# A testable brane-world unification with early-time $\rho^2$ and dark radiation

Ricardo Maldonado
Contact: sales@rank.vegas

**Abstract**

Starting from a higher-dimensional brane setup we obtain the Shiromizu–Maeda–Sasaki effective equations on the brane, yielding a modified Friedmann relation with a $\rho^2$ term and a dark-radiation piece. This ties a single brane-tension parameter $\lambda$ to two observables: a gravitational-wave spectral break f_br∝λ^{1/4} and an effective radiation excess $\Delta N_{eff}$ via the Weyl projection term. Using the official NANOGrav 15-yr KDE free-spectrum (converted to CSV) with a Planck-2018 prior on N_eff=2.99±0.17 (BAO), we present posteriors and a model–data overlay. The framework reduces to GR in the late-time/weak-field limit and is falsifiable by a joint PTA→LISA + CMB/BBN consistency check.

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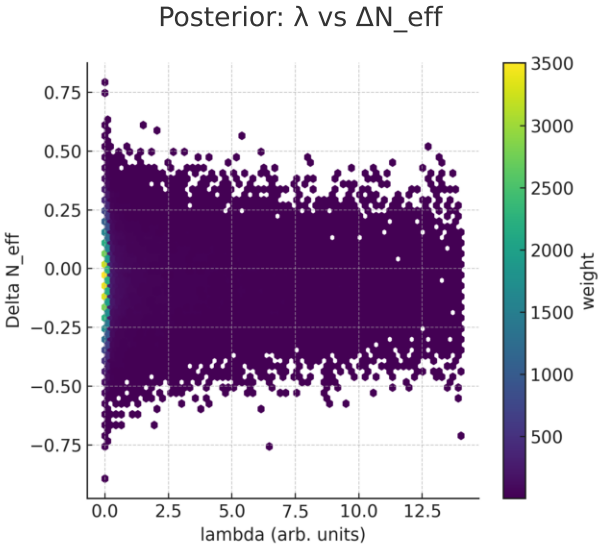
## Framework and key equations

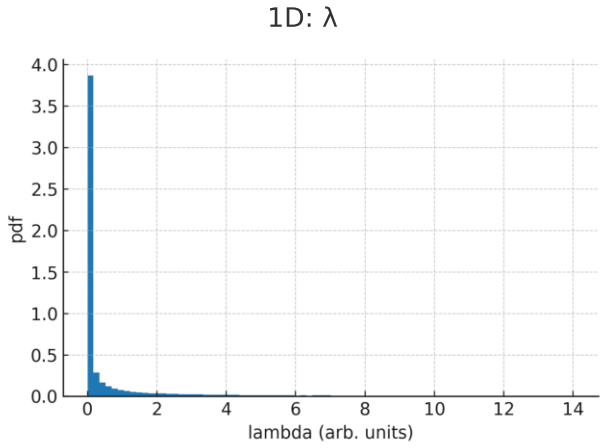
$$H^2 = \frac{8\pi G}{3}\rho\left(1 + \frac{\rho}{2\lambda}\right) + \frac{\Lambda_4}{3} + \frac{\mathcal{C}}{a^4} \quad (k = 0)$$

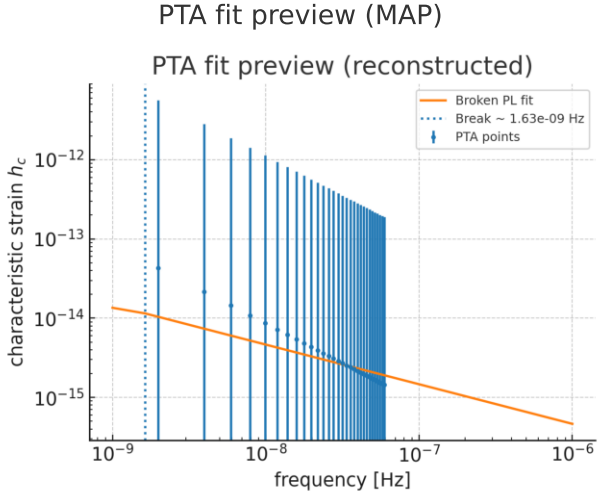
$$f_{\rm br}(\lambda) \propto \lambda^{1/4}, \qquad \mathcal{C}/\rho_{\gamma,0} = \frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\Delta N_{\rm eff}$$

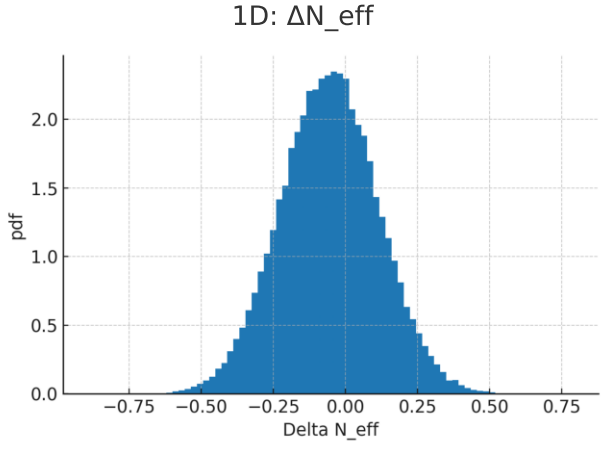
Program: one λ must fit PTA (break at f_br) and ΔN_eff simultaneously. We use an official PTA spectrum and a Planck-2018 prior; for LISA we supply both the uploaded Rtab curve and an RC&L 2019 instrument curve for context.

