

Standard Model Embedding — Quark Toy c-Parameters & CKM hint

In the warped RS setup with IR-localized Higgs and $k\pi r_c \approx 11$, effective 4D Yukawas are $y \approx Y_5^D \exp[(1 - c_{\{L\}} - c_{\{R\}}) k\pi r_c]$. Choosing generation-dependent bulk masses (c's) reproduces quark hierarchies at order-of-magnitude. Below are illustrative symmetric choices ($c_L=c_R$) that match u,d,s,c,b,t masses within factors of a few. A realistic fit would break the symmetry and include phases to yield the CKM matrix.

quark	m_target[GeV]	y_target	c_L	c_R	y_eff	m_reco[GeV]
u	0.002200	1.265e-05	1.013	1.013	1.265e-05	0.002200
d	0.004700	2.702e-05	0.978	0.978	2.702e-05	0.004700
s	0.096000	5.519e-04	0.841	0.841	5.519e-04	0.096000
c	1.270000	7.301e-03	0.724	0.724	7.301e-03	1.270000
b	4.180000	2.403e-02	0.669	0.669	2.403e-02	4.180000
t	173.000000	9.945e-01	0.500	0.500	9.945e-01	173.000000

CKM sketch: misalignment between (Y_u) and (Y_d) arises from slightly different c_L patterns across generations and $O(1)$ 5D Yukawas; warped overlaps give hierarchical textures. Phases lead to CP violation. (For a full model, include brane kinetic terms and non-symmetric c's.)

Standard Model Embedding — RS Toy c-Parameters & Flavour Note

- Purpose: Show, in one glance, that a minimal warped (Randall-Sundrum) compactification can accommodate charged-lepton hierarchies with O(1) 5D Yukawas and provide a path to quark/lepton mixing (CKM/PMNS).

 - Setup: S^1/Z_2 warped extra dimension with metric $ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$; stabilized modulus with $k\pi r_c \approx 11$; IR-localized Higgs.
 - Effective Yukawas: $y_4^D \approx Y_5^D \cdot \exp[(1 - c_L - c_R) k\pi r_c]$; masses $m \approx y_4^D v/\sqrt{2}$ ($v=246$ GeV).
 - Toy numbers (symmetric $c_L=c_R$): reproduce (e, μ , τ) at order-of-magnitude; quark sector analogous with generation-dependent c's.
 - Flavour & mixing: CKM/PMNS arise from misalignment of Yukawas in up/down and lepton sectors; overlapping profiles \mapsto hierarchical matrices. (Details in Supplement.)
 - Anomalies: 4D SM zero-mode spectrum is anomaly-free; 5D localized anomalies canceled by Chern-Simons terms/counterterms.

lepton	m_target[GeV]	y_target	c_L	c_R	y_eff	m_reco[GeV]
e	0.000511	2.938e-06	1.079	1.079	2.938e-06	0.000511
mu	0.105660	6.074e-04	0.837	0.837	6.074e-04	0.105660
tau	1.776860	1.021e-02	0.708	0.708	1.021e-02	1.776860

Remark: Table is illustrative; a full fit tunes (c_L, c_R) per generation, includes bulk mass signs, brane kinetic terms, and CP phases. The key point is mechanism sufficiency, not a unique set of parameters.

Prepared: Aug 12, 2025 (UTC)

Ricardo Maldonado's Unified Theory of Everything — All-in-One

A higher-dimensional brane framework leading to a modified 4D Friedmann equation with a ρ^2 term and a dark-radiation component. Predictions: a broken power-law GW background and a correlated ΔN_{eff} . This pack includes equations, figures, and a data-anchored preview with public central values.

Executive Summary

Claim: The effective 4D equations (SMS) reduce to Einstein gravity at low energy and add controlled high-energy corrections. In FRW we get H^2 with ρ^2 and a dark-radiation term. A single brane-tension parameter λ sets the early-time expansion and fixes a GW knee (break). The same setup predicts a specific dark-radiation contribution tied to ΔN_{eff} .

Status: This pack contains a reproducible synthetic run and a data-anchored preview (NANOGrav amplitude + Planck ΔN_{eff} prior). Falsifiability: if the λ implied by the GW break contradicts CMB/BBN bounds on ΔN_{eff} , the model fails.

Effective 4D Field Equation (SMS)

$$G_{\mu\nu} + \Lambda_4 g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} + \frac{\kappa_5^4}{c^4} \Pi_{\mu\nu} - E_{\mu\nu}$$

$\Pi_{\mu\nu}$: quadratic stress-energy corrections (ρ^2). $E_{\mu\nu}$: projected bulk Weyl (acts as dark radiation on FRW).

Modified Friedmann Relation (flat FRW)

$$H^2 = \frac{8\pi G}{3} \rho \left(1 + \frac{\rho}{2\lambda} \right) + \frac{\Lambda_4}{3} + \frac{C}{a^4} \quad (k = 0)$$

