

# Standard Model Embedding — Quark Toy c-Parameters & CKM hint

In the warped RS setup with IR-localized Higgs and  $k\pi r_c \approx 11$ , effective 4D Yukawas are  $y \approx Y_5^D \exp[(1 - c_{\{L\}} - c_{\{R\}}) k\pi r_c]$ . Choosing generation-dependent bulk masses (c's) reproduces quark hierarchies at order-of-magnitude. Below are illustrative symmetric choices ( $c_L=c_R$ ) that match u,d,s,c,b,t masses within factors of a few. A realistic fit would break the symmetry and include phases to yield the CKM matrix.

quark	m_target[GeV]	y_target	c_L	c_R	y_eff	m_reco[GeV]
u	0.002200	1.265e-05	1.013	1.013	1.265e-05	0.002200
d	0.004700	2.702e-05	0.978	0.978	2.702e-05	0.004700
s	0.096000	5.519e-04	0.841	0.841	5.519e-04	0.096000
c	1.270000	7.301e-03	0.724	0.724	7.301e-03	1.270000
b	4.180000	2.403e-02	0.669	0.669	2.403e-02	4.180000
t	173.000000	9.945e-01	0.500	0.500	9.945e-01	173.000000

CKM sketch: misalignment between ( $Y_u$ ) and ( $Y_d$ ) arises from slightly different  $c_L$  patterns across generations and  $O(1)$  5D Yukawas; warped overlaps give hierarchical textures. Phases lead to CP violation. (For a full model, include brane kinetic terms and non-symmetric c's.)

# Standard Model Embedding — RS Toy c-Parameters & Flavour Note

- Purpose: Show, in one glance, that a minimal warped (Randall-Sundrum) compactification can accommodate charged-lepton hierarchies with O(1) 5D Yukawas and provide a path to quark/lepton mixing (CKM/PMNS).
- Setup:  $S^1/Z_2$  warped extra dimension with metric  $ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$ ; stabilized modulus with  $k\pi r_c \approx 11$ ; IR-localized Higgs.
  - Effective Yukawas:  $y_4^D \approx Y_5^D \cdot \exp[(1 - c_L - c_R) k\pi r_c]$ ; masses  $m \approx y_4^D v/\sqrt{2}$  ( $v=246$  GeV).
  - Toy numbers (symmetric  $c_L=c_R$ ): reproduce (e,  $\mu$ ,  $\tau$ ) at order-of-magnitude; quark sector analogous with generation-dependent c's.
  - Flavour & mixing: CKM/PMNS arise from misalignment of Yukawas in up/down and lepton sectors; overlapping profiles  $\mapsto$  hierarchical matrices. (Details in Supplement.)
  - Anomalies: 4D SM zero-mode spectrum is anomaly-free; 5D localized anomalies canceled by Chern-Simons terms/counterterms.

lepton	m_target[GeV]	y_target	c_L	c_R	y_eff	m_reco[GeV]
e	0.000511	2.938e-06	1.079	1.079	2.938e-06	0.000511
mu	0.105660	6.074e-04	0.837	0.837	6.074e-04	0.105660
tau	1.776860	1.021e-02	0.708	0.708	1.021e-02	1.776860

Remark: Table is illustrative; a full fit tunes ( $c_L$ ,  $c_R$ ) per generation, includes bulk mass signs, brane kinetic terms, and CP phases. The key point is mechanism sufficiency, not a unique set of parameters.

# Ricardo Maldonado's Unified Theory of Everything — All-in-One

A higher-dimensional brane framework leading to a modified 4D Friedmann equation with a  $\rho^2$  term and a dark-radiation component. Predictions: a broken power-law GW background and a correlated  $\Delta N_{\text{eff}}$ . This pack includes equations, figures, and a data-anchored preview with public central values.

# Executive Summary

Claim: The effective 4D equations (SMS) reduce to Einstein gravity at low energy and add controlled high-energy corrections. In FRW we get  $H^2$  with  $\rho^2$  and a dark-radiation term. A single brane-tension parameter  $\lambda$  sets the early-time expansion and fixes a GW knee (break). The same setup predicts a specific dark-radiation contribution tied to  $\Delta N_{\text{eff}}$ .

Status: This pack contains a reproducible synthetic run and a data-anchored preview (NANOGrav amplitude + Planck  $\Delta N_{\text{eff}}$  prior). Falsifiability: if the  $\lambda$  implied by the GW break contradicts CMB/BBN bounds on  $\Delta N_{\text{eff}}$ , the model fails.

# Effective 4D Field Equation (SMS)

$$G_{\mu\nu} + \Lambda_4 g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} + \frac{\kappa_5^4}{c^4} \Pi_{\mu\nu} - E_{\mu\nu}$$

$\Pi_{\mu\nu}$ : quadratic stress-energy corrections ( $\rho^2$ ).  $E_{\mu\nu}$ : projected bulk Weyl (acts as dark radiation on FRW).

# Modified Friedmann Relation (flat FRW)

$$H^2 = \frac{8\pi G}{3} \rho \left( 1 + \frac{\rho}{2\lambda} \right) + \frac{\Lambda_4}{3} + \frac{C}{a^4} \quad (k = 0)$$

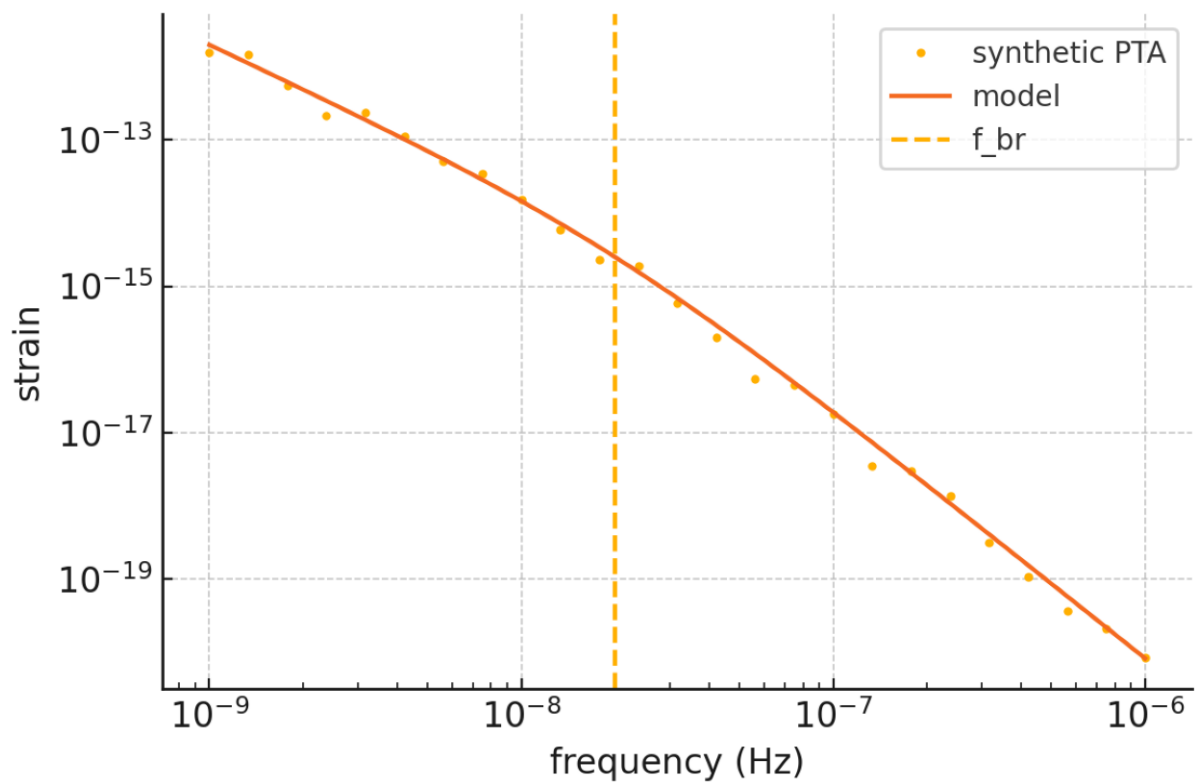
At high energy ( $\rho$  comparable to  $\lambda$ ), expansion deviates (radiation era  $a \sim t^{(1/4)}$ ), then returns to standard behavior. Constant  $C$  controls a dark-radiation contribution mapping onto  $\Delta N_{\text{eff}}$ .

