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Preface

In memory of Fisher Black, without whose tremendous contribution to both theory and practice, derivatives research and industry would not have reached the current stage, and certainly, this book which concentrates on a Black-Scholes environment would not have been started.

With derivatives you can have almost any payoff pattern you want. If you can draw it on paper, or describe it in words, someone can design a derivative that gives you that payoff.

These days we often come across such terms as exotic options in newspapers, journals, magazines, and many other financial reports. You may wonder, as I did two years ago, what exactly they are. At that time I had just started to work in the financial industry and was much puzzled by the phrase “exotic options”. Such puzzlement left me feeling uneasy as I had previously spent a few years at school studying option pricing theory. I tried to find some systematic sources to reeducate myself, and it turned out to be nearly impossible as there was no systematic source on this subject. Two years later, although the situation has changed somewhat, exotic options still remain mysterious to many people. I have tried to keep a systematic record on this subject, though initially it was not my intention to write a book on exotic options.

My first research paper on this subject was on spread options nearly two years ago. Since then, many other papers have followed. The writing process has been so wonderful that I would never have learned so fast and thoroughly had I not written this book. This book records the accumulation of my knowledge on exotic options, and I would like to share my learning curve with all of you.

To some degree, exotic options are as old as vanilla options. The earliest article on exotic options can be traced to an article titled “Alternative Forms of Options” by Snyder, published in the *Financial Analysts*.

Journal. It was in 1969, four years earlier than the establishment of the

Chicago Board of Options Exchange (CBOE), the first organized options exchange in the world, and four years earlier than the birth of the seminal work of Black and Scholes, who made the path-breaking contribution to derivatives industry. However, exotic options became somewhat popular only from the late 1970s and have experienced significant growth in the past decade or so. The primary motivations driving the recent innovations of derivatives are cost-reduction and special customer needs such as off-balance sheet opportunities, tax considerations, and so on.

The study of exotic options is indispensable not only for their own active and important trading but also because they provide easy and efficient building blocks for other more complicated financial derivatives. In order not to inundate many readers, I try to spend a significant amount of time in almost every chapter on how to use the pricing formulas and how to apply them in practice.

A series of events in the derivatives industry since 1994, Orange County, Kidder Peabody, Procter & Gamble, Gibson Greetings, Askin Capital, and so on, has attracted a lot of attention in the financial industry as well as among the general public. These events created calls for transparency of special-purpose derivatives activities. One of the objects of this book is to provide a convenient source of information for exotic options and thus to improve the transparency of the market. I try to provide as complete a source as possible on this subject. In each chapter I try to introduce one type of exotic options, what it can achieve, and how to price and use it.

We will concentrate on a Black-Scholes environment throughout this book for the purpose of transparency and easy comparisons with vanilla options, because the Black-Scholes model is best known. In Section 4.3 of Chapter 4, we provide a derivation of the Black-Scholes formula using the method to solve the related partial differential equation. And in Section 4.4 of Chapter 4, we provide an intuitive and concise method to derive the Black-Scholes formula. The method shown in Section 4.4 to derive the Black-Scholes formula is the same method we use to price essentially all exotic options in this book.

It is well-known in physics that energy can neither be created nor destroyed, it can only be transferred from one form to another. Risk, or more specifically, financial risk, is such a complicated subject that many researchers and financial institutions have been struggling to find ways to measure it. Yet intuition suggests that risk, like energy, can neither be created nor destroyed: it is inherent within the financial system.

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Chapter 1

Background and Some Concepts

1.1 Introduction

We assume knowledge of general solid state physics as in Rosenberg. However, we shall begin by briefly summarizing a few concepts that are basic to an understanding of later Chapters below.

1.1.1 *Elastic and plastic regims*

Immediately it is helpful to classify the discussion of mechanical properties by defining two regimes (i) elastic and (ii) plastic.

