CIS 511: Theory of Computation **Spring 2017**

Lecture 1:

Professor Sampath Kannan *Zach Schutzman*

NB: *These notes are from CIS511 at Penn. The course followed Michael Sipser's Introduction to the Theory of Computation (3ed) text.*

1.1 Introduction

Why theory?

1. Minimal approach to understanding the idea of **computation**.
2. What makes computation tick?
3. Theory anticipates technology.
4. Models of computation are interesting.

1.2 Mathematics!

Should know:

1. Sets
2. Functions
3. Relations
4. Logic
5. Proofs
6. Graphs

Definition 1.1 *An* ***alphabet*** *is a non-empty, finite set of characters.*

Definition 1.2 *A* ***string*** s *(over an alphabet* Σ*) is a finite ordered sequence of elements of* Σ*.*

Definition 1.3 *The* **empty string**, ϵ, *is the sequence of no symbols, and is in fact a valid string.*

Definition 1.4 *Let* Σ^* *be the set of all strings over* Σ*.*

Definition 1.5 *A* ***language*** *over* Σ *is any subset of* Σ^**.*

The empty set is a language. This is not the same as the language only containing the empty string.

1.3 Finite State Machines: A First Model

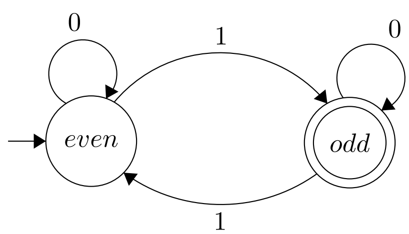
Scalability (asymptotics) is a requirement for any interesting model.

To compute on larger and larger inputs, a computer needs memory. What is the minimum amount of memory you need to do something interesting?

Definition 1.6 *A **finite state machine** will be a model with a constant amount of memory.*

The states of an FSM correspond to memory (a machine with k states can have 2^k 'bits' of memory).

Example: define the language $\mathcal{L} = \{s \in \Sigma^* | s \text{ has an odd number of } 1s\}$.



Definition 1.7 *A **deterministic finite automaton (DFA),** M**, is a 5-tuple** $(Q, \Sigma, \delta, q_0, F)$**.***

Q, the set of states.

Σ, the alphabet

$\delta : Q \times \Sigma \to Q$, the transition function

q_0, the start state

F, the accept states

Definition 1.8 *M **accepts** a string $s = s_1 s_2 \dots s_k$ if there is a sequence of states in M starting with q_0 and ending in a final state $q_0 q_1 \dots q_k$ such that $\delta(q_i, s_{i+1}) = q_{i+1}$.*

Definition 1.9 *If $\mathcal{L} = \{s | M \text{ accepts } s\}$, the we say M **recognizes** $\mathcal{L}$.*

Definition 1.10 *If a language $\mathcal{L}$ is recognized by some DFA, then it is **regular**.*

Definition 1.11 *A **nondeterministic finite automaton (NFA),** M**, is a 5-tuple** $(Q, \Sigma, \delta, q_0, F)$**.***

Q, the set of states.

Σ, the alphabet

$\delta : Q \times \Sigma \cup \{\epsilon\} \to \mathcal{P}(Q)$, the transition function

q_0, the start state

F, the accept states

Now, δ maps the current state and input character or ϵ to some subset of states.