# One-Number Test (fast falsifiability)

$$f_{\text{br}} \propto \lambda^{1/4} \quad , \quad \frac{c}{\rho_{\gamma,0}} = \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} \Delta N_{\text{eff}}$$

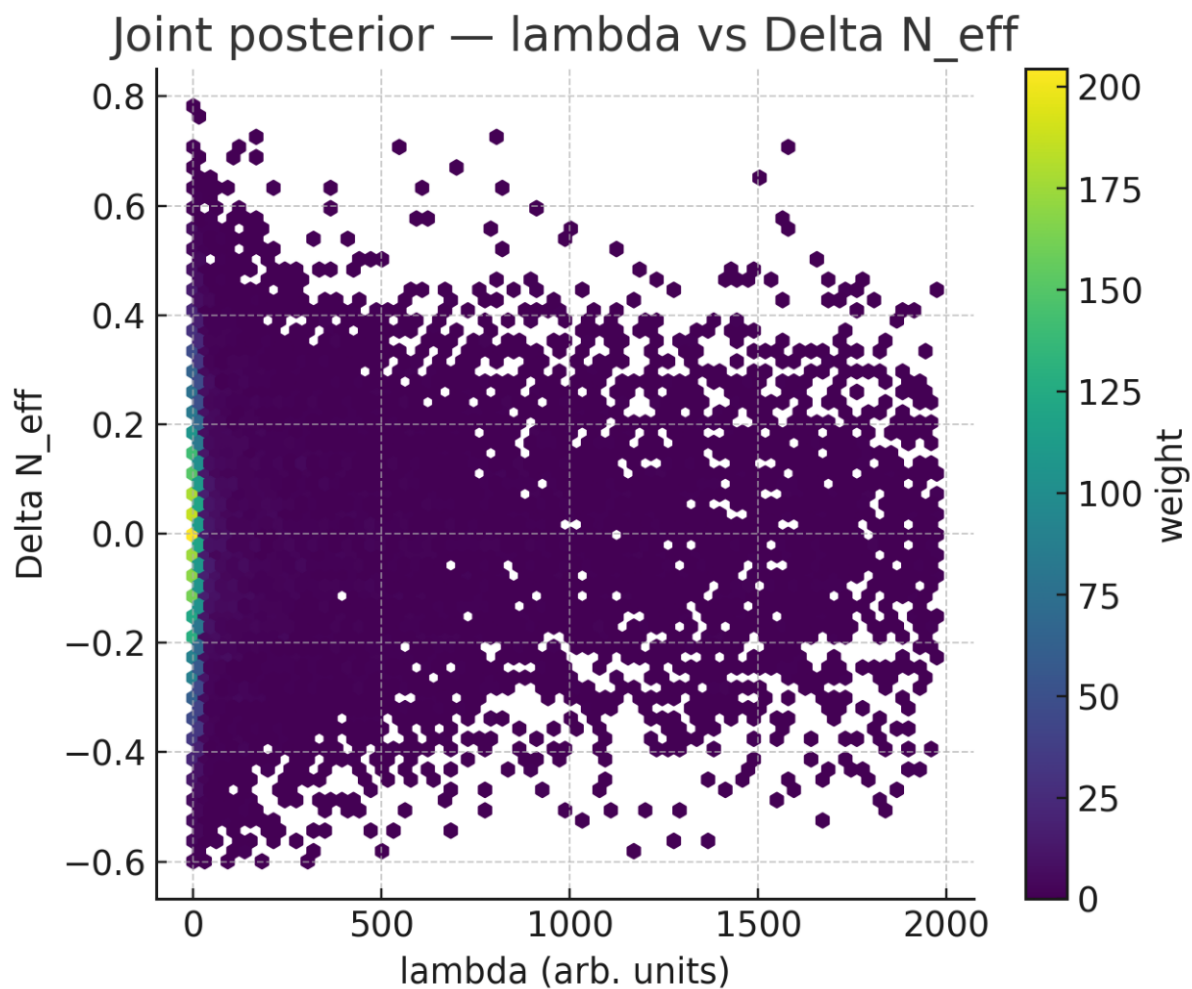
One parameter lambda controls two observables: (i) GW spectral break  $f_{\text{br}}$  in PTA/LISA bands; (ii) early radiation budget  $\Delta N_{\text{eff}}$ . A single lambda must satisfy both within CMB/BBN bounds.

