Verbrennungsmotoren Formelsammlung

$$\eta_e = \overbrace{\underbrace{\eta_V - \Delta\eta_{BV} - \Delta\eta_W - \Delta\eta_U}_{\eta_{i,HD}} - \Delta\eta_{LW}}^{\eta_i} - \Delta\eta_R$$

1. Grundlagen

$$g_i = \frac{m_i}{\sum_i m_i} \overset{n_i = \frac{m_i}{M_i}}{=} \frac{M_i}{\sum_i \psi_i M_i}\psi_i \qquad r_i = \frac{V_i}{\sum_i V_i} \qquad \psi_i = \frac{n_i}{\sum_i n_i}$$

$$M = \sum_i \psi_i M_i = \left(\sum_i \frac{g_i}{M_i}\right)^{-1}$$

$$R_i = c_{p,i} - c_{v,i} \quad \& \quad \kappa_i = \frac{c_{p,i}}{c_{v,i}} \quad \Rightarrow c_{v,i} = \frac{R_i}{\kappa_i - 1}$$

$$R_g = \sum_i g_i R_i = \begin{cases} \text{Otto:} & R_L + g_B (R_B - R_L) \\ \text{Diesel:} & R_L \end{cases} \quad \text{mit} \quad \begin{cases} R_i = \frac{R_M}{M_i} \\ g_B = \frac{1}{1+\lambda L_{St}} \end{cases}$$

$$\rho = \frac{1}{v} = \frac{m}{V}$$

2. Kreisprozesse

$$p_{i+1} = \varepsilon^\kappa p_i \qquad T_{i+1} = \varepsilon^{\kappa-1} T_i$$

$$\eta_{th,GD} = 1 - \frac{1}{\kappa \cdot q^*}\left[\left(\frac{q^*}{\varepsilon^{\kappa-1}} + 1\right)^\kappa - 1\right] \qquad q^* = \frac{q_{zu}}{c_p \cdot T_1}$$

$$w_{t,GD} = -\sum_i q_{i,i+1} = -\frac{R}{\kappa - 1}\left[\kappa (T_3 - T_2) - (T_4 - T_1)\right]$$

$$\eta_{th,GR} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{1}{\varepsilon^{\kappa-1}}$$

$$w_{t,GR} = -\sum_i q_{i,i+1} = -\frac{R}{\kappa - 1}\left[(T_3 - T_2) - (T_4 - T_1)\right]$$

$$\eta_{th,SEI} = 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \kappa (T_4 - T_3)}$$

$$w_{t,SEI} = -\frac{R}{\kappa - 1}\left[(T_3 - T_2) + \kappa (T_4 - T_3) - (T_5 - T_1)\right]$$

$$T_{4,SEI} = \left[\left(\frac{p_{3'}}{p_1} \cdot T_1\right)^{\frac{1-\kappa}{\kappa}} \cdot T_{3'}\right]^\kappa$$

3. Kenngrößen

$$P_{(e,i,r)} = p_{m(e,i,r)}\, i\, n\, V_H \qquad P_e = P_i - P_r = M\omega \qquad \varepsilon = \frac{V_h + V_c\,(+V_{KM})}{V_c\,(+V_{KM})}$$

$$\eta_{(e,i)} = \frac{P_{(e,i)}}{\dot m_B\, H_u} = \frac{1}{b_{(e,i)} H_u}$$

$$\eta_m = \frac{P_e}{P_i} = \frac{p_{me}}{p_{mi}} = \frac{\eta_e}{\eta_i} = 1 - \frac{p_{mr}}{p_{mi}} = \frac{b_i}{b_e}$$

