Vector Algebra

Representation of a vector $\vec{A}$ by use of orthonormal unit vectors $\vec{e}_1, \vec{e}_2, \vec{e}_3$:

$$\vec{A} = A_1\vec{e}_1 + A_2\vec{e}_2 + A_3\vec{e}_3 = \left(A_1, A_2, A_3\right)$$

A_1, A_2, A_3 are the components of $\vec{A}$ with respect to $\vec{e}_1, \vec{e}_2, \vec{e}_3$.

Inner product, scalar product:

$$\vec{A}\cdot\vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\cos\left(\measuredangle\vec{A},\vec{B}\right)$$

$$\vec{A}\cdot\vec{B} = A_1B_1 + A_2B_2 + A_3B_3$$

Outer product, vector product, cross product:

$$\vec{A}\times\vec{B} = \left|\begin{pmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{pmatrix}\right| = \left(A_2B_3 - A_3B_2\right)\vec{e}_1 + \left(A_3B_1 - A_1B_3\right)\vec{e}_2 + \left(A_1B_2 - A_2B_1\right)\vec{e}_3$$

$$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$$

$$\left|\vec{A}\times\vec{B}\right| = \left|\vec{A}\right|\left|\vec{B}\right|\sin\left(\measuredangle\vec{A},\vec{B}\right)$$

Parallelepidial product (deutsch: Spatprodukt):

$$\left(\vec{A}\times\vec{B}\right)\cdot\vec{C} = \begin{vmatrix} C_1 & C_2 & C_3 \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

$$\left(\vec{A}\times\vec{B}\right)\cdot\vec{C} = \left(\vec{B}\times\vec{C}\right)\cdot\vec{A} = \left(\vec{C}\times\vec{A}\right)\cdot\vec{B} = -\left(\vec{C}\times\vec{B}\right)\cdot\vec{A} = -\left(\vec{B}\times\vec{A}\right)\cdot\vec{C} = -\left(\vec{A}\times\vec{C}\right)\cdot\vec{B}$$

Double cross product:

$$\vec{A}\times\left(\vec{B}\times\vec{C}\right) = \left(\vec{A}\cdot\vec{C}\right)\vec{B} - \left(\vec{A}\cdot\vec{B}\right)\vec{C}$$

$$\left(\vec{A}\times\vec{B}\right)\times\vec{C} = \left(\vec{A}\cdot\vec{C}\right)\vec{B} - \left(\vec{B}\cdot\vec{C}\right)\vec{A}$$

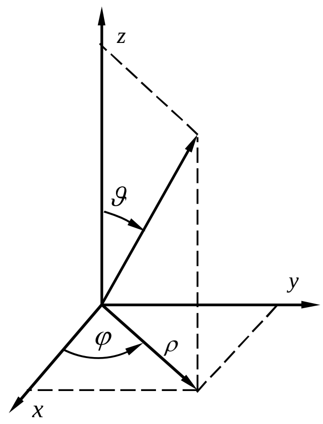
Multiple cross products:

$$\left(\vec{A}\times\vec{B}\right)\cdot\left(\vec{C}\times\vec{D}\right) = \left(\vec{A}\cdot\vec{C}\right)\left(\vec{B}\cdot\vec{D}\right) - \left(\vec{A}\cdot\vec{D}\right)\left(\vec{B}\cdot\vec{C}\right)$$

$$\left(\vec{A}\times\vec{B}\right)\times\left(\vec{C}\times\vec{D}\right) = \left[\left(\vec{A}\times\vec{B}\right)\cdot\vec{D}\right]\vec{C} - \left[\left(\vec{A}\times\vec{B}\right)\cdot\vec{C}\right]\vec{D}$$

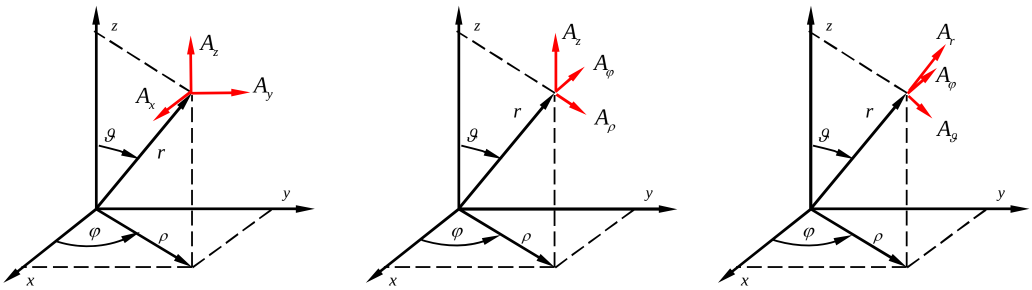
Relations between Cartesian, Cylindrical and Spherical Coordinates

