Curriculum Vitae

Dr. Lateef Ahmad Wani

Personal Information

Name: Lateef Ahmad Wani

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Institute Email: lwani@ma.iitr.ac.in

Current Address: Department of Mathematics, Indian Institute of Technology Roorkee, Uttarakhand, India–247667

D.O.B: 14 April, 1989

Mobile No.: +91 9675250863

Marital Status: Married

Permanent Address: Palpora (Wasoora), Pulwama, Kashmir, Jammu & Kashmir–192301

Academic Qualifications

- **Research Associate (Current Position)** in the Department of Mathematics, Indian Institute of Technology Roorkee (**IITR**), Roorkee, Uttarakhand, India (November 9, 2020 onwards). **1 Year, 5 Months**
- **Ph.D. (Mathematics)** from Indian Institute of Technology Roorkee (**IITR**), Roorkee, Uttarakhand, India (2015–2020).
  - Date of Registration: `December 28, 2015`
  - Date of Submission of Thesis: `June 24, 2020`
  - Date of Final Viva: `October 23, 2020`
  - Date of Award (Convocation): `November 29, 2020`
  - Title of Thesis: *Geometric properties of starlike functions associated with a nephroid domain.*
  - Supervisor: `Professor A. Swaminathan`
- **Post Graduation (Mathematics)** from University of Kashmir, Hazratbal, Srinagar, Jammu & Kashmir (2010–2012).
- **Graduation (Mathematics, Economics, English & Computer Applications)** from University of Kashmir, Hazratbal, Srinagar, Jammu & Kashmir (2007–2010).

National Level Exams

- **CSIR-NET** June 2015 with All India Rank (AIR) 26
- **GATE-2015** with All India Rank (AIR) 334

Awards & Scholarships

- **Senior Research Fellowship (MHRD)** from Indian Institute of Technology Roorkee (IITR), India (2018–2020)
- **Junior Research Fellowship (MHRD)** from Indian Institute of Technology Roorkee (IITR), India (2016–2018)

Postdoctoral Fellowships

- National Board for Higher Mathematics (**NBHM**) Post-Doctoral Fellowship 2020 (Part II).

Fields of Specializations & Interests

➤ **Geometric Function Theory** (GFT): *A branch of* **Complex Analysis** *dealing with the geometric properties of analytic functions.*

Areas of Interest

- ➤ **Semigroups in Geometric Function Theory**
- ➤ **Geometric Function Theory in Higher Dimensions**

Research Information

Research Papers Published in Refereed Journals (CLICK HERE)	7
Research Papers Accepted for Publication in Refereed Journals	2
Research Papers Communicated for Publication in Refereed Journals	1
Chapters in Research Volumes/Conference Proceedings/Monographs	2
Contributed Talks (CLICK HERE)	7
Conferences/Workshops Attended (CLICK HERE)	3

Teaching Experiences

- Teaching Assistant in the Department of Mathematics at *Govt. Degree College Kupwara* (J & K) from 03-May-2013 to 28-Feb-2014 (**10 Months**) and taught the following courses:
  - ➤ Real Analysis
  - ➤ Modern Algebra
  - ➤ Differential and Integral Calculus
- Teaching Assistant for undergraduate students at *IIT Roorkee* during 2016-2018 for the following courses:
  - ➤ Complex Analysis I
  - ➤ Complex Analysis II
  - ➤ Mathematical Methods

Professional Memberships

- Life Member: **Indian Mathematical Society** (IMS).
- Member (01-Jan-2021 to 31-Dec-2021): **American Mathematical Society** (AMS).
- Life Member: **International Association of Engineers** (IAENG)
- Life Member: **MathTech Thinking Foundation** (MTTF), An International Association for Science, Technology, Engineering and Mathematics (STEM) professionals.

