École Polytechnique Fédérale de Lausanne

Master Thesis

Turning Relaxed Radix Balanced Vector from Theory into Practice for Scala Collections

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ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Abstract

School of Computer and Communications
Computer Science

Master in Computer Science

Turning Relaxed Radix Balanced Vector from Theory into Practice for Scala Collections

by Nicolas Stucki

The Thesis Abstract is written here (and usually kept to just this page). The page is kept centered vertically so can expand into the blank space above the title too…

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Abbreviations

JIT	**J**ust **I**n **T**ime
RB	**R**adix **B**alanced
RRB	**R**elaxed **R**adix **B**alanced

I

Chapter 1

Introduction

1.1 Main Section 1

1.2 Main Section 2

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Chapter 2

Vector Structure and Operations

2.1 Radix Balanced Vectors

2.1.1 Tree structure

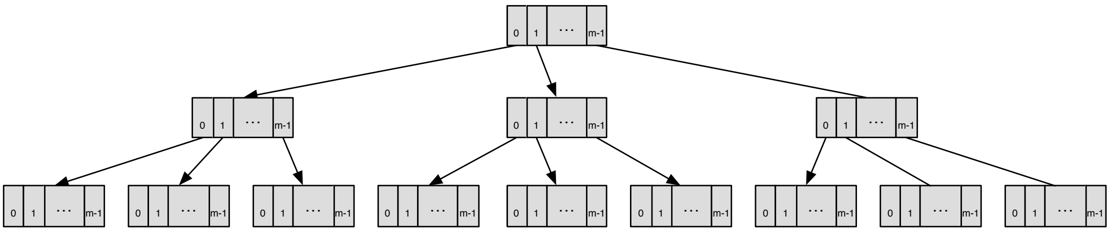

Figure 2.1: Radix Balanced Tree Structure

2.1.2 Operations

2.1.2.1 Apply

```
def apply(index: Int): A = {
  def getElem(node: Array[AnyRef], depth: Int): A = {
    val indexInNode = // get subindex
    if(depth == 1) node(indexInNode)
    else getElem(node(indexInNode), depth-1)
  }
  getElem(vectorRoot, vectorDepth)
}
```

2.1.2.2 Updated

```
def updated(index: Int, elem: A) = {
  def updatedNode(node: Array[AnyRef], depth: Int) = {
    val indexInNode = // compute index
    val copy = clone(node)
    if(depth == 1) {
      copy(indexInNode) = elem
    } else {
      copy(indexInNode) =
        updatedNode(node(indexInNode), depth-1)
    }
    copy
  }
  new Vector(updatedNode(vectorRoot, vectorDepth), ...)
}
```

2.1.2.3 Additions

Append

Prepend

Concatenation and Insert

2.1.2.4 Splits

2.2 Parallel Vectors

2.2.1 Splitter (Iterator)

To divide the work into tasks for thread pool, a splitter is used to iterate over all elements of the collection. Splitters are a special kind of iterator that can be split at any time into some partition of the remaining elements. In the case of sequences the splitter should retain the original order. The most common implementation consists in dividing the remaining elements into two half.

The current implementation of the immutable parallel vector [1] uses the common division into 2 parts for it splitter. The drop and take operations are used divide the vector for the two new splitters.

2.2.2 Combiner (Builder)

Combiners are used to merge the results from different tasks (in methods like map, filter, collect, ...) into the new collection. Combiners are a special kind of builder that is able to merge to partial results efficiently. When it's impossible to implement efficient combination operation, usually a lazy combiner is used. The lazy combiner is one keeps all the it's sub-combiners in an array buffer and only when the end result is needed they are combined. This is a fairly efficient implementation but does not take full advantage of parallelism.

The current implementation of the immutable parallel vector [1] use the lazy approach because of it's inefficient concatenation operation. One of the consequences of this is that the parallel operations will always be bounded by this sequential combination of elements, which can be beaten by the sequential version in many cases.

