École Polytechnique Fédérale de Lausanne

Master Thesis

Relaxed Radix Balanced Trees as Imutable Vectors Scala

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ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Abstract

School of Computer and Communications
Computer Science

Master in Computer Science

Relaxed Radix Balanced Trees
as Imutable Vectors Scala

by Nicolas Stucki

The Thesis Abstract is written here (and usually kept to just this page). The page is kept centered vertically so can expand into the blank space above the title too…

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Abbreviations

JIT	**J**ust **I**n **T**ime
RB	**R**adix **B**alanced
RRB	**R**elaxed **R**adix **B**alanced

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Chapter 1

Introduction

1.1 Main Section 1

1.2 Main Section 2

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Chapter 2

Vector Structure

2.1 Radix Balanced Vectors

2.1.1 Tree structure

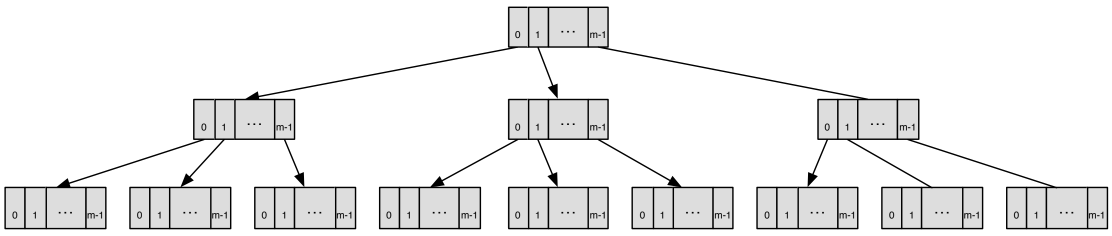

Figure 2.1: Radix Balanced Tree Structure

2.1.2 Operations

2.1.2.1 Apply

```
def apply(index: Int): A = {
  def getElem(node: Array[AnyRef], depth: Int): A = {
    val indexInNode = // compute index
    if(depth == 1) node(indexInNode)
    else getElem(node(indexInNode), depth-1)
  }
  getElem(vectorRoot, vectorDepth)
}
```

2.1.2.2 Updated

```
def updated(index: Int, elem: A) = {
  def updatedNode(node: Array[AnyRef], depth: Int) = {
    val indexInNode = // compute index
    val copy = clone(node)
    if(depth == 1) {
      copy(indexInNode) = elem
    } else {
      copy(indexInNode) =
        updatedNode(node(indexInNode), depth-1)
    }
    copy
  }
  new Vector(updatedNode(vectorRoot, vectorDepth), ...)
}
```