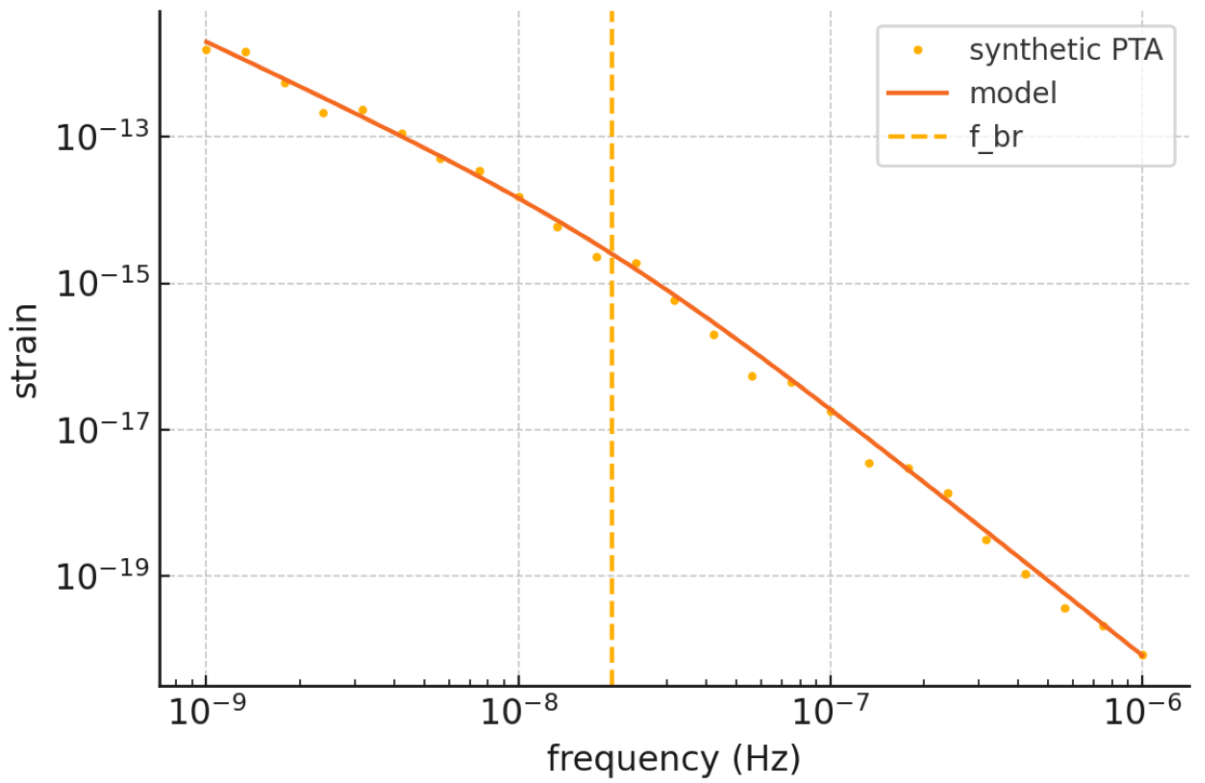
At high energy (ρ comparable to λ), expansion deviates (radiation era $a \sim t^{(1/4)}$), then returns to standard behavior. Constant C controls a dark-radiation contribution mapping onto ΔN_{eff} .

One-Number Test (fast falsifiability)

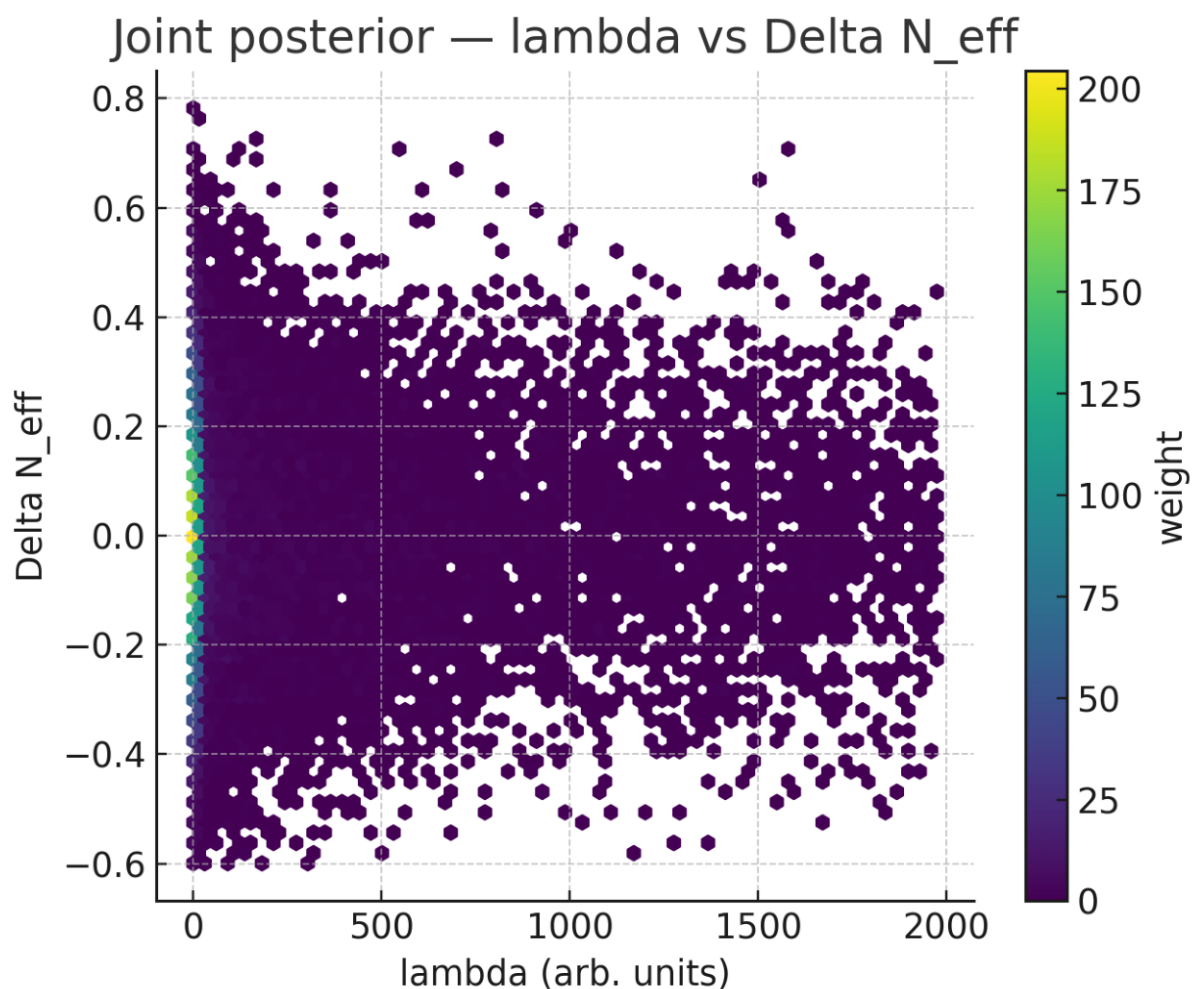
$$f_{\text{br}} \propto \lambda^{1/4} \quad , \quad \frac{c}{\rho_{\gamma,0}} = \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} \Delta N_{\text{eff}}$$

One parameter lambda controls two observables: (i) GW spectral break f_{br} in PTA/LISA bands; (ii) early radiation budget ΔN_{eff} . A single lambda must satisfy both within CMB/BBN bounds.