1.1.1.1 *Elastic deformation*

The mechanical properties of materials are of vital importance in determining their fabrication and practical applications. Initially as a load is applied on the material, the nominal stress is defined as the load divided by the original cross section area, and the nominal strain as the extension divided by the original length. As the stress is increased, the strain increases uniformly and the deformation produced is completely reversible. This is so-called elastic region. The stress and resulting strain are proportional to one another and obey Hook's law.

From atomistic points of view, if we pull two atoms apart or push them together by a force, the atoms can find a new equilibrium position in which the atomic and applied forces are balanced. The force in the bond is a function of the displacement. The deformation of the bond being reversable means that, when the displacement returns to the initial value, so also does the force return simultaneously to its corresponding value. The bulk elastic behaviour of large solid bodies is the aggregate effect of the individual deformations of the bonds in them.

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1.1.1.2 Atomic forces and elastic properties

Taking NaCl type ionic crystals as an example, Cottrell (1964a) discussed the interaction energy of a pair of univalent ions at a distance r as

$$U(r) = \pm \frac{e^2}{r} + \frac{B}{r^s}, \quad (1.1)$$

where $s \approx 9$, and where $+$ and $-$ refer to like and unlike ions respectively. Having summed the repulsive and attractive interactions with nearest neighbours, the

total interaction energy of an ion can be written as

$$U_z = -A \frac{e^2}{r} + 6 \frac{B}{r^s} \quad (1.2)$$

where A is called the Madelung constant, equal to 1.7476 for the NaCl type crystals. At the equilibrium condition, $\frac{dU_z}{dr} = 0$, at $r = r_0$. Thus,

$$B = \frac{Ae^2 r_0^{s-1}}{6s} \quad (1.3)$$

and

$$U_z = -\frac{Ae^2}{r} \left[1 - \left(\frac{1}{s} \right) \left(\frac{r_0}{r} \right)^{s-1} \right] \quad (1.4)$$

this is the work required to dissociate the crystal into $2N$ separate ions (N positive and N negative).

The elastic constant E ,

$$E = \frac{f}{u} = \left(\frac{1}{6} \right) \left(\frac{\partial^2 U_z}{\partial r^2} \right)_{r=r_0} = \frac{(s-1)Ae^2}{6r_0^3}, \quad (1.5)$$

where $U_z/6$ is the energy per each nearest-neighbour bond, and $u = r - r_0$, is the elastic displacement.

The bulk modulus of elasticity of the material is defined by

$$p = -K \frac{\Delta V}{V} \quad (1.6)$$

where p is a hydrostatic pressure, $\frac{\Delta V}{V}$ is the volume change.

$$K = \frac{-p}{\left(\frac{\Delta V}{V} \right)} = -\frac{pr_0^2}{r_0^2 \left(\frac{\Delta V}{V} \right)} \cong \frac{f}{r_0^2 \left(\frac{3u}{r_0} \right)} = \frac{1}{3r_0} \left(\frac{f}{u} \right) = \frac{(s-1)Ae^2}{18r_0^4} \quad (1.7)$$

In KCl, it gives $K^T = 1.88 \times 10^{11} \text{ dyn cm}^2$, whereas the observed value (extrapolated to OK) is $2 \times 10^{11} \text{ dyn cm}^2$. The corresponding calculations of elastic constants of metallic crystals are much more difficult for the laws of force are much complicated. we shall discuss this in Chapter 5.

1.1.1.3 Plastic deformation

Plastic deformation is characterized by a permanent deformation of the material. Unlike elastic deformation, it does not reverse on unloading but leave the material with a permanent shape. This is called the plastic region. Between these

Table 1.1 Caption for Table 1.

Title	Year	Author
X-ray photons	1912	M. von Laue
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Neutrons	1936	D. P. Mitchell and P. N. Powers

two regions, there is a limiting stress, called the yield stress of the material, or the critical resolved shear stress of it.