2.3 Relaxed Radix Balanced Vectors

2.3.1 Relaxed Tree structure

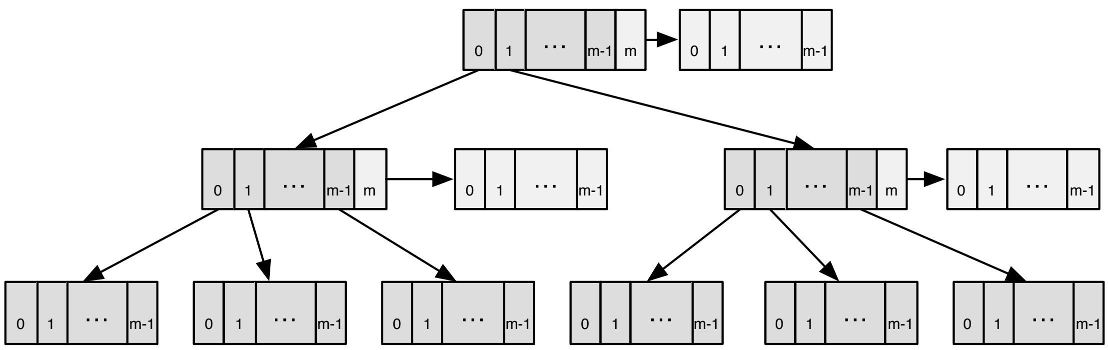

Figure 2.2: Radix Balanced Tree

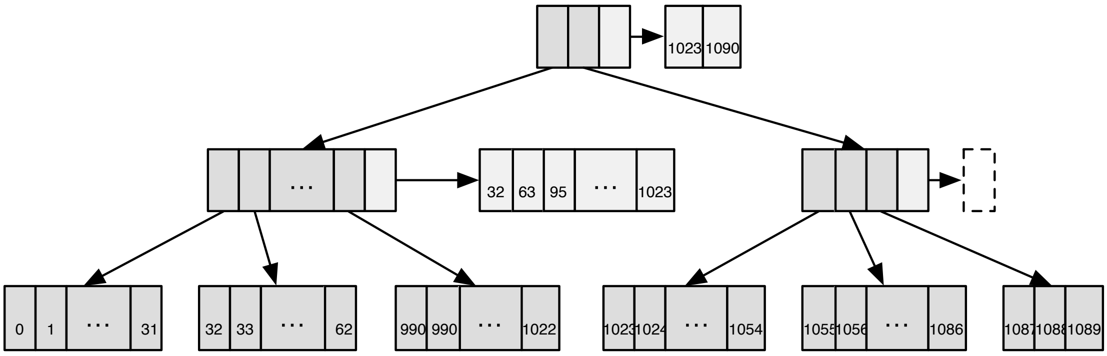

FIGURE 2.3: Relaxed radix example

2.3.2 Relaxed Operations

2.3.2.1 Apply (get element at index)

2.3.2.2 Updated

2.3.2.3 Additions

Append

Prepend

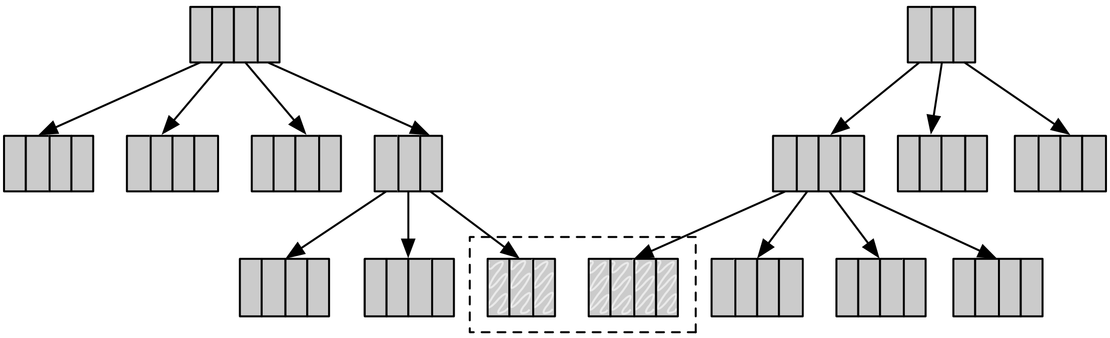

FIGURE 2.4: Concatenation example with blocks of size 4: Rebalancing level 0

Concatenation

Insert

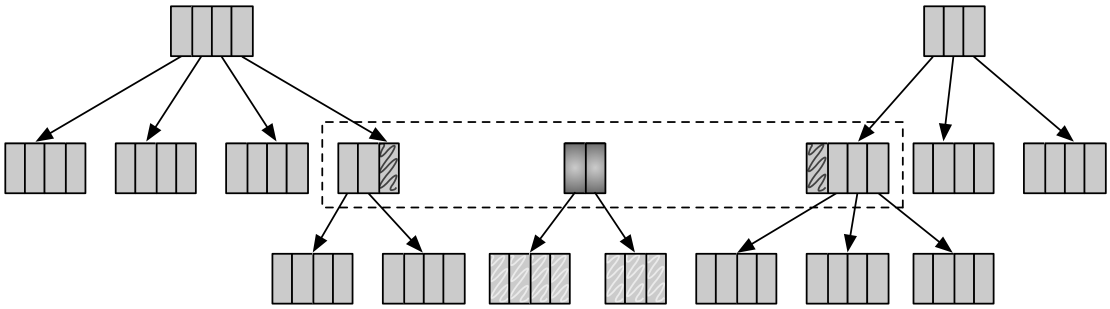

Figure 2.5: Concatenation example with blocks of size 4: Rebalancing level 1

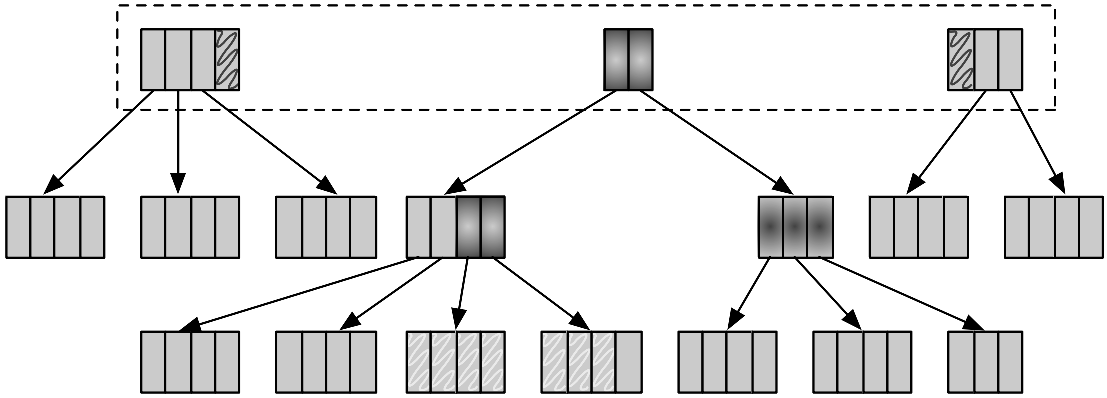

Figure 2.6: Concatenation example with blocks of size 4: Rebalancing level 2

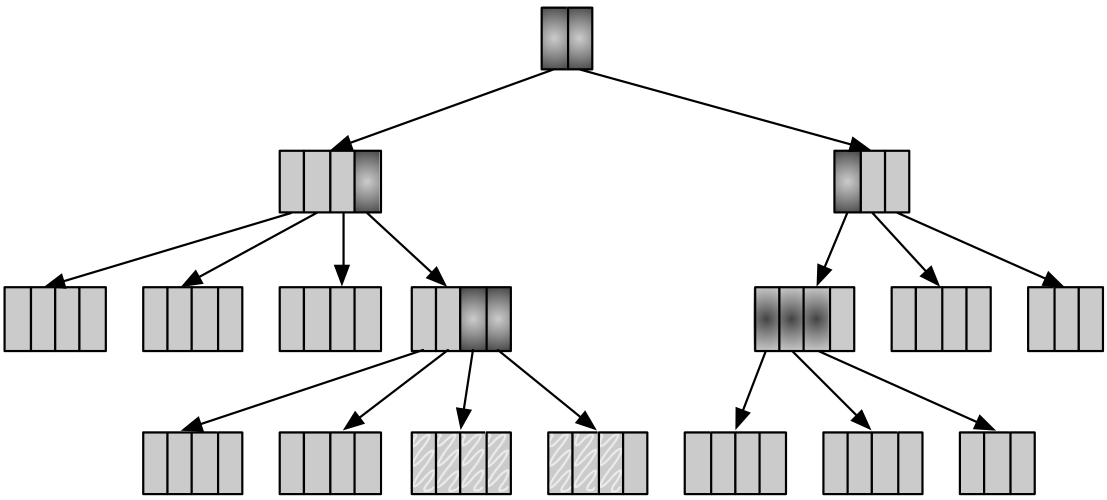

Figure 2.7: Concatenation example with blocks of size 4: Rebalancing level 3

2.3.2.4 Splits

2.3.2.5 Parallel Vector

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Chapter 3

Optimizations

3.1 Where is time spent?

3.1.1 Arrays

