Table 3: RMSE on MovieLens datasets

RMSE	LR	FM	CFM	DiFacto	PQR (k=500)	PQR (k=1000)	PQR (k=2000)
ML100K	1.0429	$1.0316 \pm 1.2e\text{-}3$	1.0326	$1.0312 \pm 9.0e\text{-}4$	1.0225	1.0215	**1.0215**
ML1M	1.0487	$1.0434 \pm 5.0e\text{-}4$	1.0403	$1.0411 \pm 9.0e\text{-}4$	1.0321	1.0250	**1.0190**
ML10M	0.9651	$0.9605 \pm 1.7e\text{-}3$	0.9616	$0.9572 \pm 1.1e\text{-}3$	0.9547	0.9541	**0.9536**

Table 4: AUC, LogLoss, Training Time, and Model Size of high-dimensional datasets

Avazu	LR	FM	DiFacto	PQR (k=2000)	PQR (k=4000)	PQR (k=8000)
AUC	0.7562	$0.7752 \pm 1.8e\text{-}4$	$0.7743 \pm 1.1e\text{-}4$	0.7757	0.7777	**0.7785**
LogLoss	0.3939	$0.3840 \pm 1.1e\text{-}4$	$0.3840 \pm 1.2e\text{-}4$	0.3830	0.3818	**0.3812**
Training Time	$1\times$	$58.5\times$	$58.0\times$	$3.3\times$	$3.6\times$	$3.8\times$
Model Size	16.37K	1.17M	0.29M	0.68M	1.32M	1.97M

Criteo	LR	FM	DiFacto	PQR (k=200)	PQR (k=500)	PQR (k=1000)
AUC	0.7151	$0.7218 \pm 1.6e\text{-}4$	$0.7216 \pm 1.6e\text{-}4$	0.7207	0.7220	**0.7221**
LogLoss	0.5116	$0.5068 \pm 2.0e\text{-}4$	$0.5070 \pm 8.0e\text{-}5$	0.5076	0.5067	**0.5066**
Training Time	$1\times$	$20.1\times$	$20.6\times$	$17.1\times$	$18.3\times$	$19.2\times$
Model Size	1.09K	17K	14.6K	18.30K	77.68K	98.61K

KDD2012	LR	FM	DiFacto	PQR (k=20)	PQR (k=50)	PQR (k=100)
AUC	0.7944	$0.7968 \pm 1.9e\text{-}4$	$0.7955 \pm 3.0e\text{-}4$	0.8013	0.8015	**0.8016**
LogLoss	0.1541	$0.1535 \pm 5.1e\text{-}5$	$0.1534 \pm 6.5e\text{-}5$	0.1524	0.1524	**0.1523**
Training Time	$1\times$	$10.4\times$	$11.8\times$	$3.1\times$	$3.1\times$	$3.2\times$
Model Size	9.11M	91.6M	72.2M	9.70M	9.71M	9.71M

For ML10M, since there is no demographic information of users, we just use the user ids as the user feature instead.

The rank of FM is selected from the set of $\{2, 4, 8, 16, 32, 64, 128, 256, 512\}$. The coefficient of regularization term and the learning rate are tuned in a range $[0.0001, 10]$. We list the RMSE of all compared algorithms in Table 3. From our observation, PQR achieves higher prediction accuracy than all the baselines, which illustrates the advantage of PQR.

4.3 Online Binary Classification

In many real applications, feature vectors are usually extremely high-dimensional hence sparse representation is used instead, i.e., only nonzero key-value pairs are stored. We demonstrate the performance of PQR as well as other approaches for this case using high-dimensional sparse datasets: Avazu, Criteo, and KDD2012. Due to the high-dimensionality, CFM is impractical. So we compare PQR with LR, vanilla FM and DiFacto in two aspects: accuracy (measured by AUC and LogLoss) and efficiency (measured by training time and model size).

The rank for vanilla FM and DiFacto is selected from the set of $\{4, 8, 16, 32, 64\}$. The coefficient of regularization term and the learning rate are selected in a range of $[0.0001, 10]$. We list the results in Table 4. In most cases, PQR outperforms the baseline algorithms in both accuracy and efficiency. The accuracy demonstrates that PQR has better expressiveness than linear model and low-rank FM models. The efficiency illustrates the theoretical computation cost of PQR model. Finally, for FM, there is no theory about

how to choose a proper rank. However, the order of PQR has a clear meaning which is good for parameter tuning.

5 Conclusion and Future Work

In this paper, we propose a projective quadratic regression (PQR) model under the online learning settings. It meets the requirements in online convex optimization framework and the global optimal solution can be achieved due to its convexity. In addition, we show that the computation cost of PQR can be low if we choose a suitable order. Finally, we demonstrate its effectiveness by comparing with state-of-the-art approaches. For future work, a more scientific approach to select the order for PQR is an attractive direction.

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