Professional Services

- **Reviewer**: Mathematical Reviews (AMS)–**04** reviews available on MATHSCINET

- **Reviewer**: Turkish Journal of Mathematics
- **Reviewer**: Advances and Applications in Mathematical Sciences (Milli Publications)
- **Reviewer**: Journal of Function Spaces (Hindawi)

Professional Skills

- Extensive Knowledge of **LaTeX**
- Expertise in **Mathematica Software**

Languages

- English (Proficiency)
- Urdu/Hindi
- Kashmiri (Mother Tongue)

References (click here)

Links to Research/Social Networking Services

Service Name	Service Link	Citations
Google Scholar	CLICK HERE	**49**
ResearchGate	CLICK HERE	**29**
ELSEVIER Scopus	CLICK HERE	**26**
MathSciNet (AMS)	CLICK HERE	**7**
ORCID	CLICK HERE	
arXiv	CLICK HERE	

List of Publications

(1). **L. A. Wani** and A. Swaminathan, ***Starlike and convex functions associated with a nephroid domain***, Bulletin of the Malaysian Mathematical Sciences Society, **44** (1), 79–104 (2021). https://doi.org/10.1007/s40840-020-00935-6

Journal Metrics (2020)

Indexing	Impact Factor	Quartile	SJR	H-Index
Science Citation Index Expanded (SCIE)	1.554	Q2	0.53	27

(2). **L. A. Wani** and A. Swaminathan, ***Radius problems for functions associated with a nephroid domain***, Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales. Serie A. Matemáticas (RACSAM) **114** (2020), no. 4, Paper No. 178, 20 pp. https://doi.org/10.1007/s13398-020-00913-4

Indexing	Impact Factor	Quartile	SJR	H-Index
SCIE	2.169	Q1	0.84	23

(3). **L. A. Wani** and A. Swaminathan, ***Certain geometric properties of starlike functions related to a limacon domain***, Mathematics in Engineering, Science and Aerospace (MESA), **12** (2021), No. 2, 323–342.

Indexing	Impact Factor	Quartile	SJR	H-Index
Scopus	NA	Q4	0.2	6

(4). **L. A. Wani** and A. Swaminathan, ***Inclusion properties of hypergeometric type functions and related integral transforms***, Studia Universitatis Babeş-Bolyai Mathematica, **65** (2020), No. 2, 211–227. http://dx.doi.org/10.24193/subbmath.2020.2.04

Indexing	Impact Factor	Quartile	SJR	H-Index
Scopus	NA	Q3	0.34	8

(5). A. Swaminathan and **L. A. Wani**, ***Sufficiency for Nephroid Starlikeness using Hypergeometric Functions***, Mathematical Methods in the Applied Sciences, **(2022)**. https://doi.org/10.1002/mma.8113

Indexing	Impact Factor	Quartile	SJR	H-Index
SCIE	2.321	Q1	0.72	65

(6). A. Swaminathan and **L. A. Wani**, ***Sufficient conditions and radii problems for a starlike class involving a differential inequality***, Bulletin of the Korean Mathematical Society, **57** (2020), No. 6, pp. 1409–1426. https://doi.org/10.4134/BKMS.b191074

Indexing	Impact Factor	Quartile	SJR	H-Index
SCIE	0.59	Q3	0.3	27

(7). N. E. Cho, A. Swaminathan and **L. A. Wani**, ***Radius constants for functions associated with a limacon domain***, Journal of the Korean Mathematical Society, **59** (2022), No. 2, pp. 353–365. https://doi.org/10.4134/JKMS.j210246

Indexing	Impact Factor	Quartile	SJR	H-Index
SCIE	0.76	Q3	0.4	31

(8). A. Swaminathan and **L. A. Wani**, ***Subordination-implication problems concerning the nephroid starlikeness of analytic functions***, Mathematica Slovaca (Accepted).

Indexing	Impact Factor	Quartile	SJR	H-Index
SCIE	0.770	Q2	0.45	23

(9). **L. A. Wani**, ***On a family of p-valently analytic functions missing initial Taylor coefficients***, FILOMAT (Accepted).