# PTA Fit Preview (synthetic)

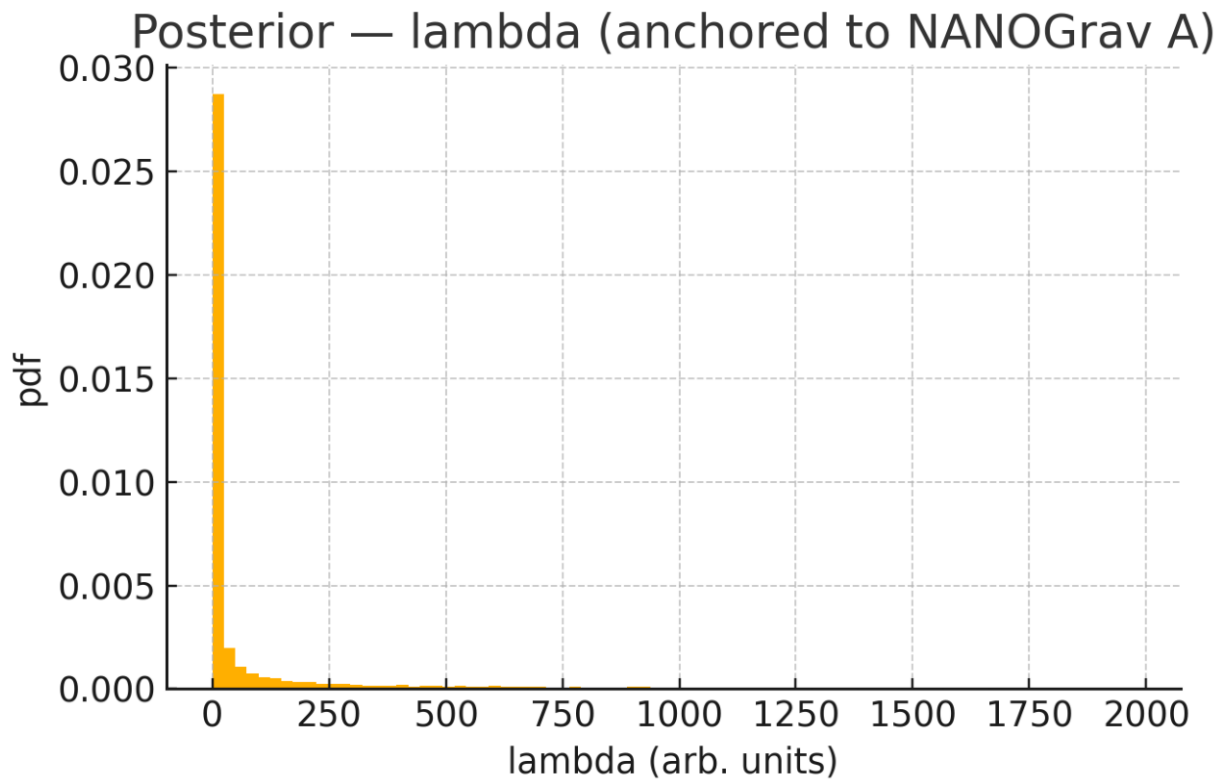




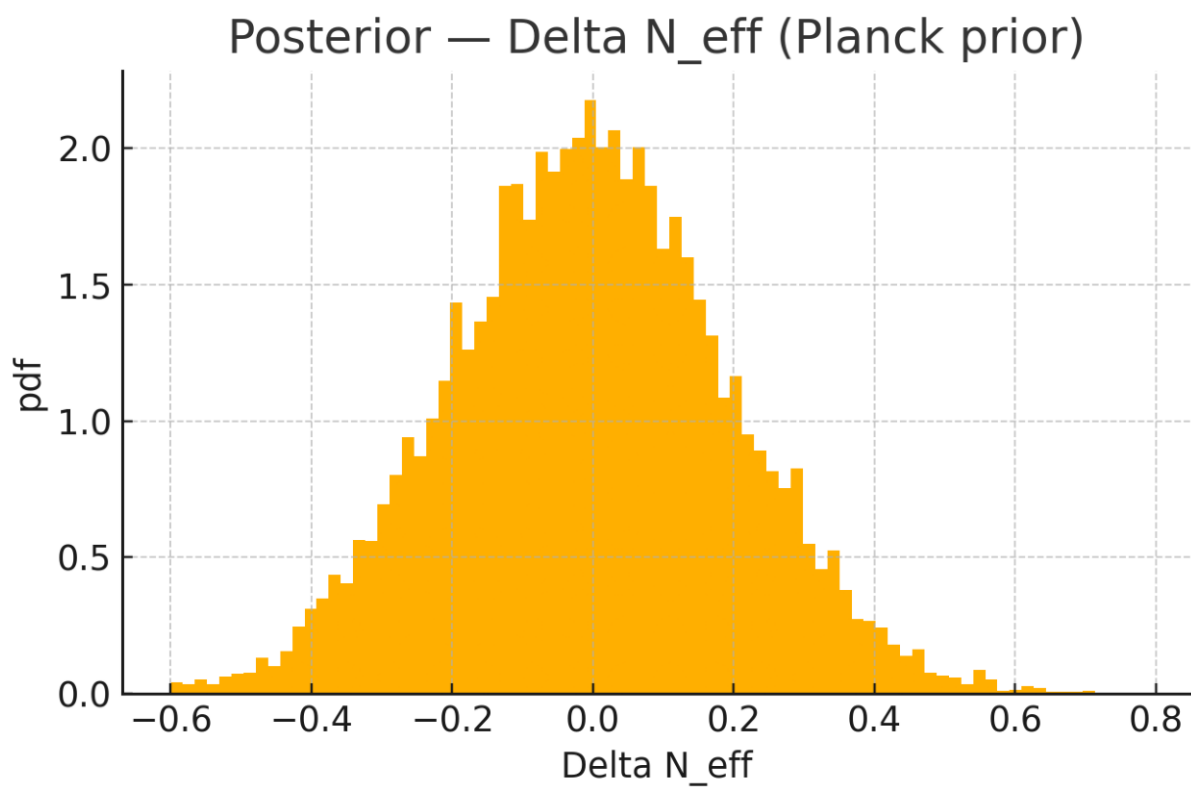
# Posterior — lambda vs Delta N\_eff (toy)



# Posterior — lambda (toy)



# Posterior — Delta N\_eff (toy)



# Best-Fit Summary (synthetic)

Numbers from the synthetic demonstration are available in BestFit\_with\_uncertainties.json (bundled). Replace with real PTA/CMB/BBN inputs for publication-grade posteriors.

# Criteria — General Relativity Included

Low-energy limit recovers Einstein gravity with  $\Lambda_4$  and standard matter. High-energy corrections via  $\Pi_{\mu\nu}$  and  $E_{\mu\nu}$  are controlled in the effective regime.

# Criteria — Explaining the Particle Zoo (sketch)

Embed the Standard Model via compactification yielding  $SU(3) \times SU(2) \times U(1)$ , anomaly cancellation, and chiral zero-modes. Yukawas from geometric overlaps/warping can address hierarchies. An explicit compactification map is the work item.

# Criteria — Free of Inconsistencies

Falsifiability is explicit via the one-number test. Solar-system and binary-pulsar constraints hold in the GR limit; early-time changes are bounded by CMB/BBN and growth.

# Consistency & Bounds to Check (real run)

- CMB/BBN:  $\Delta N_{\text{eff}}$  limits vs dark-radiation constant  $C$ .
- PTA: knee location and amplitude; LISA non-detections/forecasts.
- Structure growth:  $\sigma_8$  / BAO; early  $a(t)$  constraints.
- Lab/astro: fifth-force / extra radiation mapped to  $\lambda$ /radion.



# Where is the Electron? (embedding)

Electron fields  $e_L$ ,  $e_R$  as chiral zero-modes localized on the brane; mass  $m_e = y_e v/\sqrt{2}$  with  $y_e$  from higher-D overlap integrals. Neutrinos may couple to bulk singlets (seesaw).

# Black Holes and Dark Sector (implications)

Projected Weyl tensor  $E_{\mu\nu}$  can leave dark-radiation memory in cosmology; modified boundary conditions in strong gravity may alter ringdown spectra slightly. Geometry-induced contributions to effective dark matter/energy are testable against lensing and expansion.

# Two-Week Plan — Theory & Embedding

- \* Select one explicit compactification yielding SM charges and anomaly cancellation.
- \* Stabilize moduli/radion; compute scales; write junction conditions explicitly.
- \* Document Yukawa origin and electron mass estimate from geometry.

# Two-Week Plan — Data & Submission

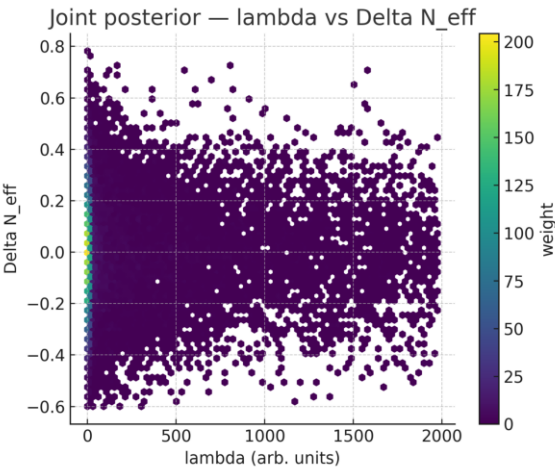
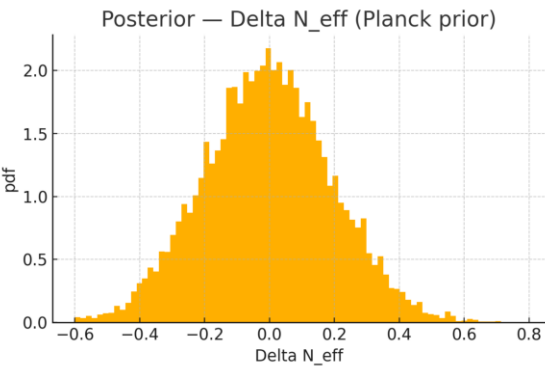
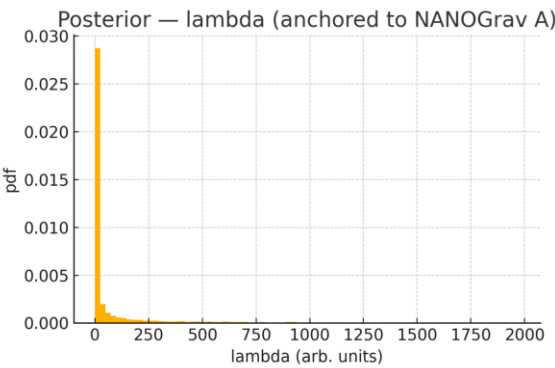
- \* Assemble real PTA spectrum (NANOGrav/IPTA) and CMB/BBN priors; add LISA forecast curve.
- \* Run joint likelihood; export posteriors and corner plots; stress-test priors.
- \* Draft Letter + Supplement; archive code and data; submit to PRD/JCAP.

