

## Ablation Study

**Number of Rays  $p$ .** Figure 6 demonstrates that the quality of the Pareto front increases with the number of rays, but up to a certain point, adding more rays no longer significantly improves the results. That means our framework doesn't require too many sampling rays to get a good performance.

**Partition.** As shown in Figure 7, partitioning algorithm makes it easier for the cosine similarity function and the HV function to cooperate and enhances PHN-HVI performance.

**Cosine Similarity.** The cosine similarity function is critical in the convergence of PHN-HVI and helps in the spread of the Pareto Front. In Figure 8, if  $\lambda$  is very large ( $\lambda = 100$ ), Pareto Front is very widely dispersed, but it is quite shallow. If  $\lambda$  is very small ( $\lambda = 0.1$ ), PHN-HVI can't generate Pareto Front. Therefore, selecting a suitable lambda that balances the HV function and the cosine similarity function is critical for the PHN-HVI to work effectively.

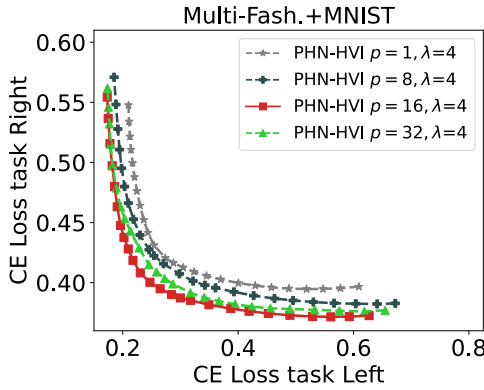


Figure 6: Performance of PHN-HVI when  $p$  varies

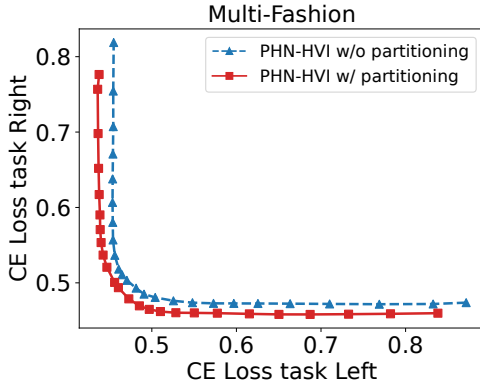


Figure 7: The effect of loss space partitioning

## 6 Conclusion and Future Work

In this paper, we propose PHN-HVI with Multi-Sample Hypernetwork, which utilizes a variety of trade-off vectors simultaneously, followed by hypervolume maximization to improve the PFL problem. This approach also opens up a wide range of potential research directions. On one hand, it is necessary to investigate theoretically for which objective functions the hypernetwork-based PFL methods will

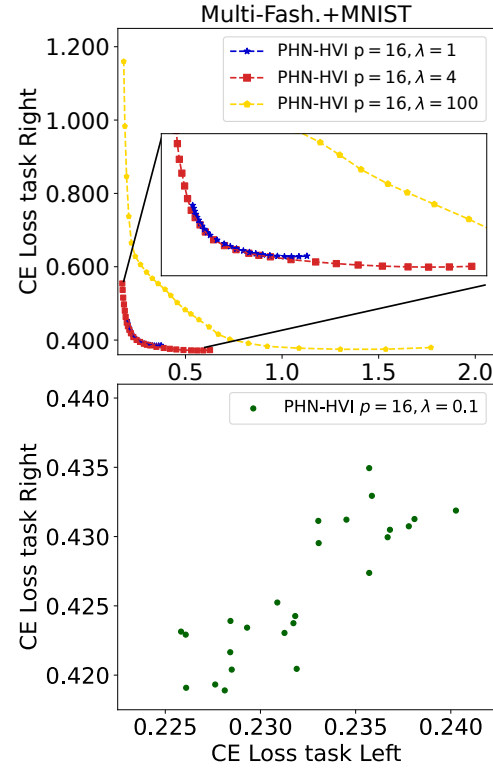


Figure 8: The impact of hyperparameter  $\lambda$

guarantee the convergence. On the other hand, it is shown that hypernetwork-based PFL can not approximate well disconnected-Pareto fronts. Hence, the question of whether PFL may be solved effectively without hypernetwork is very crucial to consider.

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