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Zentrum Mathematik
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Formelsammlung Koordinatentransformation

Es wird die Notation aus Vorlesung und Übung zur Analysis 2 (EI) verwendet.

Skalarfelder $f\colon \mathbb{R}^3 \to \mathbb{R}$

kartesische Koordinaten im Punkt $(x,y,z)^T$	Zylinderkoordinaten im Punkt $(r,\varphi,z)^T$	Kugelkoordinaten im Punkt $(r,\theta,\varphi)^T$
f	$\tilde f$	$\tilde f$
$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\widetilde{\Delta} f = \frac{\partial^2 \tilde f}{\partial r^2} + \frac{1}{r}\frac{\partial \tilde f}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \tilde f}{\partial \varphi^2} + \frac{\partial^2 \tilde f}{\partial z^2}$	$\widetilde{\Delta} f = \frac{\partial^2 \tilde f}{\partial r^2} + \frac{2}{r}\frac{\partial \tilde f}{\partial r} + \frac{\cos\theta}{r^2 \sin\theta}\frac{\partial \tilde f}{\partial \theta} + \frac{1}{r^2}\frac{\partial^2 \tilde f}{\partial \theta^2} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 \tilde f}{\partial \varphi^2}$
$\nabla f = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{pmatrix}$ bzgl. $\{e_x, e_y, e_z\}$	$\widehat{\nabla} f = \begin{pmatrix} \frac{\partial \tilde f}{\partial r} \\ \frac{1}{r}\frac{\partial \tilde f}{\partial \varphi} \\ \frac{\partial \tilde f}{\partial z} \end{pmatrix}$ bzgl. $\{e_r, e_\varphi, e_z\}$	$\widehat{\nabla} f = \begin{pmatrix} \frac{\partial \tilde f}{\partial r} \\ \frac{1}{r}\frac{\partial \tilde f}{\partial \theta} \\ \frac{1}{r\sin\theta}\frac{\partial \tilde f}{\partial \varphi} \end{pmatrix}$ bzgl. $\{e_r, e_\theta, e_\varphi\}$

Vektorfelder $g\colon \mathbb{R}^3 \to \mathbb{R}^3$

kartesische Koordinaten im Punkt $(x,y,z)^T$	Zylinderkoordinaten im Punkt $(r,\varphi,z)^T$	Kugelkoordinaten im Punkt $(r,\theta,\varphi)^T$
$g = \begin{pmatrix} g_1 \\ g_2 \\ g_3 \end{pmatrix}$ bzgl. $\{e_x, e_y, e_z\}$	$\hat g = \begin{pmatrix} \hat g_1 \\ \hat g_2 \\ \hat g_3 \end{pmatrix}$ bzgl. $\{e_r, e_\varphi, e_z\}$	$\hat g = \begin{pmatrix} \hat g_1 \\ \hat g_2 \\ \hat g_3 \end{pmatrix}$ bzgl. $\{e_r, e_\theta, e_\varphi\}$
$\operatorname{div} g = \frac{\partial g_1}{\partial x} + \frac{\partial g_2}{\partial y} + \frac{\partial g_3}{\partial z}$	$\widetilde{\operatorname{div}}\, g = \frac{1}{r}\frac{\partial (r\hat g_1)}{\partial r} + \frac{1}{r}\frac{\partial \hat g_2}{\partial \varphi} + \frac{\partial \hat g_3}{\partial z}$	$\widetilde{\operatorname{div}}\, g = \frac{1}{r^2}\frac{\partial (r^2\hat g_1)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial (\hat g_2 \sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \hat g_3}{\partial \varphi}$
$\operatorname{rot} g = \begin{pmatrix} \frac{\partial g_3}{\partial y} - \frac{\partial g_2}{\partial z} \\ \frac{\partial g_1}{\partial z} - \frac{\partial g_3}{\partial x} \\ \frac{\partial g_2}{\partial x} - \frac{\partial g_1}{\partial y} \end{pmatrix}$ bzgl. $\{e_x, e_y, e_z\}$	$\widehat{\operatorname{rot}}\, g = \begin{pmatrix} \frac{1}{r}\frac{\partial \hat g_3}{\partial \varphi} - \frac{\partial \hat g_2}{\partial z} \\ \frac{\partial \hat g_1}{\partial z} - \frac{\partial \hat g_3}{\partial r} \\ \frac{1}{r}\frac{\partial (r\hat g_2)}{\partial r} - \frac{1}{r}\frac{\partial \hat g_1}{\partial \varphi} \end{pmatrix}$ bzgl. $\{e_r, e_\varphi, e_z\}$	$\widehat{\operatorname{rot}}\, g = \begin{pmatrix} \frac{1}{r\sin\theta}\frac{\partial (\hat g_3 \sin\theta)}{\partial \theta} - \frac{1}{r\sin\theta}\frac{\partial \hat g_2}{\partial \varphi} \\ \frac{1}{r\sin\theta}\frac{\partial \hat g_1}{\partial \varphi} - \frac{1}{r}\frac{\partial (r\hat g_3)}{\partial r} \\ \frac{1}{r}\frac{\partial (r\hat g_2)}{\partial r} - \frac{1}{r}\frac{\partial \hat g_1}{\partial \theta} \end{pmatrix}$ bzgl. $\{e_r, e_\theta, e_\varphi\}$