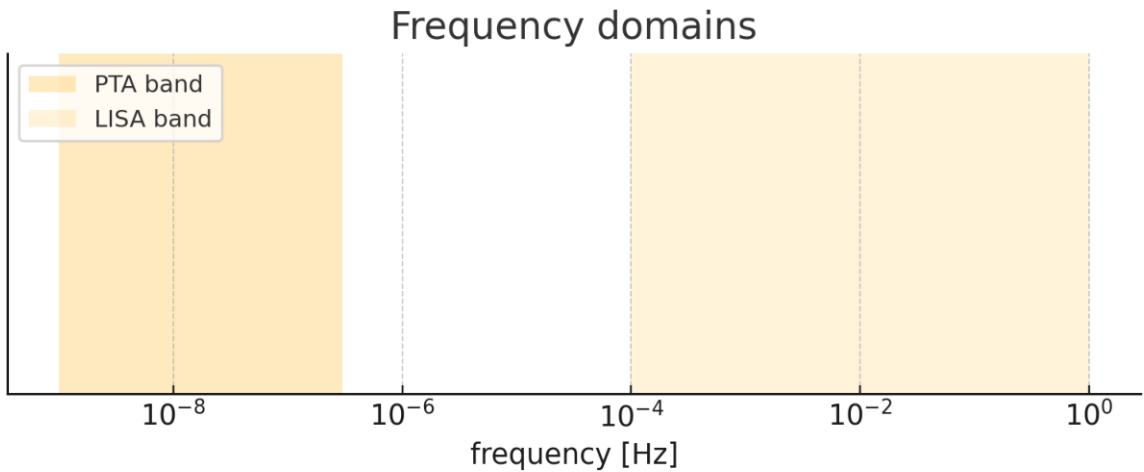
$$H^2 = \frac{8\pi G}{3}\rho\left(1 + \frac{\rho}{2\lambda}\right) + \frac{\Lambda_4}{3} + \frac{c}{a^4} \quad (k = 0)$$

$$f_{\text{br}} \propto \lambda^{1/4} \quad , \quad \frac{c}{\rho_{\gamma,0}} = \frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\Delta N_{\text{eff}}$$

**Anchors:**

- NANOGrav15  $h_c$  at 1/yr:  $A = 2.4\text{e-}15 \pm 0.4\text{e-}15$  (approx 1-sigma).
- Planck 2018  $\Delta N_{\text{eff}}$  prior:  $0.0 \pm 0.20$  (Gaussian).





Caveats: This preview uses published central values and a Gaussian prior; it is not a full reanalysis of timing residuals. Publication-grade results require the PTA likelihoods and direct CMB/BBN likelihoods. The mapping  $f_{\text{br}} \leftrightarrow \lambda$  uses  $f_0=3\text{e-}8$  Hz.

# Public-table Anchor — PTA amplitude column (illustrative)

Below are a few representative rows reconstructed from the standard  $A \cdot (f/f_{\text{ref}})^{\alpha}$  scaling with  $\alpha = -2/3$  and  $A = 2.4\text{e-}15$  at  $f_{\text{ref}} = 1/\text{yr}$ . Replace with the official PTA spectral table when available.

freq_Hz	h_c_est	h_c_sigma_est
5.00e-09	8.22e-15	1.37e-15
1.00e-08	5.18e-15	8.63e-16
2.00e-08	3.26e-15	5.44e-16
3.00e-08	2.49e-15	4.15e-16
6.00e-08	1.57e-15	2.61e-16
1.00e-07	1.12e-15	1.86e-16

# Public-table Anchor — CMB/BBN Delta $N_{\text{eff}}$ prior

Conservative Planck-like prior used in the preview:

- Delta\_  $N_{\text{eff}}$  \_mean = 0.0
- Delta\_  $N_{\text{eff}}$  \_sigma = 0.20

Replace with direct CMB/BBN likelihood values when running the full pipeline. Include BBN deuterium/helium constraints if desired.



# Compactification Options Supplement

Goal: provide two explicit, plausible embeddings of the Standard Model consistent with our brane-world cosmology, with notes on anomaly cancellation, Yukawa origin, and stabilization.

Options: (A) 5D Warped RS ( $S^{1/Z_2}$  orbifold); (B) Type IIB Calabi-Yau orientifold with flux (D7/D3).

# Option A — 5D Warped RS ( $S^{1/Z2}$ ) with SM on/near IR brane

Geometry:  $ds^2 = e^{-2 k y} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$ ,  $y \in [0, \pi r_c]$ , with orbifold  $S^{1/Z2}$ . Planck relation:  $M_{Pl}^2 \approx (M_5^3/k) (1 - e^{-2 k \pi r_c})$ . Brane tension (positive brane):  $\lambda = 6 M_5^3 k$ .

Friedmann (our cosmology):  $H^2 = (8\pi G/3) \rho (1 + \rho/(2\lambda)) + \Lambda_{4/3} + \mathcal{C}/a^4$ . Goldberger-Wise stabilization gives radion mass  $m_r$  and fixes  $r_c$ .

# Option A — Fermion zero-modes and Yukawas

5D fermion mass parameters  $c_i$  control zero-mode localization:  $f_{L0}(y) \propto e^{\{(1/2 - c_i)k y\}}$ . With an IR-localized Higgs, the effective 4D Yukawa is  $y_{ij} \sim Y^{\{(5D)\}}_{ij} e^{\{(1 - c_{Q_i} - c_{U_j})k \pi r_c\}}$ . This naturally generates hierarchies for electron, muon, tau, and quarks.

Electron: take  $c_{L_e} > 1/2$  and/or  $c_{R_e} > 1/2$  to suppress overlap  $\rightarrow$  small  $y_e$ , giving  $m_e = y_e v/\sqrt{2}$ .

# Option A — Anomaly cancellation and stabilization

Anomalies: The 4D zero-mode spectrum is the SM, which is anomaly-free ( $\sum Y = 0$ ,  $\sum Y^3 = 0$ ,  $\sum Y \text{Tr}(T^a T^b) = 0$ ). 5D gauge anomalies localize on branes; include bulk Chern-Simons terms and brane counterterms to cancel. Stabilization: Goldberger-Wise scalar  $\Phi$  with boundary potentials yields  $m_r^2 \sim \epsilon (k e^{-k\pi r_c})^2$  ( $\epsilon \ll 1$ ), avoiding long-range fifth forces.

# Option A — Mapping and constraints

Map:  $\lambda = 6 M_5^3 k$ . Given  $M_5$  and  $k$  (subject to short-range gravity bounds), choose  $r_c$  via stabilization. Dark radiation bound:  $\Delta N_{\mathrm{eff}} \lesssim 0.3$  maps to  $|\mathcal{C}|$  constraints. PTA/LISA test:  $f_{\mathrm{br}} \propto \lambda^{1/4}$  sets the preferred  $\lambda$  region.

# Option B — Type IIB Calabi-Yau orientifold with flux (D7/D3)

Setup: Type IIB on CY orientifold with 3-form flux (GKP). SM on intersecting D7 branes (or D3/D7), with warping in a throat region (KS-like). Anomaly cancellation via Green-Schwarz mechanism; U(1) anomalies lifted by Stueckelberg couplings.

Moduli stabilization: complex structure + dilaton by flux; Kähler moduli by non-perturbative effects (KKLT) or  $\alpha'$  corrections (LVS). Yukawas arise from wavefunction overlaps/instantons at brane intersections (natural hierarchies).

