



Formelsammlung Koordinatentransformation

Es wird die Notation aus Vorlesung und Übung zur Analysis 2 (EI) verwendet.

Skalarfelder $f: \mathbb{R}^3 \rightarrow \mathbb{R}$

kartesische Koordinaten im Punkt $(x, y, z)^T$	Zylinderkoordinaten im Punkt $(r, \varphi, z)^T$	Kugelkoordinaten im Punkt $(r, \varphi, \theta)^T$
f	\tilde{f}	\tilde{f}
$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\tilde{\Delta} f = \frac{\partial^2 \tilde{f}}{\partial r^2} + \frac{1}{r} \frac{\partial \tilde{f}}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \tilde{f}}{\partial \varphi^2} + \frac{\partial^2 \tilde{f}}{\partial z^2}$	$\tilde{\Delta} f = \frac{\partial^2 \tilde{f}}{\partial r^2} + \frac{2}{r} \frac{\partial \tilde{f}}{\partial r} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \tilde{f}}{\partial \varphi^2} + \frac{\cos \theta}{r^2 \sin \theta} \frac{\partial \tilde{f}}{\partial \theta} + \frac{1}{r^2} \frac{\partial^2 \tilde{f}}{\partial \theta^2}$
$\nabla f = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{pmatrix}$ bzgl. $\{e_x, e_y, e_z\}$	$\widehat{\nabla} f = \begin{pmatrix} \frac{\partial \tilde{f}}{\partial r} \\ \frac{1}{r} \frac{\partial \tilde{f}}{\partial \varphi} \\ \frac{\partial \tilde{f}}{\partial z} \end{pmatrix}$ bzgl. $\{e_r, e_\varphi, e_z\}$	$\widehat{\nabla} f = \begin{pmatrix} \frac{\partial \tilde{f}}{\partial r} \\ \frac{1}{r \sin \theta} \frac{\partial \tilde{f}}{\partial \varphi} \\ \frac{1}{r} \frac{\partial \tilde{f}}{\partial \theta} \end{pmatrix}$ bzgl. $\{e_r, e_\varphi, e_\theta\}$

Vektorfelder $g: \mathbb{R}^3 \rightarrow \mathbb{R}^3$

kartesische Koordinaten im Punkt $(x, y, z)^T$	Zylinderkoordinaten im Punkt $(r, \varphi, z)^T$	Kugelkoordinaten im Punkt r
$g = \begin{pmatrix} g_1 \\ g_2 \\ g_3 \end{pmatrix}$ bzgl. $\{e_x, e_y, e_z\}$	$\hat{g} = \begin{pmatrix} \hat{g}_1 \\ \hat{g}_2 \\ \hat{g}_3 \end{pmatrix}$ bzgl. $\{e_r, e_\varphi, e_z\}$	$\hat{g} = \begin{pmatrix} \hat{g}_1 \\ \hat{g}_2 \\ \hat{g}_3 \end{pmatrix}$ bzgl. $\{e_r, e_\varphi, e_\theta\}$
$\operatorname{div} g = \frac{\partial g_1}{\partial x} + \frac{\partial g_2}{\partial y} + \frac{\partial g_3}{\partial z}$	$\widetilde{\operatorname{div}} g = \frac{1}{r} \frac{\partial(r \hat{g}_1)}{\partial r} + \frac{1}{r} \frac{\partial \hat{g}_2}{\partial \varphi} + \frac{\partial \hat{g}_3}{\partial z}$	$\widetilde{\operatorname{div}} g = \frac{1}{r^2} \frac{\partial(r^2 \hat{g}_1)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial \hat{g}_2}{\partial \varphi} + \frac{1}{r \sin \theta} \frac{\partial(\hat{g}_3 \sin \theta)}{\partial \theta}$
$\operatorname{rot} g = \begin{pmatrix} \frac{\partial g_3}{\partial y} - \frac{\partial g_2}{\partial z} \\ \frac{\partial g_1}{\partial z} - \frac{\partial g_3}{\partial x} \\ \frac{\partial g_2}{\partial x} - \frac{\partial g_1}{\partial y} \end{pmatrix}$ bzgl. $\{e_x, e_y, e_z\}$	$\widetilde{\operatorname{rot}} g = \begin{pmatrix} \frac{1}{r} \frac{\partial \hat{g}_3}{\partial \varphi} - \frac{\partial \hat{g}_2}{\partial z} \\ \frac{\partial \hat{g}_1}{\partial z} - \frac{\partial \hat{g}_3}{\partial r} \\ \frac{1}{r} \frac{\partial(r \hat{g}_2)}{\partial r} - \frac{1}{r} \frac{\partial \hat{g}_1}{\partial \varphi} \end{pmatrix}$ bzgl. $\{e_r, e_\varphi, e_z\}$	$\widetilde{\operatorname{rot}} g = \begin{pmatrix} \frac{1}{r \sin \theta} \frac{\partial(\hat{g}_3 \sin \theta)}{\partial \theta} - \frac{1}{r} \frac{\partial \hat{g}_2}{\partial \varphi} \\ \frac{1}{r} \frac{\partial(r \hat{g}_3)}{\partial r} - \frac{1}{r} \frac{\partial \hat{g}_1}{\partial \theta} \\ \frac{1}{r \sin \theta} \frac{\partial \hat{g}_1}{\partial \varphi} - \frac{1}{r} \frac{\partial(r \hat{g}_2)}{\partial r} \end{pmatrix}$ bzgl. $\{e_r, e_\varphi, e_\theta\}$