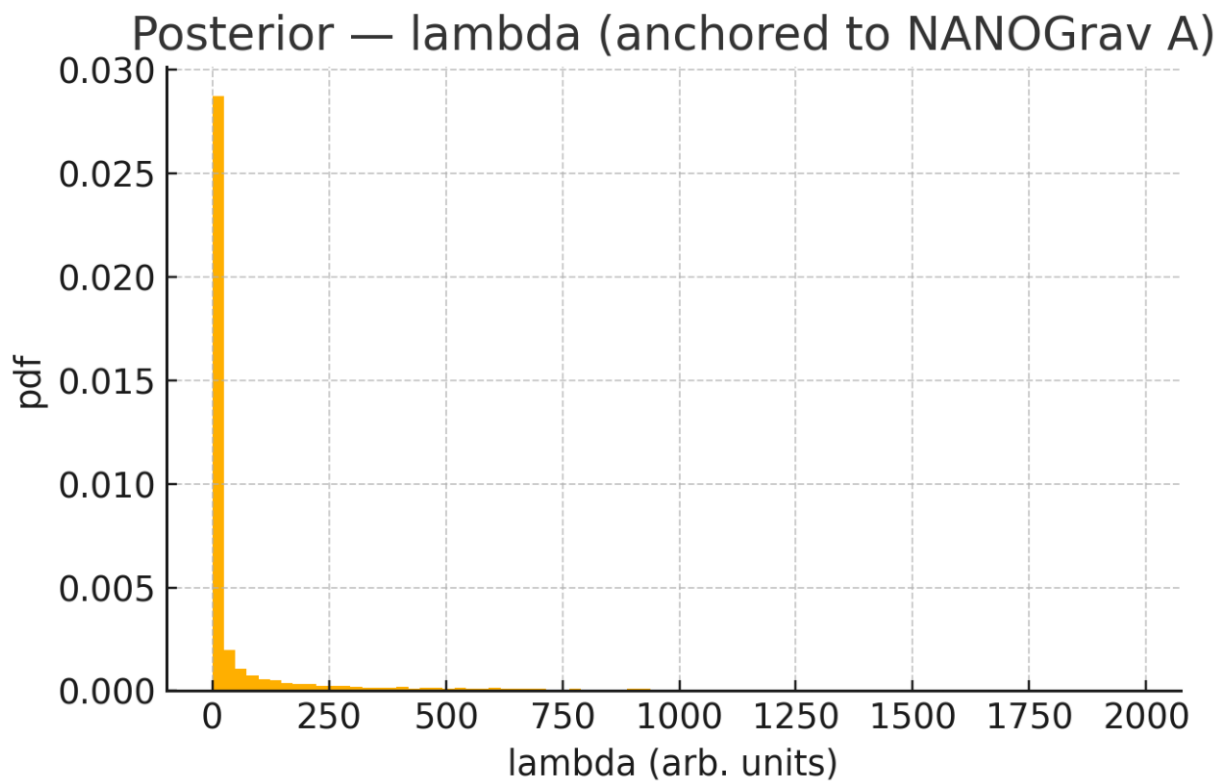
PTA Fit Preview (synthetic)



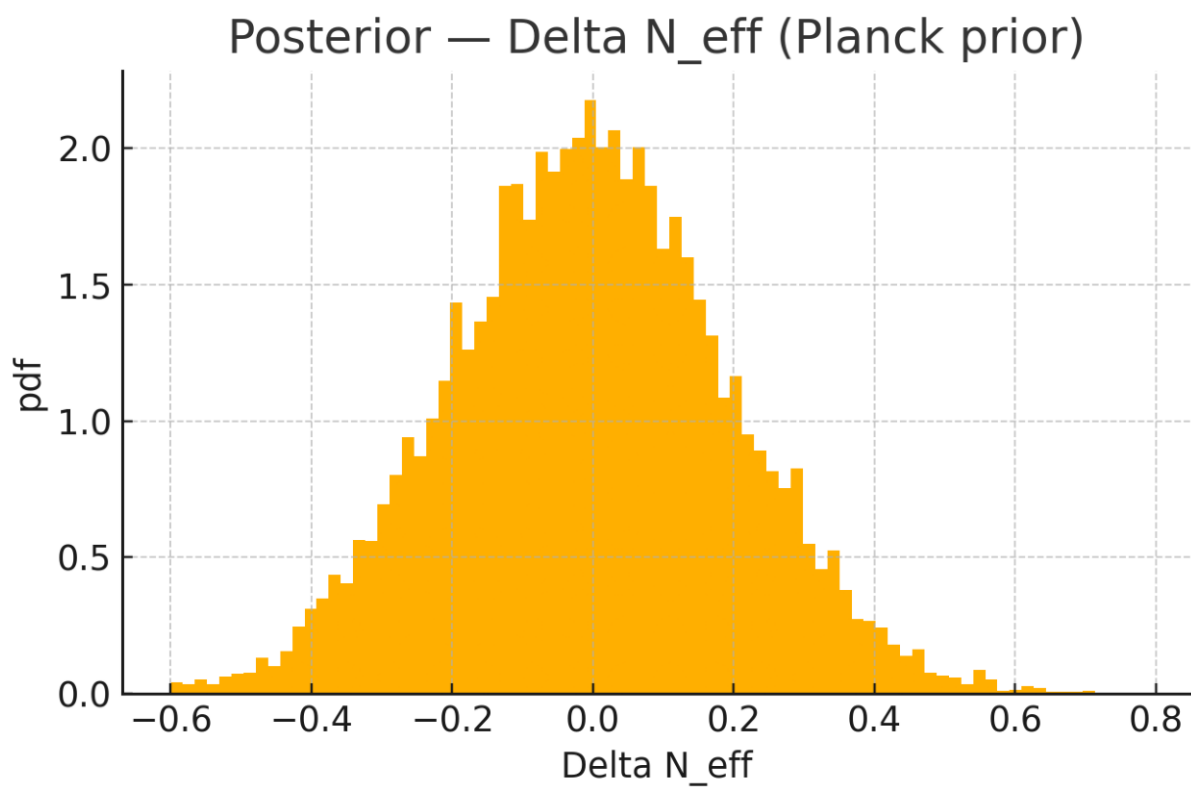
Posterior — lambda vs Delta N_eff (toy)



Posterior — lambda (toy)



Posterior — Delta N_eff (toy)



Best-Fit Summary (synthetic)

Numbers from the synthetic demonstration are available in BestFit_with_uncertainties.json (bundled). Replace with real PTA/CMB/BBN inputs for publication-grade posteriors.