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Cartesian Coordinates	x y z	$\rho\cos\varphi$ $\rho\sin\varphi$ z	$r\sin\vartheta\cos\varphi$ $r\sin\vartheta\sin\varphi$ $r\cos\vartheta$
Cylindrical Coordinates	$\sqrt{x^2+y^2}$ $\arctan\left(\frac{y}{x}\right)\pm\pi$ z	ρ φ z	$r\sin\vartheta$ φ $r\cos\vartheta$
Spherical Coordinates	$\sqrt{x^2+y^2+z^2}$ $\arctan\frac{\sqrt{x^2+y^2}}{z}\pm\pi$ $\arctan\frac{y}{x}\pm\pi$	$\sqrt{\rho^2+z^2}$ $\arctan\frac{\rho}{z}\pm\pi$ φ	r ϑ φ



Vector Components in Cartesian, Cylindrical and Spherical Coordinate Systems

	Cartesian Components	Cylindrical Components	Spherical Components
To: Cartesian Components	A_x A_y A_z	$A_\rho \cos\varphi - A_\varphi \sin\varphi$ $A_\rho \sin\varphi + A_\varphi \cos\varphi$ A_z	$A_r \sin\vartheta\cos\varphi + A_\vartheta \cos\vartheta\cos\varphi - A_\varphi \sin\varphi$ $A_r \sin\vartheta\sin\varphi + A_\vartheta \cos\vartheta\sin\varphi + A_\varphi \cos\varphi$ $A_r \cos\vartheta - A_\vartheta \sin\vartheta$
To: Cylindrical Components	$A_x \cos\varphi + A_y \sin\varphi$ $-A_x \sin\varphi + A_y \cos\varphi$ A_z	A_ρ A_φ A_z	$A_r \sin\vartheta + A_\vartheta \cos\vartheta$ A_φ $A_r \cos\vartheta - A_\vartheta \sin\vartheta$
To: Spherical Components	$A_x \sin\vartheta\cos\varphi + A_y \sin\vartheta\sin\varphi + A_z \cos\vartheta$ $A_x \cos\vartheta\cos\varphi + A_y \cos\vartheta\sin\varphi - A_z \sin\vartheta$ $-A_x \sin\varphi + A_y \cos\varphi$	$A_\rho \sin\vartheta + A_z \cos\vartheta$ $A_\rho \cos\vartheta - A_z \sin\vartheta$ A_φ	A_r A_ϑ A_φ



Differential Operators in Cartesian, Cylindrical and Spherical Coordinates

Cartesian coordinates:

$$\operatorname{grad} f = \vec{\nabla} f = \vec{e}_x \frac{\partial f}{\partial x} + \vec{e}_y \frac{\partial f}{\partial y} + \vec{e}_z \frac{\partial f}{\partial z}$$

$$\operatorname{div} \vec{A} = \vec{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\operatorname{rot} \vec{A} = \operatorname{curl} \vec{A} = \vec{\nabla} \times \vec{A} = \vec{e}_x \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \vec{e}_y \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \vec{e}_z \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

$$= \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\Delta f = \vec{\nabla} \cdot \vec{\nabla} f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

f: scalar function
$\vec{A}$: vector function

Cylindrical coordinates:

$$\operatorname{grad} f = \vec{e}_\rho \frac{\partial f}{\partial \rho} + \vec{e}_\varphi \frac{1}{\rho} \frac{\partial f}{\partial \varphi} + \vec{e}_z \frac{\partial f}{\partial z}$$

$$\operatorname{div} \vec{A} = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho A_\rho \right) + \frac{1}{\rho} \frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$$

$$\operatorname{rot} \vec{A} = \operatorname{curl} \vec{A} = \vec{e}_\rho \left(\frac{1}{\rho} \frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z} \right) + \vec{e}_\varphi \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} \right) + \vec{e}_z \frac{1}{\rho} \left(\frac{\partial}{\partial \rho} \left(\rho A_\varphi \right) - \frac{\partial A_\rho}{\partial \varphi} \right)$$

$$\Delta f = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial f}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2} = \frac{\partial^2 f}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial f}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}$$