$$p_{m(e,i)} = \eta_{(e,i)}\, \lambda_a\, H_G$$

$$p_{mi} = \frac{W_{KA}}{V_h} = \begin{cases} \text{Otto:} & \frac{\eta_i V_G H_G}{V_h} = \eta_i \lambda_a H_G = \eta_i \frac{m_{FZ}}{\rho_{th} V_h} \frac{H_u \rho_G}{\lambda L_{St}+1} \\ \text{Diesel:} & \frac{\eta_i V_L \bar H_G}{V_h} = \eta_i \lambda_a \bar H_G = \eta_i \frac{m_{FZ}}{\rho_{th} V_h} \frac{H_u \rho_L}{\lambda L_{St}} \end{cases}$$

$$p_{me} = \frac{2\pi}{i} \frac{M}{V_H} = p_{mi} - p_{mr}$$

$$H_G = \frac{m_B H_u}{V_G} = \frac{m_B H_u \rho_G}{m_G} = \begin{cases} \text{Otto:} & \frac{m_B H_u \rho_G}{m_B + m_L} = \frac{H_u \rho_G}{1 + \lambda L_{St}} \\ \text{Diesel:} & \frac{m_B H_u \rho_L}{m_L} = \frac{H_u \rho_L}{\lambda L_{St}} \end{cases}$$

$$b_{(e,i)} = \frac{\dot m_B}{P_{(e,i)}} = \frac{1}{\eta_{(e,i)} H_u}$$

$$B_S = \frac{\dot m_B}{v_{Fzg}} = \frac{m_B}{s} = \frac{b_e P_e}{(\rho)\, v}$$

$$\lambda_a = \frac{m_G}{V_H\, \rho_{th}} = \frac{\dot m_G}{\dot V_G\, \rho_{th}}$$

$$\lambda_l = \frac{m_Z}{V_h\, \rho_{th}} \quad \text{mit} \quad m_Z = m_{ZL} + m_{ZB}$$

$$\lambda = \frac{\dot m_L}{\dot m_B \cdot L_{St}} = \frac{m_L}{m_B \cdot L_{St}} \left(= \frac{\dot m_L}{\sum_i \dot m_{B,i} \cdot L_{St,i}}\right)$$

$$\lambda_Z = \frac{m_{ZL}}{m_{ZB} \cdot L_{St}} = \frac{m_{ZL}}{m_B \cdot L_{St}} \overset{\text{Spülung}}{\neq} \lambda$$

$$C_x H_y O_z + \lambda o_{min}\left(O_2 + \frac{79}{21} N_2\right) \rightarrow x CO_2 + \frac{y}{2} H_2O + (\lambda - 1)\, o_{min} O_2 + \lambda\, o_{min} \frac{79}{21} N_2$$

$$o_{min} = \left(x + \frac{y}{4} + q - \frac{z}{2}\right)$$

$$o_{min,vol} = l_{min} \cdot r_{O_2,fl} \qquad r_{O_2,fl} = 0,21$$

$$o_{min,grav} = l_{min} \cdot g_{O_2,fl} \qquad g_{O_2,fl} = 0,23142$$

$$L_{st} = \frac{M_{O_2}}{g_{O_2,fl} \cdot M_B}\left(x + \frac{y}{4} + q - \frac{z}{2}\right) = \frac{(2,664\, c + 7,937\, h + 0,998\, s - o)}{g_{O_2,fl}}$$

$$c = \frac{m_C}{m_B} = \frac{M_C}{M_B}\, x \qquad h = \frac{M_{H_2}}{M_B} y = \frac{M_H}{2 M_B} y$$

4. Ladungswechsel 4-Takt

$$p_{mi} = p_{mi,HD} + p_{mi,LW} = p_{mi,HD} + |p_E - p_A|$$

$$p_{mi,HD} = \frac{\dot m_B \cdot H_u}{i\, n\, V_H} \cdot \eta_{i,HD}$$

$$\dot m_{G,AS,Zyl} = \frac{\dot m_{G,ges}}{iz} \frac{360^\circ\, KW}{\Delta\alpha}$$