2.1.2.3 Additions

Append

Prepend

Concatenation and Insert

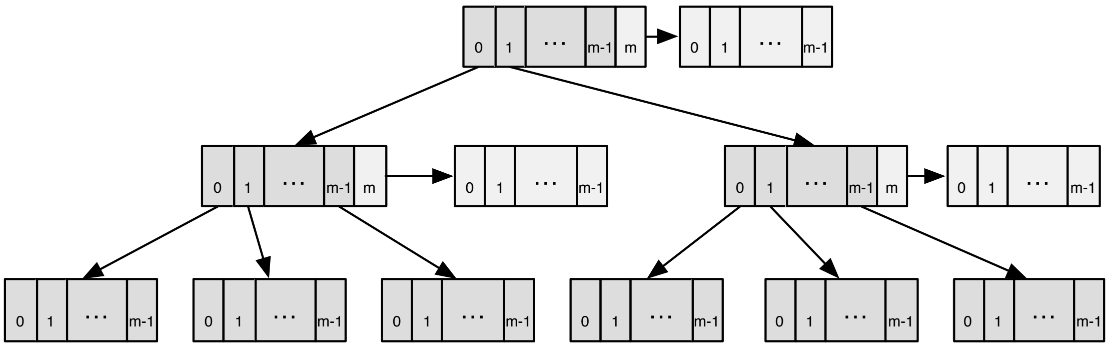

Figure 2.2: Radix Balanced Tree

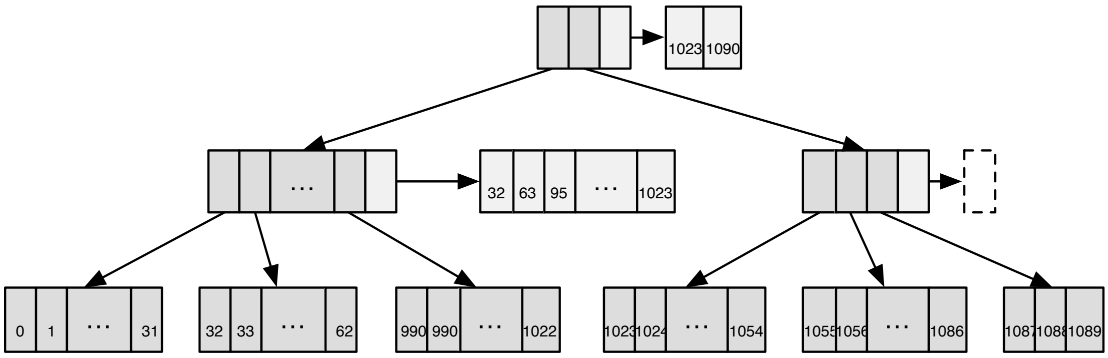

Figure 2.3: Relaxed radix example

2.1.2.4 Splits

2.2 Parallel Vectors

2.2.1 Splitter Iterator

2.2.2 Combiner Builder

2.3 Relaxed Radix Balanced Vectors

2.3.1 Relaxed Tree structure

2.3.2 Relaxed Operations

2.3.2.1 Apply (get element at index)

2.3.2.2 Updated

2.3.2.3 Additions

Append

Prepend

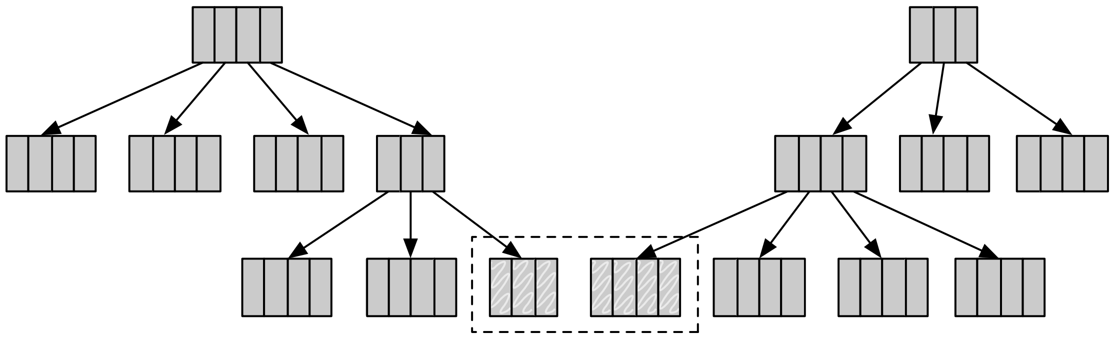

FIGURE 2.4: Concatenation example with blocks of size 4: Rebalancing level 0

Concatenation

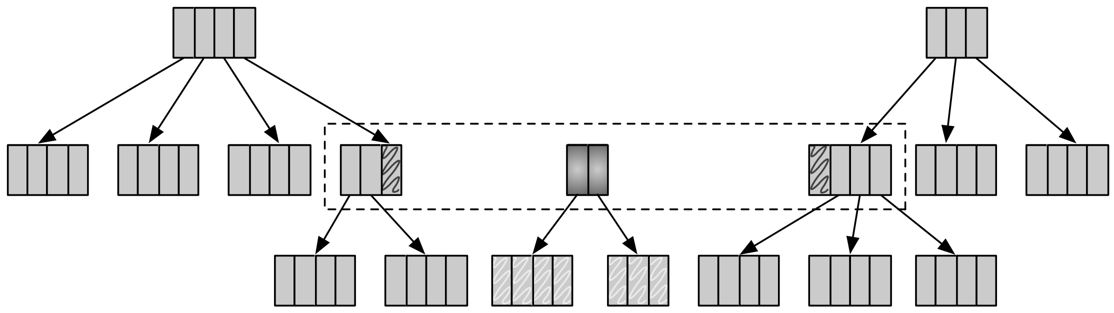

Figure 2.5: Concatenation example with blocks of size 4: Rebalancing level 1

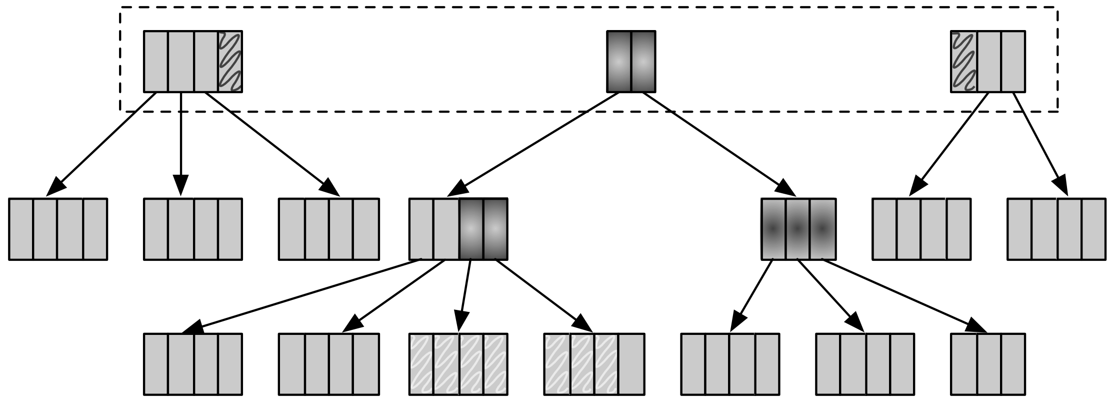

Figure 2.6: Concatenation example with blocks of size 4: Rebalancing level 2

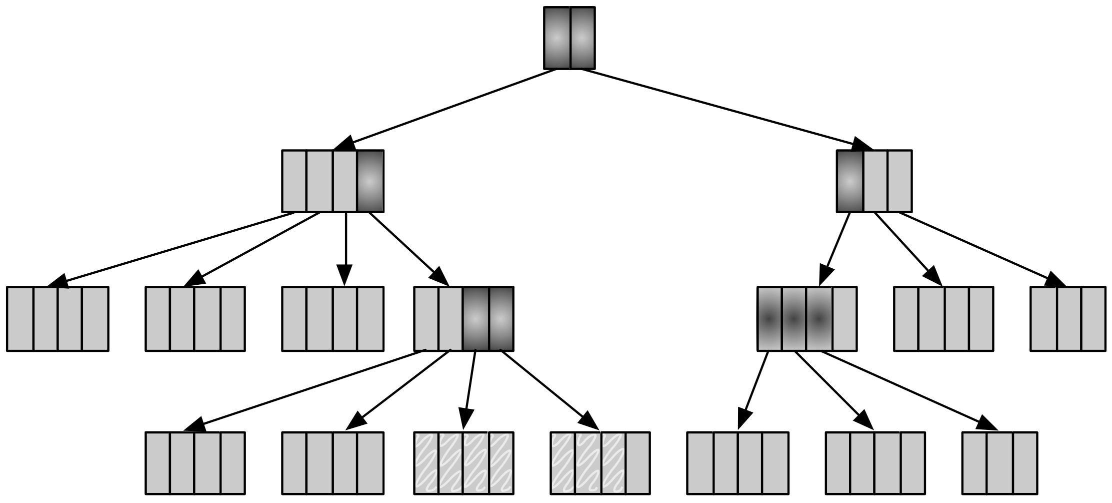

Figure 2.7: Concatenation example with blocks of size 4: Rebalancing level 3

Insert

2.3.2.4 Splits

2.3.2.5 Parallel Vector

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Chapter 3

Implementation and Optimizations

3.1 Where is time spent?

3.1.1 Arrays

3.1.2 Computing indices

$$526843 = 00\,\underbrace{00000}_{0}\,\underbrace{00000}_{0}\,\underbrace{10000}_{16}\,\underbrace{00010}_{2}\,\underbrace{01111}_{15}\,\underbrace{11011}_{27}$$

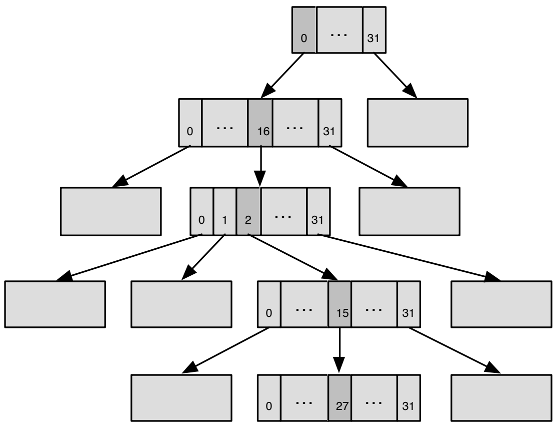

Figure 3.1: Accessing element at index 526843 in a tree of depth 5. Empty nodes represent collapses subtrees.