Spherical coordinates:

$$\operatorname{grad} f = \vec{e}_r \frac{\partial f}{\partial r} + \vec{e}_\vartheta \frac{1}{r} \frac{\partial f}{\partial \vartheta} + \vec{e}_\varphi \frac{1}{r \sin\vartheta} \frac{\partial f}{\partial \varphi}$$

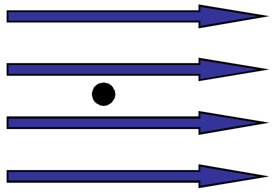
$$\operatorname{div} \vec{A} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 A_r \right) + \frac{1}{r \sin\vartheta} \frac{\partial}{\partial \vartheta} \left(A_\vartheta \sin\vartheta \right) + \frac{1}{r \sin\vartheta} \frac{\partial A_\varphi}{\partial \varphi}$$

$$\operatorname{rot} \vec{A} = \operatorname{curl} \vec{A} = \vec{e}_r \frac{1}{r \sin\vartheta} \left(\frac{\partial}{\partial \vartheta} \left(A_\varphi \sin\vartheta \right) - \frac{\partial A_\vartheta}{\partial \varphi} \right) + \vec{e}_\vartheta \frac{1}{r} \left(\frac{1}{\sin\vartheta} \frac{\partial A_r}{\partial \varphi} - \frac{\partial}{\partial r} \left(r A_\varphi \right) \right) + \vec{e}_\varphi \frac{1}{r} \left(\frac{\partial}{\partial r} \left(r A_\vartheta \right) - \frac{\partial A_r}{\partial \vartheta} \right)$$

$$\Delta f = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin\vartheta} \frac{\partial}{\partial \vartheta} \left(\sin\vartheta \frac{\partial f}{\partial \vartheta} \right) + \frac{1}{r^2 \sin^2\vartheta} \frac{\partial^2 f}{\partial \varphi^2}$$

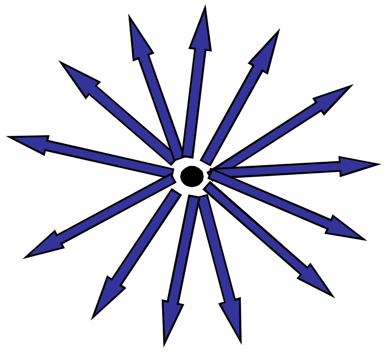
∇abla, LΔplace

Visualisation



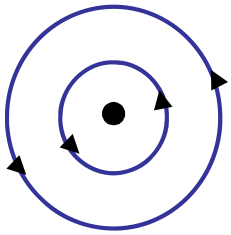
$\operatorname{div} \vec{A} = 0$ (sources at infinity)

$\operatorname{rot} \vec{A} = 0$



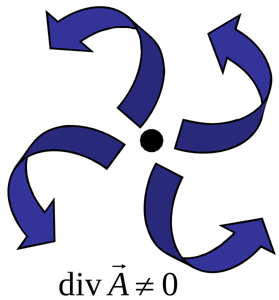
$\operatorname{div} \vec{A} \neq 0$

$\operatorname{rot} \vec{A} = 0$



$\operatorname{div} \vec{A} = 0$

$\operatorname{rot} \vec{A} \neq 0$



$\operatorname{div} \vec{A} \neq 0$

$\operatorname{rot} \vec{A} \neq 0$

Composed Operations with Gradient

$\mathrm{d}\, f = \operatorname{grad} f \cdot \mathrm{d}\vec{r} = \operatorname{grad} f \cdot \mathrm{d}\,\vec{s}$

$\operatorname{grad} f = \frac{\partial f}{\partial x}\vec{e}_x + \frac{\partial f}{\partial y}\vec{e}_y + \frac{\partial f}{\partial z}\vec{e}_z$

$\operatorname{grad}(f+g) = \operatorname{grad} f + \operatorname{grad} g$

$\operatorname{grad}(cf) = c \operatorname{grad} f;$ c = const.

