Unified Brane-Cosmology: A Testable Route to Unification

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Abstract: We propose a higher-dimensional (bulk + brane) framework in which the 4D Friedmann equation on the brane acquires a high-energy rho^2 correction and a dark-radiation term C/a^4. A single parameter (brane tension lambda) links a measurable GW spectral break f_br ~ lambda^{1/4} to Delta N_eff. We outline falsifiable predictions and a pipeline for near-term tests with PTA->LISA and CMB/BBN.

1. Framework

Consider a 5D warped bulk with a 4D brane. Standard Model fields are confined to the brane; gravity propagates in the bulk. Junction conditions relate the extrinsic curvature to the brane stress tensor and tension lambda. The effective 4D field equation on the brane contains: (i) standard Einstein tensor with Lambda4, (ii) quadratic stress term Pi_mu nu, (iii) projected bulk Weyl term E_mu nu (dark radiation).

2. Effective 4D Equation on the Brane

G_mu nu + Lambda4 g_mu nu = (8*pi*G/c^4) T_mu nu + (kappa5^4/c^4) Pi_mu nu - E_mu nu. The quadratic term Pi_mu nu captures high-energy corrections at rho >> lambda. The Weyl term E_mu nu encodes bulk memory; for FRW it yields a radiation-like contribution rho_dr ~ C/a^4.

3. Cosmology Reduction (flat FRW)

For a perfect fluid on the brane with pressure p and density rho, we obtain H^2 = (8*pi*G/3) rho (1 + rho/(2*lambda)) + Lambda4/3 + C/a^4 - k/a^2. At early times (rho >> lambda) the rho^2 term dominates, giving a(t) ~ t^{1/4} in radiation era, distinct from standard a ~ t^{1/2}.

4. Observables and Single-Parameter Link

The brane tension lambda sets the energy scale where the rho^2 term turns off and the spectrum of primordial tensor modes transitions, producing a broken-power-law stochastic GW background. The same physics contributes an effective dark radiation density parameterized as Delta N_eff, hence a correlation f_br(lambda) <-> Delta N_eff.

5. Minimal Likelihood

We forecast by fitting a broken power-law SGWB to PTA points and a prior on Delta N_eff from BBN/CMB, with parameters {A_low, A_high, f_br(lambda), slopes}. The mapping f_br ~ lambda^{1/4} closes the model; a single lambda must satisfy both datasets. We compute posteriors for E_lambda (proxy for lambda) and Delta N_eff.

6. Comparison to Prior Work

Our setup follows the brane-cosmology literature (Randall-Sundrum, Binetruy-Deffayet-Langlois, Maartens) but emphasizes a single measurable link (f_br <-> lambda) and a joint fit with Delta N_eff. We discuss overlaps and differences in parameterization and phenomenology.

7. Limitations and Open Tasks

We do not yet present a concrete compactification reproducing SU(3)xSU(2)xU(1) and full spectra/Yukawas. Quantum completion and moduli stabilization are stated as future work. The present draft focuses on near-term falsifiable signals and a data-driven test.

8. Roadmap

Step A: replace synthetic CSVs with public PTA data; rerun likelihood; publish code+preprint. Step B: extend with LISA priors and structure-growth constraints; Step C: compactification map linking (k, M5, lambda) to low-energy observables, then submit to PRD/JCAP.

References (selected)

Key brane cosmology references (to be inserted with full citations): Randall & Sundrum (1999); Binetruy, Deffayet & Langlois (2000); Shiromizu, Maeda & Sasaki (2000); Maartens (2004) and subsequent developments on brane-world cosmology and dark radiation constraints.

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