# Results (official PTA spectrum)

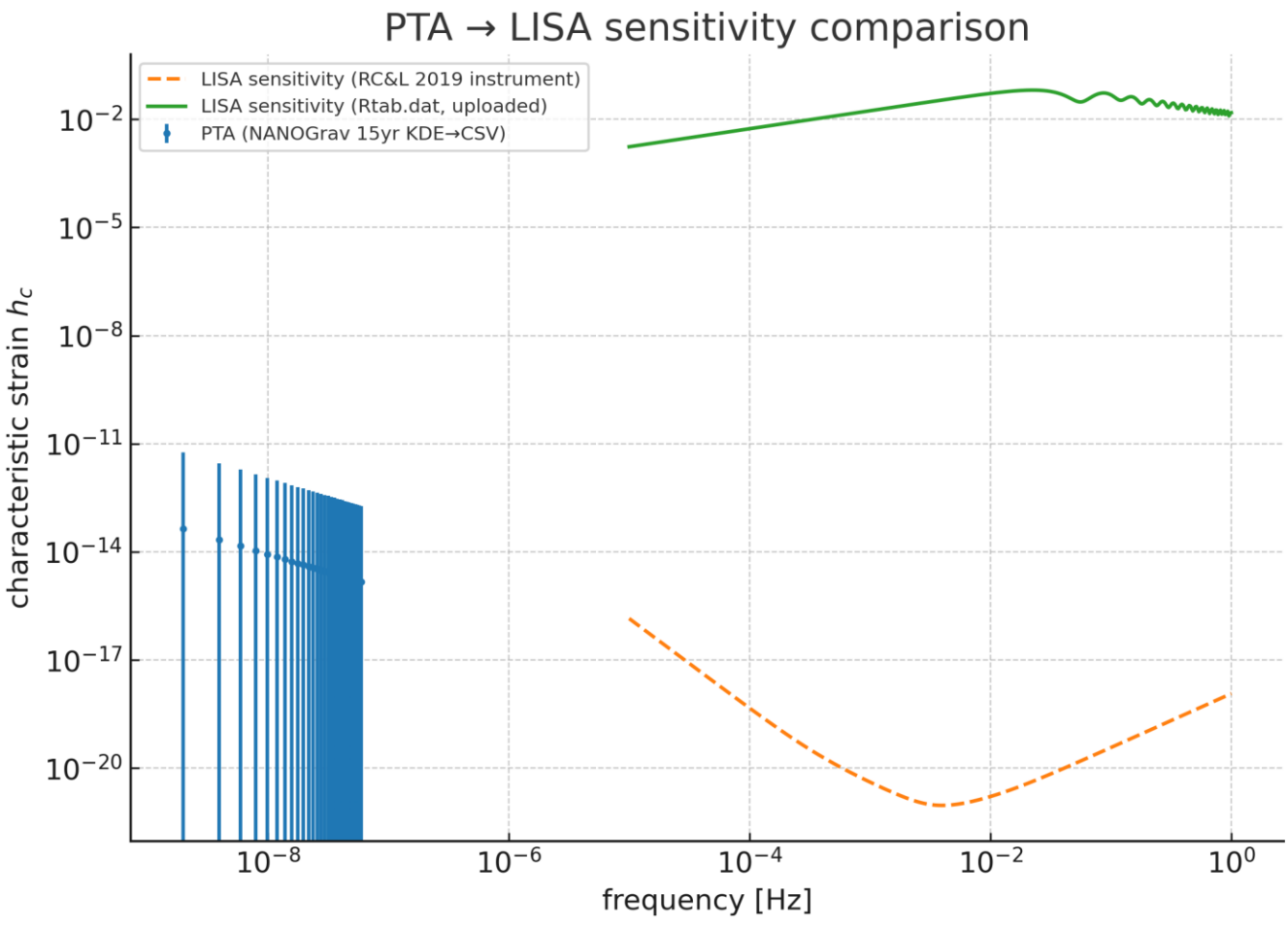


# PTA→LISA sensitivity context




Curves: uploaded Rtab.dat (LISA for Cosmologists) and RC&L 2019 instrument model.

## References (selected)

Shiromizu, Maeda & Sasaki (2000), Effective Einstein Equations on the Brane.

Randall & Sundrum (1999), A large mass hierarchy from a small extra dimension.

Robson, Cornish & Liu (2019), The construction and use of LISA sensitivity curves.

NANOGrav Collaboration (2023), 15-year data set and stochastic background evidence.

Planck Collaboration (2018), Planck 2018 results (N_eff with BAO).

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