

```
=====
Numpy covariance matrix:
```

```
[[ 1.00005258  0.77055255  0.70249069 -0.63103226 -0.59817658 -
0.36857587
- 0.11975342  0.01338994 -0.00877795  0.41848807]
[ 0.77055255  1.00005258  0.71755458 -0.60981073 -0.58117128 -0.26697495
- 0.17624349  0.0397457  0.06606485  0.33683382]
[ 0.70249069  0.71755458  1.00005258 -0.85089453 -0.80887754 -0.15987136
 0.0951622  0.01545556 -0.18668498  0.43706374]
[-0.63103226 -0.60981073 -0.85089453  1.00005258  0.97646319  0.11227813
- 0.12190575 -0.01129466  0.23528427 -0.32834957]
[-0.59817658 -0.58117128 -0.80887754  0.97646319  1.00005258  0.10016449
- 0.11877515 -0.01096618  0.22981062 -0.3046407 ]
[-0.36857587 -0.26697495 -0.15987136  0.11227813  0.10016449  1.00005258
 0.27405923  0.00255297 -0.05569155 -0.2067405 ]
[-0.11975342 -0.17624349  0.0951622  -0.12190575 -0.11877515  0.27405923
 1.00005258 -0.0171976  -0.18628439  0.03702712]
[ 0.01338994  0.0397457  0.01545556 -0.01129466 -0.01096618  0.00255297
- 0.0171976  1.00005258  0.00465937  0.01142721]
[-0.00877795  0.06606485 -0.18668498  0.23528427  0.22981062 -0.05569155
- 0.18628439  0.00465937  1.00005258 -0.22056717]
[ 0.41848807  0.33683382  0.43706374 -0.32834957 -0.3046407  -0.2067405
 0.03702712  0.01142721 -0.22056717  1.00005258]]
```

```
Manual covariance matrix:
```

```
      0      1      2      ...      7      8      9
0  1.000000  0.770512  0.702454  ...  0.013389 -0.008777  0.418466
1  0.770512  1.000000  0.717517  ...  0.039744  0.066061  0.336816
2  0.702454  0.717517  1.000000  ...  0.015455 -0.186675  0.437041
3 -0.630999 -0.609779 -0.850850  ... -0.011294  0.235272 -0.328332
4 -0.598145 -0.581141 -0.808835  ... -0.010966  0.229799 -0.304625
5 -0.368556 -0.266961 -0.159863  ...  0.002553 -0.055689 -0.206730
6 -0.119747 -0.176234  0.095157  ... -0.017197 -0.186275  0.037025
7  0.013389  0.039744  0.015455  ...  1.000000  0.004659  0.011427
8 -0.008777  0.066061 -0.186675  ...  0.004659  1.000000 -0.220556
9  0.418466  0.336816  0.437041  ...  0.011427 -0.220556  1.000000
```

```
[10 rows x 10 columns]
```

```
=====
Dominant Eigenvalue (Numpy):
```

```
4.224212992481706
```

```
Dominant Eigenvector (Numpy):
```

```
[-0.40827678 -0.39505705 -0.44892193  0.44161246  0.42923779  0.14468798
- 0.00519836 -0.01235159  0.09717497 -0.2566813 ]
```

```
Dominant Eigenvalue:
```

```
4.224212905204961
```

Dominant Eigenvector:

```
[ 0.40823591  0.39501337  0.44893924 -0.4416472  -0.42927394 -0.1445977
 0.00530784  0.01234826 -0.097255    0.2566815 ]
```

Normalized Dominant Eigenvector:

```
[ 0.40823591  0.39501337  0.44893924 -0.4416472  -0.42927394 -0.1445977
 0.00530784  0.01234826 -0.097255    0.2566815 ]
```

=====

Dominant Eigenvectors (first two):

```
[[-0.40827678  0.22569874]
 [-0.39505705  0.24528158]
 [-0.44892193 -0.09331051]
 [ 0.44161246  0.1883758 ]
 [ 0.42923779  0.19595182]
 [ 0.14468798 -0.48690288]
 [-0.00519836 -0.59959056]
 [-0.01235159  0.03147723]
 [ 0.09717497  0.45670351]
 [-0.2566813  -0.01246824]]
```