Indexing	Impact Factor	Quartile	SJR	H-Index
SCIE	0.844	Q2	0.45	34

(10). **L. A. Wani**, ***Hypergeometric functions and a parametric family of cardioid domains***, Journal of Mathematical Analysis and Applications (Under Review).

Indexing	Impact Factor	Quartile	SJR	H-Index
SCIE	1.583	Q1	0.95	142

❖ Book/Monograph Chapters

(1). **L. A. Wani** and A. Swaminathan, ***Sufficient conditions concerning the unified class of starlike and convex functions***. In: Mathematical Analysis and Applications. Springer Proceedings in Mathematics & Statistics, vol 381. Springer, Singapore. https://doi.org/10.1007/978-981-16-8177-6_3

(2). A. Swaminathan and **L. A. Wani**, ***A survey of Ma-Minda type function families***, Current Research in Mathematical and Computer Sciences III, University of Warmia and Mazury in Olsztyn, University of Rzeszów, Lodz University of Technology, Lublin University of Technology, Olsztyn Poland; **(2022)**, ISBN 978-83-8100-332-2.

Contributed Talks

(1) ***Inclusion Properties and Subordination Results for the Ma-Minda type Starlike Class Associated with a Limacon Domain***, *International Conference on Mathematical Analysis and Its Applications* (**ICMAA-2019**), Department of Mathematics, South Asian University, New Delhi (December 14-16, 2019).

(2) ***Certain Geometric Properties of Starlike Functions Associated with a Limacon Domain***, **Research Scholars Day**, Department of Mathematics, IIT Roorkee, India (January 17, 2020).

(3) ***Sufficient Conditions Concerning the Unified Class of Starlike and Convex Functions***, *International Conference on Mathematical Analysis and Applications* (**MAA-2020**), Department of Mathematics, National Institute of Technology Jamshedpur (Jharkhand), India (November 02-04, 2020).

(4) ***Radius Constants For Functions Associated with a Limacon Domain***, *VI International Conference of Mathematics and Computer Science* "**Congressio-Mathematica**", University of Warmia and Mazury in Olsztyn, University of Rzeszów, Lodz University of Technology, Lublin University of Technology, Olsztyn Poland (November 21-21, 28-29, 2020).

(5) ***On a Subclass of Analytic Functions Related to Conic Domains and Having Negative Coefficients***, **EUROASIA** *Congress on Scientific Researches and Recent Trends-VII*, Baku Eurasia University, Azerbaijan (December 6-9, 2020).

(6) ***Analytic Functions of Complex Order Involving Hadamard Product***, *86th Annual Conference of the Indian Mathematical Society (***IMS-2020***) - An International Meet*, VIT Vellore, (December 17-20, 2020).

(7) ***Hypergeometric functions and a parametric family of cardioid domains***, *87th Annual Conference of the Indian Mathematical Society-An International Meet* (**IMS-2021**), MGM University Aurangabad, (December 04-07, 2021).

Conferences/Workshops Attended

❶ ***International Conference on Mathematical Analysis and its Applications*** **ICMAA-2016**, Department of Mathematics, Indian Institute of Technology Roorkee, India (November 28 - December 02, 2016).

❷ **TEQIP** ***Short Term Course on Complex Analysis, Fourier Analysis and Special Functions*** (*with outline on mathematical software techniques*), Department of Mathematics, Indian Institute of Technology Roorkee, India (March 06-10, 2017).

❸ ***International Conference on Special Functions and Applications*** **ICSFA-2017** (*16th Annual Conference of the Society for Special Functions & their Applications (SSFA)*) and ***Symposium on Applications of Mathematical Sciences in Engineering Problems***, Department of Mathematics, Govt. College of Engineering & Technology, Bikaner (Rajasthan), India (November 02-04, 2017).

