Method	Multi-MNIST	Multi-Fashion	Fash.+MNIST	Drug	Jura	SARCOS
PHN-EPO	2.868	2.238	2.815	1.226	0.933	0.932
PHN-LS	2.859	2.219	2.764	1.208	0.932	0.934
COSMOS	2.959	2.324	2.838	NA	0.933	0.830
PHN-HVI	**3.012**	**2.408**	**2.967**	**1.294**	**0.946**	**0.949**

Table 1: Results compared to the state-of-the-art methods on Hypervolume

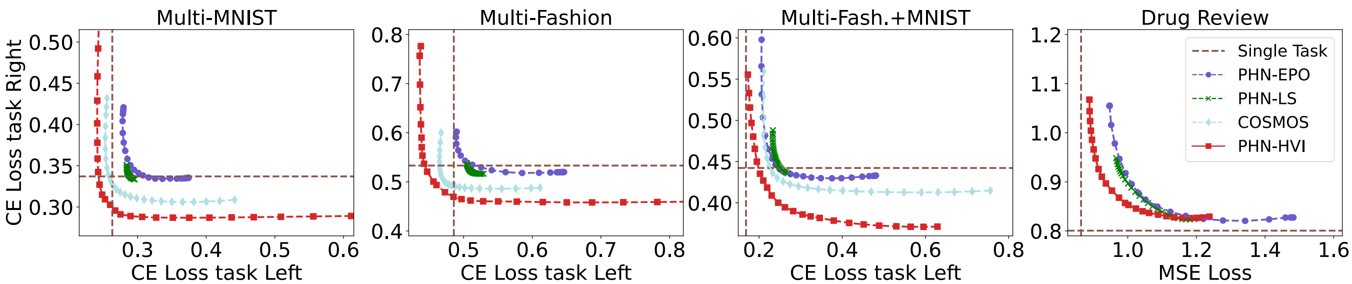

Figure 5: Pareto fronts are generated by methods

profile the Pareto front by removing the dominated solutions.

We use the sigmoid function to address constraints in Problem 4. Using the sampling technique which is described in Section 3, we use 231 uniform 3D rays. In Figure 4, PHN-LS entirely fails, COSMOS and PHN-EPO offer solutions that are widespread but a little chaotic, while PHN-HVI produces solutions that result in a pretty uniform distribution.

Image Classification

Three benchmark datasetsMulti-MNIST, Multi-Fashion, and Multi-Fashion+MNIST (**?**) are used in our evaluation. For each dataset, we have a two-objective multitask learning problem that asks us to categorize the top-left (task left) and bottom-right (task right) items (task right). Each dataset has 20,000 test set instances and 120,000 training examples. 10% of the training data are used for the validation split. Multi-LeNet (**?**) is the network that all approaches aim to reach. We set $p = 16, \lambda = 5$ on the Multi-MNIST dataset, $p = 16, \lambda = 4$ on the Multi-Fashion dataset, and Multi-Fashion+MNIST dataset for PHN-HVI .

In Quadrant I, we evaluate every methods using 25 uniformly distributed preference vectors. The results are shown in Table 1 and Figure 5. The HV was calculated by reference point (2, 2). The Pareto front of PHN-HVI outperforms and covers the baselines entirely. The Pareto Front is generated by other techniques, especially PHN-LS, that are not widely dispersed. There are numerous solutions on the Pareto front of PHN-HVI that may be achieved that are superior to a single task.

Text Classification and Regression

In this investigation, we concentrated on the Drug Review dataset (**?**). This dataset comprises user evaluations of particular medications, details on pertinent ailments, and a user score that shows general satisfaction. We examine two tasks: (1) regression-based prediction of the drug's rating and (2) classify the patient's condition.

This dataset consists of 215063 samples. 10% of the data which has conditions with insufficient user feedback are removed. Following that are 100 condition labels and 188155 samples. The dataset has a ratio of 0.65/0.10/0.25 for train/-val/test. Target network is TextCNN (**?**). The hyperparameters for the PHN-HVI model are $p = 16, \lambda = 4$. Test rays is 25 evenly preference vectors. The reference point for hypervolume is (2, 2).

In this case, PHN-HVI has a greater hypervolume than previous techniques and still permits the Pareto front to move deeper. For COSMOS, due to the mapping from a preference vector r, to the huge dimensionality of the embedding feature, it does not converge.

Multi-Output Regression

To demonstrate the viability of our strategy in high-dimensional space, we conduct experiments on 2 datasets:

- **Jura** (**?**): In this experiment, the goal variables are zinc, cadmium, copper, and lead (4 tasks), whereas the predictive features are the other metals, the type of land use, the type of rock, and the position coordinates at 359 different locations. The dataset has a ratio of 0.65/0.15/0.20 for train/val/test.