3.1.3 Abstractions

3.2 Displays

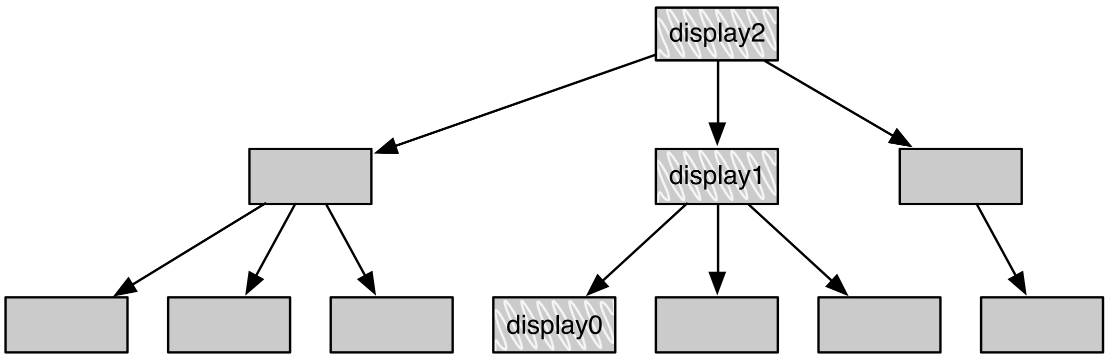

FIGURE 3.2: Displays

3.2.1 As cache

3.2.2 For transient states

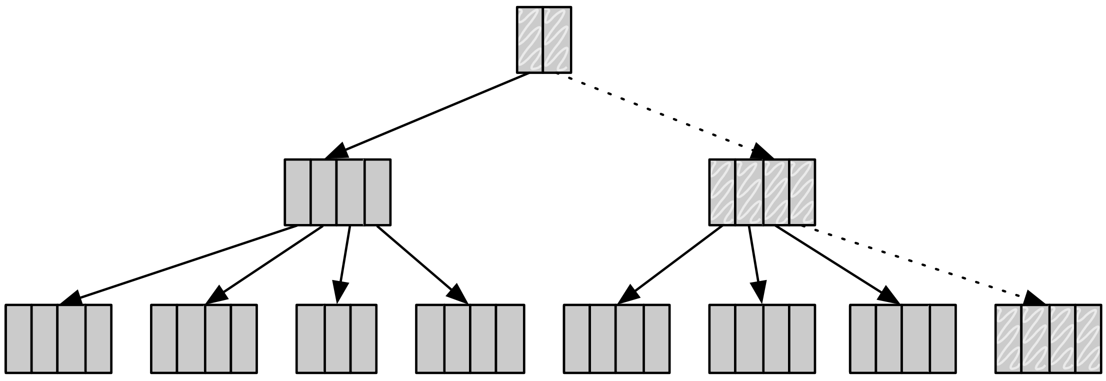

FIGURE 3.3: Radix Balanced Tree Transient state

3.3 Builder

3.4 Iterator

3.5 Relaxing the Radix

3.5.1 Relaxing Displays

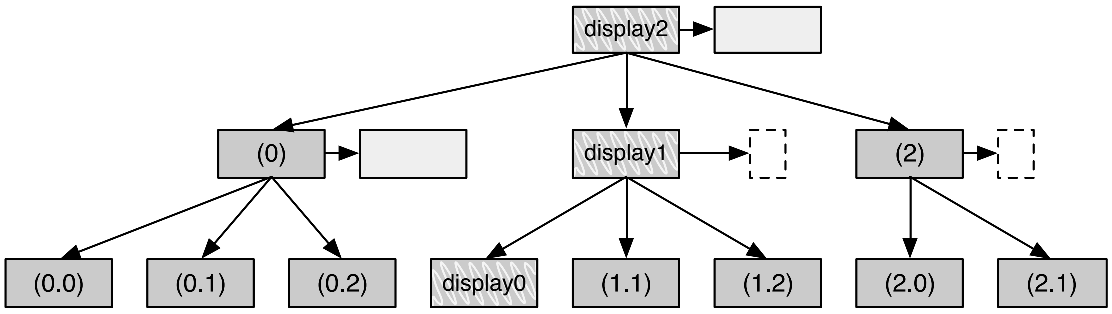

FIGURE 3.4: Radix Balanced Tree

3.5.2 Relaxing the Builder

3.5.3 Relaxing Iterator

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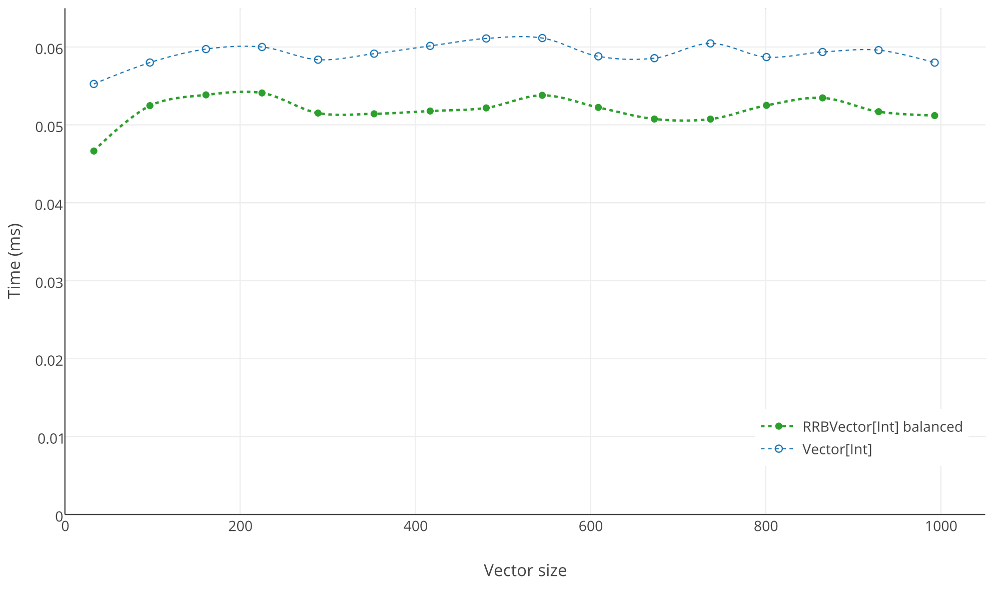


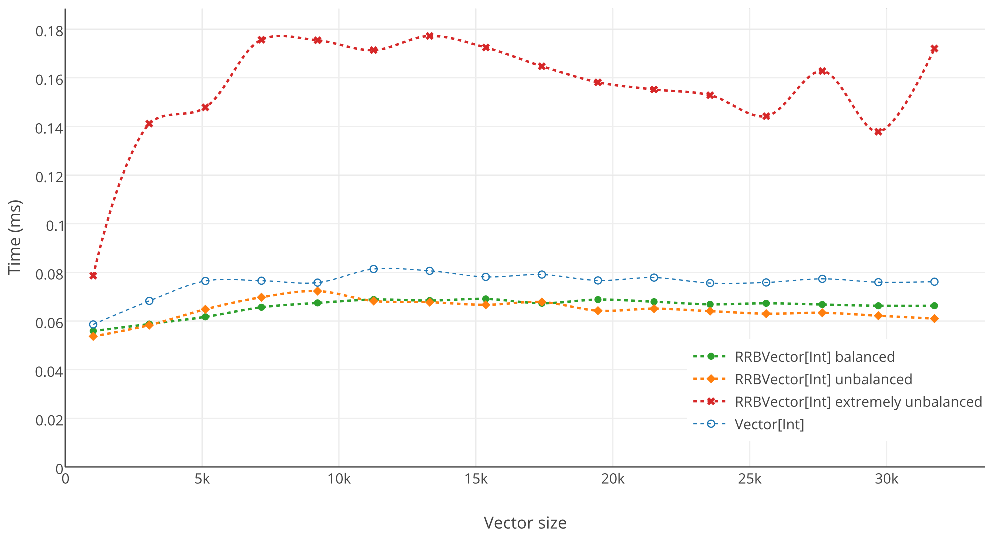


FIGURE 4.1: Time to execute 10k apply operations on sequential indices.

Chapter 4

Performance

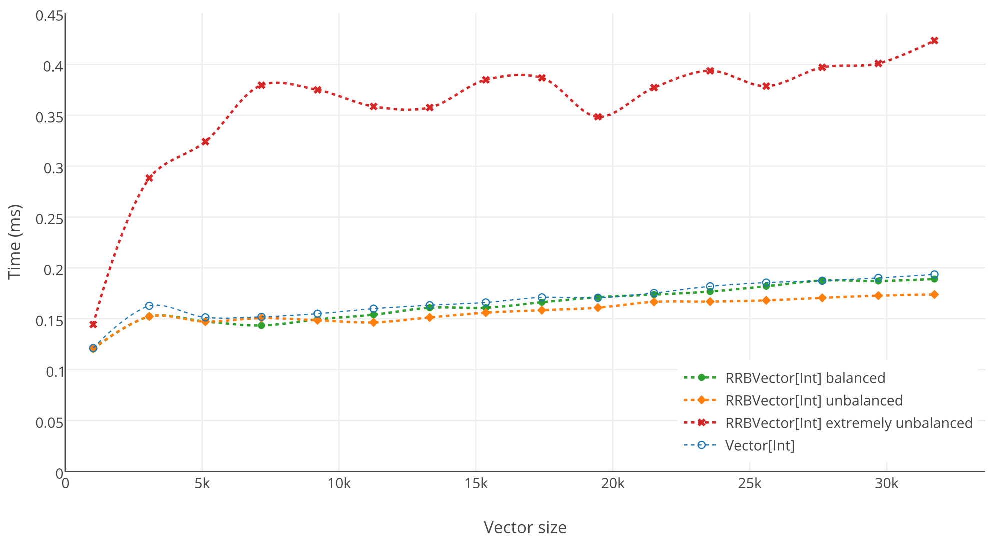


FIGURE 4.2: Time to execute 10k apply operations on random indices.