References

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RACSAM (2020) 114:178
https://doi.org/10.1007/s13398-020-00913-4

ORIGINAL PAPER

Radius problems for functions associated with a nephroid domain

Lateef Ahmad Wani[1] · **Anbhu Swaminathan**[1]

Received: 1 June 2020 / Accepted: 23 July 2020

Abstract
Let $\mathcal{S}^*_{Ne}$ be the collection of all analytic functions $f(z)$ defined on the open unit disk $\mathbb{D}$ and satisfying the normalizations $f(0) = f'(0) - 1 = 0$ such that the quantity $zf'(z)/f(z)$ assumes values from the range of the function $\varphi_{Ne}(z) := 1 + z - z^3/3$, $z \in \mathbb{D}$, which is the interior of the nephroid given by

$$\left((u-1)^2 + v^2 - \frac{4}{9}\right)^3 - \frac{4v^2}{3} = 0.$$

In this work, we find sharp $\mathcal{S}^*_{Ne}$-radii for several geometrically defined function classes introduced in the recent past. In particular, $\mathcal{S}^*_{Ne}$-radius for the starlike class $\mathcal{S}^*$ is found to be $1/4$. Moreover, radii problems related to the families defined in terms of ratio of functions are also discussed. Sharpness of certain radii estimates are illustrated graphically.

Keywords Starlike functions · Subordination · Radius problem · Bernoulli and Booth lemniscates · Cardioid · Nephroid

Mathematics Subject Classification 30C45 · 30C80

1 Introduction

Let $\mathcal{A}$ be the class of all analytic functions satisfying the conditions $f(0) = 0$ and $f'(0) = 1$ in the open unit disc $\mathbb{D} := \{z : |z| < 1\}$. Clearly, for each $f \in \mathcal{A}$, the function $\mathcal{Q}_f(z) : \mathbb{D} \to \mathbb{C}$ given by

$$\mathcal{Q}_f(z) := \frac{zf'(z)}{f(z)} \tag{1.1}$$

is analytic and satisfies $\mathcal{Q}_f(0) = 1$. Let $\mathcal{S} \subset \mathcal{A}$ be the family of univalent functions, and $\mathcal{S}^*(\alpha) \subset \mathcal{S}$ be the family of starlike functions of order α $(0 \le \alpha < 1)$ given by the analytic

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SUFFICIENT CONDITIONS AND RADII PROBLEMS FOR A STARLIKE CLASS INVOLVING A DIFFERENTIAL INEQUALITY

Anbhu Swaminathan and Lateef Ahmad Wani

Abstract. Let $\mathcal{A}_n$ be the class of analytic functions $f(z)$ of the form $f(z) = z + \sum_{k=n+1}^{\infty} a_k z^k$, $n \in \mathbb{N}$ defined on the open unit disk $\mathbb{D}$, and let

$$\Omega_n := \left\{ f \in \mathcal{A}_n : \left| z f'(z) - f(z) \right| < \frac{1}{2},\ z \in \mathbb{D} \right\}.$$

In this paper, we make use of differential subordination technique to obtain sufficient conditions for the class Ω_n. Writing $\Omega := \Omega_1$, we obtain inclusion properties of Ω with respect to functions which map $\mathbb{D}$ onto certain parabolic regions and as a consequence, establish a relation connecting the parabolic starlike class $\mathcal{S}_P$ and the uniformly starlike UST. Various radius problems for the class Ω are considered and the sharpness of the radii estimates is obtained analytically besides graphical illustrations.

1. Introduction

Let $\mathbb{C}$ be the set of complex numbers and let $\mathcal{H} := \mathcal{H}(\mathbb{D})$ be the totality of functions $f(z)$ that are analytic in the open unit disc $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$. For $a \in \mathbb{C}$ and $n \in \mathbb{N} := \{1, 2, 3, \ldots\}$, we define the function classes $\mathcal{H}_n(a)$ and $\mathcal{A}_n$ as follows:

$$\mathcal{H}_n(a) := \left\{ f \in \mathcal{H} : f(z) = a + \sum_{k=n}^{\infty} a_k z^k,\ a_k \in \mathbb{C} \right\}$$

and

$$\mathcal{A}_n := \left\{ f \in \mathcal{H} : f(z) = z + \sum_{k=n+1}^{\infty} a_k z^k,\ a_k \in \mathbb{C} \right\}.$$

In particular, we write $\mathcal{A} := \mathcal{A}_1$. For $0 \leq \alpha < 1$, let $\mathcal{S}^*(\alpha)$ and $\mathcal{C}(\alpha)$ be the subclasses of $\mathcal{A}$ which consist of functions that are, respectively, starlike and

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2010 *Mathematics Subject Classification.* 30C45, 30C80.
Key words and phrases. Differential subordination, parabolic and uniform starlikenss, radius problems, cardioid.

