Table 3: RMSE on MovieLens datasets

RMSE	LR	FM	CFM	DiFacto	PQR (k=500)	PQR (k=1000)	PQR (k=2000)
ML100K	1.0429	1.0316 ± 1.2e-3	1.0326	1.0312 ± 9.0e-4	1.0225	1.0215	**1.0215**
ML1M	1.0487	1.0434 ± 5.0e-4	1.0403	1.0411 ± 9.0e-4	1.0321	1.0250	**1.0190**
ML10M	0.9651	0.9605 ± 1.7e-3	0.9616	0.9572 ± 1.1e-3	0.9547	0.9541	**0.9536**

Table 4: AUC, LogLoss, Training Time, and Model Size of high-dimensional datasets

Avazu	LR	FM	DiFacto	PQR (k=2000)	PQR (k=4000)	PQR (k=8000)
AUC	0.7562	0.7752 ± 1.8e-4	0.7743 ± 1.1e-4	0.7757	0.7777	**0.7785**
LogLoss	0.3939	0.3840 ± 1.1e-4	0.3840 ± 1.2e-4	0.3830	0.3818	**0.3812**
Training Time	1×	58.5×	58.0×	3.3×	3.6×	3.8×
Model Size	16.37K	1.17M	0.29M	0.68M	1.32M	1.97M
Criteo	LR	FM	DiFacto	PQR (k=200)	PQR (k=500)	PQR (k=1000)
AUC	0.7151	0.7218 ± 1.6e-4	0.7216 ± 1.6e-4	0.7207	0.7220	**0.7221**
LogLoss	0.5116	0.5068 ± 2.0e-4	0.5070 ± 8.0e-5	0.5076	0.5067	**0.5066**
Training Time	1×	20.1×	20.6×	17.1×	18.3×	19.2×
Model Size	1.09K	17K	14.6K	18.30K	77.68K	98.61K
KDD2012	LR	FM	DiFacto	PQR (k=20)	PQR (k=50)	PQR (k=100)
AUC	0.7944	0.7968 ± 1.9e-4	0.7955 ± 3.0e-4	0.8013	0.8015	**0.8016**
LogLoss	0.1541	0.1535 ± 5.1e-5	0.1534 ± 6.5e-5	0.1524	0.1524	**0.1523**
Training Time	1×	10.4×	11.8×	3.1×	3.1×	3.2×
Model Size	9.11M	91.6M	72.2M	9.70M	9.71M	9.71M

gether. For ML10M, since there is no demographic information of users, we just use the user ids as the user feature instead. The rank of FM is selected from the set of $\{2, 4, 8, 16, 32, 64, 128, 256, 512\}$. The coefficient of regularization term and the learning rate are tuned in a range $[0.0001, 10]$. We list the RMSE of all compared algorithms in Table **??**. From our observation, PQR achieves higher prediction accuracy than all the baselines, which illustrates the advantage of PQR.

4.3 Online Binary Classification

In many real applications, feature vectors are usually extremely high-dimensional hence sparse representation is used instead, i.e., only nonzero key-value pairs are stored. We demonstrate the performance of PQR as well as other approaches for this case using high-dimensional sparse datasets: Avazu, Criteo, and KDD2012. Due to the high-dimensionality, CFM is impractical. So we compare PQR with LR, vanilla FM and DiFacto in two aspects: accuracy (measured by AUC and LogLoss) and efficiency (measured by training time and model size).

The rank for vanilla FM and DiFacto is selected from the set of $\{4, 8, 16, 32, 64\}$. The coefficient of regularization term and the learning rate are selected in a range of $[0.0001, 10]$. We list the results in Table **??**. In most cases, PQR outperforms the baseline algorithms in both accuracy and efficiency. The accuracy demonstrates that PQR has better expressiveness than linear model and low-rank FM models. The efficiency illustrates the theoretical computation cost of PQR model. Finally, for FM, there is no theory about how to choose a proper rank. However, the order of PQR has a clear meaning which is good for parameter tuning.

5 Conclusion and Future Work

In this paper, we propose a projective quadratic regression (PQR) model under the online learning settings. It meets the requirements in online convex optimization framework and the global optimal solution can be achieved due to its convexity. In addition, we show that the computation cost of PQR can be low if we choose a suitable order. Finally, we demonstrate its effectiveness by comparing with state-of-the-art approaches. For future work, a more scientific approach to select the order for PQR is an attractive direction.

Voluptate optio expedita facere aliquam blanditiis maiores neque quisquam inventore, inventore odio doloremque laudantium cumque vitae error esse in pariatur commodi, illum maiores sint officia, suscipit ipsum cum quia quisquam obcaecati excepturi tenetur quidem laboriosam nobis, porro quisquam quo commodi numquam ut nesciunt.Quaerat officia consequatur iste, rerum consequatur magnam cum numquam.Eos temporibus suscipit amet quo voluptatum id cumque sed obcaecati quos, voluptatem fugit et aliquam itaque ex cupiditate ipsam veritatis, odio dolores numquam fugit fuga officia reprehenderit quos unde.Laboriosam laborum qui aspernatur in quasi iure, hic fugit cupiditate iste neque vero placeat dicta saepe eligendi deserunt incidunt, autem dolores nulla perspiciatis error omnis necessitatibus minima.Debitis maxime itaque suscipit repudiandae ipsam alias accusantium cum quibusdam animi dicta, molestias illum saepe nemo nihil suscipit, fugiat delectus cumque labore cum neque?Esse quas enim, quos similique nemo magnam ea facilis ad voluptatibus natus atque asperiores,