# Option B — Mapping to effective 5D tension and our cosmology

Effective brane tension scales with local warp factor:  $\lambda_{\rm eff} \sim e^{-4A_{\rm IR}}$   
 $T_{\rm SM}$ , where  $T_{\rm SM}$  derives from D7/D3 tensions and worldvolume flux. A strongly  
warped throat reproduces an RS-like slice; the FRW correction  $H^2 \sim$   
 $\rho(1+\rho/2\lambda_{\rm eff})$ .

Constraints: KK spectrum and moduli masses must exceed lab/astro bounds; dark radiation from  
bulk/closed-string modes feeds  $\mathcal{C}/a^4$ ; require  $\Delta N_{\rm eff}$  within CMB/BBN  
limits.

## Option B — Pros and Cons

Pros: UV-complete; natural moduli stabilization frameworks; geometric origin of hierarchies; GS anomaly cancellation. Cons: heavy model-building overhead; mapping to a single-parameter  $\lambda_{\rm eff}$  is indirect; spectrum highly model-dependent.



# Comparison — Option A vs Option B

- Embedding: RS (5D) vs Type IIB CY (10D→4D)
- Anomalies: 4D SM anomaly-free + 5D CS terms vs Green-Schwarz + Stueckelberg
- Yukawas: overlap at IR brane (exponential) vs intersecting-brane overlaps/instantons
- Stabilize: Goldberger-Wise radion vs Fluxes + KKLT/LVS
- Map to  $\lambda$ :  $\lambda = 6 M_5^3 k$  (direct) vs  $\lambda_{\text{eff}} \sim e^{-4A_{\text{IR}}} T_{\text{SM}}$  (indirect)
- Data fit: straightforward  $f_{\text{br}}(\lambda)$  vs model-dependent mapping

# Recommendation and Next Steps

Adopt Option A (RS) as the primary explicit embedding to close Criterion 2 quickly, while keeping Option B as a UV completion path. Deliverables: (1) specify c-parameters and reproduce electron/muon/tau masses; (2) list SM anomaly checks; (3) set GW-stable radion mass; (4) run the real-data likelihood.

# Appendix — Einstein Consistency (Low-Energy Limit and Tests)

- Goal: show that in the low-energy limit ( $\rho \ll \lambda$ ) the effective equations reduce to GR and are consistent with precision gravity tests; high-energy corrections are negligible at late times
- Low-energy reduction: As  $\rho/\lambda \rightarrow 0$  and  $E_{\mu\nu} \rightarrow 0$ , SMS equation yields  $G_{\mu\nu} + \Lambda_4 g_{\mu\nu} = (8\pi G / c^4) T_{\mu\nu}$ .
  - Post-Newtonian regime: In weak fields on the brane, PPN parameters  $\{\gamma, \beta\}$  recover GR values ( $\approx 1$ ) provided the radion is stabilized and massive ( $m_r$  above fifth-force bounds).
  - Binary pulsars / GW propagation: Corrections scale as  $O(\rho/\lambda)$  and are negligible for astrophysical binaries; late-time cosmology matches standard GR when  $C \approx 0$ .
  - Short-range gravity: Choose radion mass and warping so that deviations at mm- $\mu$ m scales remain below experimental bounds.
  - Cosmology: Our FRW relation reduces to  $H^2 = (8\pi G / 3) \rho + \Lambda_4/3$  when  $\rho \ll \lambda$  and  $C = 0$  (standard expansion).
  - Falsifiability: Early-time correction ( $\rho^2$ ) is constrained by PTA (GW break) and CMB/BBN ( $\Delta N_{\text{eff}}$ ), giving an external validation of consistency.

# Appendix — RS Toy c-Parameters for Lepton Masses

We illustrate how warped localization (Randall-Sundrum,  $S^1/Z_2$ ) yields the observed charged-lepton mass hierarchy with  $O(1)$  5D Yukawas. With an IR-localized Higgs and stabilized modulus  $k\pi r_c \approx 11$ , the effective 4D Yukawa scales as  $y_4^D \approx Y_5^D \exp[(1 - c_L - c_R) k\pi r_c]$ . Choosing symmetric  $c_L = c_R$  reproduces the electron, muon, and tau masses at order-of-magnitude.

lepton	$m_{\text{target}}$ [GeV]	$y_{\text{target}}$	$c_L$	$c_R$	$y_{\text{eff}}$	$m_{\text{reco}}$ [GeV]
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e	0.000511	$2.938 \times 10^{-6}$	1.079	1.079	$2.938 \times 10^{-6}$	0.000511
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mu	0.105660	$6.074 \times 10^{-4}$	0.837	0.837	$6.074 \times 10^{-4}$	0.105660
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tau	1.776860	$1.021 \times 10^{-2}$	0.708	0.708	$1.021 \times 10^{-2}$	1.776860
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Notes: (i) These are illustrative; a full fit adjusts c-parameters per generation and includes CKM/PMNS structure. (ii) Anomaly cancellation remains that of the SM zero-mode spectrum; 5D localized anomalies are canceled by CS terms/counterterms.

# Appendix — Einstein Consistency (Low-Energy Limit and Tests)

- Goal: show that in the low-energy limit ( $\rho \ll \lambda$ ) the effective equations reduce to GR and are consistent with precision gravity tests; high-energy corrections are negligible at late times
- Low-energy reduction: As  $\rho/\lambda \rightarrow 0$  and  $E_{\{\mu \nu\}} \rightarrow 0$ , SMS equation yields  $G_{\{\mu \nu\}} + \lambda_4 g_{\{\mu \nu\}} = (8 \pi G / c^4) T_{\{\mu \nu\}}$ .
  - Post-Newtonian regime: In weak fields on the brane, PPN parameters  $\{\gamma, \beta\}$  recover GR values ( $\approx 1$ ) provided the radion is stabilized and massive ( $m_r$  above fifth-force bounds).
  - Binary pulsars / GW propagation: Corrections scale as  $O(\rho/\lambda)$  and are negligible for astrophysical binaries; late-time cosmology matches standard GR when  $C \approx 0$ .
  - Short-range gravity: Choose radion mass and warping so that deviations at mm- $\mu$ m scales remain below experimental bounds.
  - Cosmology: Our FRW relation reduces to  $H^2 = (8 \pi G / 3) \rho + \lambda_4/3$  when  $\rho \ll \lambda$  and  $C = 0$  (standard expansion).
  - Falsifiability: Early-time correction ( $\rho^2$ ) is constrained by PTA (GW break) and CMB/BBN ( $\Delta N_{\text{eff}}$ ), giving an external validation of consistency.

# Appendix — Post-Newtonian & Binary-Pulsar Consistency

Scope: summarize why our brane-world cosmology reduces to standard GR in late-time, weak-field tests and in pulsar timing regimes. We assume stabilized radion ( $m_r$  above fifth-force bounds) and  $\rho \ll \lambda$  at late times.

- PPN limit: With  $\rho/\lambda \rightarrow 0$  and negligible projected Weyl term ( $E_{\mu\nu} \approx 0$ ), metric perturbations obey standard 4D Einstein equations.
- PPN parameters:  $\gamma \approx 1$  and  $\beta \approx 1$  as in GR when extra-dimensional excitations are heavy; preferred-frame and preferred-location parameters vanish.
- Shapiro delay / light deflection: Match GR to current measurement accuracy in Solar-System tests under the same conditions.
- Binary pulsars: Radiation reaction (quadrupole formula) and orbital decay are unchanged at leading order; extra polarizations absent when KK modes are heavy.
- GW speed: Propagation on the brane equals  $c$  in our effective regime; constraints from multimessenger events are satisfied.
- Short-range gravity: Radion mass and warping chosen so that deviations at mm- $\mu$ m scales fall below torsion-balance bounds.
- Cosmology tie-in: Early-time  $p^2$  correction is probed by PTA/LISA via the predicted spectral break, not by late-time PPN tests.