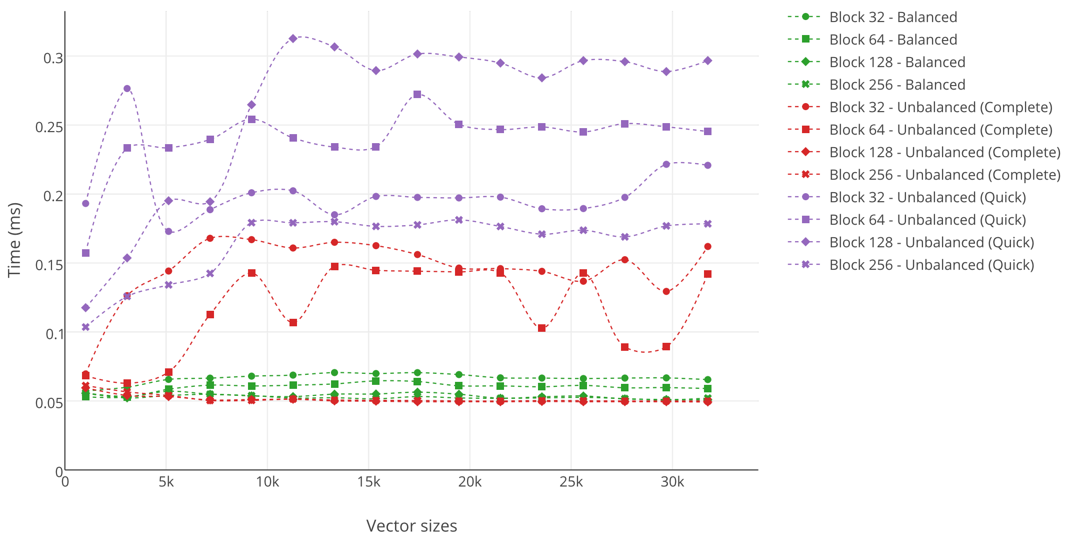
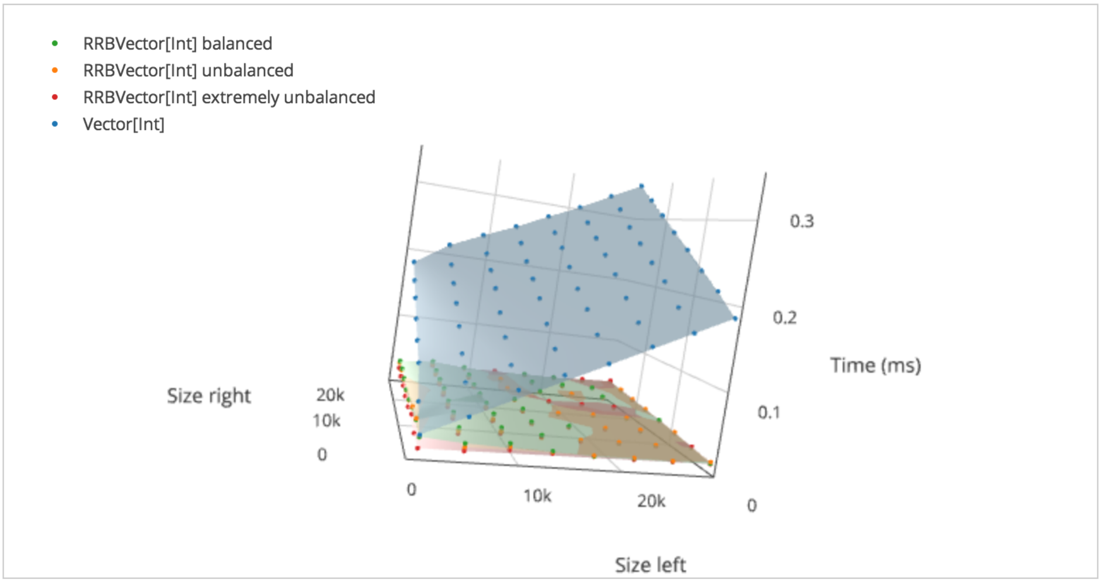


FIGURE 4.3: Time to execute 10k apply operations on sequential indices. Comparing performances for different block sizes and different implementation of the concatenation inner branch rebalancing (Complete/Quick).



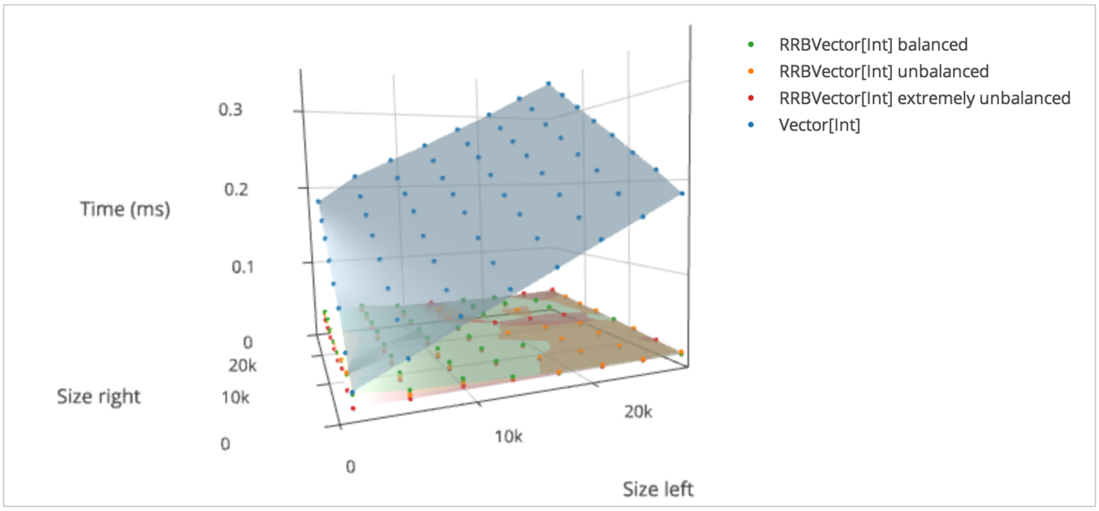
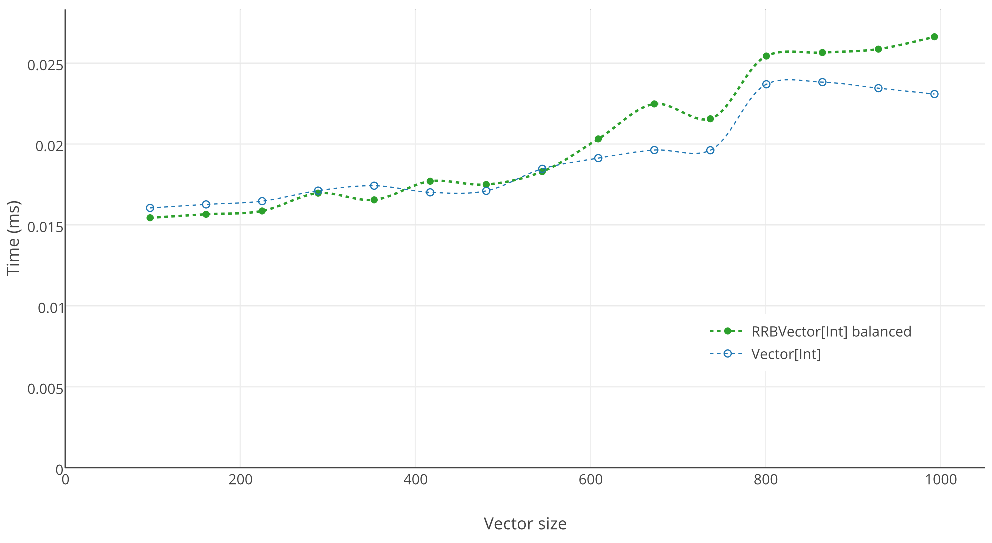


FIGURE 4.4: Execution time for a concatenation operation on two vectors. In theory (and in practice) Vector concatenation is $O(left + right)$ and the rrbVector concatenation operation is $O(log_{32}(left + right))$.



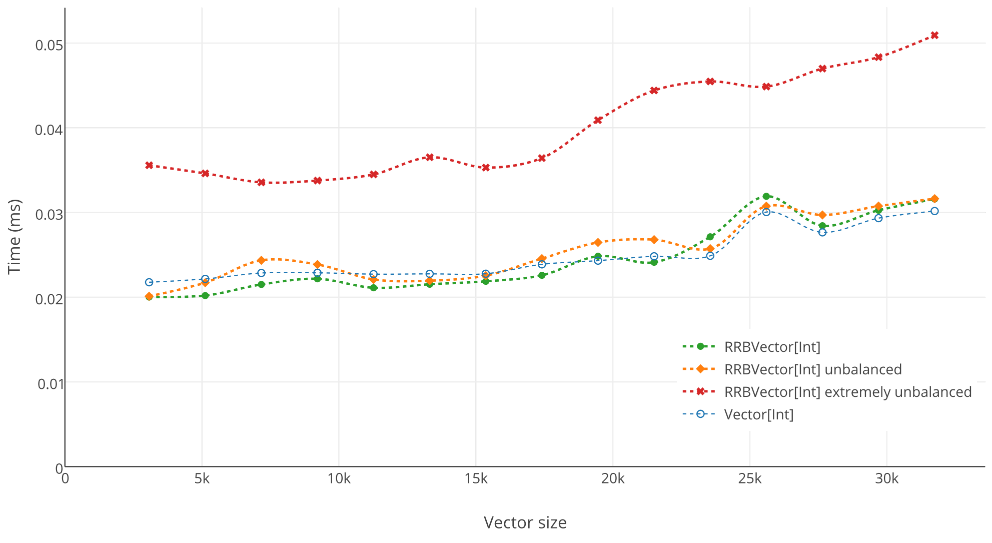


FIGURE 4.5: Time to execute 256 append operations. This shows the amortized cost of the append operation.

