We will exploit this finding in the derivation of the conditional distribution induced by the probabilistic circuit $\gamma = (Y \wedge \alpha) \vee (\neg Y \wedge \beta)$.

$$\begin{aligned}
&\Pr{}_{\gamma}(Y = 1 \mid \mathbf{x}) \\
&= \frac{\Pr_{\gamma}(Y=1)\Pr_{\alpha}(\mathbf{x})}{\Pr_{\gamma}(Y=0)\Pr_{\beta}(\mathbf{x}) + \Pr(Y=1)\Pr_{\alpha}(\mathbf{x})} \\
&= \frac{1}{1 + \frac{\Pr_{\gamma}(Y=0)\Pr_{\beta}(\mathbf{x})}{\Pr_{\gamma}(Y=1)\Pr_{\alpha}(\mathbf{x})}} \\
&= \frac{1}{1 + \frac{\Pr_{\gamma}(Y=0)\prod_{(n,c)\in\mathcal{W}_{\beta}} f_{\beta}(n,\mathbf{x},c)\theta^{\beta}_{(n,c)}}{\Pr_{\gamma}(Y=1)\prod_{(n,c)\in\mathcal{W}_{\alpha}} f_{\alpha}(n,\mathbf{x},c)\theta^{\alpha}_{(n,c)}}}
\end{aligned}$$

As stated in Proposition 6 and shown in Figure 5, subcircuits α and β share the same structure. Therefore, we can further simplify this equation as follows.

$$\begin{aligned}
&\Pr{}_{\gamma}(Y = 1 \mid \mathbf{x}) \\
&= \frac{1}{1 + \frac{\Pr_{\gamma}(Y=0)}{\Pr_{\gamma}(Y=1)}\prod_{(n,c)\in\mathcal{W}_{\alpha}} f_{\vee\alpha}(n,\mathbf{x},c)^{\frac{\theta^{\beta}_{(n,c)}}{\theta^{\alpha}_{(n,c)}}}} \\
&= \frac{1}{1 + \exp\left[-g(\mathbf{x})\right]} = \Pr{}_{\vee\alpha}(Y = 1 \mid \mathbf{x})
\end{aligned}$$

where

$$g(\mathbf{x}) = \log\frac{\Pr_{\gamma}(Y=1)}{\Pr_{\gamma}(Y=0)} + \sum_{(n,c)\in\mathcal{W}_{\alpha}} f_{\vee\alpha}(n,\mathbf{x},c)\log\frac{\theta^{\alpha}_{(n,c)}}{\theta^{\beta}_{(n,c)}} \tag{2}$$

$$= \theta^{\vee\alpha}_{root} + \sum_{(n,c)\in\mathcal{W}_{\alpha}} f_{\vee\alpha}(n,\mathbf{x},c)\cdot\theta^{\vee\alpha}_{(n,c)}. \tag{3}$$

The transformation from Equation 2 to 3 expresses the logistic circuit parameters as the log-ratios of probabilistic circuit probabilities. For example, the class priors captured in the output wires of α and β are now combined as a log-ratio to form the bias term for $\vee\alpha$, expressed by the root parameter. This proof also provides us with a new perspective to understand the semantics of the learned parameters. The parameters represent the log-odds ratio of the features given different classes. Note that by Bayes' theorem, a naive Bayes model would derive its induced distribution in a sequence of steps similar to the ones above, resulting in Equation 2. Given this correspondence, one can also view our proposed structure learning method as a way to construct meaningful features for a naive Bayes classifier. We know that after training, naive Bayes classifiers are equivalent to logistic regression classifiers (as in Equation 3).

7 Related Work

? (**?**) proposed the first parameter learning algorithm for discriminative SPNs, using MPE inference as a sub-routine. Without the support of the determinism property, parameter learning of general SPNs is a relatively harder question than its logistic circuit counterpart, since it is non-convex. **?** (**?**) boost the accuracy of SPNs on MNIST to 97.6% by extracting more representative features from raw inputs based on the Hilbert-Schmidt independence measure. **?** (**?**) further improved the classification ability of SPNs by drastically simplifying SPN structure requirements and utilizing a loss objective that hybrids cross-entropy (discriminative learning) with log-likelihood (generative learning).

? (**?**) developed a discriminative structure learning algorithm for arithmetic circuits. The method updates the circuit that represents a corresponding conditional random field (CRF) model by adding features conditioned on arbitrary evidence to the model. This work further relaxes decomposability and smoothness properties of ACs for a more compact representation. However, it targets the setting where there are a large number of output variables, not single-variable classification.

All the aforementioned literature conforms to a common trend of abandoning properties of the chosen circuit representations for easier structure learning and better prediction accuracy. They argue that those special syntactic restrictions complicate the learning process. On the contrary, this paper chooses perhaps the most structure-restrictive circuit as the target representation. Instead of relaxing the target representation's syntactical requirements, our proposed method fully leverages the valuable properties that stem from these restrictions, and in particular convexity.

8 Conclusions

We have presented logistic circuits, a novel circuit-based classification model with convex parameter learning and a simple structure learning procedure based on local search. Logistic circuits outperform much larger classifiers and perform well in a limited data regime. Compared to other symbolic, circuit-based approaches, logistic circuits present a leap in performance on image classification benchmarks. Future work includes support for convolution, parameter tying, and structure sharing in the logistic circuits framework.

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