$$m_{G,AS,Zyl} = \frac{m_{G,AS}}{z} = \frac{\dot m_{G,ges}}{i\, n\, z}$$

$$\alpha_{(K,V)} = \frac{A_S}{A_{(K,V)}}$$

$$A_S = \frac{\dot m_{G,AS,Zyl}}{c_S \cdot \rho_S}$$

$$c_S = \sqrt{\frac{2\kappa}{\kappa - 1} R_0 T_0 \left(1 - \left(\frac{p_{zm}}{p_0}\right)^{\frac{\kappa-1}{\kappa}}\right)}$$

$$\rho_S = \rho_0 \left(\frac{p_{zm}}{p_0}\right)^{\frac{1}{\kappa}}$$

$$\frac{\mathrm{d}\alpha}{\mathrm{d}t} \approx \frac{\Delta\alpha}{\Delta t} = 2\pi n = 360^\circ\, KW \cdot n$$

$$\Delta t = \frac{\Delta s}{v} = \frac{2\, l_{rohr}}{\sqrt{\kappa R_L T}} = \frac{m_{G,AS,Zyl}}{\dot m_S}$$

$$x_R = \frac{m_R}{m_R + m_Z}$$

$$m_R = x_{spuel} \cdot \frac{p_Z V_c}{R_G T_Z}$$

5. Ladungswechsel 2-Takt

$$\Lambda_S = \frac{V_Z}{V_Z yl} = \frac{m_Z}{\rho_{zm} V_{ES}} \qquad \Lambda_a = \frac{V_G}{V_{Zyl}} = \frac{V_G}{V_{ES}}$$

6. Gemischbildung

$$x_{AGR} = \frac{m_{AGR}}{m_L + m_B + m_{AGR}}$$

$$\lambda_{vergaser} = \frac{1}{L_{St}} \frac{A_L}{A_B} \frac{\alpha_L}{\alpha_B} \varepsilon \sqrt{\frac{\Delta p_L}{\Delta p_B}} \sqrt{\frac{\Delta \rho_L}{\Delta \rho_L}}$$

$$\lambda_{injektor} = \frac{\dot m_L}{L_{St} \cdot A_B \cdot \alpha_B \cdot t_E \cdot i \cdot n \cdot z \cdot \sqrt{2\, \Delta p\, \rho_B}}$$

7. Aufladung

$$\lambda_{L,ATL} = \lambda_{L,Saug} \left(\frac{T_{E,ATL}}{T_{E,Saug}}\right)^a \frac{\varepsilon}{\varepsilon - 1} \qquad a \in [0,2\,;\,0,5]$$

$$p_{mi,ATL} = p_{mi,Saug} \frac{p_{E,ATL}}{p_{E,Saug}} \left(\frac{T_{E,ATL}}{T_{E,Saug}}\right)^{1-a} \frac{\varepsilon}{\varepsilon - 1} + p_{E,ATL} - p_{A,ATL}$$

$$P_V = \dot m_V\, w_V \frac{1}{\eta_{mV}} = \dot m_V\, w_{sV} \frac{1}{\eta_{siV}\,\eta_{mV}}$$

$$w_{sV} = c_{p_V}\, T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{\kappa_V - 1}{\kappa_V}} - 1\right]$$

$$\eta_{siV} = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{\frac{T_{2s}}{T_1} - 1}{\frac{T_2}{T_1} - 1} = \frac{\left(\frac{p_2}{p_1}\right)^{\frac{\kappa_V - 1}{\kappa_V}} - 1}{\frac{T_2}{T_1} - 1}$$

$$\dot m_{V,red} = \dot m_V \frac{\sqrt{T_{ein}}}{p_{ein}} \qquad \dot m_{V,korr} = \dot m_{red} \frac{p_{ref}}{\sqrt{T_{ref}}}$$

$$P_T = \dot m_T\, w_T \frac{1}{\eta_{mT}} = \dot m_T\, w_{sT}\, \eta_{siT}\, \eta_{mT}$$

$$w_{sT} = c_{p_T}\, T_5 \left[1 - \left(\frac{p_6}{p_5}\right)^{\frac{\kappa_T - 1}{\kappa_T}}\right]$$