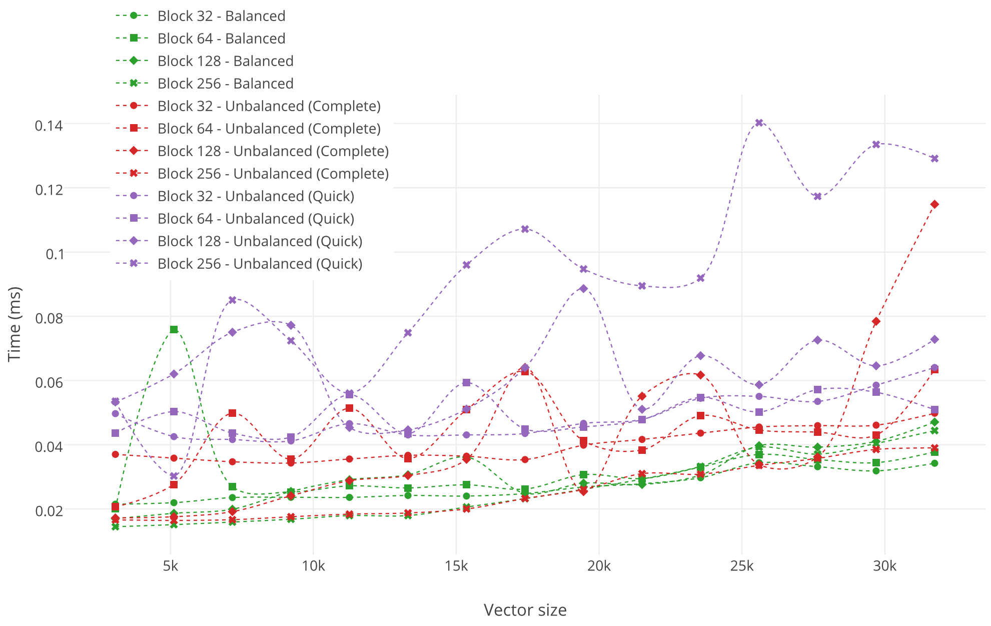
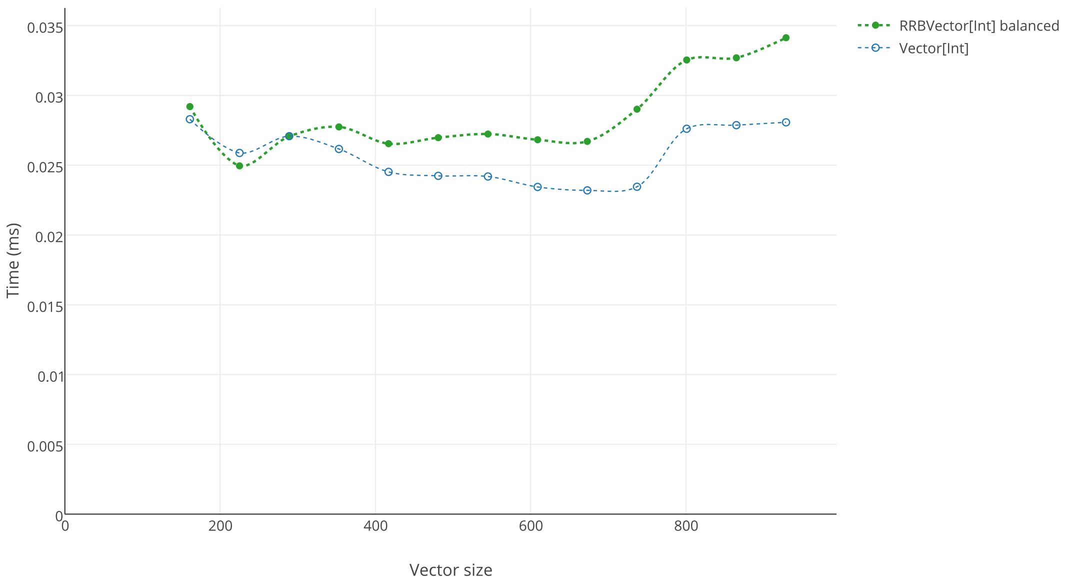


FIGURE 4.6: Time to execute 256 append operations. This shows the amortized cost of the append operation. Comparing performances for different block sizes and different implementation of the concatenation inner branch rebalancing (Complete/Quick).



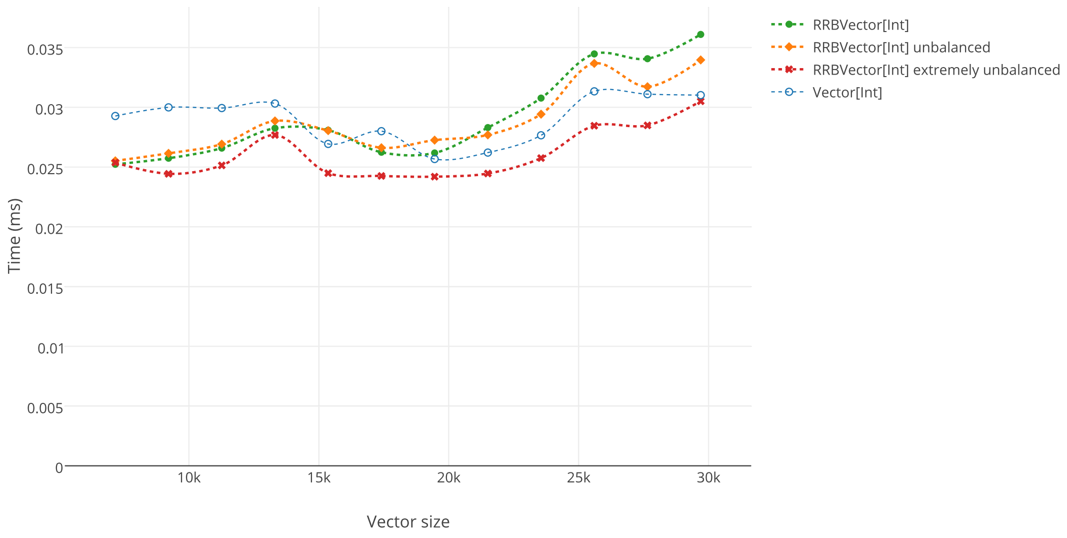


FIGURE 4.7: Time to execute 256 prepend operations. This shows the amortized cost of the prepend operation.

