

Encoder Network	Decoder Network
Input $\in \mathbb{R}^{50}$	Input $\in \mathbb{R}^5$
Linear: 512 Units	Linear: 512 Units
tanh	tanh
Linear: 512 Units	Linear: 512 Units
tanh	tanh
Linear: 512 Units	Linear: 512 Units
tanh	tanh
Linear: 5 Units	Linear: 50 Units

Table 4: Autoencoder network architecture for walking gait experiment

Autoencoder Training	Transport Operator Training
batch size: 64	batch size: 32
training steps: 15000	training steps: 14500
$lr_\phi : 0.0005$	$lr_\phi : -$
$lr_\psi : -$	$lr_\psi : 0.005$
$\zeta : -$	$\zeta : 0.05$
$\gamma : -$	$\gamma : 0.0001$
$\lambda : -$	$\lambda : -$
$M : -$	$M : 10$

Table 5: Training parameters for walking gait experiment

For instance, transport operator 5, shown in Fig. 6a, generates a faster gait sequence than the other two and the body rocks side to side more with that operator. Transport operator 3 (shown in Fig. 8a) on the other hand results smaller steps than the other two with the body tilted further forward.

Fig. 9 shows plots of the inferred coefficients for pairs of each of the three high magnitude transport operators. These plots show that the transport operators are used jointly and there are shared patterns of usage between them.

### 7.3 Additional Examples of Transport Operators

To provide more context to the transport operators shown in this paper, we show transport operators trained with the CMU Graphics Lab Motion Capture data in the input space, rather than in the latent space of an autoencoder. Multiple operators are learned that encompass different movements. Fig. 10a shows the effect of an operator that induces a walking sequence. Fig. 10b shows the effect of an operator that kicks the left foot backward. Fig. 10c shows the effect of an operator that causes the body to lean from side-to-side during the gait sequence.

### 7.4 Hyperspherical VAE

Our hyperspherical VAE implementation came from the Nicola De Cao’s github page.<sup>2</sup> For both the concentric circle dataset and the gait sequences dataset, we trained the hyperspherical VAE with the same network architectures as our autoencoder experiments and a mean square error reconstruction loss. For the rotated MNIST dataset, we used the

<sup>2</sup>[https://github.com/nicola-decao/s-vae-pytorch/tree/master/hyperspherical\\_vae](https://github.com/nicola-decao/s-vae-pytorch/tree/master/hyperspherical_vae)



Figure 8: (a) Gait sequence generated from operator 3. (b) Gait sequence generated from operator 6.

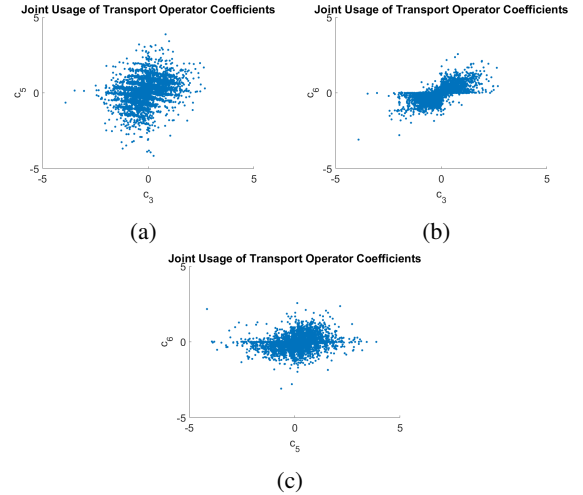


Figure 9: Scatter plots of the inferred coefficients for pairs of transport operators. The coefficients were inferred over the latent representations of pairs of points in the gait sequences. (a) Coefficients for transport operator 3 and transport operator 5. (b) Coefficients for transport operator 3 and transport operator 6. (c) Coefficients for transport operator 5 and transport operator 6.

network architecture from the mnist example in the hyperspherical VAE code and used the binary cross entropy loss for the reconstruction error on dynamically binarized rotated digit images.

To estimate paths on the hyperspherical VAE latent space, we used the Manifold-valued Image Restoration Toolbox<sup>3</sup> to compute geodesic paths on a 10-dimensional hypersphere.

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<sup>3</sup><https://ronnybergmann.net/mvirt/>

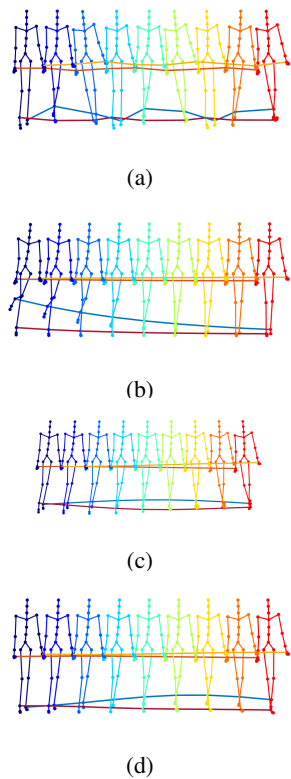


Figure 10: The effect of applying four example transport operators trained on gait data in the input space.

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