MTH 377/577 CONVEX OPTIMIZATION
Winter Semester 2023
Indraprastha Institute of Information Technology Delhi
Coding Assignment
Submission Time: Feb 12 (Sunday midnight); Total Points: 20

Instructions

1. No relaxation of deadline will be entertained. Late submissions will attract a penalty per my discretion.
2. Please upload your code as a jupyter notebook on Google Classrooom. Written work, if any, should be scanned as a pdf and then uploaded. You should preferably do your written work in the same jupyter notebook using markdown cells.
3. Please run your code and show the output by printing meaningful output statements.
4. Readability of code is a criterion for grading. So write code with ample comments.
5. Contact the TAs in case of any doubts regarding the submission formalities.

Problem 1 (5 points). (Taylor approximation) Consider the univariate function

$$f(x) = x^2 + \log x$$

Suppose we want to approximate the function values in the vicinity of $x = 1$ using Taylor polynomials. Lets call the linear approximation function (linear Taylor polynomial) as $L(x)$ and the quadratic approximation function (quadratic Taylor polynomial) as $Q(x)$.

(a) Plot $f(x)$, $L(x)$ and $Q(x)$ in the interval $[0, 2]$ in the same graph.

(b) Let the error associated with the linear approximation be given by $e_L(x) = f(x) - L(x)$, and the error associated with the quadratic approximation be given by $e_Q(x) = f(x) - Q(x)$. Plot $\frac{e_L(x)}{x-1}$ and $\frac{e_Q(x)}{(x-1)^2}$ in the interval $[0, 2]$ in the same graph.

Problem 2 (6 points). (Combination descent algorithm) Write a code to compute the unconstrained minimum of the following optimization problem by implementing a combination descent algorithm with initial point $(1, 1)$.

$$\min_{x,y} f(x, y) = (x^2 - 3y^2)^2 + \sin^2(x^2 + y^2)$$

Problem 3 (9 points). (Combination descent algorithm) Suppose S is a subset of $\mathbb{R}^n$ that is defined by a collection of inequalities:

$$S = \{\mathbf{x} \in \mathbb{R}^n : \text{ for every } j = 1, \ldots, m, \quad g_j(\mathbf{x}) \geq 0\}$$

Associated with any such set S is the potential function defined as

$$\Psi(\mathbf{x}) = -\sum_{j=1}^{m} \log g_j(\mathbf{x})$$

The analytic center of the set S is the vector that minimizes the associated potential function i.e. the vector $\mathbf{x}$ that solves

$$\min_{\mathbf{x}} \quad \Psi(\mathbf{x})$$

As an example instance, suppose S is a subset of $\mathbb{R}^2$ described as above by three linear inequalities given by

$$\begin{aligned} g_1(x_1, x_2) &= 2x_2 - x_1 \\ g_2(x_1, x_2) &= 2x_1 - x_2 \\ g_3(x_1, x_2) &= 1 - x_1 - x_2 \end{aligned}$$

Frame the problem as an unconstrained optimization problem and write a code that uses the combination descent algorithm with initial point $(0.25, 0.25)$ to compute the analytic center of S.

(Hint: Since log is a monotonically increasing function, minimizing $\log f(x)$ is equivalent to minimizing $f(x)$)