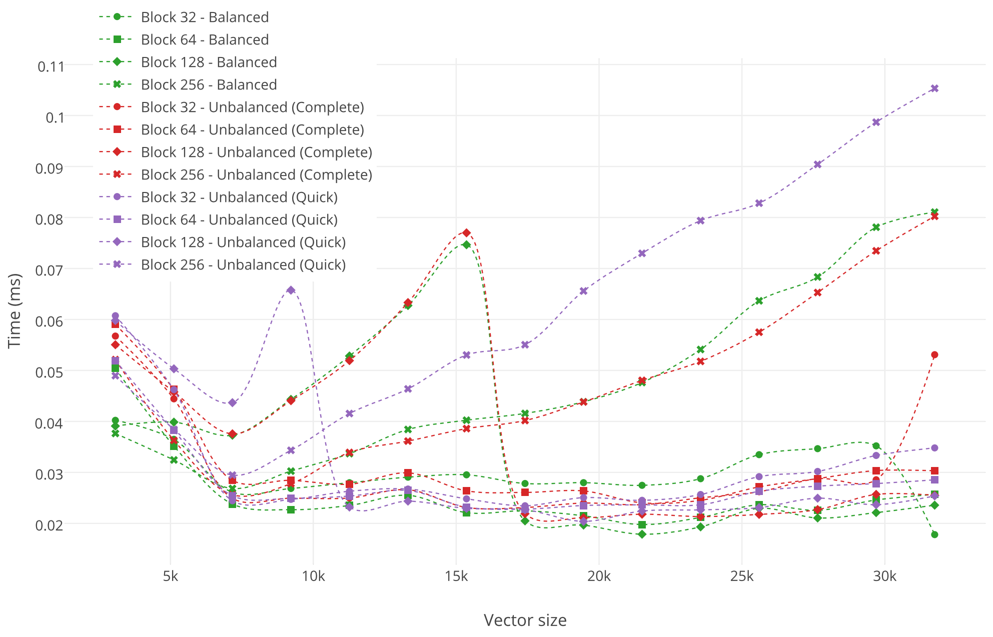
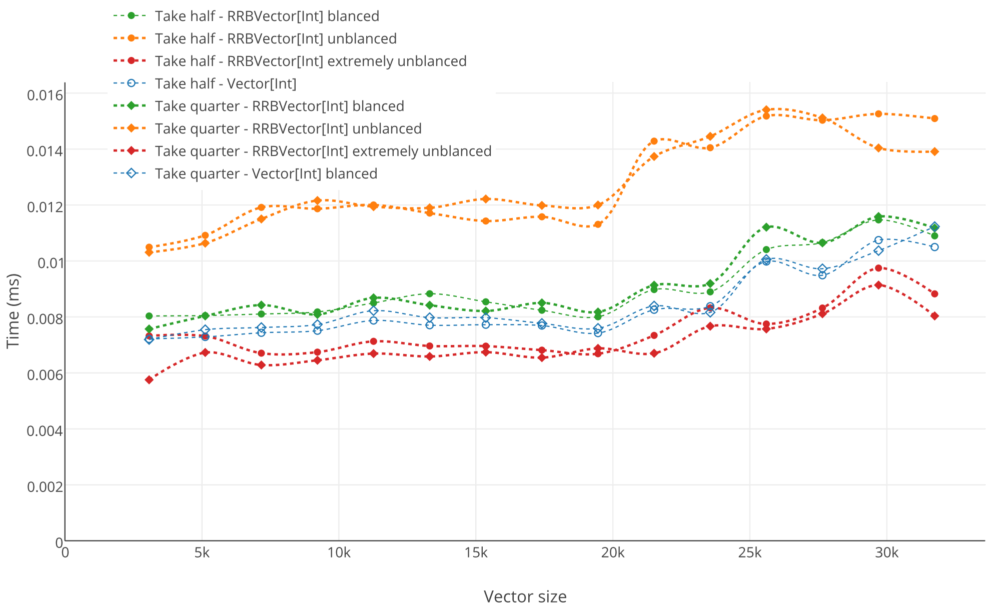


FIGURE 4.8: Time to execute 256 prepend operations. This shows the amortized cost of the append operation. Comparing performances for different block sizes and different implementation of the concatenation inner branch rebalancing (Complete/Quick).



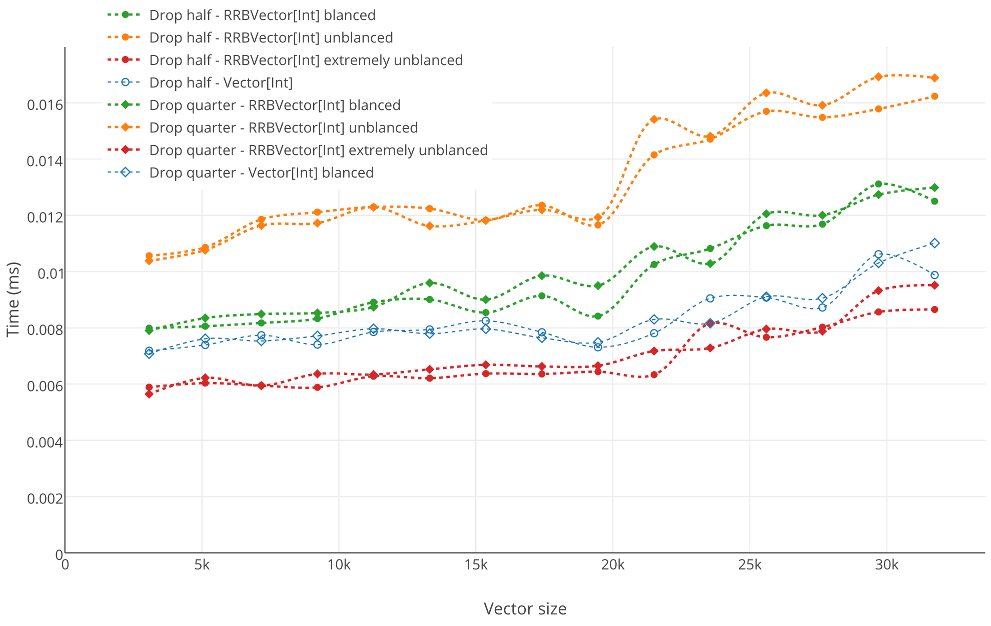
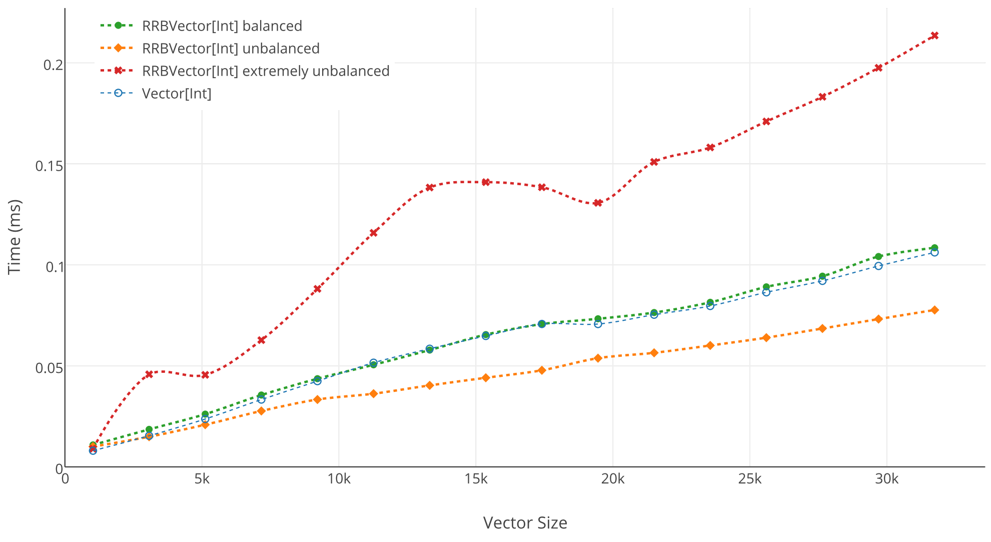


FIGURE 4.9: Execution time of take and drop.



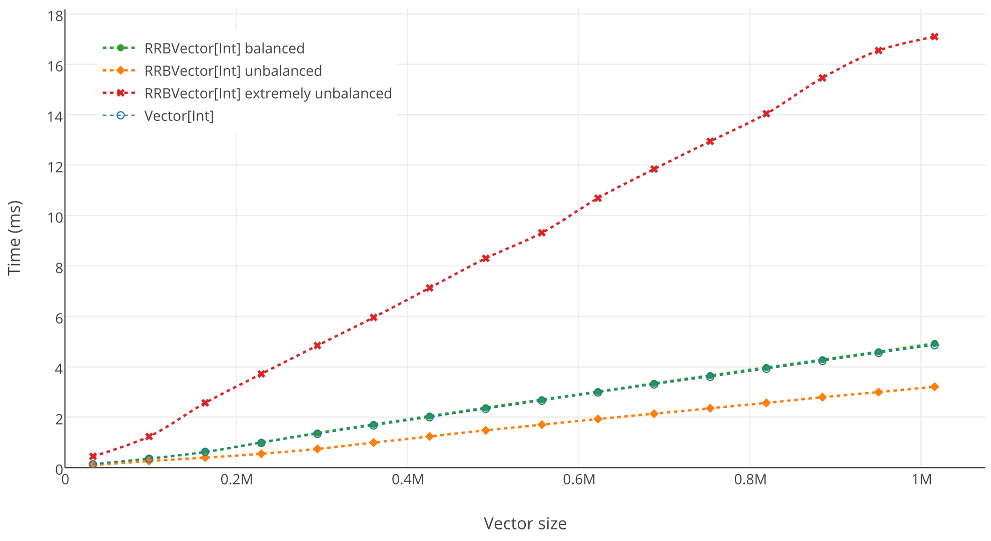


FIGURE 4.10: Excecution time to iterate through all the elements of the vector.