Most of the memory used in the vector data structure is composed of arrays. The three key operations used on this arrays: array creation, array update and array access. The arrays are used as immutable arrays, as such the update operations are only allowed when the array is initialised. This also implies that each time there is a modification on some part of an array, a new array must be created and all the old elements copied.

The size of the array will affect the performance of the vector. With larger blocks the access times will be reduced because the depth of the tree will decrease. But, on the other hand, increasing the size of the block will make slow down the update operations. This is a direct consequence of the need to copy the entire array for a single update.

3.1.2 Computing indices

Computing the indices in each node while traversing or modifying the vector is key in performance. This performances is gained by using low level binary computations on the indices in the case where the tree is balanced. And, using precomputed sizes in the case where the balance is relaxed.

Radix Assuming that the tree is full, elements are fetched from the tree using radix search on the index. As each node has a branching of 32, the index can be split bitwise in blocks of 5 ($2^5 = 32$) and used to know the path that must be taken from the root down to the element. The indices at each level L can be computed with $(index >> (5 \cdot L))\&31$. For example the index 526843 would be:

$$526843 = 00\,\underbrace{00000}_{0}\,\underbrace{00000}_{0}\,\underbrace{10000}_{16}\,\underbrace{00010}_{2}\,\underbrace{01111}_{15}\,\underbrace{11011}_{27}$$

```
def getSubIndex(indexInTree: Int, level: Int): Int =
  (index >> (5*level)) & 31
```

This scheme can be generalised to any block size m where $m = 2^i$ for $0 < i \leq 31$. The formula would be $(index >> (m \cdot L))\&((1 << m) - 1)$. It is also possible to generalise for other values of m using the modulo, division and power operations. In that case the formula would become $(index/(m^L))\%m$. This last generalisation is not used because it reduces sightly the performance and it complicates other index manipulations.

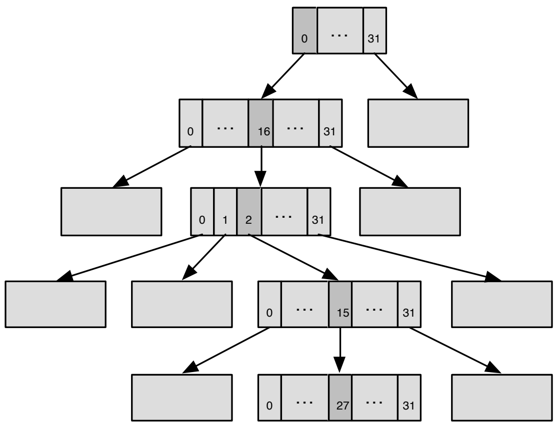


FIGURE 3.1: Accessing element at index 526843 in a tree of depth 5. Empty nodes represent collapses subtrees.