$\operatorname{grad}(f \cdot g) = f \operatorname{grad} g + g \operatorname{grad} f$

$\operatorname{grad}\left(\vec{A}\cdot\vec{B}\right) = \left(\vec{A}\cdot\operatorname{grad}\right)\vec{B} + \left(\vec{B}\cdot\operatorname{grad}\right)\vec{A} + \vec{A}\times\operatorname{rot}\vec{B} + \vec{B}\times\operatorname{rot}\vec{A}$

$\operatorname{grad} r = \frac{\vec{r}}{r} = \vec{e}_r$

$r = \left|\vec{r}\right| = \sqrt{x^2+y^2+z^2}$

$\operatorname{grad}\frac{1}{r} = -\frac{\vec{r}}{r^3} = -\frac{1}{r^2}\vec{e}_r$

$\operatorname{grad} f\left(g\left(\vec{r}\right)\right) = \frac{\mathrm{d}\, f}{\mathrm{d}\, g}\cdot\operatorname{grad} g\left(\vec{r}\right)$ implicit derivative

$\operatorname{grad} f(r) = \frac{df}{dr}\vec{e}_r;$ e.g.: $\operatorname{grad}\left[\ln(r)\right] = \frac{\vec{r}}{r^2} = \frac{1}{r}\vec{e}_r$

$\operatorname{grad}\left(\vec{a}\cdot\vec{r}\right) = \vec{a};$ $\vec{a}$ constant vector

$\left(\vec{A}\cdot\operatorname{grad}\right)\vec{B} = \left(\vec{A}\cdot\operatorname{grad} B_x\right)\vec{e}_x + \left(\vec{A}\cdot\operatorname{grad} B_y\right)\vec{e}_y + \left(\vec{A}\cdot\operatorname{grad} B_z\right)\vec{e}_z$

$\left(\vec{A}\cdot\operatorname{grad}\right)\vec{r} = \vec{A}$

$\left(\vec{A}\cdot\operatorname{grad}\right)\vec{e}_r = \frac{\vec{A}}{r} - \frac{A_r}{r}\vec{e}_r$

$\left(\vec{a}\cdot\operatorname{grad}\right) f\vec{A} = f\left(\vec{a}\cdot\operatorname{grad}\right)\vec{A} + \vec{A}\left(\vec{a}\cdot\operatorname{grad} f\right)$

Composed Operations with Divergence

$\operatorname{div}\vec{B}(\vec{r}) = \lim_{\Delta V \to 0} \frac{1}{\Delta V} \oiint_{A(V)} \vec{B}(\vec{r}) \cdot d\vec{A};$ — $d\vec{A}$ = differential area element

$\operatorname{div}\vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}$

$\operatorname{div}\left(\vec{C} + \vec{B}\right) = \operatorname{div}\vec{C} + \operatorname{div}\vec{B}$

$\operatorname{div}\left(c\vec{B}\right) = c \operatorname{div}\vec{B};$ — c constant

$\operatorname{div}\left(f\vec{B}\right) = f \operatorname{div}\vec{B} + \vec{B} \cdot \operatorname{grad} f;$ — product theorem

$\operatorname{div}\left(\vec{C} \times \vec{B}\right) = \vec{B} \cdot \operatorname{rot}\vec{C} - \vec{C} \cdot \operatorname{rot}\vec{B};$ — product theorem

$\operatorname{div}\left(\operatorname{rot}\vec{C}\right) = 0$

$\operatorname{div}\vec{r} = 3$

$\operatorname{div}\vec{e}_r = \frac{2}{r}$

$\operatorname{div}\left[\vec{B}\left(f(\vec{r})\right)\right] = \frac{d\vec{B}(\vec{r})}{df} \cdot \operatorname{grad} f(\vec{r});$ — implicit derivative

$\operatorname{div}\left[\vec{B}(r)\right] = \frac{d\vec{B}}{dr} \cdot \vec{e}_r;$ — implicit derivative

$\operatorname{div}\left[f(r)\vec{r}\right] = 3f(r) + rf'(r)$

$\operatorname{div}\left(\vec{a} \times \vec{r}\right) = 0;$ — $\vec{a}$ constant vector