- **SARCOS** (**?**): The goal is predict pertinent 7 joint torques (7 tasks) from a 21-dimensional input space (7 joint locations, 7 joint velocities, 7 joint accelerations). There are 4449 and 44484 examples on testing/training set. As validation set, 10% of the training data are used.

The target network in both experiments is a Multi-Layer Perceptron with 4 hidden layers containing 256 units. We set $p = 8, \lambda = 0.001$ for PHN-HVI on both two datasets. The reference point for calculating HV is $(1, 1, \dots, 1)$. PHN-HVI outperforms all other baselines in terms of Hypervolume.

Ablation Study

Number of Rays p. Figure 6 demonstrates that the quality of the Pareto front increases with the number of rays, but up to a certain point, adding more rays no longer significantly improves the results. That means our framework doesn't require too many sampling rays to get a good performance.

Partition. As shown in Figure 7, partitioning algorithm makes it easier for the cosine similarity function and the HV function to cooperate and enhances PHN-HVI performance.

Cosine Similarity. The cosine similarity function is critical in the convergence of PHN-HVI and helps in the spread of the Pareto Front. In Figure 8, if λ is very large ($\lambda = 100$), Pareto Front is very widely dispersed, but it is quite shallow. If λ is very small ($\lambda = 0.1$), PHN-HVI can't generate Pareto Front. Therefore, selecting a suitable lambda that balances the HV function and the cosine similarity function is critical for the PHN-HVI to work effectively.

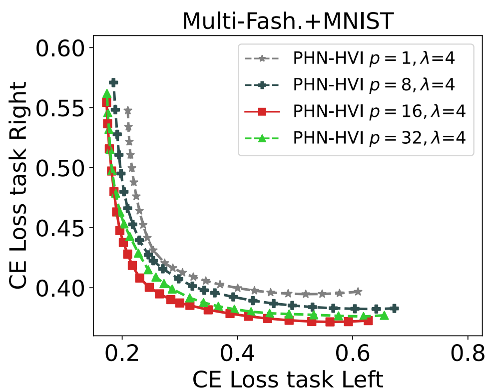

Figure 6: Performance of PHN-HVI when p varies

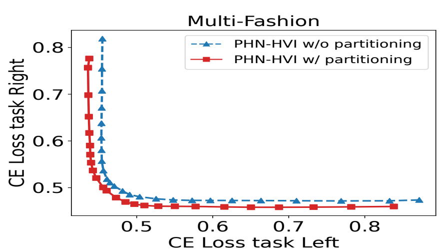

Figure 7: The effect of loss space partitioning

6 Conclusion and Future Work

In this paper, we propose PHN-HVI with Multi-Sample Hypernetwork, which utilizes a variety of trade-off vectors simultaneously, followed by hypervolume maximization to improve the PFL problem. This approach also opens up a wide range of potential research directions. On one hand, it is necessary to investigate theoretically for which objective functions the hypernetwork-based PFL methods will guarantee the convergence. On the other hand, it is shown that hypernetwork-based PFL can not approximate well disconnected-Pareto fronts. Hence, the question of whether

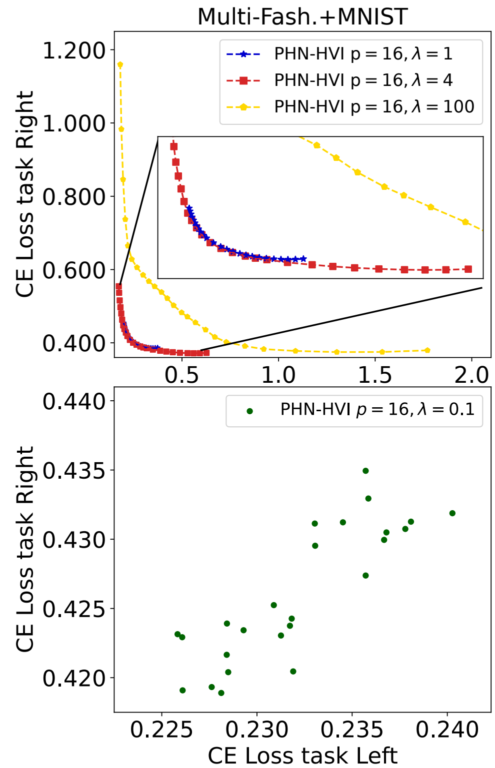

Figure 8: The impact of hyperpareter λ

PFL may be solved effectively without hypernetwork is very crucial to consider.

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