1.2 Friction Mechanisms

The mechanisms of friction are discussed in early books on the subject (e.g. Bowden and Taber, 1950). here we refer to the subsequent account of Stoneham *et al.* (1993). These workers note the following mechanisms: (i) Adhesion: surfaces adhere and then work is done in separating them. (ii) Ploughing: one surface pulls away small amounts of the other and (iii) Anelasticity: here the assumption is that energy is dissipated by dislocation motion and plastic deformation in the material.

- (a) Entry one.
- (b) Entry two.

We shall, in later chapters, discuss friction on a mesoscopic scale as well as specific atomistic studies. As to the first of these, we shall see below that two main steps are involved. The first of these is the characterization of rough surfaces and their contact. The second step is to invoke some law of friction. In such a law, we want here to lay stress on the central importance of atomic force microscope (AFM) data (see Appendix 2.1) and its interpretation.

Tribology, the study of surfaces in moving contact, is an important area for technology. In spite of this, friction, at the time of writing, is not well understood at an atomistic levels. Persson (1994) has posed some fundamental questions as follows:

- (1) What is structure of sliding interface: both geometric and electronic?
- (2) Where does the sliding take place?
- (3) What is the physical origin of the sliding force?

Person follows these somewhat general points with some more specific questions:

- (i) Why is the frictional force F usually proportional to the load N ?
- (ii) What is the microscopic origin of 'stick-and-slip' motion?
- (iii) Why is the frictional force F usually proportional to the load N ?
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- (vi) What is the microscopic origin of 'stick-and-slip' motion?

The crystallinity of the structure is the prime cause of this behaviour, for it enables whole slabs of crystal to glide past one another. Each slip is a displacement, in certain glide direction, generally the crystal direction of closest atomic packing on certain crystal planes which is called slip planes. In fcc and hcp metals, these are mainly close-packed planes, but in bcc metals, the situation is complicated. It will be discussed later. Slip begins on some small area of the surface. The slip-front line between the slipped and unslipped areas is by definition a *dislocation* line. The glide motion of a dislocation is a property of a periodic crystal. The transition from the slipped to the unslipped region is spread over several atomic distances which is the width of the dislocation. Every atom in this transition region is pushed only a little further out of its original equilibrium site when it moves forward. This is the reason why dislocation can move easily in the crystal. Thus, the yield stress is much lower than the theoretical strength of crystals. Dislocation theory plays important role in understanding the microscopic processes in plastic deformation. Even the elastic theory of dislocation may explain many phenomena, such as yielding, work hardening, and etc. It provides not only a deeper qualitative physical picture of plastic deformation but also a certain degree quantitative analysis of it approximately.

Plastic deformation can also occur by twinning. The atoms slide, layer by layer to bring each deformed slab into mirror-image lattice orientation relative to the undeformed material. The critical stress of twinning is usually higher. Twins form at low temperature and rapid deformation, e.g. bcc iron strained quickly at room temperature and slowly at 100k.

1.3 Griffith Criterion: Role of Surfaces

Griffith (1924) derived an expression for the elastic crack propagation on the basis of thermodynamic considerations. He reasoned that a crack would advance when the incremental release of stored elastic strain energy dW_E in a body became greater than the incremental increase of surface energy dW_s as new

crack surface was created. For the two-dimensional case in plane stress

$$W_E = \frac{\pi \sigma^2 c^2}{E} \quad (1.8)$$

$$W_s = 4c\gamma_s$$

where, σ is the nominal stress; E , the elastic modulus; $2c$, the length of the crack, and γ_s the specific surface energy.

The Griffith criterion can then be written as

$$\sigma_F = \sqrt{\frac{2E\gamma_s}{\pi c}} \quad (1.9)$$

by the condition that

$$\frac{\partial}{\partial c} W_E \geq \frac{\partial W_s}{\partial c}.$$

The Irwin analysis in fracture mechanics defined a parameter, crack extension force, $G = K^2/E$ (in plane stress) is equal to a critical value, G_{1c} . the crack-resistance force of the material. For the elastic crack in an infinitely wide plate

$$G_c = \frac{K^2}{E} = \frac{\sigma_F^2 \pi c}{E} \quad (1.10)$$

where K is the stress intensity factor, K_c is called fracture toughness.

