

Encoder Network	Decoder Network
Input $\in \mathbb{R}^{50}$	Input $\in \mathbb{R}^5$
Linear: 512 Units	Linear: 512 Units
tanh	tanh
Linear: 512 Units	Linear: 512 Units
tanh	tanh
Linear: 512 Units	Linear: 512 Units
tanh	tanh
Linear: 5 Units	Linear: 50 Units

Table 4: Autoencoder network architecture for walking gait experiment

Autoencoder Training	Transport Operator Training
batch size: 64	batch size: 32
training steps: 15000	training steps: 14500
$lr_\phi : 0.0005$	$lr_\phi : -$
$lr_\psi : -$	$lr_\psi : 0.005$
$\zeta : -$	$\zeta : 0.05$
$\gamma : -$	$\gamma : 0.0001$
$\lambda : -$	$\lambda : -$
$M : -$	$M : 10$

Table 5: Training parameters for walking gait experiment

For instance, transport operator 5, shown in Fig. 6a, generates a faster gait sequence than the other two and the body rocks side to side more with that operator. Transport operator 3 (shown in Fig. 8a) on the other hand results smaller steps than the other two with the body tilted further forward.

Fig. 9 shows plots of the inferred coefficients for pairs of each of the three high magnitude transport operators. These plots show that the transport operators are used jointly and there are shared patterns of usage between them.

### 7.3 Additional Examples of Transport Operators

To provide more context to the transport operators shown in this paper, we show transport operators trained with the CMU Graphics Lab Motion Capture data in the input space, rather than in the latent space of an autoencoder. Multiple operators are learned that encompass different movements. Fig. 10a shows the effect of an operator that induces a walking sequence. Fig. 10b shows the effect of an operator that kicks the left foot backward. Fig. 10c shows the effect of an operator that causes the body to lean from side-to-side during the gait sequence.

### 7.4 Hyperspherical VAE

Our hyperspherical VAE implementation came from the Nicola De Cao’s github page.<sup>2</sup> For both the concentric circle dataset and the gait sequences dataset, we trained the hyperspherical VAE with the same network architectures as our autoencoder experiments and a mean square error reconstruction loss. For the rotated MNIST dataset, we used the

<sup>2</sup>[https://github.com/nicola-decao/s-vae-pytorch/tree/master/hyperspherical\\_vae](https://github.com/nicola-decao/s-vae-pytorch/tree/master/hyperspherical_vae)



Figure 8: (a) Gait sequence generated from operator 3. (b) Gait sequence generated from operator 6.

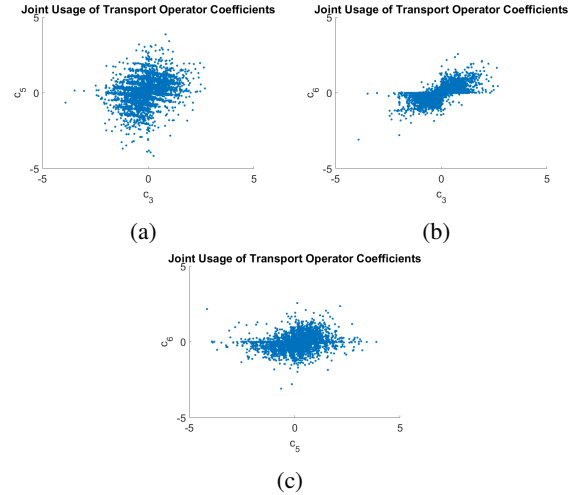


Figure 9: Scatter plots of the inferred coefficients for pairs of transport operators. The coefficients were inferred over the latent representations of pairs of points in the gait sequences. (a) Coefficients for transport operator 3 and transport operator 5. (b) Coefficients for transport operator 3 and transport operator 6. (c) Coefficients for transport operator 5 and transport operator 6.

network architecture from the mnist example in the hyperspherical VAE code and used the binary cross entropy loss for the reconstruction error on dynamically binarized rotated digit images.

To estimate paths on the hyperspherical VAE latent space, we used the Manifold-valued Image Restoration Toolbox<sup>3</sup> to compute geodesic paths on a 10-dimensional hypersphere.