Relaxing the Radix When the tree is relaxed it is not possible to know the subindices from index. That is why we keep the sizes array in the unbalanced nodes. This array keeps the accumulated sizes to make the computation of subindices as trivial as possible. The subindex is the same as the first index in the sizes array where $index < sizes[subindex]$. The simplest way to find this subindex is by a linearly scanning the array.

```
def getSubIndex(sizes: Array[Int], indexInTree: Int): Int = {
```

```
  var is = 0
  while (sizes(is) <= indexInTree)
    is += 1
  is
}
```

For small arrays (like blocks of size 32) this will take be faster than a binary search because it takes advantage of the cache lines. If we would consider using bigger block sizes it would be better to use a hybrid between binary and linear search.

To traverse the tree down to the leaf where the index is, the subindices are computed from the sizes as long as the tree node is unbalanced. If the node is balanced, then the more efficient radix based method is used from there to the leaf. To avoid the need of accessing and scanning an additional array in each level.

3.1.3 Abstractions

3.2 Displays

As base for optimizations, the vector object keeps a set of fields to track one branch of the tree. They are named with using the level number from 0 up to the maximum possible level. In the case of blocks of size 32 the maximum level used is 5 [1], they are allocated by default and nulled if the tree if shallower. The highest non null display is and replaces the root field. All displays bellow the root are never null. This implies that the vector will always be focused on some branch.

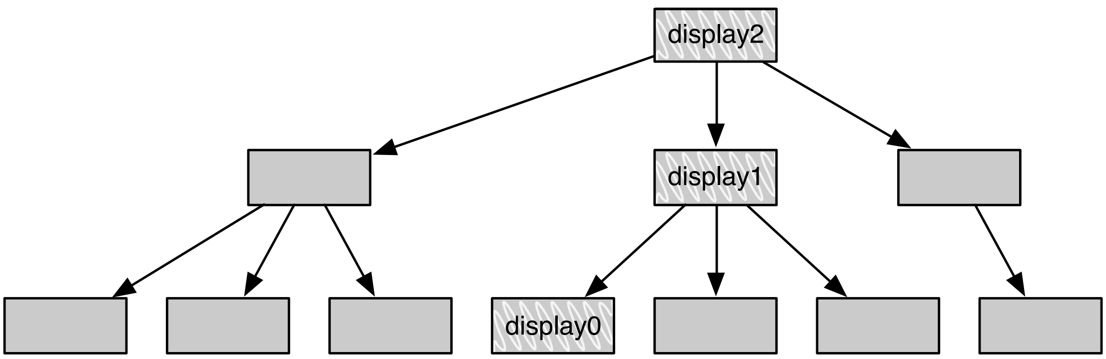


FIGURE 3.2: Displays

[1] As in practice, only the 30 bits of the index are used.

To know on which branch the vector is focused there is also a `focus` field with an index. This index is the index of any element in the current `display0`. This index represents the radix indexing scheme of node subindices described in 3.1.2.

To follow the simple implementations scheme of immutable objects in concurrent contexts, the focus is also immutable. Therefore each vector object will have a single focused branch during its existence[2]. Each method that creates a new vector must decide which focus to set. Two heuristics are used for this: If there was an update operation on some branch where that operations could be used again, that branch is used as focus. If the first one cant be applied, the display is set to the first element as this helps key collection operations such as `iterator`.