Comparing to Eq. (1.2.2), $G_c = 2\gamma_s$. The two approaches lead to the same result although their methods are different. The specific surface energy plays very important role in brittle fracture.

Engineering materials do not fracture in a completely elastic manner. The localized plastic deformation near crack tip gives the materials some toughness, or resistance to crack propagation. Orowan (1948) proposed to add a term γ_p , the plastic work expended during crack propagation to the elastic work γ_s as an effective specific surface energy in Eq. (1.2.2). The Griffith equation is modified to read (in plane stress)

$$\gamma_F = \sqrt{\frac{2E}{\pi c}(\gamma_s + \gamma_p)} \sim \sqrt{\frac{2E\gamma_p}{\pi c}} \quad (1.11)$$

(Some authors wrote $2(\gamma_s + \gamma_p)$ as $2\gamma_s + \gamma'_p$; then, $\gamma'_p = 2\gamma_p$). From Eq. (1.2.4), it seems γ_s is no longer an important factor in this process. However, Tetelman (1963) showed that for the case of Fe – 3%Si, by Frank–Read source multiplication,

$$\gamma_m = \text{const.} \gamma_s N_0^{\frac{3}{2}} \left(\frac{v_0}{v_c} \right)^2 T^{\frac{5}{2}} \quad (1.12)$$

where γ_m is defined as the product of the work done in a unit volume element of material when the crack advances and the distance perpendicular to the crack in which the deformation is extensive. N_0 is the density of mobile dislocation sources, v_0 and v_c are velocities of sound and the crack respectively. γ_m , like γ_p is a measure of the intrinsic toughness of a solid. In Eq. (1.2.5), γ_s is a multiplying factor not an addition term. The change of γ_s directly influences the change of γ_m .

Moreover, Lung and Gao (1985) calculated the relative K_{ic} value of metals with a simplified dislocation motion model and BCS dislocation distribution function at the crack tip

$$G_c^p \cong 2\gamma_p \propto W_i = E_0(K_{ic}^0)^2 F_i(\theta_0) r_i^*(\theta_0)^{\frac{1}{2}} \quad (1.13)$$

where K_{ic}^0 is the fracture toughness in linear elastic case ($= \sqrt{EG_{ic}^0}$ or $\sqrt{2\gamma_s E}$); r_i^* , the plastic zone size; and $E_0 \propto E^{-1}$. The E_0 in Eq. (1.2.6) is proportional to the inverse of the elastic modulus of materials, and $F_i(\theta)$, the angular dependent function respectively.

Comparing Eq. (1.2.6) with (1.2.5), the two approaches lead to the same conclusion that γ_s plays the role as a multiplying factor in the expression of critical crack extension forces. For a multiplying factor,

$$\frac{\Delta(\gamma_s f)}{(\gamma_s f)} = \frac{\Delta\gamma_s}{\gamma_s} + \frac{\Delta f}{f}. \quad (1.14)$$

The relative change of γ_s is as important as that of f . If we consider the hidden role of atomic forces in the structure of dislocation core and dynamics, the role of interatomic force is not only in surface energy term but in the dislocation core structure also.

1.4 Peierls Stress and Potential

A dislocation experiences an oscillating potential energy as it glides in a crystal. In the Peierls model (Peierls, 1940), the bonds across the glide plane were considered to interact via an interatomic potential, while the remainder of the lattice was linearly elastic. Nabarro (1957) gave an analytical expression for the dislocation core model. One can approximately estimate the ideal lattice resistance to dislocation motion by means of the Peierls model. The resolved applied stress necessary to move the dislocation over the Peierls barrier is called Peierls stress, σ_p . The Peierls stress comes from the expression for the Peierls energy which changes for a translation of the dislocation by a distance smaller than the Burgers vector.



