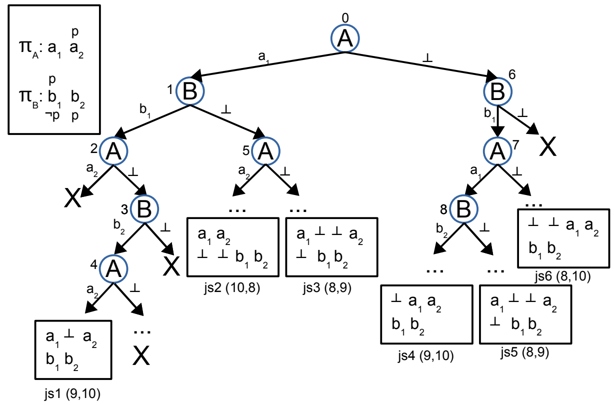

Figure 1: Tree example

joint schedule profile $js1$, or its equivalent $js4$. This schedule profile reports the highest possible utility for agent B, and a penalty of one unit (generic penalty) for agent A. Let's see how the backward induction algorithm obtains the SPE in this example. The payoffs of $js1$ are back up to node 2, where they will be compared with the values of node 5. The joint schedule $js2$ is backed up to node 5 because agent A is who chooses at node 5. Then, in node 1 agent B chooses between node 2 and node 5 and hence, $js1$ is chosen. In the other branches, in node 8 $js4$ will prevail over $js5$ and then, when compared in node 7 with $js6$, the choice of agent A is $js4$. This results in agent A choosing at node 0 between $js1$ and $js4$, both with the same payoffs, and so both are equivalent SPE solutions. If the tree is developed following a different agent order the SPE solution will be the same.

Experimental results

In this section, we present some experimental results in order to validate and discuss our approach. As several factors can affect the solutions of the general game, we show different examples of game situations.

We implemented a program to generation the extensive-form tree and apply the backward induction algorithm. The NE in the normal-form game is computed with the tool Gambit (**?**).

For the experiments we used problems of the well-known Zeno-Travel domain from the International Planning Competition (IPC-3)[4]. However, for simplicity and the sake of clarity, we show generic actions in the figures.

The experiments were carried out for two agents, A and B. Both agents have a set of individual plans that solve one or more goals. The more goals achieved by a plan, the more the benefit of the plan. In addition, the benefit of a plan depends on the makespan of such plan. Given a plan π, which earliest plan execution is denoted by ψ_0, $\beta_i(\pi)$ is calculated as follows: $\beta_i(\pi) = nGoals(\pi) * 10 - makespan(\psi_0)$, where $nGoals(\pi)$ represents the number of goals solved by π and $makespan(\psi_0)$ represents the minimum duration schedule for π.

[4]http://ipc.icaps-conference.org/

The utility of a particular schedule $\psi \in \Psi_\pi$ is a function of $\beta_i(\pi)$ and the number of time units that the actions of π are delayed in ψ with respect to the earliest plan execution ψ_0; in other words, the difference in the makespan of ψ and ψ_0. Thus, $\mu_i(\psi) = \beta_i(\pi)$ if $\psi = \psi_0$. Otherwise, $\mu_i(\psi) = \beta_i(\pi) - delay(\psi)$, where $delay(\psi)$ is the delay in the makespan of ψ with respect to the makespan of ψ_0.

Problem	Agent	Plan	nAct(π)	$\beta_i(\pi)$
1	A	$\pi_{A1}(g_1 g_2)$	3	17
		$\pi_{A2}(g_1)$	2	8
		$\pi_{A3}(g_2)$	1	9
	B	$\pi_{B1}(g_1 g_2)$	2	18
		$\pi_{B2}(g_1)$	1	9
		$\pi_{B3}(g_2)$	1	9
2	A	$\pi_{A1}(g_1 g_2)$	2	18
		$\pi_{A2}(g_1 g_2)$	4	16
		$\pi_{A3}(g_1)$	2	8
		$\pi_{A4}(g_2)$	1	9
	B	$\pi_{B1}(g_1 g_2)$	2	18
		$\pi_{B2}(g_1 g_2)$	4	16
		$\pi_{B3}(g_1)$	1	9
		$\pi_{B4}(g_2)$	1	9

Table 1: Problems description

Table 1 shows the problems used in these experiments: the set of initial plans of each agent, the number of actions of each plan and its utility.

	π_{B1}	π_{B2}	π_{B3}
π_{A1}	15,16 (2,2)	17,7 (0,2)	17,9 (0,0)
π_{A2}	8,16 (0,2)	8,7 (0,2)	7,9 (1,0)
π_{A3}	7,18 (2,0)	9,9 (0,0)	8,9 (1,0)

Table 2: Problem 1

In Table 2 we can see the results of the general game for problem 1. Each cell is the result of a joint plan schedule game that combines a plan of agent A and a plan of agent B. In each cell, we show the payoff of π_{Ax} and π_{By} as well as the values of $delay(\psi)$ for each plan (delay values are shown between parenthesis). The values in each cell are the result of the schedule profile returned by the internal game.