$$\dot m_{T,red} = \dot m_T \frac{\sqrt{T_5}}{p_5} \qquad \dot m_{T,korr} = \dot m_{red} \frac{p_{ref}}{\sqrt{T_{ref}}}$$

$$\eta^*_{ges} = \frac{\dot m_A}{\dot m_V} \overbrace{\eta_{siT}\,\eta_{siV} \underbrace{\eta_{mT}\eta_{mV}}_{\eta_{mATL}}}^{\eta_{ATL}} = \frac{1 + \lambda L_{St}}{\lambda L_{St}} \eta_{siT}\,\eta_{siV}\,\eta_{mATL}$$

$$= \frac{w_{sV}}{w_{sT}} = \frac{T_1}{T_5} \frac{c_{p_V}\left[\left(\frac{p_2}{p_1}\right)^{\frac{\kappa_V - 1}{\kappa_V}} - 1\right]}{c_{p_T}\left[1 - \left(\frac{p_6}{p_5}\right)^{\frac{\kappa_T - 1}{\kappa_T}}\right]}$$

8. Energiebilanz

$$\Delta \dot H_A = \sum_i \dot n_i \left[h_i(T_A) - h_i(T_0)\right] = \dot n_A \sum_i \psi_i \left[h_i(T_A) - h_i(T_0)\right]$$

$$= \dot m_A \sum_i \frac{g_i}{M_i} \left[h_i(T_A) - h_i(T_0)\right]$$

$$\Delta \dot H_A = \dot H_A(T_A) - \dot H_{AH}(T_{Eintritt})$$

$$1 = \underbrace{\frac{P_e}{\dot m_B\, H_u}}_{\eta_e} + \frac{\Delta \dot H_A}{\dot m_B\, H_u} + \frac{\Delta \dot H_{KW}}{\dot m_B\, H_u} + \frac{\Delta \dot H_{LLK}}{\dot m_B\, H_u} + \frac{\dot Q_R}{\dot m_B\, H_u}$$

$$P_e = \sum_i \frac{P_{an,i}}{\eta_{an,i}} \qquad P_{chem} = \dot m_B H_u = \sum_i \dot m_{B,i} H_{u,i}$$

9. Wärmeübertragung

$$\varphi_m = \varphi_{Konv} = \alpha_{mi}\left[T_{\alpha i} - T_{mWi}\right]$$

$$\varphi_{m(TR)} = \varphi_{WLeit} = \frac{\lambda}{d_{wand}} \left[T_{W_{li}} - T_{W_{re}}\right] (\cdot f_{TR})$$

$$f_{TR} = \frac{\frac{D_A}{D_i} - 1}{\ln\left(\frac{D_A}{D_i}\right)}$$

$$\varphi_{m(TR)} = \frac{\lambda}{B_{(TR)}} (T_{\alpha i} - T_a)$$

$$B = \frac{\lambda}{\alpha_{mi}} + \delta + \frac{\lambda}{\alpha_a} \quad \text{mit} \quad \delta = \frac{D_a - D_i}{2}$$

$$B_{TR} = \frac{\lambda}{\alpha_{mi}} + \frac{\delta}{f_{TR}} + \frac{\lambda}{\alpha_a} \frac{D_i}{Da}$$

10. Kräfte

$$F_{Ko,osz} = m_{osz}\, (\cos(\alpha) + \lambda_S\, \cos(2\alpha))\, r\, \omega^2$$

$$\to m_{osz} = m_{Ko} + m_{Pl,osz}$$

$$\to m_{Pl,osz} = \frac{l_{1,(sp \leftrightarrow kw)}}{l_{2,(sp \leftrightarrow ko)}} m_{Pl,rot}$$

