



**Figure 2:** An empirical demonstration illustrating the convergence of the parameters of  $Q(s, o)$ ,  $\pi(a|s, o)$ , and  $\pi(o|s)$ . We have randomly selected one parameter from each function approximator and plotted its value against the number of steps.

where  $\mu_\Omega$  is the stationary distribution of the Markov chain defined by the hierarchical policy, and  $P_{\pi, \beta}$  is the probability while at the next state, and terminating the options for the last state, that the agent arrives at a particular new set of option selections.

*Proof.* The proof for this theorem is in the Appendix.  $\square$

## Two-Timescale Convergence

Next, we prove that the aforementioned parameters,  $\theta$ , asymptotically converge to their optimal values, when employing a linear approximation  $\forall Q_\Omega$ . We analyze our framework using the ordinary differential equation (ODE) approach, delineated by ? (?), and study its asymptotic properties using the fixed points of the derived ODE.

**Theorem 2 (Convergence Proof).** *For the parameter iterations of the global set of shared parameters defined in Algorithm 1, we have  $(\hat{J}_t, v_t, \theta_t) \rightarrow \{(J(\theta^*)_t, v^*, \theta^*) | \theta^* \in \mathcal{Z}\}$  as  $t \rightarrow \infty$  with probability one, where  $\mathcal{Z}$  corresponds to the set of local maxima of a performance function whose gradient is  $E[\delta_t^\pi \psi(s_t, a_t) | \theta]$*

*Proof.* The proof for this theorem is in the Appendix.  $\square$

## Empirical Results

Finally, we look at the susceptibility of our framework to traps, and compare it to the DR setting proposed by ? (?). Figure 3(b) depicts a grid world environment characterized by sparse rewards. An agent must navigate to either one of the pickup locations,  $P_1$  or  $P_2$ , in order to retrieve a parcel; and must subsequently deliver the parcel to the drop off location. The agent gets a reward of +100 for every parcel from  $P_2$ , and +50 for every parcel from  $P_1$ . The optimal policy for an agent would naturally involve picking up the parcels from  $P_2$ .

We introduce a trap<sup>1</sup> at the green-blue junction to entice the DR-RL agents into picking up the parcels from  $P_1$ . Once

<sup>1</sup>The reward of +20 was primarily chosen for illustrating the potential pitfalls when employ a  $\gamma \leq 0.9$ . Similar traps can be created for any  $\gamma \leq 1$ .



**Figure 3:** (a) A trap that employs delayed rewards to fool DR-RL agents into learning incorrect credit assignments. (b) A grid-world navigation experiment where the reward at the drop off point depends upon which pickup location was previously visited (50 for  $P_1$  and 100 for  $P_2$ ). The trap at the blue-green junction misguides agents towards the sub-optimal pickup location,  $P_1$ .

the agent reaches the blue zone, it obtains a reward of +20 as opposed to a reward of +10 at the red-green junction. In Figure 1, we plot the rewards obtained per cycle for both the AR-RL agent and a DR-RL agent, and show that the hierarchical AR policy gradient performs better than its DR counterpart proposed by ? (?). Finally, we illustrate the asymptotic convergence of the actor and critic parameters in Figure 2.

## Conclusion and Future Work

In this work, we propose a novel method for maximizing the long term steady-state reward, by learning intra-option policies, termination functions, and value functions end-to-end. These algorithms can be used in infinite-horizon control problems that exhibit an inherent cyclic structure, like inventory-management, queuing and traffic light control. A detailed empirical analysis for a cyclical infinite-horizon application would be necessary to demonstrate the viability of our approach in complex environments. Additionally, while the proofs provided here leverage a linear approximation for each of the  $Q_\Omega(s, o^{0:\ell})$ , it would also be interesting to investigate the convergence properties of a non-linear critic.

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