Composed Operations with Curl

$\vec{n} \cdot \operatorname{rot}\vec{B}(\vec{r}) = \lim_{\Delta F \to 0} \frac{1}{\Delta A} \oint_{C(\Delta A)} \vec{B}(\vec{r}) \cdot d\vec{s};$ — $d\vec{s}$ = differential line element

$$\operatorname{rot}\vec{B} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix}$$

$\operatorname{rot}\left(\vec{C} + \vec{B}\right) = \operatorname{rot}\vec{C} + \operatorname{rot}\vec{B}$

$\operatorname{rot} c\vec{B} = c \operatorname{rot}\vec{B};$ — c constant

$\operatorname{rot}\left(f\vec{B}\right) = f \operatorname{rot}\vec{B} + \operatorname{grad} f \times \vec{B} = f \operatorname{rot}\vec{B} - \vec{B} \times \operatorname{grad} f;$ — product theorem

$\operatorname{rot}\left(\operatorname{grad} f\right) = 0$

$\operatorname{rot}\operatorname{rot}\vec{B} = \operatorname{grad}\operatorname{div}\vec{B} - \Delta\vec{B}$

$\operatorname{rot}\left(\vec{C} \times \vec{B}\right) = \vec{C} \operatorname{div}\vec{B} - \vec{B} \operatorname{div}\vec{C} + \left(\vec{B} \cdot \operatorname{grad}\right)\vec{C} - \left(\vec{C} \cdot \operatorname{grad}\right)\vec{B}$

$\operatorname{rot}\vec{r} = 0$

$\operatorname{rot}\left[f(r)\vec{r}\right] = 0$

$\operatorname{rot}\left[f(r)\vec{a}\right] = f'(r)\frac{\vec{r}}{r} \times \vec{a} = f'(r)\vec{e}_r \times \vec{a};$ — $\vec{a}$ constant vector

$\operatorname{rot}\vec{B}\left(f(\vec{r})\right) = -\frac{d\vec{B}(\vec{r})}{df} \times \operatorname{grad} f(\vec{r});$ — implicit derivative

$\operatorname{rot}\vec{B}(r) = -\frac{d\vec{B}}{dr} \times \vec{e}_r$

$\operatorname{rot}\left(\vec{a} \times \vec{r}\right) = 2\vec{a}$

Integral Theorems

$$\iiint_V \operatorname{div} \vec{B} dv = \oiint_{A(V)} \vec{B} \cdot d\vec{A}$$ integral theorem of Gauss — dv = differential volume element

$$\iint_A \operatorname{rot} \vec{B} \cdot d\vec{A} = \oint_{C(A)} \vec{B} \cdot d\vec{s}$$ integral theorem of Stokes

$$\iint_A \operatorname{div}_s \vec{B}\, dA = \oint_{C(A)} \vec{B} \cdot \left(d\vec{s} \times \vec{n}\right) - \iint_A J\left(\vec{B} \cdot \vec{n}\right) dA$$ Gauss theorem on surface

div_s surface divergence; $J = \frac{1}{R_1} + \frac{1}{R_2}$; R_1, R_2 principal radii of curvature; $\vec{n}, d\vec{s}$ related by right-hand rule

$$\iint_A \vec{n} \times \operatorname{grad} f dA = \oint_{C(A)} f \cdot d\vec{s}$$

$$\iiint_V \operatorname{grad} f\, dv = \oiint_{A(V)} f\, d\vec{A}$$

$$\iiint_V \operatorname{rot} \vec{B} dv = \oiint_{A(V)} d\vec{A} \times \vec{B} = \oiint_{A(V)} \left[\vec{n} \times \vec{B}\right] dA$$

$$\iiint_V \left[\vec{F} \operatorname{div} \vec{G} + \left(\vec{G} \cdot \operatorname{grad}\right) \vec{F}\right] dv = \oiint_{A(V)} \vec{F}\left(\vec{G} \cdot d\vec{S}\right) = \oiint_{A(V)} \vec{F}\left(\vec{G} \cdot \vec{n}\right) dA$$