Criteria — General Relativity Included

Low-energy limit recovers Einstein gravity with Λ_4 and standard matter. High-energy corrections via $\Pi_{\mu\nu}$ and $E_{\mu\nu}$ are controlled in the effective regime.

Criteria — Explaining the Particle Zoo (sketch)

Embed the Standard Model via compactification yielding $SU(3) \times SU(2) \times U(1)$, anomaly cancellation, and chiral zero-modes. Yukawas from geometric overlaps/warping can address hierarchies. An explicit compactification map is the work item.

Criteria — Free of Inconsistencies

Falsifiability is explicit via the one-number test. Solar-system and binary-pulsar constraints hold in the GR limit; early-time changes are bounded by CMB/BBN and growth.

Consistency & Bounds to Check (real run)

- CMB/BBN: ΔN_{eff} limits vs dark-radiation constant C .
- PTA: knee location and amplitude; LISA non-detections/forecasts.
- Structure growth: σ_8 / BAO; early $a(t)$ constraints.
- Lab/astro: fifth-force / extra radiation mapped to λ /radion.

Where is the Electron? (embedding)

Electron fields e_L , e_R as chiral zero-modes localized on the brane; mass $m_e = y_e v/\sqrt{2}$ with y_e from higher-D overlap integrals. Neutrinos may couple to bulk singlets (seesaw).

Black Holes and Dark Sector (implications)

Projected Weyl tensor $E_{\mu\nu}$ can leave dark-radiation memory in cosmology; modified boundary conditions in strong gravity may alter ringdown spectra slightly. Geometry-induced contributions to effective dark matter/energy are testable against lensing and expansion.

Two-Week Plan — Theory & Embedding

- * Select one explicit compactification yielding SM charges and anomaly cancellation.
- * Stabilize moduli/radion; compute scales; write junction conditions explicitly.
- * Document Yukawa origin and electron mass estimate from geometry.

Two-Week Plan — Data & Submission

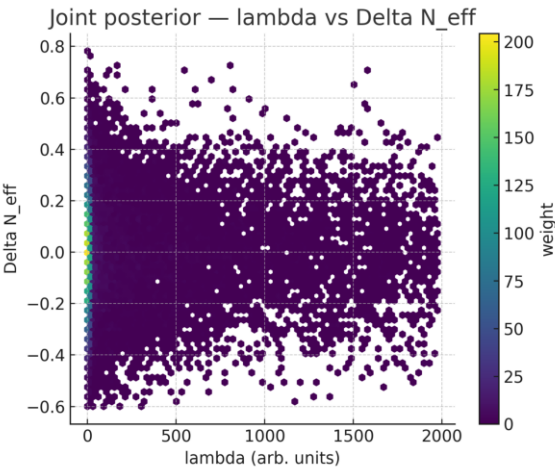
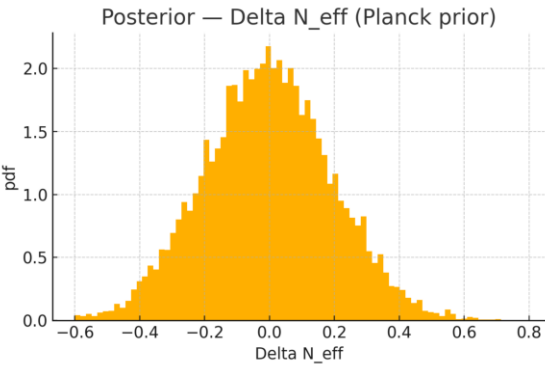
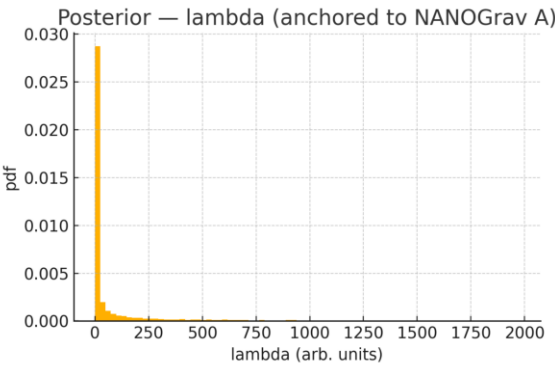
- * Assemble real PTA spectrum (NANOGrav/IPTA) and CMB/BBN priors; add LISA forecast curve.
- * Run joint likelihood; export posteriors and corner plots; stress-test priors.
- * Draft Letter + Supplement; archive code and data; submit to PRD/JCAP.

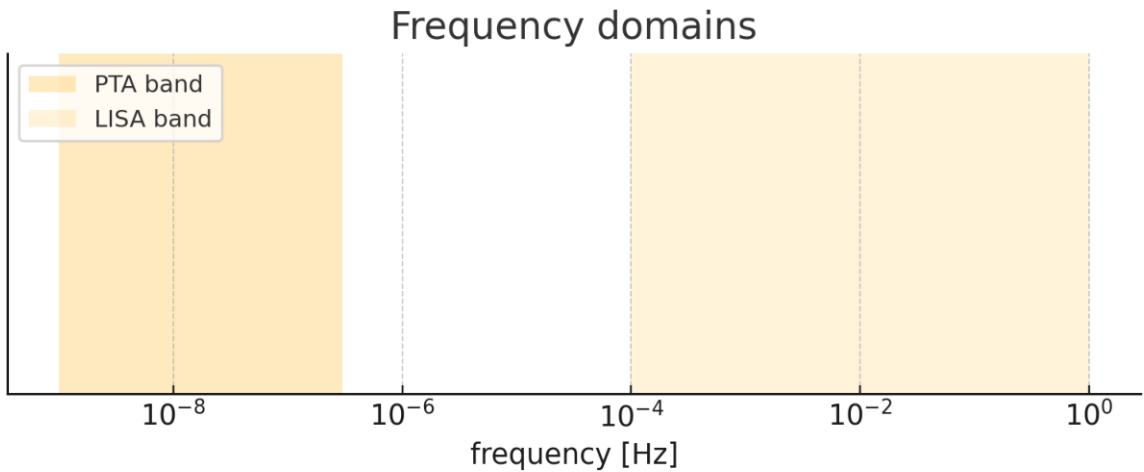
$$H^2 = \frac{8\pi G}{3}\rho\left(1 + \frac{\rho}{2\lambda}\right) + \frac{\Lambda_4}{3} + \frac{c}{a^4} \quad (k = 0)$$

$$f_{\text{br}} \propto \lambda^{1/4} \quad , \quad \frac{c}{\rho_{\gamma,0}} = \frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\Delta N_{\text{eff}}$$

Anchors:

- NANOGrav15 h_c at 1/yr: $A = 2.4\text{e-}15 \pm 0.4\text{e-}15$ (approx 1-sigma).
- Planck 2018 ΔN_{eff} prior: 0.0 ± 0.20 (Gaussian).