The NE of this problem is the combination of π_{A1} and π_{B1}, with an utility of (15,16) for agent A and B, respectively. Agent A uses the plan that solves its goals g_1 and g_2 delayed two time steps. Agent B uses the plan that solves its goals g_1 and g_2, also delayed two time steps. The solution for both agents is to use the plan that solves more goals (with a higher initial benefit) a bit delayed. This can be a typical situation if there are not many conflicts and if the delay is not very punishing to the agents. The schedule of this solution is shown in Figure 2. We can see in the figure that agent A starts the execution of its plan π_{A1} at $t = 0$, but after having scheduled its first two actions, the strategy of agent A introduces a delay of two time steps (empty actions) until it can finally execute its final action without causing a mutex with the actions of agent B. Regarding agent B, its first action in π_{B1} is delayed two time units to avoid the conflict with agent A. In this example, both agents have a conflict with each other (both have an action which deletes a condition that the other agent needs).

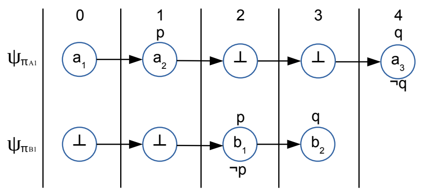

Figure 2: Schedule example

Table 3 represents the game in normal-form of problem 2 shown in Table 1. In this case, we find three different equilibria: (π_{A1}, π_{B2}) with payoffs (15,14) and delays (3,2) for agent A and B, respectively; another NE is (π_{A2}, π_{B1}), with payoffs (14,15) and a delay of (2,3) time steps, respectively; the last NE is a mixed strategy with probabilities 0.001 and 0.999 for π_{A1} and π_{A2} of agent A, and probabilities 0.001 and 0.999 for strategies π_{B1} and π_{B2} of agent B. In this problem we have a cell with $-\infty$ as payoff of the two agents. This payoff represents that there does not exist a valid joint schedule for the plans due to an unsolvable conflict as the one shown in Figure 3.

	π_{B1}	π_{B2}	π_{B3}	π_{B4}
π_{A1}	$-\infty$,$-\infty$	15,14 (3,2)	18,7 (0,2)	17,9 (1,0)
π_{A2}	14,15 (2,3)	14,14 (2,2)	16,6 (0,3)	16,9 (0,0)
π_{A3}	8,16 (0,2)	8,16 (0,0)	8,7 (0,2)	8,8 (0,1)
π_{A4}	7,18 (2,0)	6,16 (3,0)	9,9 (0,0)	8,9 (1,0)

Table 3: Problem 2

The game in Table 4 is the same game as the one in Table 3 but, in this case, the agents suffer a delay penalty of 3.5 (instead of 1) per each action delayed in their plan schedules. Under this new evaluation, we can see how this affects the general game. In this situation, the only NE solution is (π_{A2}, π_{B2}) with utility values (9,9) and a delay of two time steps for each agent. Note that this solution is neither Pareto-optimal (solution (16,9) is Pareto-optimal) nor it maximizes the social welfare. However, these two solution concepts can be applied in case of multiple NE.

In conclusion, our approach simulates how agents behave with several strategies and it returns an equilibrium solution that is stable for all of the agents. All agents participate in the schedule profile solution and their utilities are dependent on the strategies of the other agents regarding the conflicts that appear in the problem.

Conclusions and future work

In this paper, we have presented a complete game-theoretic approximation for non-cooperative agents. The strategies of

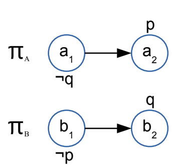

Figure 3: Unsolvable conflict

	π_{B1}	π_{B2}	π_{B3}	π_{B4}
π_{A1}	$-\infty$,$-\infty$	7.5,9 (3,2)	18,2 (0,2)	14.5,9 (1,0)
π_{A2}	9,7.5 (2,3)	9,9 (2,2)	16,-1.5 (0,3)	16,9 (0,0)
π_{A3}	8,11 (0,2)	8,16 (0,0)	8,2 (0,2)	8,5.5 (0,1)
π_{A4}	2,18 (2,0)	-1.5,16 (3,0)	9,9 (0,0)	5.5,9 (1,0)

Table 4: Problem 2b, more delay penalty to the utility

the agents are determined by the different ways of solving mutex actions at a time instant and the loss of utility of the solutions in the plan schedules. We also present some experiments carried out in a particular planning domain. The results show that the SPE solution of the extensive-form game in combination with the NE of the general game return a stable solution that responds to the strategic behavior of all of the agents. As for future work, we intend to explore two different lines of investigation. The exponential cost of this approach represents a major limitation for being used as a general MAP method for self-interested agents. Our combination of a general+internal game can be successively applied in subproblems of the agents. Considering that this approach solves a subset of goals of an agent, the agent could get engaged in a new game to solve the rest of his goals, and likewise for the rest of agents. Then, a MAP problem can be viewed as solving a subset of goals in each repetition of the whole game. In this line, the utility functions of the agents can be modeled not only to consider the benefit of the current schedule profile but also to predict the impact of this strategy profile in the resolution of the future goals. That is, we can define payoffs as a combination of the utility gained in the current game plus an estimate of how the joint plan schedule would impact in the resolution of the remaining goals. Another line of investigation is to extend this approach to cooperative games, allowing the formation of coalitions of agents if the coalition represents a more advantageous strategy than playing *alone*.

Acknowledgments

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