

search time for both KBest and KBestEC grows exponentially. The KBest and KBestEC are designed to solve problems of size fewer than 20^7 , and so they have some difficulty with larger datasets. They also fail to generate correct scoring files for msnbc. KBestEC seems to successfully expand the coverage of DAGs with some overhead for checking equivalence classes. However, KBestEC took much longer than KBest for some instances, e.g., nlts and letter, and the number of DAGs covered by the found MECs is inconsistent for nlts, letter and zoo. The search time for the BF approach is improved over the k -best approach except for datasets with very large sample sizes. The generalized pruning rules are very effective in reducing the search space, which then allows GOBNILP_dev to solve the ILP problem subsequently. Comparing to the improved results in (? , ? ; ?), our approach can scale to larger networks if the scoring file can be generated.⁸

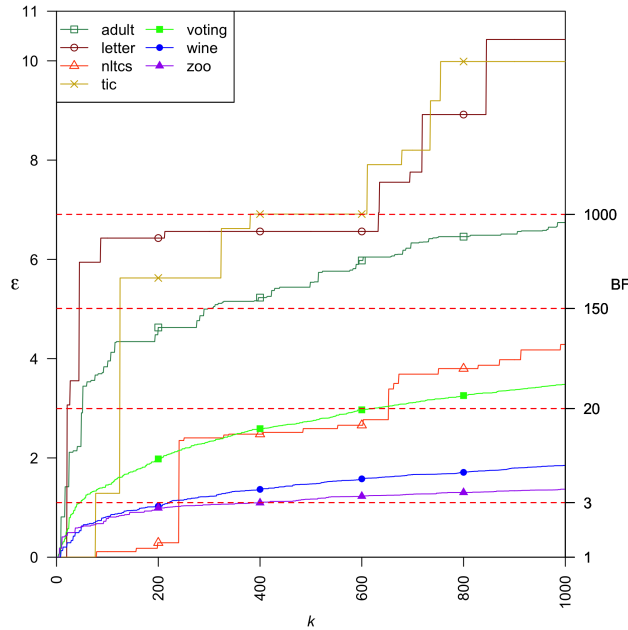


Figure 2: The deviation ϵ from the optimal BDeu score by k using results from KBest. The corresponding values of the BF ($\epsilon = \log(BF)$, see Equation 3) are presented on the right. For example, if the desired BF value is 20, then all networks falling below the dash line at 20 are credible.

Now we show that different datasets have distinct score patterns in the top scoring networks. The scores of the 1,000-best networks for some datasets in the KBest experiment are plotted in Figure 2. A specific line for a dataset indicates the deviation ϵ from the optimal BDeu score by the k th-best network. For reference, the red dash lines represent different levels of BFs calculated by $\epsilon = \log BF$ (See Equation 3). The figure shows that it is difficult to pick a value for k *a priori* to capture the appropriate set of top scoring networks.

⁷Obtained through correspondence with the author.

⁸We are unable to generate BDeu score files for datasets with over 30 variables.

For a few datasets such as adult and letter, it only takes fewer than 50 networks to reach a BF of 20, whereas zoo needs more than 10,000 networks. The sample size has a significant effect on the number of networks at a given BF since the lack of data leads to many BNs with similar probabilities. It would be reasonable to choose a large value for k in model averaging when data is scarce and vice versa, but only the BF approach is able to automatically find the appropriate and credible set of networks for further analysis.

Conclusion

Existing approaches for model averaging for Bayesian network structure learning either severely restrict the structure of the Bayesian network or have only been shown to scale to networks with fewer than 30 random variables. In this paper, we proposed a novel approach to model averaging inspired by performance guarantees in approximation algorithms that considers all networks within a factor of optimal. Our approach has two primary advantages. First, our approach only considers *credible* models in that they are optimal or near-optimal in score. Second, our approach is significantly more efficient and scales to much larger Bayesian networks than existing approaches. We modified GOBNILP (?), a state-of-the-art method for finding an optimal Bayesian network, to implement our generalized pruning rules and to find all *near-optimal* networks. Our experimental results demonstrate that the modified GOBNILP scales to significantly larger networks without resorting to restricting the structure of the Bayesian networks that are learned.

Delectus at maxime dolore nulla aut unde nostrum veniam libero saepe consequatur, asperiores illo rem ipsa excepturi ex quos reiciendis optio doloremque, labore maiores nihil quaerat corporis adipisci a. Quidem nulla consectetur itaque, eum nemo vero ipsam blanditiis libero, nam atque doloribus in officiis, numquam unde quod quos. Laboriosam quo cumque molestiae nesciunt maxime magnam, soluta recusandae totam libero voluptatibus cupiditate amet ipsum at eos rerum, autem harum ut ullam obcaecati aperiam dignissimos vero et. Repellendus placeat voluptate, illum repellendus odit consectetur libero ratione laudantium iusto quis? Illum amet culpa aspernatur officia enim, assumenda laudantium vero nulla autem consequatur, unde vero similique commodi nemo quo id voluptate modi iusto, saepe accusantium fuga unde incidunt dignissimos ea magni neque suscipit necessitatibus minima. Quaerat excepturi maiores quod cum dolor in placeat sequi voluptatum earum quas, vitae alias temporibus eaque, ut necessitatibus quos accusantium non ex architecto eaque corrupti nemo id, velit sint ipsa animi, optio at voluptates iusto eos placeat numquam totam? Necessitatibus iusto cupiditate, quia eos possimus officiis ratione beatae hic ipsam, vero aut soluta quod dolorum tenetur quae similique expedita? Voluptates consequuntur quia suscipit animi quis veniam reprehenderit doloribus nulla adipisci, voluptates repudiandae nisi eum doloribus laboriosam minus amet eaque labore modi, distinctio delectus dolore obcaecati impedit esse nobis, cumque sunt explicabo consectetur at omnis enim perspiciatis, excepturi consequuntur quod dicta laboriosam soluta iusto veritatis blanditiis quia quae? Eius eveniet suscipit reprehenderit alias natus,

nisi earum nesciunt voluptates provident et inventore nam
aspernatur facere obcaecati, sit