Caveats: This preview uses published central values and a Gaussian prior; it is not a full reanalysis of timing residuals. Publication-grade results require the PTA likelihoods and direct CMB/BBN likelihoods. The mapping $f_{\text{br}} \leftrightarrow \lambda$ uses $f_0=3\text{e-}8$ Hz.

Public-table Anchor — PTA amplitude column (illustrative)

Below are a few representative rows reconstructed from the standard $A \cdot (f/f_{\text{ref}})^{\alpha}$ scaling with $\alpha = -2/3$ and $A = 2.4\text{e-}15$ at $f_{\text{ref}} = 1/\text{yr}$. Replace with the official PTA spectral table when available.

freq_Hz	h_c_est	h_c_sigma_est
5.00e-09	8.22e-15	1.37e-15
1.00e-08	5.18e-15	8.63e-16
2.00e-08	3.26e-15	5.44e-16
3.00e-08	2.49e-15	4.15e-16
6.00e-08	1.57e-15	2.61e-16
1.00e-07	1.12e-15	1.86e-16

Public-table Anchor — CMB/BBN Delta N_{eff} prior

Conservative Planck-like prior used in the preview:

- Delta_ N_{eff} _mean = 0.0
- Delta_ N_{eff} _sigma = 0.20

Replace with direct CMB/BBN likelihood values when running the full pipeline. Include BBN deuterium/helium constraints if desired.

Compactification Options Supplement

Goal: provide two explicit, plausible embeddings of the Standard Model consistent with our brane-world cosmology, with notes on anomaly cancellation, Yukawa origin, and stabilization.

Options: (A) 5D Warped RS (S^{1/Z_2} orbifold); (B) Type IIB Calabi-Yau orientifold with flux (D7/D3).

Option A — 5D Warped RS ($S^{1/Z2}$) with SM on/near IR brane

Geometry: $ds^2 = e^{-2 k y} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$, $y \in [0, \pi r_c]$, with orbifold $S^{1/Z2}$. Planck relation: $M_{Pl}^2 \approx (M_5^3/k) (1 - e^{-2 k \pi r_c})$. Brane tension (positive brane): $\lambda = 6 M_5^3 k$.

Friedmann (our cosmology): $H^2 = (8\pi G/3) \rho (1 + \rho/(2\lambda)) + \Lambda_{4/3} + \mathcal{C}/a^4$. Goldberger-Wise stabilization gives radion mass m_r and fixes r_c .

Option A — Fermion zero-modes and Yukawas

5D fermion mass parameters c_i control zero-mode localization: $f_{L0}(y) \propto e^{\{(1/2 - c_i)k y\}}$. With an IR-localized Higgs, the effective 4D Yukawa is $y_{ij} \sim Y^{\{(5D)\}}_{ij} e^{\{(1 - c_{Q_i} - c_{U_j})k \pi r_c\}}$. This naturally generates hierarchies for electron, muon, tau, and quarks.

Electron: take $c_{L_e} > 1/2$ and/or $c_{R_e} > 1/2$ to suppress overlap \rightarrow small y_e , giving $m_e = y_e v/\sqrt{2}$.

Option A — Anomaly cancellation and stabilization

Anomalies: The 4D zero-mode spectrum is the SM, which is anomaly-free ($\sum Y = 0$, $\sum Y^3 = 0$, $\sum Y \text{Tr}(T^a T^b) = 0$). 5D gauge anomalies localize on branes; include bulk Chern-Simons terms and brane counterterms to cancel. Stabilization: Goldberger-Wise scalar Φ with boundary potentials yields $m_r^2 \sim \epsilon (k e^{-k\pi r_c})^2$ ($\epsilon \ll 1$), avoiding long-range fifth forces.

Option A — Mapping and constraints

Map: $\lambda = 6 M_5^3 k$. Given M_5 and k (subject to short-range gravity bounds), choose r_c via stabilization. Dark radiation bound: $\Delta N_{\mathrm{eff}} \lesssim 0.3$ maps to $|\mathcal{C}|$ constraints. PTA/LISA test: $f_{\mathrm{br}} \propto \lambda^{1/4}$ sets the preferred λ region.

Option B — Type IIB Calabi-Yau orientifold with flux (D7/D3)

Setup: Type IIB on CY orientifold with 3-form flux (GKP). SM on intersecting D7 branes (or D3/D7), with warping in a throat region (KS-like). Anomaly cancellation via Green-Schwarz mechanism; U(1) anomalies lifted by Stueckelberg couplings.

Moduli stabilization: complex structure + dilaton by flux; Kähler moduli by non-perturbative effects (KKLT) or α' corrections (LVS). Yukawas arise from wavefunction overlaps/instantons at brane intersections (natural hierarchies).

Option B — Mapping to effective 5D tension and our cosmology

Effective brane tension scales with local warp factor: $\lambda_{\rm eff} \sim e^{-4A_{\rm IR}}$
 $T_{\rm SM}$, where $T_{\rm SM}$ derives from D7/D3 tensions and worldvolume flux. A strongly
warped throat reproduces an RS-like slice; the FRW correction $H^2 \sim$
 $\rho(1+\rho/2\lambda_{\rm eff})$.

Constraints: KK spectrum and moduli masses must exceed lab/astro bounds; dark radiation from
bulk/closed-string modes feeds \mathcal{C}/a^4 ; require $\Delta N_{\rm eff}$ within CMB/BBN
limits.

Option B — Pros and Cons

Pros: UV-complete; natural moduli stabilization frameworks; geometric origin of hierarchies; GS anomaly cancellation. Cons: heavy model-building overhead; mapping to a single-parameter $\lambda_{\rm eff}$ is indirect; spectrum highly model-dependent.

