Unified Brane-Cosmology: A Testable Route to Unification

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Abstract — A higher-dimensional (bulk + brane) framework yields a 4D Friedmann equation on the brane with a high-energy ρ^2 correction and a dark-radiation term C/a^4. A single parameter (brane tension λ) links a measurable stochastic-GW spectral break $f_{br} \propto \lambda^{1/4}$ to ΔN_{eff}. We outline falsifiable predictions and a minimal pipeline for near-term tests with PTA→LISA and CMB/BBN.

1. Framework

We consider a warped 5D bulk with a 4D brane. Standard-Model fields reside on the brane; gravity propagates in the bulk. Israel junction conditions relate the extrinsic curvature to the brane stress tensor and brane tension λ. The effective 4D equation contains: (i) Einstein tensor + Λ_4, (ii) quadratic stress $\Pi_{\mu\nu}$, (iii) projected bulk Weyl $E_{\mu\nu}$ (dark-radiation).

2. Effective 4D Equation

$G_{\mu\nu} + \Lambda_4 g_{\mu\nu} = (8\pi G/c^4) T_{\mu\nu} + (\kappa_5^4/c^4) \Pi_{\mu\nu} - E_{\mu\nu}$. The $\Pi_{\mu\nu}$ term controls high-energy corrections ($\rho \gg \lambda$). The Weyl piece $E_{\mu\nu}$ encodes bulk memory and yields a radiation-like $\rho_{dr} \propto C/a^4$ in FRW.

3. Cosmology Reduction (flat FRW)

For a perfect fluid (ρ, p), the brane Friedmann equation reduces to $H^2 = (8\pi G/3)\, \rho\, (1 + \rho/(2\lambda)) + \Lambda_4/3 + C/a^4 - k/a^2$. At early times ($\rho \gg \lambda$), the ρ^2 term dominates, giving $a(t) \propto t^{1/4}$ in the radiation era.

4. Observables and the Single-Parameter Link

The brane tension λ sets the transition scale where the ρ^2 term turns off, imprinting a broken-power-law SGWB with a break frequency $f_{br}(\lambda)$. The same physics contributes to dark radiation parametrized by ΔN_{eff} via C/a^4. Hence $f_{br}(\lambda) \leftrightarrow \Delta N_{eff}$ provides a joint, falsifiable target.

5. Minimal Likelihood (preview)

We fit a broken power-law SGWB to PTA data with slopes (n_1, n_2) and a break $f_{br}(\lambda) \propto \lambda^{1/4}$, adding a Gaussian prior on ΔN_{eff} from CMB/BBN. A single λ must satisfy the PTA band and the ΔN_{eff} prior. The present draft includes a real-anchored preview run.

6. Relation to Prior Work

Within the brane-cosmology literature (Randall–Sundrum; Binetruy–Deffayet–Langlois; Shiromizu–Maeda–Sasaki; Maartens), our emphasis is an explicit, near-term falsifiable λ-link that spans PTA→LISA while respecting CMB/BBN.

7. Limitations and Open Tasks

We have not yet specified a compactification reproducing SU(3)×SU(2)×U(1) with correct spectra/Yukawas and stabilized moduli. The present focus is the cosmology link and testable predictions.

8. Roadmap

Next: swap in official PTA likelihood/products; add LISA curve; perform joint posteriors with goodness-of-fit; release code and submit to PRD/JCAP.

Master Relation (cosmology-reduced)

$H^2 = (8\pi G/3)\cdot\rho\cdot(1 + \rho/(2\lambda)) + \Lambda_4/3 + C/a^4$.

Unicode draft (Δ, λ, ρ). A LaTeX version can be produced on request.

References — Unified Brane-Cosmology (curated)

[1] Randall, L., Sundrum, R. 'A Large Mass Hierarchy from a Small Extra Dimension.' Phys. Rev. Lett. 83, 3370 (1999). DOI:10.1103/PhysRevLett.83.3370

[2] Binetruy, P., Deffayet, C., Langlois, D. 'Brane cosmological evolution in a bulk with cosmological constant.' Phys. Lett. B 477, 285 (2000).

[3] Shiromizu, T., Maeda, K., Sasaki, M. 'The Einstein equations on the 3-brane world.' Phys. Rev. D 62, 024012 (2000). DOI:10.1103/PhysRevD.62.024012

[4] Maartens, R. 'Brane-World Gravity.' Living Rev. Relativity 7, 7 (2004); update arXiv:1004.3962

[5] NANOGrav Collaboration (Agazie et al.). 'The NANOGrav 15-year Data Set: Evidence for a Gravitational-Wave Background.' ApJL 951, L8 (2023); arXiv:2306.16213

[6] EPTA+InPTA Collaboration (Antoniadis et al.). 'The second data release from the European Pulsar Timing Array (DR2): Search for an isotropic GWB.' A&A 678, A48 (2023).

[7] PPTA Collaboration (Reardon et al.). 'Search for an isotropic gravitational-wave background with the Parkes PTA.' (2023) arXiv:2306.16215

[8] Planck Collaboration (Aghanim et al.). 'Planck 2018 results. VI. Cosmological parameters.' A&A 641, A6 (2020). arXiv:1807.06209

[9] PDG Review: 'Big Bang Nucleosynthesis.' Review of Particle Physics, 2022 update (Fields et al.); PDG chapter.

[10] Cooke, R. J., Pettini, M., Steidel, C. C. 'One percent determination of the primordial deuterium abundance.' ApJ 855, 102 (2018).

[11] Robson, T., Cornish, N. J., Liu, C. 'The construction and use of LISA sensitivity curves.' Class. Quantum Grav. 36, 105011 (2019). arXiv:1803.01944

[12] Babak, S., Hewitson, M., Petiteau, A. 'LISA Sensitivity and SNR Calculations.' (2021) LISA-LCST-SGS-TN-001.