Supplement: Derivations, Junctions, and Term Map

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A. Action and Junctions

Start from a bulk action S5 = (1/2 kappa5^2) ∫ d^5x sqrt(-g5) (R5 - 2 Lambda5) + ∫_brane d^4x sqrt(-g) (-lambda + L_matter). Apply Israel junction conditions across the brane: [K_mu nu] = -kappa5^2 (T_mu nu - (1/3) g_mu nu (T - lambda)).

B. Quadratic Term Pi_mu nu

Pi_mu nu = -(1/4) T_mu alpha T^alpha_nu + (1/12) T T_mu nu + (1/8) g_mu nu T_alpha beta T^alpha beta - (1/24) g_mu nu T^2. This generates the high-energy rho^2 correction when reduced to FRW.

C. Projected Weyl E_mu nu

E_mu nu = C_ABCD n^A n^C g_mu^B g_nu^D, with E^mu_mu = 0. For FRW, this yields an effective radiation term rho_dr = C/a^4 ("dark radiation").

D. Friedmann Reduction

For a flat FRW brane with perfect fluid p = w rho, the modified Friedmann equation becomes H^2 = (8*pi*G/3) rho (1 + rho/(2 lambda)) + Lambda4/3 + C/a^4. Early-time solution in radiation era: a(t) ~ t^{1/4}.

E. Observable Map

Break frequency f_br(lambda) scales with the crossover energy density ~ lambda; use f_br ~ k0 * lambda^{1/4}. Relate C/a^4 to Delta N_eff via rho_dr/rho_gamma = (7/8) (4/11)^{4/3} * Delta N_eff. Provide bounds priors from BBN/CMB.

F. Symbols and Units

lambda: brane tension (energy density). C: dark-radiation constant. M5: 5D Planck mass. k: warp parameter. H: Hubble parameter. a: scale factor. rho: energy density. All c = 1 conventions unless otherwise stated.