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DOI: 10.24193/subbmath.2020.2.04

Inclusion properties of hypergeometric type functions and related integral transforms

Lateef Ahmad Wani and Swaminathan Anbhu

Abstract. In this work, conditions on the parameters a, b and c are given so that the normalized Gaussian hypergeometric function $zF(a, b; c; z)$, where

$$F(a, b; c; z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n (1)_n} z^n, \quad |z| < 1,$$

is in certain class of analytic functions. Using Taylor coefficients of functions in certain classes, inclusion properties of the Hohlov integral transform involving $zF(a, b; c; z)$ are obtained. Similar inclusion results of the Komatu integral operator related to the generalized polylogarithm are also obtained. Various results for the particular values of these parameters are deduced and compared with the existing literature.

Mathematics Subject Classification (2010): 30C45, 33C45, 33A30.

Keywords: Univalent, convex, starlike, close-to-convex functions, Gaussian hypergeometric functions, incomplete beta functions, Komatu integral operator, polylogarithm.

1. Introduction

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \tag{1.1}$$

analytic in the open unit disk $\mathbb{D} = \{z : |z| < 1\}$, and $\mathcal{S}$ denote the subclass of $\mathcal{A}$ that contains functions univalent in $\mathbb{D}$. A function $f \in \mathcal{A}$ is called starlike, denoted by $f \in \mathcal{S}^*$, if $tw \in f(\mathbb{D})$ whenever $w \in f(\mathbb{D})$ and $t \in [0, 1]$. The class of all convex functions, denoted by $\mathcal{C}$, consists of the functions $f \in \mathcal{A}$ such that zf' is starlike. A function $f \in \mathcal{A}$ is said to be *close-to-convex* with respect to a fixed starlike function $g \in S^*$ if and only if $\mathrm{Re}\left(e^{i\lambda} \frac{zf'(z)}{g(z)}\right) > 0$ for $z \in \mathbb{D}$ and $\lambda \in \mathbb{R}$. . Let $\mathcal{K}$ denote the

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Certain geometric properties of starlike functions related to a limacon domain

Lateef Ahmad Wani[1,⋆], A. Swaminathan[1]

[1] Department of Mathematics, Indian Institute of Technology, Roorkee-247667, Uttarakhand, India

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Abstract. Let $\mathcal{ST}_{lim}$ be the family of starlike functions associated with $\varphi_{lim}(z) = 1 + \sqrt{2}z + z^2/2$, the function which sends the open unit disc $\mathbb{D}$ univalently onto a region bounded by the dimpled limacon curve $\left(9u^2+9v^2-18u+5\right)^2 - 16\left(9u^2+9v^2-6u+1\right) = 0$. In this paper, we study certain geometrical properties along with some inclusion results related to the function class $\mathcal{ST}_{lim}$. Further, given some analytic function $\mathcal{P}(z)$ having nice geometrical properties, we obtain sharp estimates on the real β such that the first-order differential subordination

$$1+\beta\frac{zp'(z)}{p^j(z)} \prec \mathcal{P}(z), \quad j \in \{0,1,2\},$$

implies $p(z) \prec \varphi_{lim}(z)$, where p is an analytic function satisfying $p(0) = 1$, and the symbol $\prec$ denotes subordination. As a consequence, we establish several sufficient conditions for the family $\mathcal{ST}_{lim}$.