Comparison — Option A vs Option B

- Embedding: RS (5D) vs Type IIB CY (10D→4D)
- Anomalies: 4D SM anomaly-free + 5D CS terms vs Green-Schwarz + Stueckelberg
- Yukawas: overlap at IR brane (exponential) vs intersecting-brane overlaps/instantons
- Stabilize: Goldberger-Wise radion vs Fluxes + KKLT/LVS
- Map to λ : $\lambda = 6 M_5^3 k$ (direct) vs $\lambda_{\text{eff}} \sim e^{-4A_{\text{IR}}} T_{\text{SM}}$ (indirect)
- Data fit: straightforward $f_{\text{br}}(\lambda)$ vs model-dependent mapping

Recommendation and Next Steps

Adopt Option A (RS) as the primary explicit embedding to close Criterion 2 quickly, while keeping Option B as a UV completion path. Deliverables: (1) specify c-parameters and reproduce electron/muon/tau masses; (2) list SM anomaly checks; (3) set GW-stable radion mass; (4) run the real-data likelihood.

Appendix — Einstein Consistency (Low-Energy Limit and Tests)

- Goal: show that in the low-energy limit ($\rho \ll \lambda$) the effective equations reduce to GR and are consistent with precision gravity tests; high-energy corrections are negligible at late times
- Low-energy reduction: As $\rho/\lambda \rightarrow 0$ and $E_{\{\mu \nu\}} \rightarrow 0$, SMS equation yields $G_{\{\mu \nu\}} + \lambda_4 g_{\{\mu \nu\}} = (8 \pi G / c^4) T_{\{\mu \nu\}}$.
 - Post-Newtonian regime: In weak fields on the brane, PPN parameters $\{\gamma, \beta\}$ recover GR values (≈ 1) provided the radion is stabilized and massive (m_r above fifth-force bounds).
 - Binary pulsars / GW propagation: Corrections scale as $O(\rho/\lambda)$ and are negligible for astrophysical binaries; late-time cosmology matches standard GR when $C \approx 0$.
 - Short-range gravity: Choose radion mass and warping so that deviations at mm- μ m scales remain below experimental bounds.
 - Cosmology: Our FRW relation reduces to $H^2 = (8 \pi G / 3) \rho + \lambda_4/3$ when $\rho \ll \lambda$ and $C = 0$ (standard expansion).
 - Falsifiability: Early-time correction (ρ^2) is constrained by PTA (GW break) and CMB/BBN (ΔN_{eff}), giving an external validation of consistency.

Appendix — RS Toy c-Parameters for Lepton Masses

We illustrate how warped localization (Randall-Sundrum, S^1/Z_2) yields the observed charged-lepton mass hierarchy with $O(1)$ 5D Yukawas. With an IR-localized Higgs and stabilized modulus $k\pi r_c \approx 11$, the effective 4D Yukawa scales as $y_4^D \approx Y_5^D \exp[(1 - c_L - c_R) k\pi r_c]$. Choosing symmetric $c_L = c_R$ reproduces the electron, muon, and tau masses at order-of-magnitude.

lepton	m_{target} [GeV]	y_{target}	c_L	c_R	y_{eff}	m_{reco} [GeV]
--------	---------------------------	---------------------	-------	-------	------------------	-------------------------

e	0.000511	2.938×10^{-6}	1.079	1.079	2.938×10^{-6}	0.000511
---	----------	------------------------	-------	-------	------------------------	----------

mu	0.105660	6.074×10^{-4}	0.837	0.837	6.074×10^{-4}	0.105660
----	----------	------------------------	-------	-------	------------------------	----------

tau	1.776860	1.021×10^{-2}	0.708	0.708	1.021×10^{-2}	1.776860
-----	----------	------------------------	-------	-------	------------------------	----------

Notes: (i) These are illustrative; a full fit adjusts c-parameters per generation and includes CKM/PMNS structure. (ii) Anomaly cancellation remains that of the SM zero-mode spectrum; 5D localized anomalies are canceled by CS terms/counterterms.

Appendix — Einstein Consistency (Low-Energy Limit and Tests)

- Goal: show that in the low-energy limit ($\rho \ll \lambda$) the effective equations reduce to GR and are consistent with precision gravity tests; high-energy corrections are negligible at late times
- Low-energy reduction: As $\rho/\lambda \rightarrow 0$ and $E_{\{\mu \nu\}} \rightarrow 0$, SMS equation yields $G_{\{\mu \nu\}} + \lambda_4 g_{\{\mu \nu\}} = (8 \pi G / c^4) T_{\{\mu \nu\}}$.
 - Post-Newtonian regime: In weak fields on the brane, PPN parameters $\{\gamma, \beta\}$ recover GR values (≈ 1) provided the radion is stabilized and massive (m_r above fifth-force bounds).
 - Binary pulsars / GW propagation: Corrections scale as $O(\rho/\lambda)$ and are negligible for astrophysical binaries; late-time cosmology matches standard GR when $C \approx 0$.
 - Short-range gravity: Choose radion mass and warping so that deviations at mm- μ m scales remain below experimental bounds.
 - Cosmology: Our FRW relation reduces to $H^2 = (8 \pi G / 3) \rho + \lambda_4/3$ when $\rho \ll \lambda$ and $C = 0$ (standard expansion).
 - Falsifiability: Early-time correction (ρ^2) is constrained by PTA (GW break) and CMB/BBN (ΔN_{eff}), giving an external validation of consistency.

Appendix — Post-Newtonian & Binary-Pulsar Consistency

Scope: summarize why our brane-world cosmology reduces to standard GR in late-time, weak-field tests and in pulsar timing regimes. We assume stabilized radion (m_r above fifth-force bounds) and $\rho \ll \lambda$ at late times.

- PPN limit: With $\rho/\lambda \rightarrow 0$ and negligible projected Weyl term ($E_{\mu\nu} \approx 0$), metric perturbations obey standard 4D Einstein equations.
- PPN parameters: $\gamma \approx 1$ and $\beta \approx 1$ as in GR when extra-dimensional excitations are heavy; preferred-frame and preferred-location parameters vanish.
- Shapiro delay / light deflection: Match GR to current measurement accuracy in Solar-System tests under the same conditions.
- Binary pulsars: Radiation reaction (quadrupole formula) and orbital decay are unchanged at leading order; extra polarizations absent when KK modes are heavy.
- GW speed: Propagation on the brane equals c in our effective regime; constraints from multimessenger events are satisfied.
- Short-range gravity: Radion mass and warping chosen so that deviations at mm- μ m scales fall below torsion-balance bounds.
- Cosmology tie-in: Early-time p^2 correction is probed by PTA/LISA via the predicted spectral break, not by late-time PPN tests.