3.2.1 As cache

3.2.2 For transient states

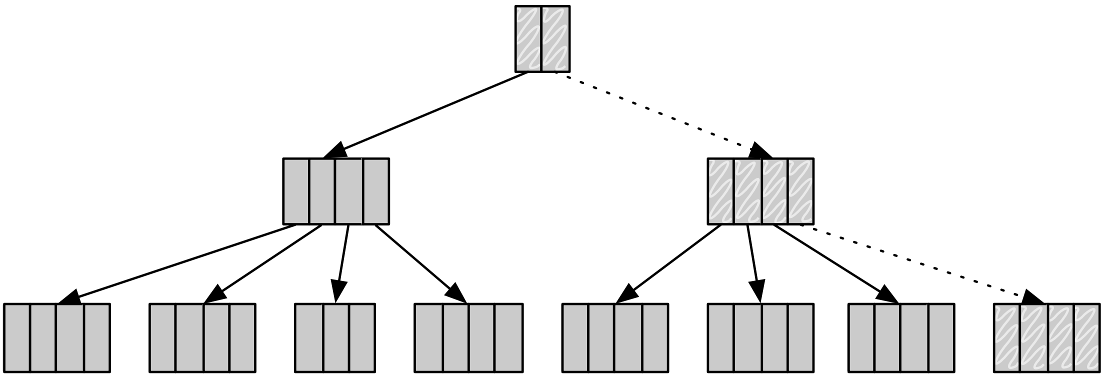


FIGURE 3.3: Radix Balanced Tree Transient state

3.2.3 Relaxing the Displays

3.3 Builder

Relaxing the Builder

[2]The display focus may change during the initialisation of the object as optimisation of some methods

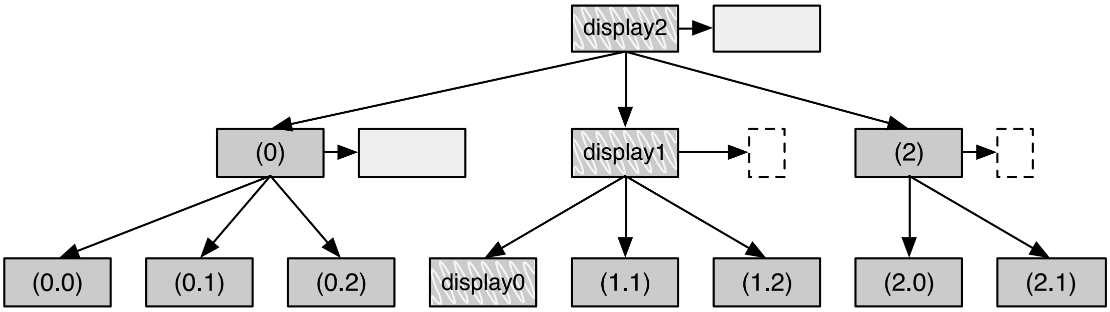
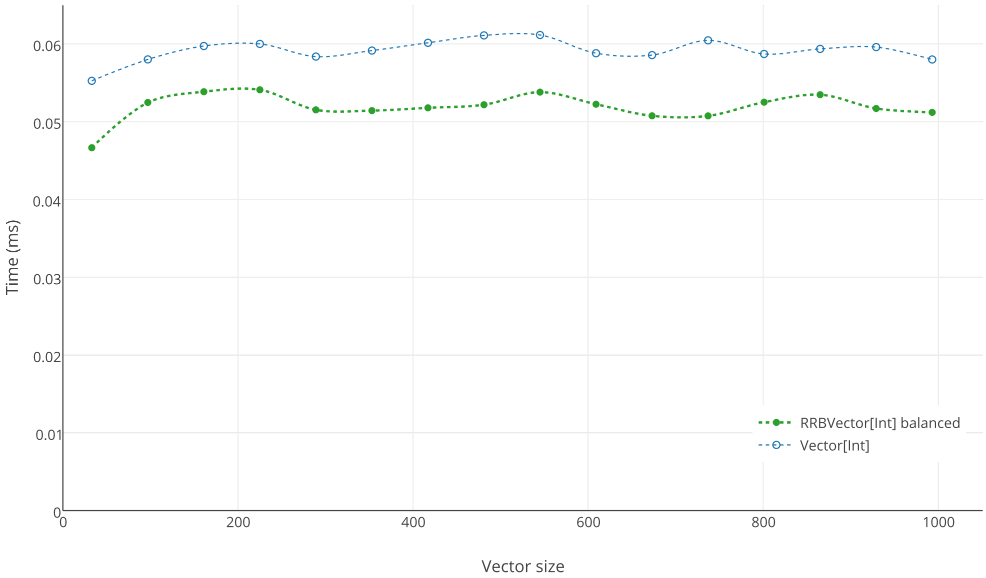


