As it turns out, this is equivalent to the *staircase* mechanism for local privacy which is the optimal local differential privacy mechanism for low privacy regime (**?**, Theorem 14). The trade-off between utility and privacy is controlled by ϵ.

Using the corruption parameters from Eq. (2) with Corollary 1, we arrive at the following upper bound.

Corollary 2 *At time T, the regret of* SW-KLUCB-CF *with ϵ-locally differentially private bandit feedback corruption scheme given by Eq.* (2) *is* $\tilde{O}\left(\sum_{a\in A}\sqrt{L_T T} + \sum_{i=1}^{L_T}\sum_{a\neq a_*(i)} \frac{\log\left(\sqrt{\frac{T}{L_T}}\right)}{\left(\frac{e^\epsilon-1}{e^\epsilon+1}\right)^2}\right)$.

The term $\left(\frac{e^\epsilon-1}{e^\epsilon+1}\right)^2$ in the above expression conveys the relationship of the regret with the level of local differential privacy symbolized by ϵ. For low values of ϵ, $\left(\frac{e^\epsilon-1}{e^\epsilon+1}\right) \approx \epsilon/2$. This is in line with other bandit algorithms providing differential privacy (e.g., **?**).

Elements of Mathematical Analysis

Here, we provide a proof outline for Theorem 1. Please refer to the Appendix for the complete proof.

We start by bounding the expected number of times a suboptimal arm (i.e., an arm other than the optimal arm at the time of selection) is pulled by the algorithm till horizon T. Recall that, at any time step t, SW-KLUCB-CF pulls an arm maximizing an index defined as

$$\begin{aligned}&\mathrm{Index}_a(t)\\ &\coloneqq \max\left\{q:\ N_a(t,w)\cdot d\left(\hat{\lambda}_a(t,w), g_a(q)\right) \le f\left(t\wedge w\right)\right\}\\ &= \max g_a^{-1}\left(\left\{q: N_a(t,w)\cdot d\left(\hat{\lambda}_a(t,w), q\right) \le f\left(t\wedge w\right)\right\}\right).\end{aligned}$$

We further decompose the computation of index as follows,

$$\mathrm{Index}_a(t) \coloneqq \begin{cases} g_a^{-1}(\ell_a(t)) & \text{if } g_a \text{ is decreasing,}\\ g_a^{-1}(u_a(t)) & \text{if } g_a \text{ is increasing}\end{cases}$$

where,

$$\ell_a(t) \coloneqq \min\left\{q: N_a(t,w)\cdot d\left(\hat{\lambda}_a(t,w), q\right) \le f\left(t\wedge w\right)\right\},$$

$$u_a(t) \coloneqq \max\left\{q: N_a(t,w)\cdot d\left(\hat{\lambda}_a(t,w), q\right) \le f\left(t\wedge w\right)\right\}.$$

The interval $[\ell_a(t), u_a(t)]$ is a KL-based confidence interval on the mean feedback $\lambda_{a,t}$ of arm a at time t. This is in contrast to kl-UCB (**?**) where a confidence interval is placed on the mean reward. Furthermore, This differs from kl-UCB-CF (**?**) where the mean feedback of an arm remains the same for all the time steps and f does not feature w.

In our analysis, we use the fact that when an arm a is picked at time $t+1$ by SW-KLUCB-CF, one of the following is true: Either the mean feedback of the optimal arm $a_{*,t}$ with mean reward $\mu_{*,t}$ is outside its confidence interval (i.e., $g_{a_{*,t}}(\mu_{*,t}) < \ell_{a_{*,t}}(t)$ or $g_{a_{*,t}}(\mu_{*,t}) > u_{a_{*,t}}(t)$) which is unlikely. Or, the mean feedback of the optimal arm is where it should be, and then the fact that arm a is selected indicates that the confidence interval on λ_a cannot be too small as either $(u_a(t) \ge g_a(\mu_{*,t}))$ or $(\ell_a(t) \le g_a(\mu_{*,t}))$. The previous statement follows from considering various cases depending on whether the corruption functions g_a and $g_{a_{*,t}}$ are increasing or decreasing. We then need to control the two terms in the decomposition of the expected number of draws of arm a. The term regarding the "unlikely" event, is bounded using the same technique as in the kl-UCB analysis, however with some added challenges due to the use of a sliding window. In particular, the analysis of a typical upper confidence bound algorithm for bandits relies on the fact that the confidence interval for any arm is always non-increasing, however this is not true while using a sliding window. To control the second term, depending on the monotonicity of the corruption functions g_a and $g_{a_{*,t}}$, we need to meticulously adapt the arguments in **?** to control the number of draws of a suboptimal arm, as can be seen in the Appendix.

Concluding Remarks

In this work, we proposed the setting of non-stationary stochastic corrupt bandits for preserving privacy while still maintaining high utility in sequential decision making in a changing environment. We devised an algorithm called SW-KLUCB-CF and proved its regret upper bound which is near-optimal in the number of time steps and matches the best known bound for analogous problems in terms of the number of time steps and the number of changes. Moreover, we provided an optimal corruption scheme to be used with our algorithm in order to attain the dual goal of achieving high utility while maintaining the desired level of privacy.

Interesting directions for future work include:

1. Complete an empirical evaluation of the proposed algorithm on simulated as well as real-life data.
2. Characterize the changes in the environment by a variation budget (as done in **?** for classical bandits) instead of the number of changes.
3. Incorporate contextual information in the learning process.
4. Propose a Bayesian algorithm for non-stationary stochastic corrupt bandits.
5. Propose a (near-)optimal differentially private algorithm which does not need to know the number of changes.

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