1 Engineering Mathematics, MATLAB matrices, autumn 2017

1.1 Reflexions and rotations

By the following commands you can create matrices (triangle with the vertices $(2,4)$, $(4,2)$ and $(6,4)$; M is the reflexion and R rotation matrix):

» v=[2,4,6,2;4,2,4,4]
» M=[1,0;0,-1]
» R=[0,1;-1,0]

Then the matrix products determine the reflexion and rotation:

» v1=M*v
» v2=R*v

And then the plots

» figure(1)
» plot(v(1,:),v(2,:))
» axis([-10 10 -8 8])
» figure(2)
» plot(v(1,:),v(2,:),v1(1,:),v1(2,:))
» axis([-10 10 -8 8])
» figure(3)
» plot(v(1,:),v(2,:),v2(1,:),v2(2,:),0,0,'+')
» axis([-10 10 -8 8])

1.2 Matrices and MATLAB

1. Creation of matrices

Let

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{pmatrix}.$$

By the following commands you can create these matrices:

» A=[1,1,1;1,2,3;1,3,6]
» B=[8,1,6;3,5,7;4,9,2]

A row vector $c = (1\,2\,3)$ by the command:

» c=[1,2,3]

And a column vector

$$\mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

by the command

» c=[1,2,3]'

or

» c=[1;2;3]

2. Basic operations

Transpose A^T, B^T

» A'
» B'

Addition$A + B$, subtraction $A - B$

» A+B
» A-B

Determinant $\det(A)$ or $|A|$

» $\det(A)$

Inverse A^{-1}

» inv(A)

Matrix products AB

» A*B

3. Simultaneous equations

Consider

$$\begin{cases} x+y+z=1 \\ x+2y+3z=2 \\ x+3y+6z=3. \end{cases}$$

This can be written in the matrix form $A\bar{x}=\bar{b}$, where

$$\mathbf{A}=\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{pmatrix} \quad \text{and} \quad \bar{x}=\begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad \text{and} \quad \bar{b}=\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

Solving by the method of inverse matrix $\bar{x}=A^{-1}\bar{b}$

» A=[1,1,1;1,2,3;1,3,6]
» b=[1,2,3]'
» x=inv(A)*b

Gaussian elimination

» $x=A\backslash b$

4. Eigenvalues and eigenvectors

Eigenvalues can be found by the command eig( ):

» eig(A)

And eigenvectors by:

» [V1,D1]=eig(A)

This command returns

$$V1=\begin{matrix} -0.54 & -0.82 & 0.19 \\ 0.78 & -0.41 & 0.47 \\ -0.31 & 0.41 & 0.86 \end{matrix}$$

$$D1=\begin{matrix} 0.127 & 0 & 0 \\ 0 & 1.00 & 0 \\ 0 & 0 & 7.87 \end{matrix}$$

which means that the eigenvector $\begin{pmatrix} -0.54 \\ 0.78 \\ -0.31 \end{pmatrix}$ corresponds the eigenvalue $\lambda_1 = 0.127$, the eigenvector $\begin{pmatrix} -0.82 \\ -0.41 \\ 0.41 \end{pmatrix}$ corresponds the eigenvalue $\lambda_2 = 1.00$ and the eigenvector $\begin{pmatrix} 0.19 \\ 0.47 \\ 0.86 \end{pmatrix}$ corresponds the eigenvalue $\lambda_3 = 7.87$.

Another method to find eigenvalues is the following. The eigenvalues are roots of the charasteristic equation $\det(A - \lambda I) = 0$. The corresponding cofficients of this polynomial equation can be found by the command:

» poly(A)

and then the roots by

» roots(poly(A))

1.3 MATLAB exercises

1. Solve (a) by the method of inverse matrix, (b) by Gaussian elimination

$$\begin{cases} x_1 + x_2 + x_3 + x_4 + x_5 = 7 \\ 2x_1 + 3x_2 - 4x_3 + x_4 + 10x_5 = 5 \\ -x_1 + 4x_2 + x_3 + 6x_4 + 7x_5 = 0 \\ 3x_1 - 3x_2 + 2x_3 + x_4 - 8x_5 = -1 \\ 5x_1 + 7x_2 - 11x_3 + x_4 + 3x_5 = 3 \end{cases}$$

2. Let

$$\mathbf{B} = \begin{pmatrix} 17 & 24 & 1 & 8 & 15 \\ 23 & 5 & 7 & 14 & 16 \\ 4 & 6 & 13 & 20 & 22 \\ 10 & 12 & 19 & 21 & 3 \\ 11 & 18 & 25 & 2 & 9 \end{pmatrix}$$

Find B^T, $det(B)$, B^{-1}, eigenvalues and eigenvectors of B.

2 Engineering Mathematics, MATLAB differentiation/integration, autumn 2017

2.1 Newton-Raphson

Use ten iterations of the Newton-Raphson technique to find an improved estimate of the root of

$3\sin(t) = t$, given $t_0 = 2.5$ is an approximate root.

MATLAB program

```
» t=2.5;
» for   n=1:10
t(n+1)=t(n)-(3*sin(t(n))-t(n))./(3*cos(t(n))-1);
end
» t
```

2.2 Riemann sum

Find a numerical estimate to the integral

$$\int_{-1}^{1} \frac{1}{x^2+1} dx$$

using Riemann sum with 10 subintervals (i.e. the definition of definite integral with $n = 10$).

MATLAB program

```
» n = 10
» dx = (1 - (-1))/n
» x = -1 + dx/2 : dx : 1;
» y = 1./(x.^2 + 1);
» I = sum(y)*dx
```

2.3 MATLAB exercises

1. **MATLAB**

   Use ten iterations of the Newton-Raphson technique to find an improved estimate of the root of

   (a) $t^3 = e^t$, given $t = 1.8$ is an approximate root.

   (b) $\ln(x) = \frac{1}{x}$, given $x = 1.6$ is an approximate root.

2. **MATLAB**

   Find a numerical estimate to the following integrals

   $$\text{(a)} \quad \int_1^2 \frac{1}{x}dx, \quad \text{(b)} \quad \int_{-1}^1 te^{2t}dt, \quad \text{(c)} \int_{-1}^1 \frac{1}{x^4+1}dx$$

   using Riemann sum with 10, 100 and 1000 subintervals (i.e. the definition of definite integral with $n = 10$, $n = 100$, $n = 1000$).

   In items (a) and (b) compare the numerical estimates to the accurate values.