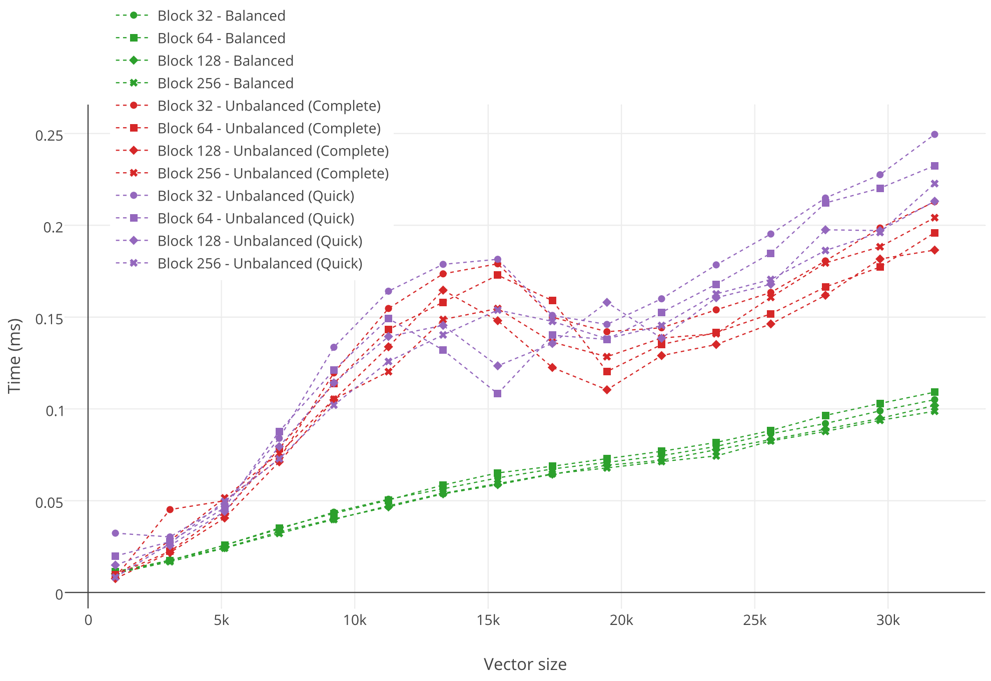
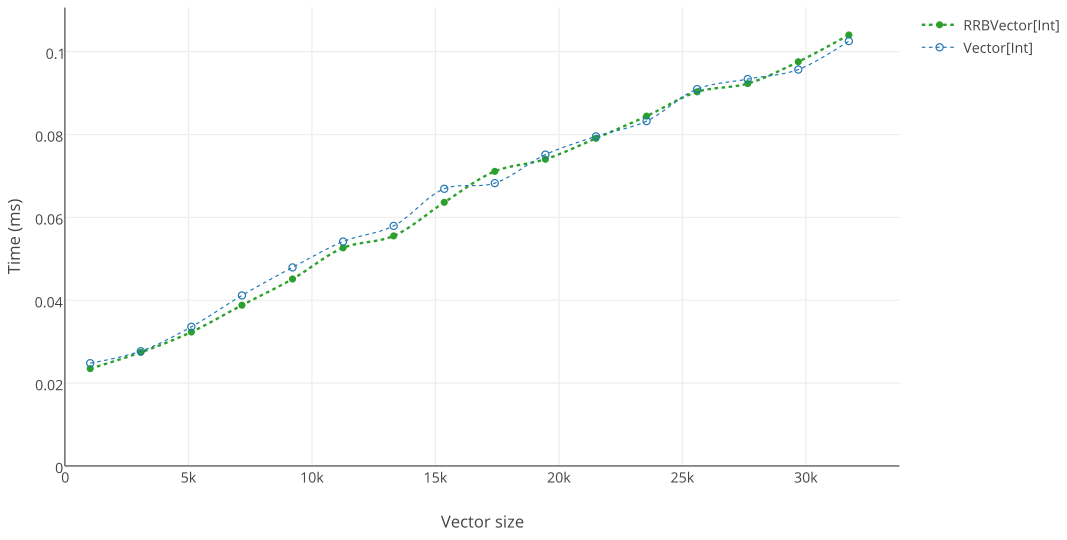


FIGURE 4.11: Excecution time to iterate through all the elements of the vector. Comparing performances for different block sizes and different implementation of the concatenation inner branch rebalancing (Complete/Quick).



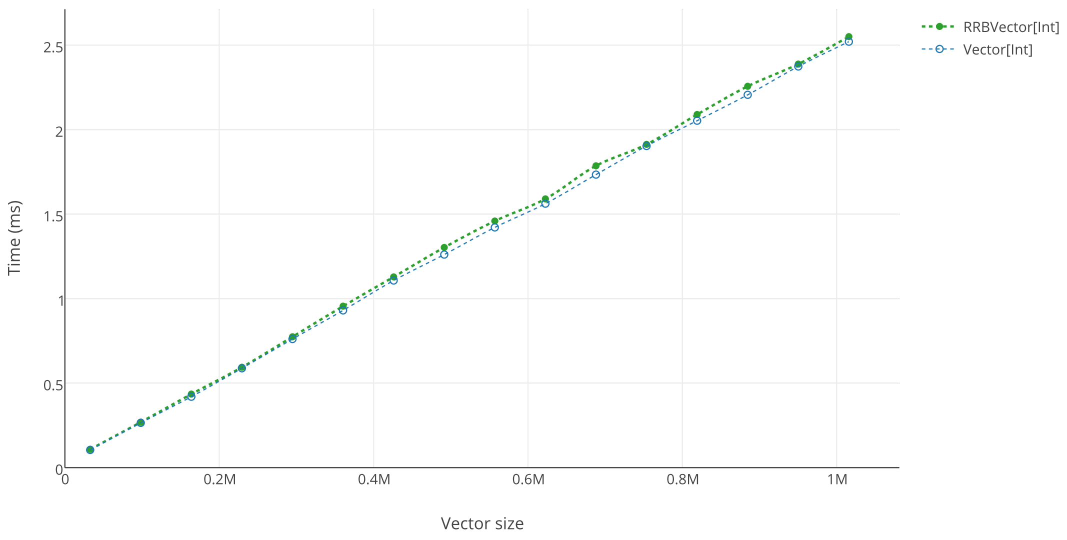


FIGURE 4.12: Execution time to build a vector of a given size.

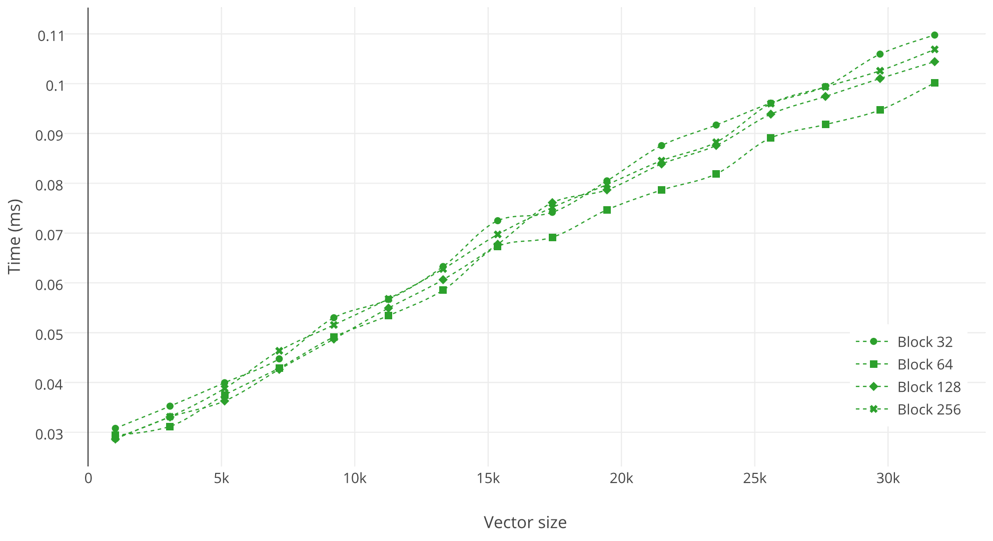


FIGURE 4.13: Execution time to build a vector of a given size. Comparing performances for different block sizes.

