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THEORY

1) $p \rightarrow$ Universe will simply exist as it is
$q \rightarrow$ Universe will end in a heat death
$r \rightarrow$ There was a big bang
$s \rightarrow$ Universe is expanding
$t \rightarrow$ Universe is accelerated

(a) (1) The universe either will simply exist as it is or end in a heat death.

$$\downarrow$$

$$(p \vee q)$$

(2) If and only if the universe is expanding, then there was a big bang.

$$\downarrow$$

$$s \leftrightarrow r$$

(3) If there was no big bang, then the universe simply existed.

$$\downarrow$$

$$(\neg r) \rightarrow p$$

(4) If the universe is expanding and accelerated, then it will end in a heat death.

$$\downarrow$$

$$(s \wedge t) \rightarrow q$$

(b) (1) Contrapositive of $(p \vee q)$ is same as contrapositive of $\neg p \rightarrow q$:
$$\neg q \rightarrow p$$

(2) $(\neg r) \leftrightarrow (\neg s)$

(3) $(\neg p) \rightarrow r$ (since $\neg(\neg r) \equiv r$)

(4) $(\neg q) \rightarrow (\neg(s \wedge t)) \equiv (\neg q) \rightarrow (\neg s \vee \neg t)$

(c) $R_1 : (\neg(p \wedge q)) \wedge (p \vee q)$

$R_2 : s \leftrightarrow r \equiv (r \rightarrow s) \wedge (s \rightarrow r) \equiv (s \wedge r) \vee (\neg s \wedge \neg r)$

$R_3 : \neg r \rightarrow p \equiv r \vee \neg p \equiv p \rightarrow r$

$R_4 : (s \wedge t) \rightarrow q \equiv (\neg s \vee \neg t) \vee q$

It can be inferred that either of p or q must be true. Both cannot be false.

The inverse of each statement can be inferred. For example, we can infer:

R_5: If there is a big bang, the universe may or may not simply exist.

$R_5 : (r \rightarrow p) \vee (r \rightarrow \neg p)$

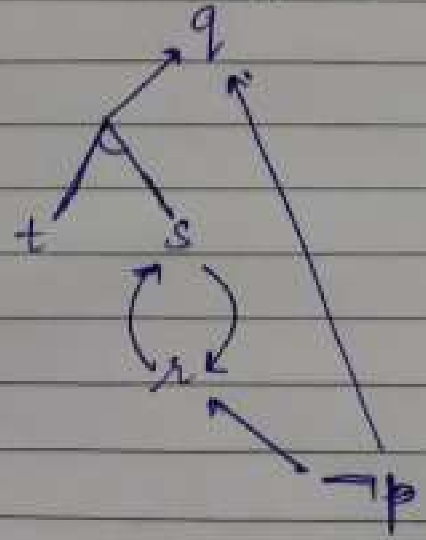
The truth value and relation between p and s cannot be inferred from this knowledge base, since the truth values of the original propositions are unknown. For example, a relation between p and s such as $(p \vee \neg s)$ cannot be inferred from this knowledge base.

