Logarithms

"**Logarithm**" is a word made up by Scottish mathematician John Napier (1550-1617), from the Greek word logos meaning "proportion, ratio or word" and *arithmos* meaning "number".

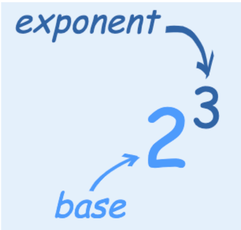
Practical: **How many of one number do we multiply to get another number?*

Example: **2 × 2 × 2 = 8** therefore, logarithm is 3 => $\log_2(8) = 3$

The number we multiply is called the "base", so we can say:

- "the logarithm of 8 with base 2 is 3"
- or "log base 2 of 8 is 3"
- or "the base-2 log of 8 is 3"

Exponents

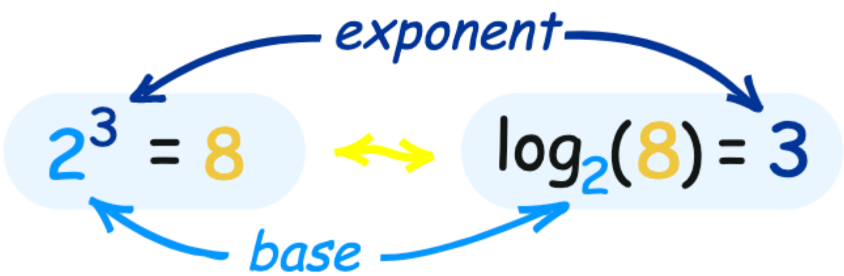


The **exponent** says **how many times** to use the number in a multiplication.

In this example: $\mathbf{2^3 = 2 \times 2 \times 2 = 8}$

(2 is used 3 times in a multiplication to get 8)

The logarithm tells us what the exponent is!



Common Logarithms: Base 10

Sometimes a logarithm is written **without** a base, like this: log(100)
This ***usually*** means that the base is really 10.
It is called a "common logarithm"
On a calculator it is the "log" button.

It is how many times we need to use 10 in a multiplication, to get our desired number.

Natural Logarithms: Base "e"

Base that is often used is [e (Euler's Number)] which is about 2.71828
This is called a "natural logarithm"
On a calculator it is the "ln" button.

It is how many times we need to use "e" in a multiplication, to get our desired number.

Negative Logarithms

A negative logarithm means how many times **to divide** by the number.

Properties of Logarithms

- $\log_a(m \times n) = \log_a m + \log_a n$
- $\log_a(m/n) = \log_a m - \log_a n$
- $\log_a(1/n) = -\log_a n$
- $\log_a(m^r) = r\ (\log_a m\ )$
- $\log_a(1) = 0$
- $\log_a x = \log_b x / \log_b a$ (changing the base)
- $\log_a x = 1 / \log_x a$
- $\log_a x = \ln x / \ln a$
- $\ln(e^w) = w$