$$\iiint_V \vec{G}\, dv = \oiint_{A(V)} \vec{r}\left(\vec{G} \cdot d\vec{A}\right) - \iiint_V \vec{r} \operatorname{div} \vec{G}\, dv$$

$$\iiint_V \left(f \Delta g + \operatorname{grad} f \operatorname{grad} g\right) dv = \oiint_{A(V)} f \operatorname{grad} g \cdot d\vec{S} = \oiint_{A(V)} f\ \vec{n} \cdot \operatorname{grad} g\ dA$$ 1st Green's theorem

$$\iiint_V \left(f \Delta g - g \Delta f\right) dv = \oiint_{A(V)} \left(f \operatorname{grad} g - g \operatorname{grad} f\right) \cdot \vec{n}\ dA$$ 2nd Green's theorem

Parametric Evaluation of Line, Surface and Volume Integrals

Line integrals over arbitrary contours:

$$\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \varphi(\xi) \\ \psi(\xi) \\ \chi(\xi) \end{pmatrix}$$ parametric represenation

$$d\vec{s} = \pm \frac{d\vec{r}}{d\xi} d\xi$$ differential vector line element

$$ds = \left|d\vec{s}\right|$$ differential scalar line element

$$\int_C f(\vec{r}) ds = \iint_{\xi,\eta} f\left(\varphi(\xi), \psi(\xi), \chi(\xi)\right) \left|\frac{d\vec{r}}{d\xi}\right| d\xi$$

$$\int_C \vec{B}(\vec{r}) \cdot d\vec{s} = \pm \int_\xi \vec{B}\left(\varphi(\xi), \psi(\xi), \chi(\xi)\right) \cdot \frac{d\vec{r}}{d\xi} d\xi$$

Surface integrals over arbitrary surfaces:

$$\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \varphi(\xi,\eta) \\ \psi(\xi,\eta) \\ \chi(\xi,\eta) \end{pmatrix}$$ parametric represenation

$$d\vec{A} = \pm \left(\frac{d\vec{r}}{d\xi} \times \frac{d\vec{r}}{d\eta}\right) d\xi\, d\eta$$ differential vector area element

$$dA = \left|d\vec{A}\right|$$ differential scalar area element

$$\iint_A f(\vec{r}) dA = \iint_{\xi,\eta} f\left(\varphi(\xi,\eta), \psi(\xi,\eta), \chi(\xi,\eta)\right) \left|\frac{d\vec{r}}{d\xi} \times \frac{d\vec{r}}{d\eta}\right| d\xi\, d\eta$$

$$\iint_A \vec{B}(\vec{r}) \cdot d\vec{A} = \pm \iint_{\xi,\eta} \vec{B}\left(\varphi(\xi,\eta), \psi(\xi,\eta), \chi(\xi,\eta)\right) \cdot \left(\frac{d\vec{r}}{d\xi} \times \frac{d\vec{r}}{d\eta}\right) d\xi\, d\eta$$

Volume integrals over arbitrary volumes:

$x = \varphi(\xi,\eta,\varsigma),\ y = \psi(\xi,\eta,\varsigma),\ z = \chi(\xi,\eta,\varsigma)$

$$\iiint_V f(\vec{r}) dv = \iiint_{\xi,\eta,\varsigma} f\left(\varphi(\xi,\eta,\varsigma), \psi(\xi,\eta,\varsigma), \chi(\xi,\eta,\varsigma)\right) \left|\frac{\partial(\varphi,\psi,\chi)}{\partial(\xi,\eta,\varsigma)}\right| d\xi\, d\eta\, d\varsigma$$

with $\dfrac{\partial(\varphi,\psi,\chi)}{\partial(\xi,\eta,\varsigma)} = \begin{vmatrix} \varphi_\xi & \psi_\xi & \chi_\xi \\ \varphi_\eta & \psi_\eta & \chi_\eta \\ \varphi_\varsigma & \psi_\varsigma & \chi_\varsigma \end{vmatrix}$; $\dfrac{\partial(\varphi,\psi,\chi)}{\partial(\xi,\eta,\varsigma)} \neq 0$ in V.

$$\boxed{\begin{aligned} &d\vec{s} = \text{differential line element} \\ &d\vec{A} = \text{differential area element} \\ &dv = \text{differential volume element} \end{aligned}}$$