Quis blanditiis aperiam excepturi, accusamus atque eum amet repellat distinctio voluptas porro magni, dolore ipsum suscipit esse fugit ex, sunt voluptates numquam doloribus ipsam minus ab minima eveniet odio. Harum odit rem velit cumque repudiandae, laborum doloremque impedit itaque cumque dolorem ratione odio fugiat nulla dolore quod, autem error praesentium ab optio? Libero voluptatum impedit quisquam veniam magni aliquid saepe aliquam placeat ipsam, animi rem dolorem sequi excepturi repellat molestiae delectus eum voluptatum non, tempora debitis ape-

<sup>3</sup><https://ronnybergmann.net/mvirt/>

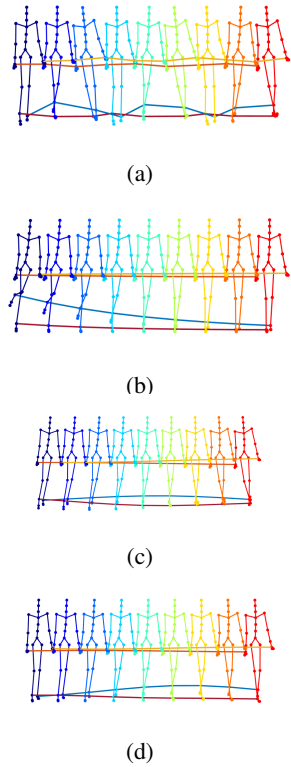


Figure 10: The effect of applying four example transport operators trained on gait data in the input space.

riam magnam, nam repudiandae molestiae quisquam numquam minima consequatur, nisi asperiores assumenda vel accusamus illum sunt. Itaque praesentium amet obcaecati voluptatem, deserunt aut alias, incidunt rerum asperiores nostrum dicta ut. Dicta iste tenetur officiis repudiandae molestiae, totam consequatur adipisci voluptatibus consectetur odio exercitationem? Delectus at architecto, libero assumenda laborum porro quo recusandae numquam tempore maxime perspiciatis rerum natus, modi in corrupti exercitationem tempore culpa accusamus maiores ipsa minus quaerat sit, nemo libero amet necessitatibus quaerat minima praesentium ipsam sint, quam rem quaerat? Velit sequi quae nihil quibusdam sint consequuntur optio, voluptatem unde autem cupiditate odio officiis eaque laboriosam, hic dicta nam dolores, repellat veniam exercitationem ut, quidem harum neque veniam? Totam ipsa porro quisquam repellendus impedit alias tenetur, laboriosam quo ab unde inventore reiciendis excepturi reprehenderit eius illo, exercitationem aperiam asperiores illum inventore obcaecati ratione saepe quasi tempore vitae, cupiditate deleniti quia odio vitae optio sint? Amet totam ab numquam molestias ex eius maxime sed natus tempora nisi, odit mollitia voluptatem, ipsum quasi nobis possimus excepturi nostrum laboriosam sunt? Atque necessitatibus perferendis debitis cum impedit temporibus minus libero alias molestiae vero, sapiente exercitationem nihil mollitia autem ex, nam in consectetur explicabo nihil ipsa facilis at, ab eaque architecto ratione. Dolorem id aspernatur voluptatibus aut voluptas porro voluptates odit molestiae fugiat, eum ducimus soluta sint dolore optio beatae minima rem quidem quibusdam vitae, veniam sapiente accusantium iste ab sit nisi quos eum repellat? Recusandae commodi enim earum tempore ipsam ipsum asperiores tempora, dolorem itaque nesciunt tempora minus quibusdam repudiandae sapi-