Projected Data: [[1.01654976 -0.36925645]

```
[ 1.67996111 -0.54922547]
 [-5.96624169  1.8313162 ]
```

...

```
[-2.76395431 -0.49118566]
 [-4.64718554  2.31055878]
 [-3.35429914  4.76432253]]
```

Variance in the Projected Subspace:

```
[4.2239909 1.5751879]
```

=====

Eigenvalues: [4.22421299 1.57527072 1.01185852 0.99406496 0.74205841
0.65012065

```
0.40756002 0.22037401 0.15476152 0.02024398]
```

Covariance Matrix in Eigen Decomposition Form:

```
[[ 1.00005258  0.77055255  0.70249069 -0.63103226 -0.59817658 -0.36857587
   - 0.11975343  0.01338994 -0.00877795  0.41848807]
 [ 0.77055255  1.00005258  0.71755458 -0.60981073 -0.58117128 -0.26697495
   - 0.17624349  0.0397457  0.06606485  0.33683382]
 [ 0.70249069  0.71755458  1.00005258 -0.85089453 -0.80887754 -0.15987136
   0.0951622  0.01545556 -0.18668498  0.43706374]
 [-0.63103226 -0.60981073 -0.85089453  1.00005258  0.97646319  0.11227813
   - 0.12190575 -0.01129466  0.23528427 -0.32834957]
 [-0.59817658 -0.58117128 -0.80887754  0.97646319  1.00005258  0.10016449
   - 0.11877515 -0.01096618  0.22981062 -0.3046407 ]
 [-0.36857587 -0.26697495 -0.15987136  0.11227813  0.10016449  1.00005258
   0.27405923  0.00255297 -0.05569155 -0.2067405 ]
 [-0.11975343 -0.17624349  0.0951622  -0.12190575 -0.11877515  0.27405923
   1.00005258 -0.0171976  -0.18628439  0.03702712]
```

```
[ 0.01338994  0.0397457   0.01545556 -0.01129466 -0.01096618  0.00255297
- 0.0171976   1.00005258  0.00465937  0.01142721]
[-0.00877795  0.06606485 -0.18668498  0.23528427  0.22981062 -0.05569155
- 0.18628439  0.00465937  1.00005258 -0.22056717]
[ 0.41848807  0.33683382  0.43706374 -0.32834957 -0.3046407  -0.2067405
 0.03702712  0.01142721 -0.22056717  1.00005258]]
```

```
=====
Mean Square Error (MSE):
```

```
Column_1      0.215663
Column_2      0.245993
Column_3      0.135020
Column_4      0.120334
Column_5      0.161268
Column_6      0.538136
Column_7      0.433592
Column_8      0.997795
Column_9      0.631563
Column_10     0.721456
```

```
dtype: float64
```

```
Sum of Eigenvalues except the First Two:
```

```
4.2010420732095195
```

```
Coordinates of the first 10 data points in the new basis:
```

```
Data Point 1: [ 47.1242565 -22.67621318 -63.30409173  58.50903858
49.02707646
 34.47105152 -23.80096097]
Data Point 2: [ 12.72224991 -40.40937819  10.69631561   1.39222913
1.19139963
- 19.81135897   3.68715757]
Data Point 3: [ -83.08880724   58.82974594 -26.08512648   67.7899386   -
30.0018109
- 174.22734268 -62.514109   ]
Data Point 4: [ 40.25366877 -21.89665459 -31.29367666  31.11263661
64.36151683
 6.83447506 -11.57116858]
Data Point 5: [-57.09639147 -15.03211487  69.57313735 -83.10622849 -
86.78935715
- 66.10771077  25.79023723]
Data Point 6: [ -5.06319122 -58.34972961   2.99996313 -19.55009551 -
28.51853288
- 44.45738768 -7.13157188]
Data Point 7: [ -3.33876289 -35.59762456  10.19139108 -22.55289728 -
16.53105432
- 6.81196473 -3.40498259]
Data Point 8: [  5.44915627 -43.37893799  22.10314184 -17.29303804 -
9.96282738
- 38.26785726 13.51823199]
Data Point 9: [-28.4723678  -94.79382797 -42.0075043  -30.68030582 -
65.19845409
- 75.84427374 -51.50856732]
```

Data Point 10: [31.19206904 -45.41992467 -39.89825338 38.36570852
40.09064867
37.17499305 -41.26701263]