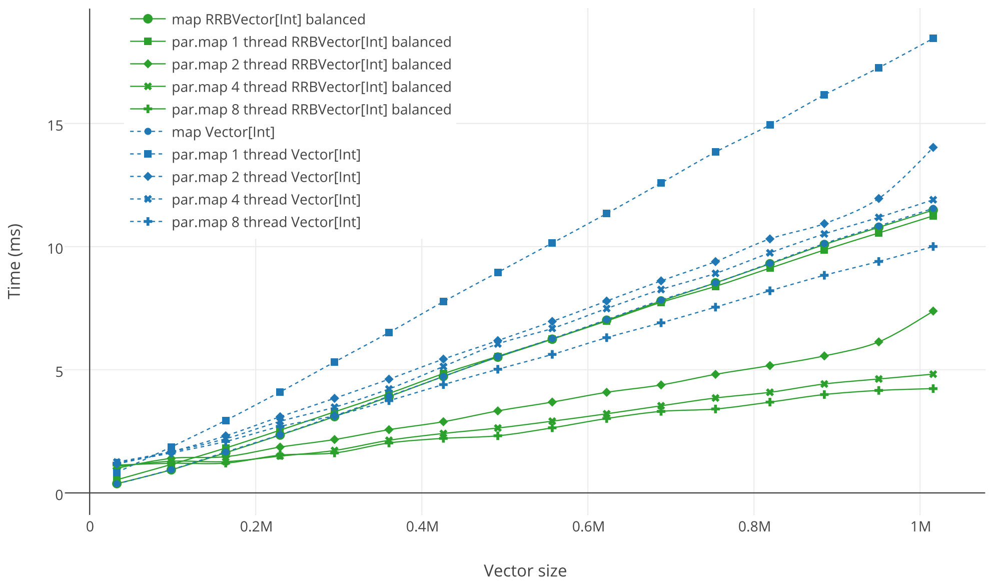


FIGURE 4.14: Benchmark on map and parallel map using the function (X=>X) to show the difference time used in the framework. This time represents the time spent in the splitters and combiners of the parallel collection (iterator and builder for the sequential version).

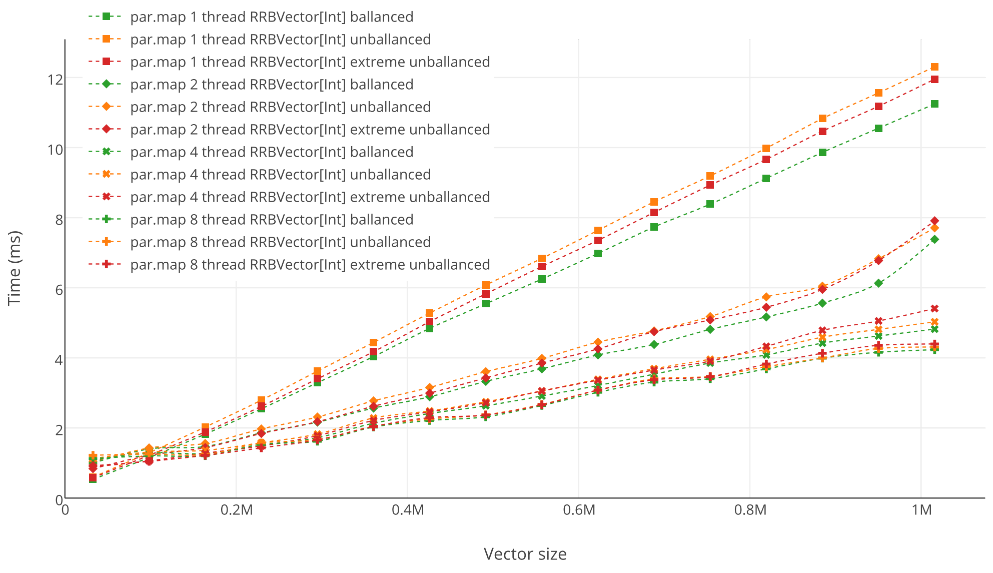
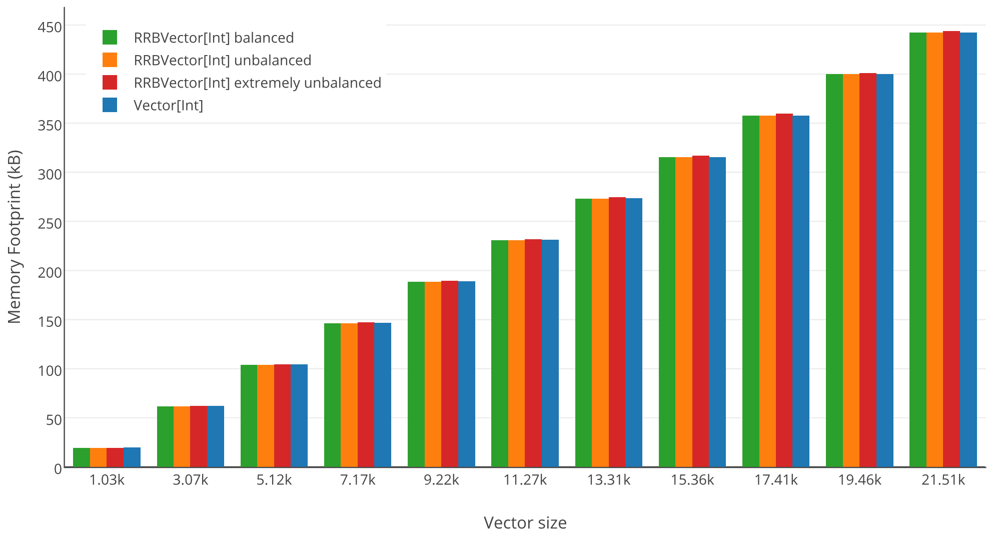


FIGURE 4.15: Benchmark on map and parallel map using the function (x=>x) to show the difference time used in the framework. This time represents the time spent in the splitters and combiners of the parallel collection.



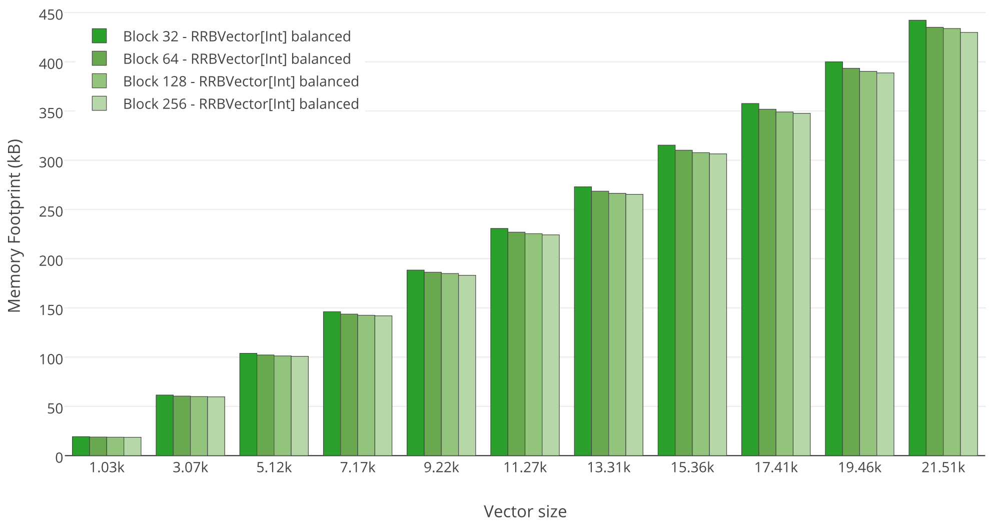


FIGURE 4.16: Memory Footprint

Chapter 5

Testing

5.1 Teststing correctness

5.1.1 Unit tests

5.1.2 Invariant Assertions

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Chapter 6

Related Work

6.1 RRB-Vectors in Clojure

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Chapter 7

Conclusions