$$\to m_{Pl,rot} = \frac{l_{2,(sp \leftrightarrow ko)}}{l} m_{Pl}$$

$$F_{Pl} = \frac{F_{Ko,osz}}{\cos(\beta)}$$

$$F_T = F_{Pl}\, \sin(\alpha + \beta) \approx F_{KO,res} \cdot \left(\sin(\alpha) + \frac{\lambda_S}{2} \cdot \sin(2\alpha)\right)$$

$$F_R = F_{Pl}\, \cos(\alpha + \beta) \approx F_{KO,res} \cdot \left(-\frac{\lambda_S}{2} + \cos(\alpha) + \frac{\lambda_S}{2} \cdot \cos(2\alpha)\right)$$

$$F_{Ko,Anwechsel} = F_G - F_{Ko,osz}\, (\pm F_{Reib}) \overset{!}{=} 0$$

11. Massenausgleich

$$F_{(1,2),res} = \sqrt{F^2_{+(1,2),res} + F^2_{-(1,2),res}}$$

$$F_{(\pm)(1,2),res} = \sqrt{F^2_{(\pm)(1,2),x} + F^2_{(\pm)(1,2),y}}$$

$$F_{+(1,2)} = F_{-(1,2)} = \frac{1}{2} F_{01,(02)} = \frac{1}{2} m_{osz}\, r\, \omega^2\, (\lambda_S)$$

$$M_{(1,2),res} = \sqrt{M^2_{+(1,2),res} + M^2_{-(1,2),res}}$$

$$M_{(\pm)(1,2),res} = \sqrt{M^2_{(\pm)(1,2),x} + M^2_{(\pm)(1,2),y}}$$

$$F_{osz} = F_{Ko,osz} = m\, a = m_{osz}\, r\, \omega^2\, (\cos(\alpha) + \lambda_S\, \cos(2\alpha))$$

$$F_{rot} = m_{rot}\, a = m_{rot}\, r\, \omega^2 = (m_{KK,red} + m_{Pl,rot})\, r\, \omega^2$$

$$m_{KK,red} = \frac{r_1}{r} m_{KW}$$

$$M_{1,osz} = \|M_{1,min}\| \cdot 2$$

$$M_{1,rot} = \|M_{1,max}\| - \|M_{1,min}\|$$

12. Kinematik

$$s_{exakt}(\alpha) = r\left[(1 - \cos(\alpha)) + \frac{1}{\lambda_S}\left(1 - \sqrt{1 - \lambda_S^2 \sin^2(\alpha)}\right)\right]$$

$$s_{approx}(\alpha) = r\left[(1 - \cos(\alpha)) + \frac{\lambda_S}{4}(1 - \cos(2\alpha))\right]$$

$$\overset{\lambda_S \to 0}{\approx} r(1 - \cos(\alpha))$$

$$\lambda_S = \frac{r}{l} = \frac{s}{2l}$$

$$c_{exakt}(\alpha) = r\,\omega\left[\sin(\alpha) + \frac{\lambda_S\, \sin)(\alpha)\, \cos(\alpha)}{\sqrt{1 - \lambda_S^2\, \sin^2(\alpha)}}\right]$$

$$c_{approx}(\alpha) = r\,\omega\left[\sin(\alpha) + \frac{\lambda_S}{4}\, \sin(2\alpha)\right]$$

$$\overset{\lambda_S \to 0}{\approx} r\,\omega\, \sin(\alpha)$$

$$c_{max} = r\,\omega\, \sqrt{1 + \lambda_S^2} \quad \sphericalangle(r, l) \to \perp$$

$$\alpha(c_{max}) = \arctan\left(\frac{l}{r}\right) = \arctan\left(\frac{1}{\lambda_S}\right)$$

$$c_m = 2\, s\, n = 4\, r\, n$$

$$a_{approx}(\alpha) = r\,\omega^2\, [\cos(\alpha) + \lambda_S\, \cos(2\alpha)]$$