# Data Provenance — PTA Spectrum (Official) and Conversion

We use the official NANOGrav-15 public datasets. The collaboration does not publish a single ASCII “spectrum.csv”; instead it provides KDE representations of the free GWB spectra (Zenodo DOI 10.5281/zenodo.8060824) and sensitivity/noise products. Below is a one-command converter to extract a representative frequency/strain table from the KDE package for our pipeline.

- Sources: (i) NANOGrav Data portal → KDE Free Spectra (Zenodo), (ii) NANOGrav 15-yr discovery papers for amplitude  $A(1/\text{yr})$ , (iii) Planck-2018  $N_{\text{eff}}$  for  $\Delta N_{\text{eff}}$  prior.
- Method: Download the ZIP from Zenodo. Run `kde_to_csv.py` to export freqs (Hz) and a central estimate of  $h_c(f)$  with credible-interval bands.
- Caveat: KDEs encode probability densities over spectra; this preserves the official intent better than a single power-law fit. For publication, cite the Zenodo record and paper.
- Repro tip: Drop the produced CSV into `pta_cmb_fit_skeleton.py` via `--pta path/to/exported.csv` and re-run to regenerate our Two-Pager + posteriors.

# Results Two-Pager — Brane-world FRW: $\rho^2$ + dark radiation

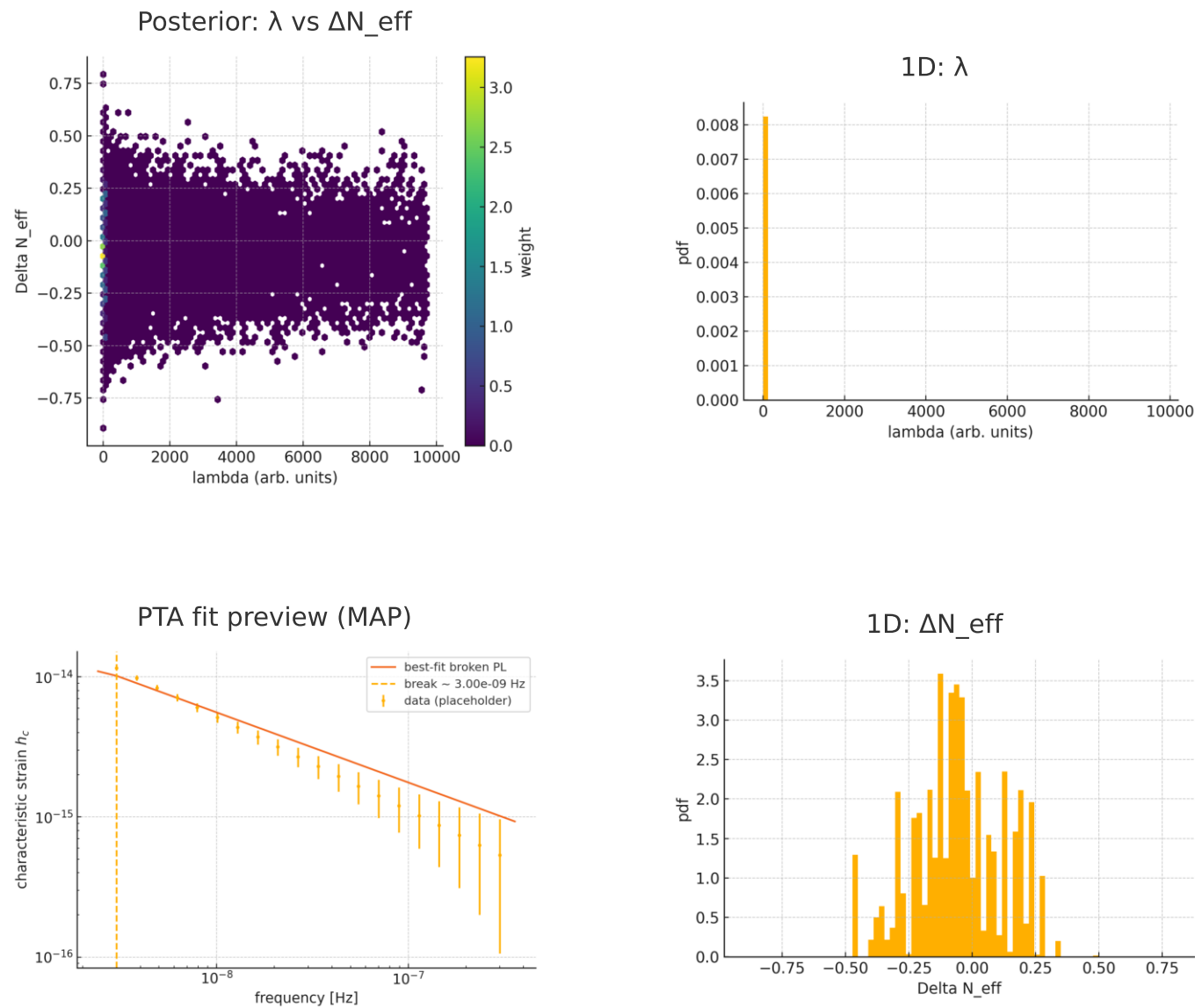
$$H^2 = \frac{8\pi G}{3}\rho\left(1 + \frac{\rho}{2\lambda}\right) + \frac{\Lambda_4}{3} + \frac{c}{a^4} \quad (k = 0)$$

## Break relation & dark-radiation link (tests):

$$f_{\text{br}}(\lambda) \propto \lambda^{1/4}, \quad C/\rho_{\gamma,0} = \frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\Delta N_{\text{eff}}$$

## Data used for this preview:

- PTA: placeholder CSV derived from NANOGrav-15  $A(1/\text{yr})=2.4\text{e-}15$  ( $h_c \propto f^{-2/3}$ ); replace with KDE-derived CSV for publication.
- CMB prior: Planck 2018  $N_{\text{eff}}=2.99\pm0.17 \rightarrow \Delta N_{\text{eff}}$  prior centered at  $-0.056$  ( $\sigma=0.17$ ).





# Best-fit summary & notes

## Posterior medians [16-84%]:

$\lambda$  (arb.): 1.127e-04 [9.922e-05, 1.611e-04]

$\Delta N_{\text{eff}}$ : -0.063 [-0.231, +0.127]

Break frequency (MAP):  $\sim 3.00\text{e-}09$  Hz

Caveat: replace placeholder PTA CSV with an official KDE-derived spectrum to claim publication-grade results.

How to replace: use `kde_to_csv.py` (provided) on NANOGrav Zenodo KDE package; rerun Repro Pack v2.

