

# PRD Letter — Title & Abstract

Title: A testable brane-world unification:  $\rho^2$  cosmology, dark radiation, and a GW spectral break

Abstract: We present a minimal higher-dimensional (brane-world) framework that yields a modified 4D Friedmann equation  $H^2 = (8\pi G/3) \rho (1 + \rho/2\lambda) + \lambda/4 + C/a^4$  (flat FRW). A single physical scale—the brane tension  $\lambda$ —controls two independent observables: a broken-power-law stochastic gravitational-wave background with break frequency  $f_{\text{br}} \propto \lambda^{1/4}$ , and an early-universe radiation excess parameterized by  $\Delta N_{\text{eff}}$  via  $C$ . Using the public NANOGrav 15-year KDE free-spectrum (HD, 30 frequencies) and a loose Planck-2018 prior on  $\Delta N_{\text{eff}}$ , we demonstrate a data-anchored fit and provide a small reproducibility pack (CSV + script). The claim is falsifiable: one value of  $\lambda$  must simultaneously place the GW break and satisfy CMB/BBN bounds. We outline an explicit RS-type toy embedding of the Standard Model on the brane and show the GR/PPN limit for  $\rho \ll \lambda$ .

# PRD Letter — Equations & Setup

SMS:  $G_{\mu\nu} + \Lambda g_{\mu\nu} = (8\pi G) T_{\mu\nu} + (\kappa_5^4) \Pi_{\mu\nu} - E_{\mu\nu}.$

FRW (flat):  $H^2 = (8\pi G/3) \rho (1 + \rho/2\Lambda) + \Lambda/3 + C/a^4.$

Early-time scale factor:  $a(t) \sim t^{(1/4)} (\rho^2 \text{ era}).$

# PRD Letter — Predictions & Falsifiability

GW break:  $f_{\text{br}} \propto \lambda^{1/4}$ ; dark radiation: C maps to  $\Delta N_{\text{eff}}$ .

One  $\lambda$  must fit both PTA→LISA context and CMB/BBN bounds.

# PRD Letter — Data-Anchored Two-Pager Summary

NANOGrav 15yr KDE (HD, 30f) + Planck-2018 Delta  $N_{\text{eff}}$  prior.

Broken-power-law fit; best-fit table in SM; LISA appendix for sensitivity context.

# PRD Letter — Discussion & Outlook

RS toy embedding (anomaly checklist, Yukawas via localization, radion stabilization).

PPN/GR consistency for  $\rho \ll \lambda$ ; next steps: full likelihood and compactification.