from $X = V$ and $p(v) = 0$ ($v \in V$), repeat the following process: Price $p(v) = \min_{X: x \in X} (f(X) - f(X \setminus x))$ for each $v \in X$, remove $\tilde{x}$ that attains the minimum from X, and then price $p(\tilde{x}) = +\infty$. This algorithm is motivated by the ascending auction (**?**).

We remark that there are no existing algorithms that are directly applicable to the optimal pricing problem with submodular valuations (see also Section **??**).

We used all the networks described above; we set $|V| = 100$, $|W| = 10000$, $d = 10$ and $q_{\max} = 0.3$ for `Uniform` and `PowerLaw`. The result is summarized in Table **??**. The proposed algorithm outperforms all compared algorithms for all datasets

(3) Scalability We evaluated the scalability of the proposed algorithm. We used `Uniform` with $|V| \in \{16, 32, \dots, 1024\}$, $|W| \in \{100, 1000, 10000, 100000\}$, $d = 10$, and $q_{\max} = 0.3$. We also conducted the same experiment on `PowerLaw` but we omit it since it yields very similar results.

The result is shown in Figure **??**. The elapsed times were (roughly) proportional to both $|V|$ and $|W|$. This is consistent with our analysis that the proposed algorithm runs in $O(|V||E|)$ time, and the number of edges is proportional to $|W|$ for these networks. Therefore, the proposed algorithm scales to moderately large networks.

(4) Number of allocated channels and activation probabilities We observe the relationship between activation probability and the obtained allocation. We used `Uniform` and `PowerLaw` with the parameters $|V| = 100$, $|W| = 10000$, and $d = 10$. We controlled the maximum activation probability $q_{\max} \in \{0.05, 0.10, \dots, 0.95\}$ and observe the number of assigned marketing channels.

The result is shown in Figure **??**. For both networks, when $q_{\max}$ was small the proposed algorithm assigned all channels, and when $q_{\max}$ was large it assigned a few channels. The number of assigned channels decreased much faster in `PowerLaw` than in `Uniform`, since there were highly correlated marketing channels in `PowerLaw`.

(5) Profit and the number of advertisers Next, we conducted experiments on the multiple advertisers case and the multiple collaborating advertisers case. Here we observe the relationship between the profit and the number of advertisers in these settings. For these experiments, we modified `Uniform` and `PowerLaw` to assign multiple probabilities $q_1(e), \dots, q_n(e)$ for each edge, each of which follows the uniform distribution on $[0, q_{\max}]$.

The result is shown in Figure **??**. By comparing two results obtained by the multiple (non-collaborating) advertisers case, the number of advertisers had little influence on the profit. On the other hand, by comparing the results obtained by collaborating advertisers, the profit increased when the number of advertisers increased. Moreover, the profits obtained from the collaborating advertisers consistently outperformed those obtained from non-collaborating advertisers. This means that

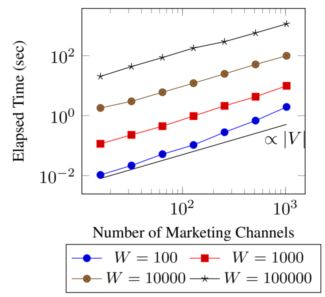

Figure 1: Scalability of the proposed algorithm.

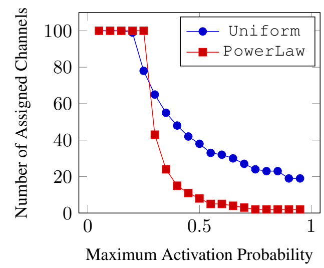

Figure 2: The number of assigned channels versus edge probability.

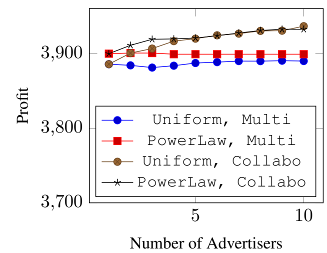

Figure 3: Profit versus the number of non-collaborating or collaborating advertisers.

collaboration of advertisers yields a better profit to the publisher. Note that we could not observe the difference between `Uniform` and `PowerLaw`.

7 Conclusion

We propose some future works. One is to develop an approximate pricing algorithm for the case that multiple buyers have limited budgets. Another one is to analyze the optimal pricing problem with multiple sellers. In this study, we assumed there is one seller; who can be regarded as a monopolist. The seller can select both the assignment and the price as long as they satisfy the stability condition. This situation is highly advantageous for the seller. Finally, in this study, we do not consider nonlinear or non-anonymous pricing. It would be interesting in future work to analyze the effect of such generalizations of pricing.

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