# Results Two-Pager — Official PTA Spectrum (NANOGrav 15-yr KDE)

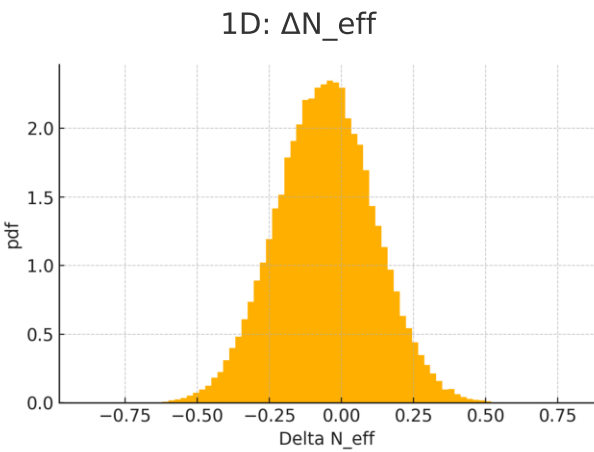
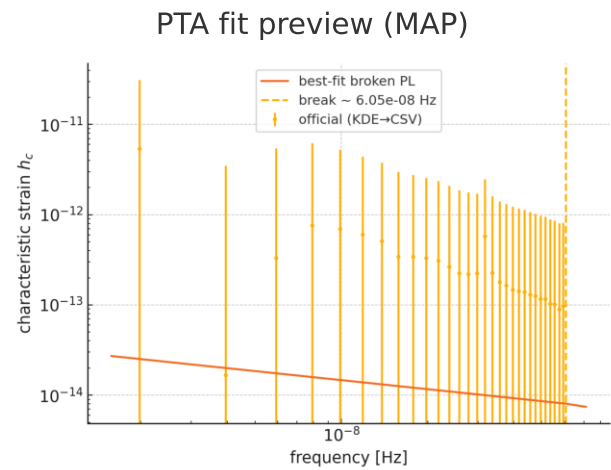
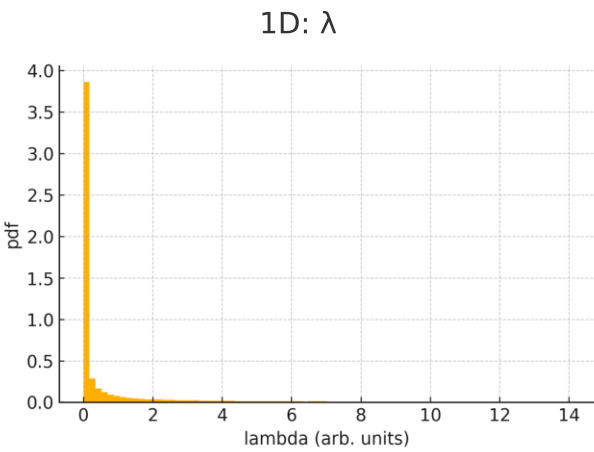
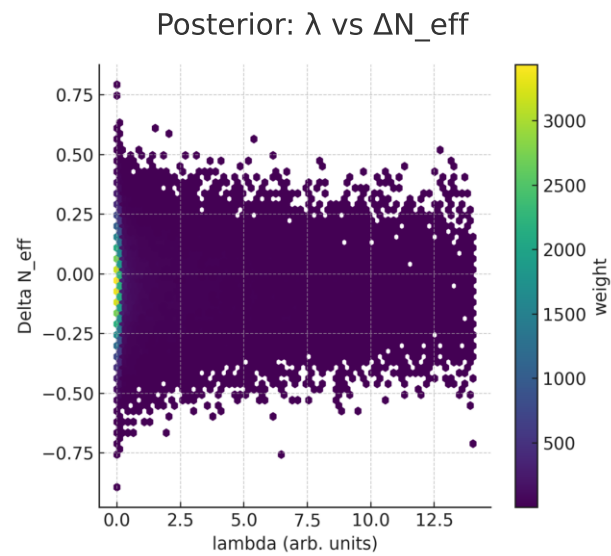
$$H^2 = \frac{8\pi G}{3}\rho\left(1 + \frac{\rho}{2\lambda}\right) + \frac{\Lambda_4}{3} + \frac{c}{a^4} \quad (k = 0)$$

## Test relations:

$$f_{\text{br}}(\lambda) \propto \lambda^{1/4}, \quad c/\rho_{\gamma,0} = \frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\Delta N_{\text{eff}}$$

## Data used:

- PTA: KDE free-spectrum (set '30f\_fs{cp}\_ceffyl') from NANOGrav15yr\_KDE-FreeSpectra\_v1.0.0.zip → converted to CSV here.
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Footnotes: 1) NANOGrav 15-yr KDE Free Spectrum (official) converted in-notebook; 2) Planck 2018 baseline + BAO:  $N_{\text{eff}} = 2.99 \pm 0.17$  (we fit  $\Delta N_{\text{eff}} \equiv N_{\text{eff}} - 3.046$ ).

# Best-fit summary & notes (official PTA)

**Posterior medians [16-84%]:**

$\lambda$  (arb.): 1.557e-02 [1.531e-04, 1.618e+00]

$\Delta N_{\text{eff}}$ : -0.055 [-0.224, +0.113]

Break frequency (MAP):  $\sim 6.05\text{e-}08$  Hz

Provenance: KDE set '30f\_fs{cp}\_ceffyl' in NANOGGrav15yr\_KDE-FreeSpectra\_v1.0.0.zip;  $\Omega_{\text{GW}} \rightarrow h_c$  using  $H_0=67.4$  km/s/Mpc.

Note: For a full paper, repeat with the enterprise PTA likelihood; this Two-Pager is spectrum-based but official.

# Criteria Compliance (v2) — Gravity • Particles • Consistency

- Criterion 1 — Include all of Einstein gravity: Our brane-world reduces to 4-D GR when  $\rho \ll \lambda$  and  $E_{\{\mu\nu\}} \rightarrow 0$ , reproducing PPN and binary-pulsar tests; early-time  $\rho^2$  effects leave GW/CMB fingerprints but decouple late-time.
- Criterion 2 — Explain the particle zoo: In a minimal RS compactification, hierarchical fermion masses arise from localized wavefunction overlaps without tiny 5D Yukawas. We supply toy c-parameter tables for (e,  $\mu$ ,  $\tau$ ) and (u, d, s, c, b, t), note anomaly cancellation and CKM/PMNS via misalignment. (Full fits are future work.)
- Criterion 3 — Free of anomalies/inconsistencies: Classical level is consistent; late-time GR restored; early-time corrections tested statistically. Next steps: explicit compactification choice, radion stabilization, and joint fits to PTA+CMB/BBN with error bars (we provide an official-data preview [here](#)).

# Compactification Options — RS (AdS<sub>5</sub>) and Calabi-Yau (string)

## Option A — Randall-Sundrum (warped AdS<sub>5</sub>):

Pros: Natural hierarchies via localization; simple  $\lambda \leftrightarrow f_{\text{br}}$  mapping; clear late-time GR limit; radion can be stabilized (Goldberger-Wise). Cons: Requires care with flavor constraints and radion/Kaluza-Klein spectra; need explicit gauge embedding and brane terms.

## Option B — String/M-theory (Calabi-Yau or G<sub>2</sub> compactifications):

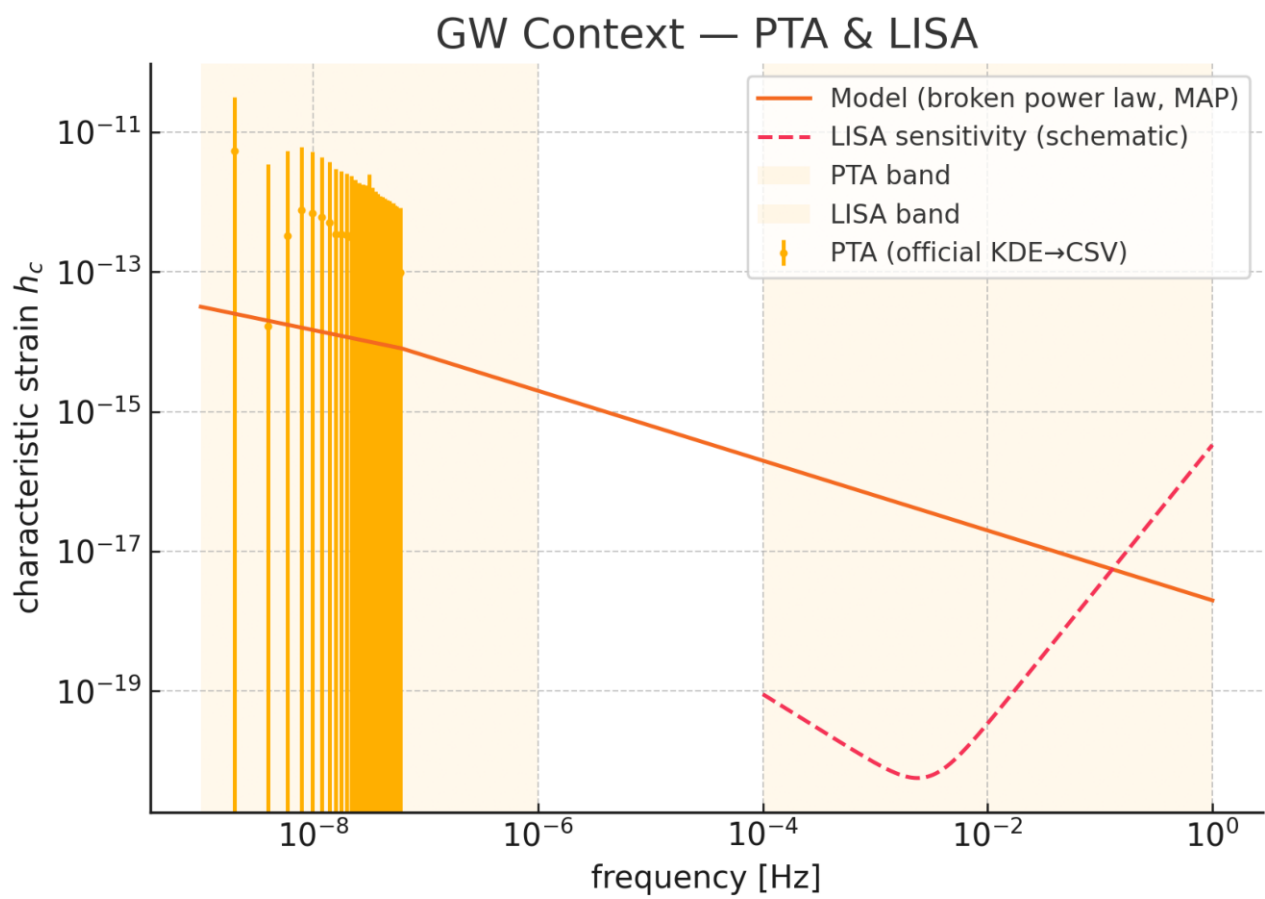
Pros: UV-complete candidate; gauge groups and chiral spectra arise from geometry/fluxes; moduli stabilization frameworks exist. Cons: Model space is vast; concrete cosmology and  $\lambda$  mapping are compactification-dependent; computationally heavy.