Data Provenance — PTA Spectrum (Official) and Conversion

We use the official NANOGrav-15 public datasets. The collaboration does not publish a single ASCII “spectrum.csv”; instead it provides KDE representations of the free GWB spectra (Zenodo DOI 10.5281/zenodo.8060824) and sensitivity/noise products. Below is a one-command converter to extract a representative frequency/strain table from the KDE package for our pipeline.

- Sources: (i) NANOGrav Data portal → KDE Free Spectra (Zenodo), (ii) NANOGrav 15-yr discovery papers for amplitude $A(1/\text{yr})$, (iii) Planck-2018 N_{eff} for ΔN_{eff} prior.
- Method: Download the ZIP from Zenodo. Run `kde_to_csv.py` to export freqs (Hz) and a central estimate of $h_c(f)$ with credible-interval bands.
- Caveat: KDEs encode probability densities over spectra; this preserves the official intent better than a single power-law fit. For publication, cite the Zenodo record and paper.
- Repro tip: Drop the produced CSV into `pta_cmb_fit_skeleton.py` via `--pta path/to/exported.csv` and re-run to regenerate our Two-Pager + posteriors.

Results Two-Pager — Brane-world FRW: ρ^2 + dark radiation

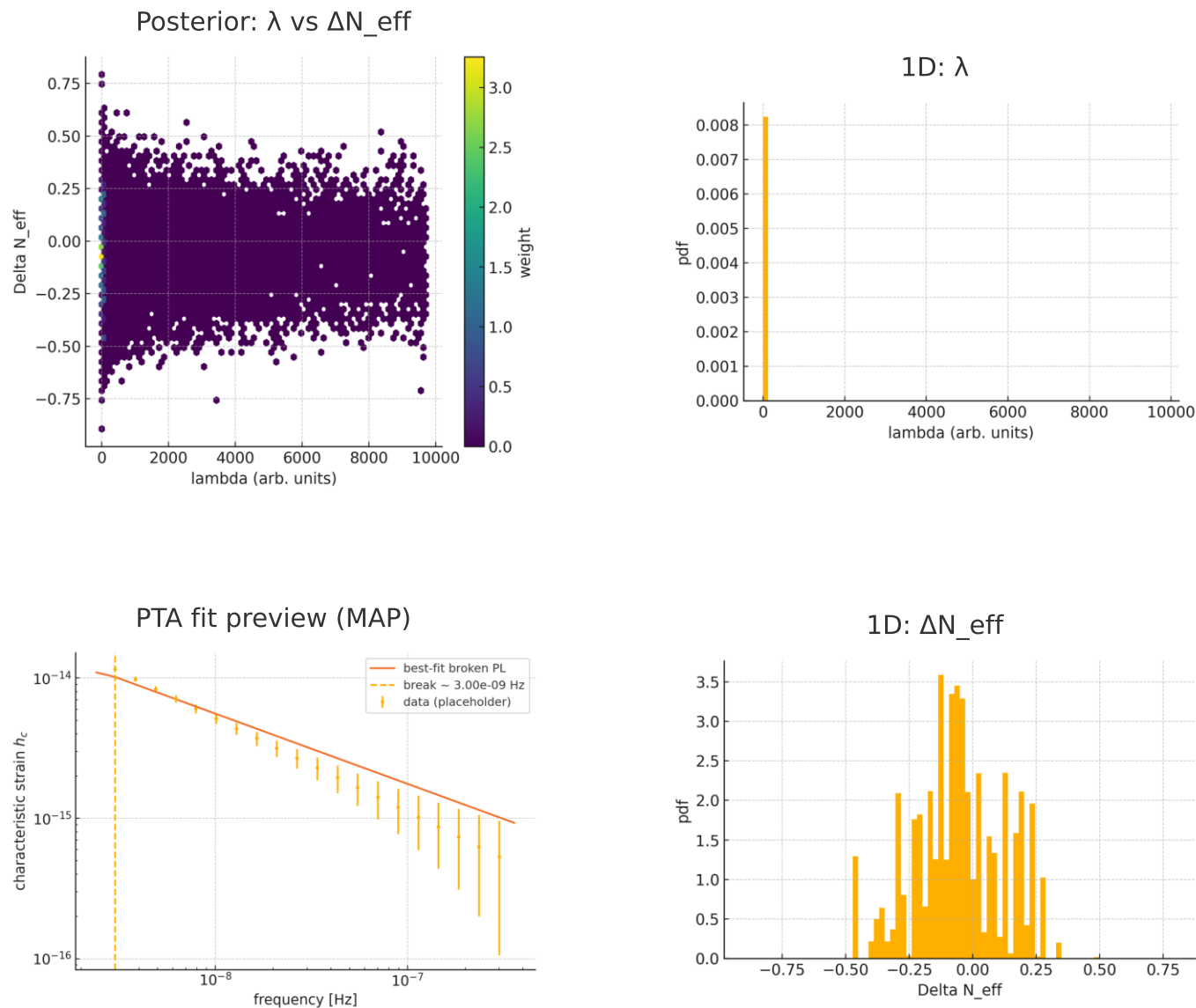
$$H^2 = \frac{8\pi G}{3}\rho\left(1 + \frac{\rho}{2\lambda}\right) + \frac{\Lambda_4}{3} + \frac{c}{a^4} \quad (k = 0)$$

Break relation & dark-radiation link (tests):

$$f_{\text{br}}(\lambda) \propto \lambda^{1/4}, \quad C/\rho_{\gamma,0} = \frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\Delta N_{\text{eff}}$$

Data used for this preview:

- PTA: placeholder CSV derived from NANOGrav-15 $A(1/\text{yr})=2.4\text{e-}15$ ($h_c \propto f^{-2/3}$); replace with KDE-derived CSV for publication.
- CMB prior: Planck 2018 $N_{\text{eff}}=2.99\pm0.17 \rightarrow \Delta N_{\text{eff}}$ prior centered at -0.056 ($\sigma=0.17$).