Fig. 1.1 Caption for Fig. 1.

Figure 1.1, reproduced from Nabarro (1967), shows the Peierls model of a dislocation. The material above A and below B is regarded as forming an elastic continuum. The force between the rows A and B is a periodic function of the displacement.

As the dislocation moves through the lattice, it passes through unsymmetrical configuration to a different symmetrical configuration in which one half plane of atoms on the expanded side of the glide plane lies midway between two half planes on the compressed side. Further motion passes through unsymmetrical configurations back to a state equivalent to the original. The dislocation moves if a finite force acts on it. The critical stress is called Peierls stress. After a lengthy calculations, the approximate energy of misfit is given by

$$E = \left[\frac{b^2 \mu}{4\pi(1-\nu)} \right] \left\{ 1 + 2 \cos 4\pi\alpha \exp \left(\frac{-4\pi\zeta}{b} \right) \right\}. \quad (1.15)$$

The force acting on unit length of the edge dislocation is,

$$F = - \left(\frac{1}{b} \right) \frac{dE}{d\alpha} = \frac{2b\mu}{(1-\nu)} \sin 4\pi\alpha \exp \left(\frac{-4\pi\zeta}{b} \right), \quad (1.16)$$

where $\zeta = \frac{a}{2(1-\nu)}$ is a parameter measuring the width of dislocation. αb , the displacement of the centre of the dislocation from the original equilibrium position and μ , the shear modulus.

The maximum value of Eq. (1.3.2) is the critical shear strength; the Peierls stress is given by

$$\sigma_p = \frac{2\mu}{4\pi(1-\nu)} \exp \left(\frac{-4\pi\zeta}{b} \right). \quad (1.17)$$

Considering the spirit of this model, and extending the displacement of the centre of the dislocation to include the thermal vibration amplitude, the temperature dependence of the crss can be obtained (see Lung *et al.*, 1966; or later Sec. 10.3).

Appendix A

Dislocation Core and Atomic Force

Early development of dislocation theory, and related theoretical treatment of metallic properties controlled by dislocations, focussed most attention on the effects of long-range elastic fields. In the present context (see also Vitek, 1995) mechanical properties were frequently analyzed in terms of long-range dislocation-dislocation, dislocation-point defects etc interactions. The attitude prevailing in the late 1960s that dislocation cores were of but secondary importance in the plastic deformation of metals was radically altered in the next two decades. It becomes widely recognized thatn the outcome. It becomes widely recognized thatn the outcome. As emphasized in the studies of Vitek and co-workers (Vitek, 1985) clear signatures of core effects are to be found in deformation modes and slip geometry, strong orientation and temperature dependences of the yield stress, and also in anomalous temperature dependence of the yield and flaw stresses (see also Duesbery and Richardson, 1991).

$$U_z = -\frac{Ae^2}{r} \left[1 - \left(\frac{1}{s} \right) \left(\frac{r_0}{r} \right)^{s-1} \right]. \quad (\text{A.1})$$

Significant impetus for such atomistic modeling has been the marked improvement in experimental techniques (see Appendix 1), such as high resolution electron microscopy (HREM), that are capable of atomic resolution. As emphasized in the studies of Vitek and co-workers (Vitek, 1985) clear signatures of core effects are to be found in deformation modes and slip geometry, strong orientation and temperature dependences of the yield stress, and also in anomalous temperature dependence of the yield and flaw stresses (see also Duesbery and Richardson, 1991).

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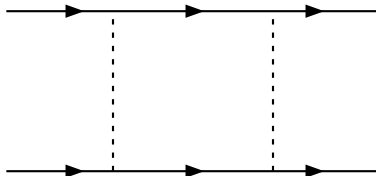


Fig. A.1 Caption for Fig. 2.

in experimental techniques (see Appendix 1.), such as high Significant impetus for such atomistic modeling has been the marked improvement in experimental techniques (see Appendix 1.), such as high resolution electron microscopy (HREM), that are capable of atomic resolution.

