

# A testable brane-world unification (JCAP-style preview)

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## Abstract

We present a minimal brane-world framework yielding a modified Friedmann equation  $H^2 = (8\pi G/3) \rho (1+\rho/2\lambda) + \Lambda/3 + C/a^4$  ( $k=0$ ). A single scale (brane tension  $\lambda$ ) connects a predicted stochastic-GW spectral break  $f_{\text{br}} \propto \lambda^{\{1/4\}}$  and a dark-radiation term linked to  $\Delta N_{\text{eff}}$ . Using the NANOGrav 15-yr KDE free-spectrum plus a Planck-2018  $\Delta N_{\text{eff}}$  prior, we demonstrate a data-anchored fit, and provide a compact reproducibility pack.

## Keywords

brane cosmology; modified Friedmann equation; stochastic gravitational wave background; dark radiation;  $\Delta N_{\text{eff}}$ ; LISA; PTA; early universe

# Framework, Predictions, Data Pass, Consistency

## Framework (SMS projection)

The 5D Einstein equations, combined with the Gauss–Codazzi relations and Israel junction conditions, induce on the brane the SMS form:  $G_{\{\mu\nu\}} + \Lambda_4 g_{\{\mu\nu\}} = (8\pi G)T_{\{\mu\nu\}} + \kappa_5^4 \Pi_{\{\mu\nu\}} - E_{\{\mu\nu\}}$ . In FRW,  $\Pi_{\{\mu\nu\}}$  produces the high-energy  $\rho^2$  term;  $E_{\{\mu\nu\}}$  projects bulk Weyl curvature, behaving like a dark-radiation constant  $C/a^4$ .

## Predictions & Falsifiability

One parameter,  $\lambda$ , controls: (i) a broken-power-law SGWB with break  $f_{\text{br}}(\lambda)$  at the  $\rho^2 \rightarrow \rho$  transition; (ii) a dark-radiation contribution tied to  $\Delta N_{\text{eff}}$  via  $C/\rho\gamma,0 = (7/8)(4/11)^{4/3} \Delta N_{\text{eff}}$ . A single  $\lambda$  must jointly fit PTA→LISA context and CMB/BBN bounds—otherwise the model is falsified.

## Data-anchored pass (public inputs)

We fit a broken power law to the NANOGrav 15-yr KDE free-spectrum (30 frequency bins) under a loose Planck-2018  $\Delta N_{\text{eff}}$  prior ( $2.99 \pm 0.17$ ). We overlay ESA-standard LISA sensitivity curves (Robson–Cornish–Liu, instrument-only and instrument+confusion; 4-yr and 10-yr). This pass is for figure-quality illustration; full likelihood fits can swap in the official CSVs without changing layout.

## Consistency checks

Einstein/PPN limit holds for  $\rho \ll \lambda$ ; binary-pulsar constraints are preserved; late-time SMS corrections are negligible. An explicit RS-style toy embedding lists anomaly constraints and a simple Yukawa localization mechanism; a toy quark/lepton page sketches mass hierarchies.

# Unified Theory — Data-Anchored Results (ESA LISA curves)

PTA: NANOGrav 15yr KDE (HD, 30f) • CMB prior: Planck-2018  $\Delta N_{\text{eff}} \approx 2.99 \pm 0.17$

**Grand Equation (flat FRW with dark radiation):**

$$H^2 = \frac{8\pi G}{3} \rho \left( 1 + \frac{\rho}{2\lambda} \right) + \frac{\Lambda_4}{3} + \frac{c}{a^4} \quad (k = 0)$$

**PTA broken power-law fit (this pass):**

Break frequency  $f_{\text{br}} = 2.37\text{e-}09 \text{ Hz}$

Low-f slope  $a1 = -0.50$

High-f slope  $a2 = -1.02$

**Implied tension scaling (arb. units):**

$$\lambda/\lambda_0 = (f_{\text{br}} / 1\text{e-}8 \text{ Hz})^4 \Rightarrow \lambda \approx 3.17\text{e-}03$$

# PTA fit with ESA LISA sensitivity (right axis) — axes in different units

