be the set of all strings over $\Sigma$.*

***Definition 1.5*** *A **language** over $\Sigma$ is any subset of $\Sigma^*$.*

*The empty set is a language. This is not the same as the language only containing the empty string.*

# Finite State Machines: A First Model

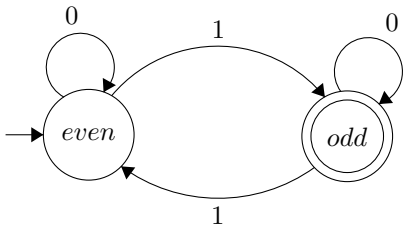
*Scalability (asymptotics) is a requirement for any interesting model.*

*To compute on larger and larger inputs, a computer needs memory. What is the minimum amount of memory you need to do something interesting?*

***Definition 1.6*** *A **finite state machine** will be a model with a constant amount of memory.*

*The states of an FSM correspond to memory (a machine with k states can have $2^k$ 'bits' of memory).*

*Example: define the language $\mathcal{L} = \{s \in \Sigma^* | s \text{ has an odd number of } 1s\}$.*




***Definition 1.7*** *A **deterministic finite automaton (DFA), $M$, is a 5-tuple** $(Q, \Sigma, \delta, q_0, F)$.*

- *$Q$, the set of states.*
- *$\Sigma$, the alphabet*
- *$\delta : Q \times \Sigma \to Q$, the transition function*
- *$q_0$, the start state*
- *$F$, the accept states*

***Definition 1.8*** *$M$ **accepts** a string $s = s_1 s_2 \dots s_k$ if there is a sequence of states in $M$ starting with $q_0$ and ending in a final state $q_0 q_1 \dots q_k$ such that $\delta(q_i, s_{i+1}) = q_{i+1}$.*

***Definition 1.9*** *If $\mathcal{L} = \{s | M \text{ accepts } s\}$, the we say $M$ **recognizes $\mathcal{L}$**.*

***Definition 1.10*** *If a language $\mathcal{L}$ is recognized by some DFA, then it is **regular**.*

***Definition 1.11*** *A **nondeterministic finite automaton (NFA), $M$, is a 5-tuple** $(Q, \Sigma, \delta, q_0, F)$.*

- *$Q$, the set of states.*
- *$\Sigma$, the alphabet*
- *$\delta : Q \times \Sigma \cup \{\epsilon\} \to \mathcal{P}(Q)$, the transition function*
- *$q_0$, the start state*
- *$F$, the accept states*

*Now, $\delta$ maps the current state and input character or $\epsilon$ to some subset of states.*

***Definition 1.12*** *A string $s$ is **accepted** by and NFA $M$ if there is some path for $s$ from the start state to a final state.*

***Definition 1.13*** *$M$ **recognizes** a language $\mathcal{L}$ consisting of all strings it accepts.*

*Example: let $\mathcal{L} = \{s | the 3rd\ last\ character\ is\ a\ 1\}$.*