(d) $R_1 : (p \vee q) \equiv \neg p \rightarrow q$

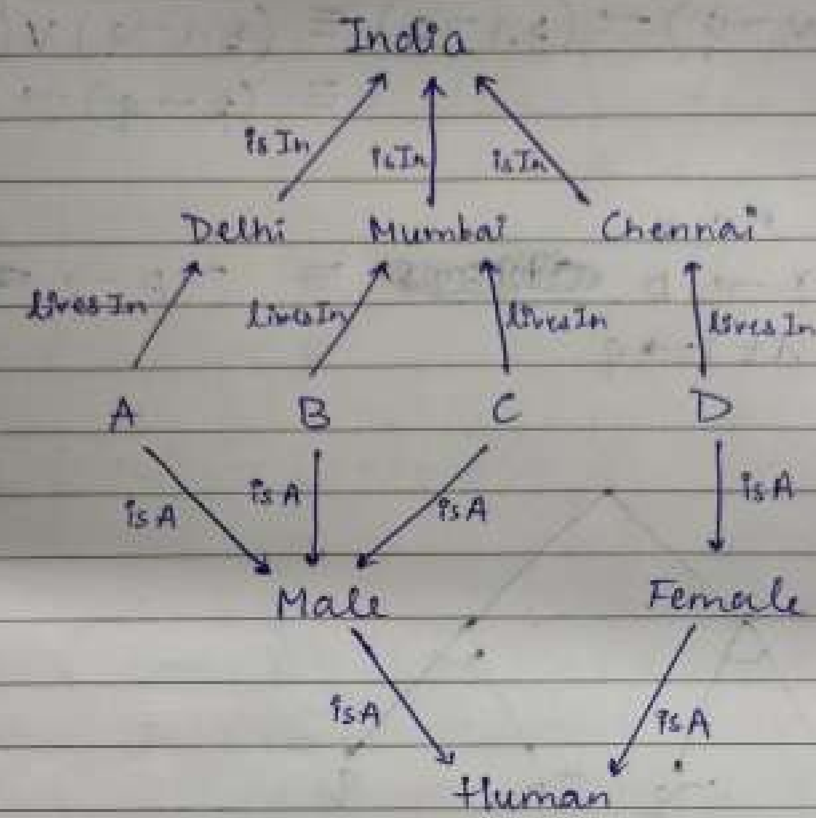
$R_2 : s \leftrightarrow r$

$R_3 : \neg r \rightarrow p \equiv \neg p \rightarrow r$

$R_4 : (s \wedge t) \rightarrow q$



2) Let the people be A, B, C, D. Their gender and location is represented as a semantic network with the relations described by text on the connectors. Inheritance is shown after the network graph.
Here, A represents me (as required) and B, C, D represent three other random individuals with their genders and locations assumed.



All A, B, C belong to the Male category. B and C belong to the Mumbai category through ~~the~~ the livesIn relation.
These are examples of Inheritance.

A, B, C belong to the Human category through the Male category. D also belongs to the Human category through the Female category.
These are examples of Multiple Inheritance.

(Multiple Inheritance - When an object / instance belongs to one category, and that category belongs to another country which implies the object/instance belongs to the parent (super) category.
Inheritance - Different objects belong to (or inherit) a category through some relation)

3) Soundness - All statements derived from true premises must also be true when derived using resolution.

Completeness - All true conclusions that follow from the given premises must be derivable using resolution under the condition of truth of these premises.

The proof by resolution approach for propositional logic is sound and complete.

Proof by Contradiction -

(1) Soundness

Assume there exist premises A and B in the knowledge base. Without loss of generality, assuming the premises are true, let C be a derived statement from A and B using the principle of resolution. Let C be false.

→ Since A and B are true, derived conclusions of the form $(A \wedge B)$ must also always be true. This is subject to the condition that A has a literal L and B has a literal $\neg L$.

→ $(A \wedge B)$ will thus be true, given A and B are individually true.

→ C, thus, must also be true given it is a derived clause and by principle of resolution, it holds true.

→ This contradicts the assumption system is not sound.

→ Q.E.D.

(2) Completeness

Assume there exists some premises $P_1, P_2, \ldots, P_n$, represented as P henceforth. Assume there exists some clause Q which can be derived from P, but resolution fails to derive it.

→ By the principle of resolution, if Q is derivable from P, Q must be representable by the literals in P.

→ There must exist complementary literals in P that represent Q as a propositional relation.

→ By our assumption, Q cannot be resolved from P i.e. there exists no reduction of P to Q.

→ But, if Q can be represented by the literals of P, there must exist reduction sequence that resolves to Q.

→ This contradicts the assumption that the system is not complete.

→ Q.E.D.