

# A testable brane-world unification: $\rho^2$ cosmology, dark radiation, GW break

We present a four-dimensional effective cosmology derived from a higher-dimensional brane setup. The resulting Friedmann equation contains a quadratic density term ( $\rho^2/2\lambda$ ) and a dark-radiation contribution ( $\propto a^4$ ). A single parameter, the brane tension  $\lambda$ , controls a gravitational-wave spectral break  $f_{\text{br}} \propto \lambda^{\{1/4\}}$  and correlates with  $\Delta N_{\text{eff}}$ . Using the official NANOGrav 15-yr KDE spectrum (converted to CSV) with a Planck-2018 prior on  $N_{\text{eff}}=2.99\pm0.17$ , we compute posteriors and provide a model-data overlay. The framework reduces to GR in the late-time/weak-field regime (PPN/binary pulsars) while offering falsifiable early-universe signatures testable by PTA→LISA and CMB/BBN.

