

search time for both KBest and KBestEC grows exponentially. The KBest and KBestEC are designed to solve problems of size fewer than  $20^7$ , and so they have some difficulty with larger datasets. They also fail to generate correct scoring files for msnbc. KBestEC seems to successfully expand the coverage of DAGs with some overhead for checking equivalence classes. However, KBestEC took much longer than KBest for some instances, e.g., nltns and letter, and the number of DAGs covered by the found MECs is inconsistent for nltns, letter and zoo. The search time for the BF approach is improved over the  $k$ -best approach except for datasets with very large sample sizes. The generalized pruning rules are very effective in reducing the search space, which then allows GOBNILP\_dev to solve the ILP problem subsequently. Comparing to the improved results in (?; ?, ?), our approach can scale to larger networks if the scoring file can be generated.<sup>8</sup>

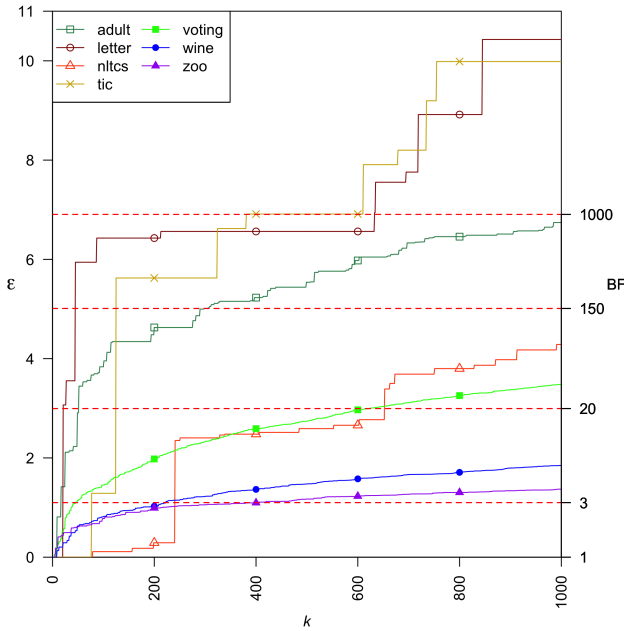


Figure 2: The deviation  $\epsilon$  from the optimal BDeu score by  $k$  using results from KBest. The corresponding values of the BF ( $\epsilon = \log(BF)$ , see Equation 3) are presented on the right. For example, if the desired BF value is 20, then all networks falling below the dash line at 20 are credible.

Now we show that different datasets have distinct score patterns in the top scoring networks. The scores of the 1,000-best networks for some datasets in the KBest experiment are plotted in Figure 2. A specific line for a dataset indicates the deviation  $\epsilon$  from the optimal BDeu score by the  $k$ th-best network. For reference, the red dash lines represent different levels of BFs calculated by  $\epsilon = \log BF$  (See Equation 3). The figure shows that it is difficult to pick a value for  $k$  *a priori* to capture the appropriate set of top scoring networks. For a few datasets such as adult and letter, it only takes fewer

than 50 networks to reach a BF of 20, whereas zoo needs more than 10,000 networks. The sample size has a significant effect on the number of networks at a given BF since the lack of data leads to many BNs with similar probabilities. It would be reasonable to choose a large value for  $k$  in model averaging when data is scarce and vice versa, but only the BF approach is able to automatically find the appropriate and credible set of networks for further analysis.

## Conclusion

Existing approaches for model averaging for Bayesian network structure learning either severely restrict the structure of the Bayesian network or have only been shown to scale to networks with fewer than 30 random variables. In this paper, we proposed a novel approach to model averaging inspired by performance guarantees in approximation algorithms that considers all networks within a factor of optimal. Our approach has two primary advantages. First, our approach only considers *credible* models in that they are optimal or near-optimal in score. Second, our approach is significantly more efficient and scales to much larger Bayesian networks than existing approaches. We modified GOBNILP (?), a state-of-the-art method for finding an optimal Bayesian network, to implement our generalized pruning rules and to find all *near-optimal* networks. Our experimental results demonstrate that the modified GOBNILP scales to significantly larger networks without resorting to restricting the structure of the Bayesian networks that are learned.

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<sup>7</sup>Obtained through correspondence with the author.

<sup>8</sup>We are unable to generate BDeu score files for datasets with over 30 variables.

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