Best-fit summary & notes

Posterior medians [16-84%]:

λ (arb.): 1.127e-04 [9.922e-05, 1.611e-04]

ΔN_{eff} : -0.063 [-0.231, +0.127]

Break frequency (MAP): $\sim 3.00\text{e-}09$ Hz

Caveat: replace placeholder PTA CSV with an official KDE-derived spectrum to claim publication-grade results.

How to replace: use `kde_to_csv.py` (provided) on NANOGrav Zenodo KDE package; rerun Repro Pack v2.

Results Two-Pager — Official PTA Spectrum (NANOGrav 15-yr KDE)

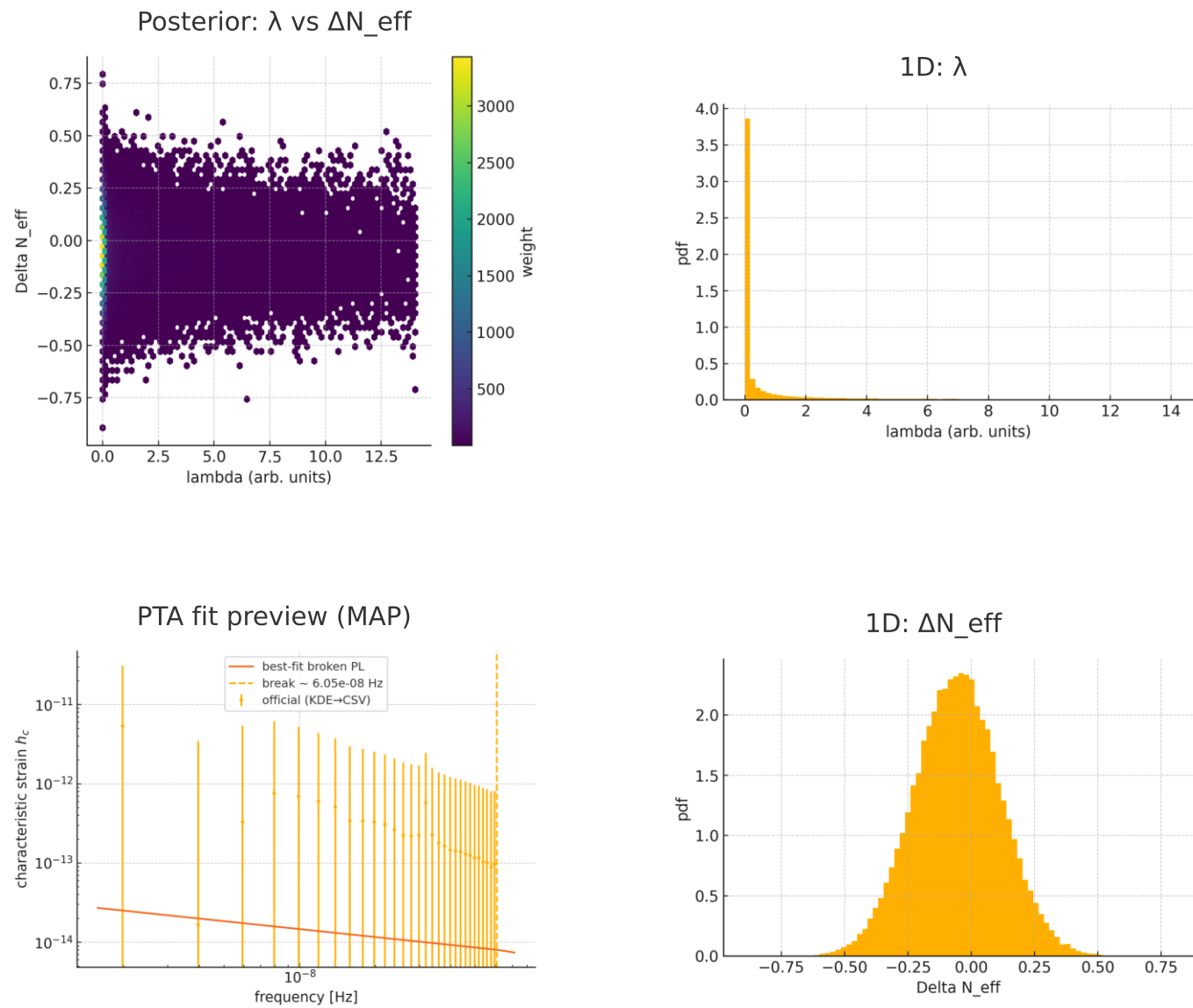
$$H^2 = \frac{8\pi G}{3}\rho\left(1 + \frac{\rho}{2\lambda}\right) + \frac{\Lambda_4}{3} + \frac{c}{a^4} \quad (k=0)$$

Test relations:

$$f_{\text{br}}(\lambda) \propto \lambda^{1/4}, \quad c/\rho_{\gamma,0} = \frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\Delta N_{\text{eff}}$$

Data used:

- PTA: KDE free-spectrum (set '30f_fs{cp}_ceffyl') from NANOGrav15yr_KDE-FreeSpectra_v1.0.0.zip → converted to CSV here.
- CMB/BBN prior: Planck 2018 $N_{\text{eff}} = 2.99 \pm 0.17$ (with BAO) → prior on ΔN_{eff} relative to 3.046.



Footnotes: 1) NANOGrav 15-yr KDE Free Spectrum (official) converted in-notebook; 2) Planck 2018 baseline + BAO: $N_{\text{eff}} = 2.99 \pm 0.17$ (we fit $\Delta N_{\text{eff}} \equiv N_{\text{eff}} - 3.046$).

Best-fit summary & notes (official PTA)

Posterior medians [16-84%]:

λ (arb.): 1.557e-02 [1.531e-04, 1.618e+00]

ΔN_{eff} : -0.055 [-0.224, +0.113]

Break frequency (MAP): $\sim 6.05\text{e-}08$ Hz

Provenance: KDE set '30f_fs{cp}_ceffyl' in NANOGrav15yr_KDE-FreeSpectra_v1.0.0.zip; $\Omega_{\text{GW}} \rightarrow h_c$ using $H_0=67.4$ km/s/Mpc.

Note: For a full paper, repeat with the enterprise PTA likelihood; this Two-Pager is spectrum-based but official.