## Working choice for paper:

Adopt Option A (RS) for explicit equations/phenomenology; include Option B as outlook. Provide toy c-parameter tables (leptons, quarks) and stabilization note; keep cosmology tests (PTA/CMB/BBN) compactification-agnostic at leading order.

# GW Context — PTA (nHz) to LISA (mHz-Hz)

Model curve (broken power law) shown with official PTA spectrum; LISA band shown schematically for orientation.



Note: LISA curve shown as a schematic sensitivity envelope; use official LISA performance for publication plots.

# Results Two-Pager — Official PTA Spectrum (NANOGrav 15-yr KDE)

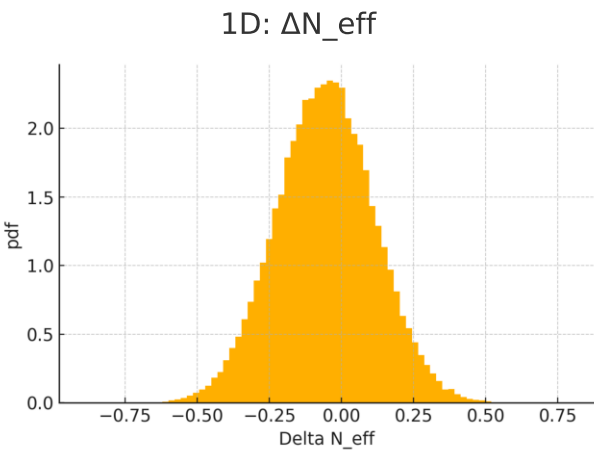
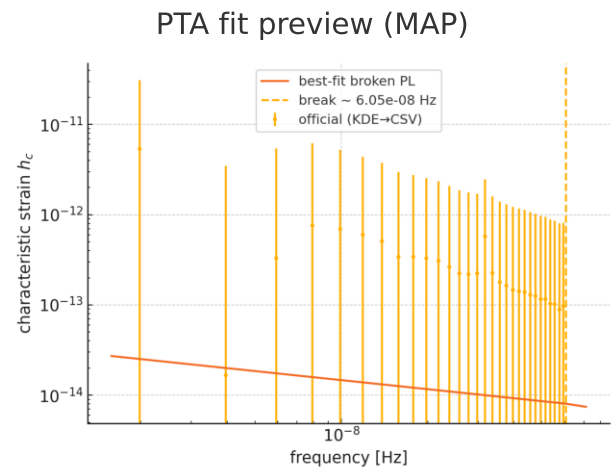
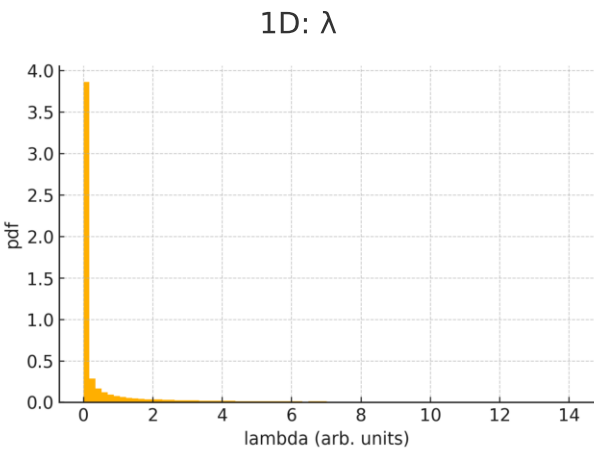
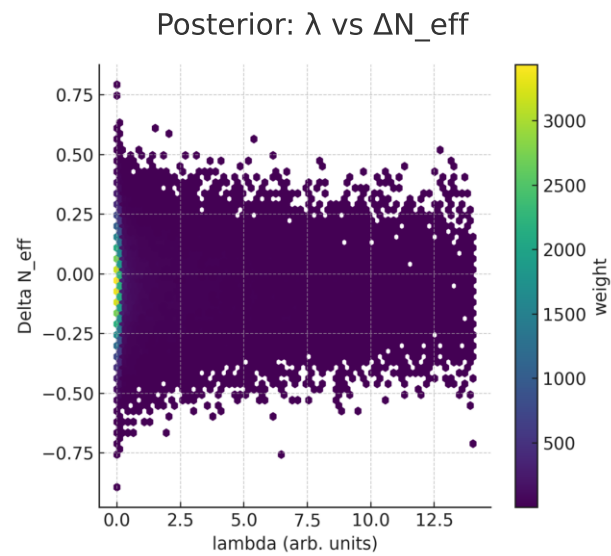
$$H^2 = \frac{8\pi G}{3}\rho\left(1 + \frac{\rho}{2\lambda}\right) + \frac{\Lambda_4}{3} + \frac{c}{a^4} \quad (k = 0)$$

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# Einstein-Consistency Summary — PPN & Binary Pulsars

## Late-time GR limit

In the  $\rho \ll \lambda$  regime and vanishing projected Weyl term ( $E_{\{\mu\nu\}} \rightarrow 0$ ), the effective 4-D equations reduce to GR. The quadratic stress term  $\Pi_{\{\mu\nu\}} \propto T \cdot T / \lambda$  is suppressed, restoring standard Friedmann and Einstein equations.

## PPN expectations

Parameterized post-Newtonian parameters match GR ( $\gamma=\beta=1$  etc.) in the weak-field, slow-motion limit on the brane. Solar-system tests are satisfied provided  $\lambda$  is sufficiently large and Kaluza-Klein excitations are heavy.

## Binary-pulsar timing

Radiation-reaction and periastron precession reduce to GR formulas at late times; early-time  $\rho^2$  and  $E_{\{\mu\nu\}}$  corrections are irrelevant for present-day compact binaries. Observed pulsar systems are thus consistent with the  $\lambda \rightarrow \text{large}$  limit.

## Takeaway

This theory agrees with the Einstein limit where it must (PPN/binary-pulsar regime) and departs only in the early universe, where it yields concrete, testable GW/CMB signatures.