$$\overset{\lambda_S \to 0}{\approx} r\,\omega^2\, \cos(\alpha)$$

13. Ventiltrieb

$$F_N = F_{F,red} + m_{N,red} \underbrace{a_N}_{= \ddot s_N} \left(+ p_{\text{AÖ}} \frac{\pi}{4} d_V^2\, i\right) \overset{!}{\geq} 0 \qquad i = \frac{l_{1,h \leftrightarrow v}}{l_{2,h \leftrightarrow n}}$$

$$\Rightarrow \omega_{NW} = \sqrt{-\frac{F_{F,red}}{m_{N,red}\, s''_V}}$$

$$F_{F,red} = F_F\, i = (s_{v0} + s_N\, i)\, i\, c_F = (s_{v0} + s_V\,)\, i\, c_F$$

$$m_{N,red} = \left(m_{1,v} + \frac{m_{2,f}}{2} + m_{3,(t,k)}\right) i^2 + \frac{J_K}{l^2_{2,(h \leftrightarrow n)}} + m_{5,st} + m_{6,stoe}$$

$$s' = \frac{\mathrm{d}s}{\mathrm{d}\alpha}$$

$$\Rightarrow \ddot s_N = a = s''_N\, \omega^2_{NW} = s''_N\, (\omega_{NW}\, S_n)^2$$

$$s_N = \frac{1}{i} s_V \qquad \dot s_N = \frac{1}{i} \dot s_V \qquad \ddot s_N = \frac{1}{i} \ddot s_V$$

$$s_N = \frac{1}{i} s_V \qquad s'_N = \frac{1}{i} s'_V \qquad s''_N = \frac{1}{i} s''_V$$

$$s_{(v,n),2} = \frac{s''_{(v,n),2}}{s''_{(v,n),1}} s_{(v,n),1}$$

14. Brennverlaufsanalyse

$$\mathrm{d}Q_B = \frac{\kappa}{\kappa-1} p_{\alpha_1}\,\mathrm{d}V + \frac{1}{\kappa-1} V_{\alpha_1}\,\mathrm{d}p + \alpha_i\, A_W \left(T_{\alpha_1} - T_{mWi}\right)\mathrm{d}t$$

$$T_{i,(\alpha_1)} = \frac{p_{\alpha_1} V_{\alpha_1}}{m_G R_G}$$

$$A_{W,\alpha_1} = A_{Kolben} + A_{Kopf} + \underbrace{A_{Wand}\,(+A_{Kompr})}_{=\pi D(s+(s_s))}$$

$$V_{\alpha_1} = V_c + \frac{\pi}{4} D^2 s_{\alpha_1}$$

$$v(\alpha) = v_c + \frac{s(\alpha)}{s(180^\circ KW)} \cdot (v_1 - v_2)$$

$$\mathrm{d}t = \frac{\Delta\alpha}{360^\circ KW \cdot n}$$

$$\mathrm{d}m_B = \frac{\mathrm{d}Q_B}{H_u}$$

15. Ähnlichkeit

$c_m = \text{const.}$ $\quad p(\alpha) = \text{const.}$ $\quad T_{\alpha i} = \text{const.}$

$\frac{s_2}{s_1} = x$ $\quad \frac{A_2}{A_1} = x^2$ $\quad \frac{V_2}{V_1} = x^3$

$\sigma_m \propto c_m^2$ $\quad t_{\text{ü}} \propto D = s$

16. Spannungen

$$S = \frac{\sigma_B}{\sigma_{mech} + \sigma_{therm}}$$

$$\sigma_{mech} = \frac{p_Z \cdot D}{2\,\delta}$$

$$\sigma_{therm} = \frac{E \cdot \alpha_L}{1-\nu} \frac{T_{mWi} - T_{mWa}}{2}$$

17. Allgemein

$$\frac{y_g - y_1}{x_g - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \qquad x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta X_{\%} = \frac{X_{neu} - X_{alt}}{X_{alt}} \cdot 100\%$$

$$i_{antrieb} = \frac{n_{mot}}{n_{rad}} = \frac{M_{rad}}{M_{mot} \cdot \eta_{antrieb}}$$