1 Introduction

Let $\mathbb{D}$ denotes the open unit disc $\{z : |z| < 1\}$ in the complex plane $\mathbb{C}$. Let $\mathcal{H} := \mathcal{H}(\mathbb{D})$ be the totality of analytic functions $f : \mathbb{D} \to \mathbb{C}$, and $\mathcal{A}$ consist of $f \in \mathcal{H}$ satisfying the normalizations $f(0) = 0$ and $f'(0) = 1$. By $\mathcal{S}$ we symbolize the family of functions $f \in \mathcal{A}$ that are univalent (*schlicht*) in $\mathbb{D}$, that is $f \in \mathcal{S}$ if it never takes a value in $\mathbb{C}$ more than once. A complex region Ω is called a starlike region with respect to a point $\xi_0 \in \Omega$ if for every point $\xi \in \Omega$ the line-segment $\{\xi_0 + t(\xi - \xi_0) : 0 \le t \le 1\}$ joining ξ_0 to ξ lies entirely within Ω. A function $f \in \mathcal{A}$ is said to be a *starlike* function with respect to a point $\xi_0 \in f(\mathbb{D})$ if f maps $\mathbb{D}$ onto a region that is starlike with respect to ξ_0. In the special case $\xi_0 = 0$, we simply say f is a starlike function. If $\mathcal{ST}$ denotes the collection of all starlike functions defined on $\mathbb{D}$, then the inclusion $\mathcal{ST} \subsetneq \mathcal{S}$ holds, i.e., every starlike function is univalent. The functions in $\mathcal{ST}$ have the following analytic characterization, see Duren [6, p. 41],

[2010] **Mathematics Subject Classification** 30C45; 30C80
Keywords: Starlike functions; differential subordination; Lemniscate of Bernoulli; Cardioid; Limacon

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RESEARCH ARTICLE

WILEY

Sufficiency for nephroid starlikeness using hypergeometric functions

Anbhu Swaminathan | **Lateef Ahmad Wani**

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Communicated by: H.M. Srivastava

Funding information
Science and Engineering Research Board, Department of Science and Technology, New Delhi, India, Grant/Award Number: CRG/2019/000200/MS

Let $\mathcal{A}$ consists of analytic functions $f : \mathbb{D} \to \mathbb{C}$ satisfying $f(0) = f'(0) - 1 = 0$. Let $\mathcal{S}^*_{Ne}$ be the recently introduced Ma–Minda type functions family associated with the two-cusped kidney-shaped *nephroid* curve $\left((u-1)^2 + v^2 - \frac{4}{9}\right)^3 - \frac{4v^2}{3} = 0$ given by

$$\mathcal{S}^*_{Ne} := \left\{ f \in \mathcal{A} : \frac{zf'(z)}{f(z)} \prec \varphi_{Ne}(z) = 1 + z - z^3/3 \right\}.$$

In this paper, we adopt a novel technique that uses the geometric properties of *hypergeometric functions* to determine sharp estimates on β so that each of the differential subordinations

$$p(z) + \beta z p'(z) \prec \begin{cases} \sqrt{1+z}; \\ 1 + z; \\ e^z; \end{cases}$$

imply $p(z) \prec \varphi_{Ne}(z)$, where $p(z)$ is analytic satisfying $p(0) = 1$. As applications, we establish conditions that are sufficient to deduce that $f \in \mathcal{A}$ is a member of $\mathcal{S}^*_{Ne}$.

KEYWORDS

differential subordination, hypergeometric functions, lemniscate of Bernoulli, nephroid domain, starlike functions

MSC CLASSIFICATION

30C45; 30C80; 33C05; 33C15

1 | INTRODUCTION

Let $\mathcal{A}$ be the family of analytic functions f defined on the open unit disk $\mathbb{D} := \{z : |z| < 1\}$ and satisfying $f(0) = f'(0) - 1 = 0$. Let $\mathcal{S} \subset \mathcal{A}$ be the family of one-one (*univalent*) functions defined on $\mathbb{D}$. Further, let $\mathcal{S}^* \subset \mathcal{S}$ and $\mathcal{C} \subset \mathcal{S}$ be, respectively, the well-known classes of *starlike* and *convex* functions defined on $\mathbb{D}$. We note that the functions in $\mathcal{S}^*$ are analytically characterized by the condition that for each $z \in \mathbb{D}$, the quantity $zf'(z)/f(z)$ lies in the interior of the half-plane

*Dedicated to Professor H. M. Srivastava on the occasion of his 80th birthday.