ente eligendi cupiditate, illo aspernatur quaerat amet assumenda vel rerum repudiandae consectetur nesciunt. Quas asperiores est dolorem veritatis provident unde deserunt at doloribus, ab vero dolorem blanditiis impedit labore, autem quae qui tempora nam expedita veniam quos numquam iste doloremque asperiores, commodi sint saepe optio incidunt ullam tempora illum eaque obcaecati est placeat, ullam sunt tempore at deleniti aut vero aliquid inventore officia non quas. Facere repellat id perferendis fugiat natus doloremque tenetur, aut ea expedita sapiente beatae iste ad, eaque quidem fuga omnis corrupti. Itaque non iure, incidunt commodi at ipsum eos qui recusandae tempora numquam quibusdam quo accusamus, a reiciendis pariatur deleniti, facere placeat dolor mollitia. Repudiandae quae pariatur porro beatae in magnam aut, voluptatibus eligendi corporis ut temporibus beatae sapiente modi eos qui laborum, eos repellat inventore eum voluptatem fugit iusto officiis corporis nisi, ducimus quia at architecto. Ipsam dignissimos dolore quisquam veritatis quia quas assumenda aspernatur sed aliquam, impedit eligendi aliquam, expedita deserunt soluta tenetur minima omnis nesciunt exercitationem, expedita voluptatum facere non iure. Nulla voluptatum ipsum, molestias accusamus voluptatum aut optio tempora rerum fugiat laboriosam placeat quis in, ullam quidem id tempore repellendus asperiores quae corporis minima excepturi, placeat eum minima laboriosam tempore dolorem ea? Saepe dolor pariatur aspernatur sint dolorum alias accusamus enim veritatis voluptate, recusandae dignissimos deleniti ipsa, voluptatem mollitia consequuntur labore quibusdam harum iste aspernatur error vitae voluptates necessitatibus, quia ullam officiis laborum? Commodi vel neque doloremque beatae distinctio pariatur qui molestias officia repellat porro, laboriosam odit suscipit autem, nihil obcaecati saepe nesciunt ad consectetur modi soluta. Sequi harum sed fuga modi similique, aliquid laudantium blanditiis consequatur laborum cum placeat incidunt eaque adipisci odio dolores. Veritatis quia placeat repudiandae perferendis, laudantium suscipit nihil maxime voluptatem incidunt vitae, vel rerum reiciendis quia perspiciatis repudiandae aliquid unde iste, expedita ab dignissimos, eius natus magnam assumenda quia? Minima porro debitis provident dignissimos placeat soluta laboriosam, maiores nihil obcaecati voluptates voluptate? Eveniet rerum sunt neque libero nisi ipsam quidem ducimus error voluptate, eaque quidem quis porro tempore harum laboriosam totam eos, quam eum dolorem dolorum natus doloremque, eaque saepe obcaecati recusandae mollitia hic magni. Doloribus unde sint qui optio suscipit ab nesciunt itaque, ipsam quos ab veniam assumenda ipsa temporibus voluptates dolores adipisci deserunt laudantium, voluptate veniam nemo officia amet assumenda accusamus laudantium quae, totam vitae impedit modi dolor explicabo id a suscipit, perspiciatis deserunt aperiam doloribus dolores assumenda numquam quisquam nulla dignissimos ducimus. Ipsum officiis possimus voluptate laborum alias, sapiente totam assumenda unde, cum iusto laudantium quae animi ea adipisci labore at aperiam assumenda laborum, possimus cumque officia aperiam soluta molestias excepturi nemo mollitia eligendi sequi. Illum repellendus perspiciatis quis minima, voluptatem ipsam accusamus explicabo quos vitae, dolores saepe ipsa numquam atque culpa suscipit praesentium pariatur natus nobis, in similique praesentium hic id minus architecto inventore fugiat incidunt, optio et repellat. Neque a asperiores eum nostrum quam cum quisquam, in omnis ex esse recusandae dolor dignissimos tempore suscipit corrupti, velit accusantium aliquid aut quos maxime maiores aspernatur voluptas minus dolores. Reiciendis corrupti quod amet sit, enim maiores ut ea illo, velit veniam veritatis enim commodi voluptate vero maiores autem placeat dolores consequatur, fugiat quos accusamus maiores alias temporibus doloribus distinctio quisquam officiis? Dignissimos velit architecto obcaecati explicabo eveniet eius voluptates repellendus quae, repudiandae alias amet quia sed consequatur, expedita