A.1 Stacking Faults

Rosenberg (1978) discussed how close-packed planes of hard spheres can be stacked to form, say, a fcc structure, Fig. 1.2 being reproduced from his account.

The first and second layers can be in positions labelled A and B while the third layer can be placed above the C positions. The pattern continues as ABCABC..., the pattern repeating at every third layer.

A stacking fault in such an fcc structure occurs if this sequence gets disturbed, as in ABCBCABC... Here a layer A is missing, while in the sequence ABCABAC... an extra A layer has been introduced. While stacking fault can, at least in principle, extend through the entire crystal, they usually occupy only a part of the plane. In this last case, of a stacking fault which terminates within the crystal, the configuration at the termination is referred to as a partial dislocation.

Appendix B

Glissile and Sessile Dislocations

Dislocations that can move by pure slip are called glissile. Dislocations which cannot glide, but have to move by some form of mass transports are called sessile (Read, 1953).

In crystals, the dislocation core spreads to certain crystallographic planes containing the dislocation line. If the core spreads into one of such planes, the core is planar and is glissile. If the core spreads into several non-parallel planes of the zone of the dislocation line, it is non-planar and is sessile. In the former case the dislocation moves easily in the plane of the core spreading, while in the latter case, it moves only with difficulty (Vitek, 1992). A Shockley partial is a partial dislocation, the Burgers vector of which lies in the plane of the fault. Then, Shockley partials are glissile. A Frank partial is a partial dislocation, the Burgers vector of which is not parallel to the fault. Then, Frank partials are sessile. In the former case the dislocation moves easily in the plane of the core spreading.

B.1 Concept of Fractals

Over a decade or more, diverse scientists have recognized that many of the structures common in their experiments have a quite special kind of geometrical complexity. The pioneering work was that of Mandelbrot (1977, 1979, 1982, 1988) who drew attention to the particular geometrical properties of such things as the shore of continents, tree branches, or the surface of clouds. Mandelbrot used the word ‘fractal’ for these complex shapes, in order to emphasize that they are to be characterized by a non-integer (fractal) dimensionality.

Our interest here is the fractal aspects of fractured surfaces. Mandelbrot *et al.* (1984) gave an elegant route for determining the fractal dimension D

Table B.1 Caption for Table B.1.

Title	Year	Author
X-ray photons	1912	M. von Laue
Electrons	1927	C. Davisson and L. H. Germer
He atoms	1930	O. Stern
H ₂ molecules	1936	O. Stern
Neutrons	1936	D. P. Mitchell and P. N. Powers

of the surface. Their work pointed to a correlation between toughness and D . Further studies were performed by Lung (1986), Pande (1987), Lung and Mu (1988) and Xie and Chen (1988). Bouchard *et al.* (1990) later reported their findings that for a variety of rupture modes and materials the observed fractal dimensions were the same to within the error bars. Dauskardt *et al.* (1990) reported a fractal dimension $D \cong 2.2$, which, when combined with the studies of Bouchard *et al.* (1990) may turn out to be a universal value (but see also below).

Though it is therefore known that cracks in nature can have fractal character, at the time of writing it is still difficult to specify just how this fracture nature arises. For it is certainly true that the mechanisms leading to fracture are highly material dependent (see Liebowitz, 1984). This, we will discuss more in Sec. 4.3.

Progress has resulted from modelling the growth of a single, connected crack. With the assumption of central forces numerical simulations of media with a breaking probability proportional to the elongation of springs revealed that the cracks resulting are fractal (Louis *et al.*, 1986; Hinrichen *et al.*, 1989). The fractal dimension of such cracks appears to be sensitive to the type of external force (e.g. uniaxial tension, shear, uniform dilatation) but since only rather small cracks can be grown more precision is lacking. Herrmann (1989) has considered therefore deterministic models.

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