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RADIUS CONSTANTS FOR FUNCTIONS ASSOCIATED WITH A LIMACON DOMAIN

Nak Eun Cho, Anbhu Swaminathan, and Lateef Ahmad Wani

Abstract. Let $\mathcal{A}$ be the collection of analytic functions f defined in $\mathbb{D} := \{\xi \in \mathbb{C} : |\xi| < 1\}$ such that $f(0) = f'(0) - 1 = 0$. Using the concept of subordination ($\prec$), we define

$$\mathcal{S}^*_\ell := \left\{ f \in \mathcal{A} : \frac{\xi f'(\xi)}{f(\xi)} \prec \Phi_\ell(\xi) = 1 + \sqrt{2}\xi + \frac{\xi^2}{2},\ \xi \in \mathbb{D} \right\},$$

where the function $\Phi_\ell(\xi)$ maps $\mathbb{D}$ univalently onto the region Ω_ℓ bounded by the limacon curve

$$\left(9u^2 + 9v^2 - 18u + 5\right)^2 - 16\left(9u^2 + 9v^2 - 6u + 1\right) = 0.$$

For $0 < r < 1$, let $\mathbb{D}_r := \{\xi \in \mathbb{C} : |\xi| < r\}$ and $\mathcal{G}$ be some geometrically defined subfamily of $\mathcal{A}$. In this paper, we find the largest number $\rho \in (0, 1)$ and some function $f_0 \in \mathcal{G}$ such that for each $f \in \mathcal{G}$

$$\mathcal{L}_f(\mathbb{D}_r) \subset \Omega_\ell \quad \text{for every} \quad 0 < r \le \rho,$$

and

$$\mathcal{L}_{f_0}(\partial\mathbb{D}_\rho) \cap \partial\Omega_\ell \neq \emptyset,$$

where the function $\mathcal{L}_f : \mathbb{D} \to \mathbb{C}$ is given by

$$\mathcal{L}_f(\xi) := \frac{\xi f'(\xi)}{f(\xi)}, \quad f \in \mathcal{A}.$$

Moreover, certain graphical illustrations are provided in support of the results discussed in this paper.

1. Introduction

Let $\mathbb{D}$ denote the open unit disc $\{\xi \in \mathbb{C} : |\xi| < 1\}$ in the complex plane $\mathbb{C}$. Let $\mathcal{H} := \mathcal{H}(\mathbb{D})$ be the collection of all holomorphic functions defined on $\mathbb{D}$.

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2020 *Mathematics Subject Classification.* 30C45, 30C80.
Key words and phrases. Subordination, radii problems, lemniscates, limacon, cardioid, nephroid.
The first author was supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (No. 2019R1I1A3A01050861). The second and third-named authors were also supported by the Project No. CRG/2019/000200/MS of Science and Engineering Research Board, Department of Science and Technology, New Delhi, India.

Sufficient Conditions Concerning the Unified Class of Starlike and Convex Functions

Lateef Ahmad Wani and A. Swaminathan

Abstract Let $\mathcal{A}_n$ be the family of analytic functions $f(\xi) = \xi + \sum_{j=n+1}^{\infty} a_j \xi^j$, defined in the open unit disk $\mathbb{D}$. We use differential subordinations to establish sufficient conditions involving third-order differential inequalities for $f \in \mathcal{A}_n$ to be in the unified class of starlike and convex functions

$$\mathcal{S}^*\mathcal{C}_n(\alpha, \beta) := \left\{ f \in \mathcal{A}_n : \Re\left(\frac{\xi f'(\xi) + \beta \xi^2 f''(\xi)}{\beta \xi f'(\xi) + (1-\beta) f(\xi)} \right) > \alpha \right\},$$

where $\alpha \in [0, 1)$ and $\beta \in [0, 1]$. As applications, we construct certain members of $\mathcal{S}^*\mathcal{C}_n(\alpha, \beta)$ involving triple-integrals and also derive conditions for the Pascu class of functions. Apart from obtaining new results, some of the already known results concerning starlikeness of $f \in \mathcal{A}_n$ are obtained as special cases.

Keywords Starlikeness · Convexity · Differential subordination · Pascu class

1 Introduction

Let $\mathcal{H}$ denotes the set of all analytic functions defined in $\mathbb{D} := \{\xi : |\xi| < 1\}$. Let $\varsigma \in \mathbb{C}$ and $n \in \mathbb{N}$. Define

$$\mathcal{H}_n(\varsigma) := \left\{ f \in \mathcal{H} : f(\xi) = \varsigma + \sum_{j=n}^{\infty} a_j \xi^j,\ a_j \in \mathbb{C} \right\}$$

and

$$\mathcal{A}_n := \left\{ f \in \mathcal{H} : f(\xi) = \xi + \sum_{j=n+1}^{\infty} a_j \xi^j,\ a_j \in \mathbb{C} \right\}.$$

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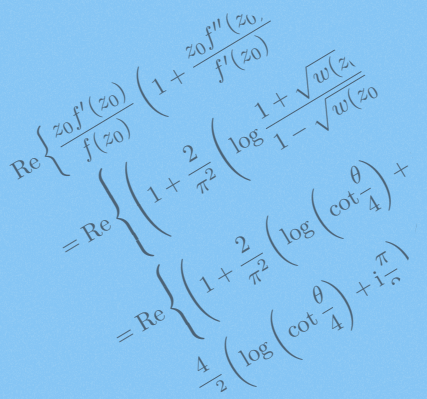

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A SURVEY OF MA-MINDA TYPE FUNCTION FAMILIES

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Abstract. Let $\mathbf{D}$ denote the open unit disk in $\mathbf{C}$ and $\mathcal{A}$ be the set of analytic functions f defined on $\mathbf{D}$ satisfying $f(0) = f'(0) - 1 = 0$. Let $\varphi : \mathbf{D} \to \mathbf{C}$ be an analytic function such that (i) $\varphi(\xi)$ is univalent with $\mathrm{Re}(\varphi) > 0$ for each $\xi \in \mathbf{D}$, (ii) $\varphi(\mathbf{D})$ is starlike with respect to $\varphi(0) = 1$, (iii) $\varphi(\mathbf{D})$ is symmetric about the real line, and (iv) $\varphi'(0) > 0$. For such a function φ, the family

$$\mathcal{S}^*(\varphi) := \left\{ f \in \mathcal{A} : \frac{\xi f'(\xi)}{f(\xi)} \prec \varphi(\xi) \right\},$$

is called a Ma-Minda type function family associated with φ.

This chapter surveys some recently introduced Ma-Minda type families $\mathcal{S}^*(\varphi)$ whose underlying function φ has certain special and interesting geometries.

1. Introduction

Geometric Function Theory (GFT), an interplay of geometry and analysis, is one of the most important and noteworthy areas of complex function theory. At the beginning of the 20^{th} century, Koebe [53], Alexander [3] and Bieberbach [15], considered GFT as a separate branch of complex analysis, and in the words of MacGregor:
"*The significance of geometric ideas and problems in complex analysis is what is suggested by the term geometric function theory. These ideas also occur in real analysis, but geometry has had a much greater impact in complex analysis and it is a very fundamental aspect of its vitality.*"

Recently, applications of GFT have been made in several other fields of science including nuclear physics, fluid mechanics, electrotechnics, etc.

1.1. The family of univalent functions $\mathcal{S}$. The main area of study in GFT is the theory of univalent functions. A complex-valued function f defined on a domain Ω is said to be *univalent* if it takes no value more than once in $\mathbf{C}$. Let

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