FIGURE 3.4: Radix Balanced Tree

3.4 Iterator

Relaxing the Iterator I



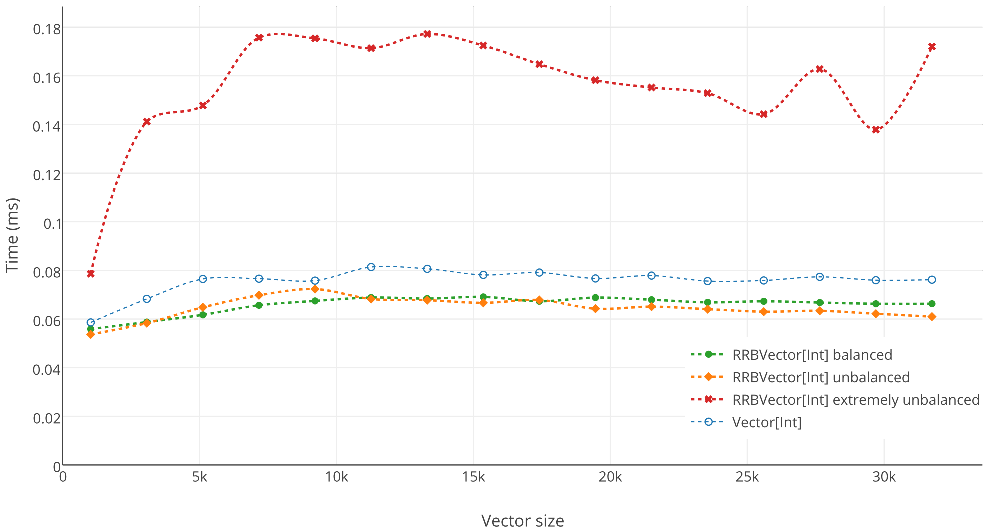


FIGURE 4.1: Time to execute 10k apply operations on sequential indices.

Chapter 4

Performance

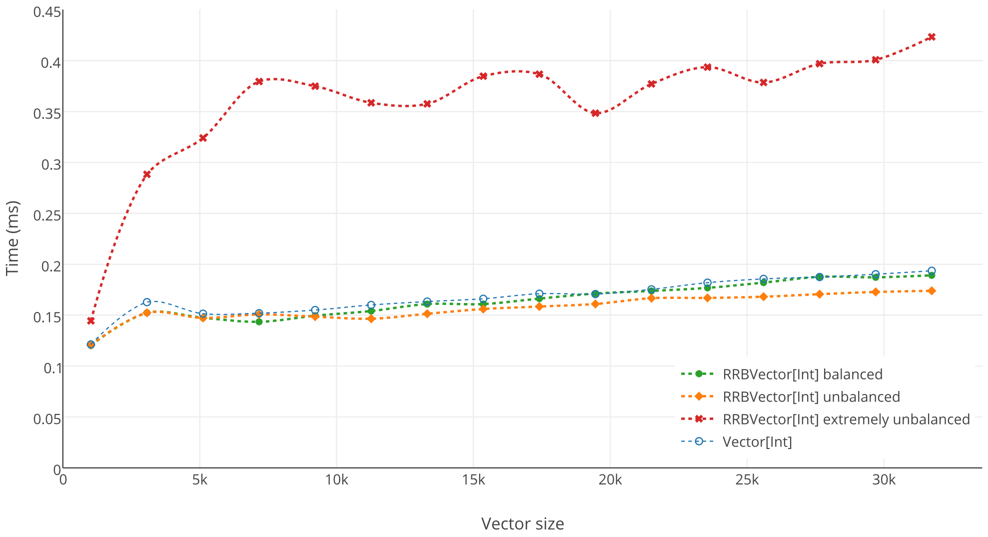


FIGURE 4.2: Time to execute 10k apply operations on random indices.

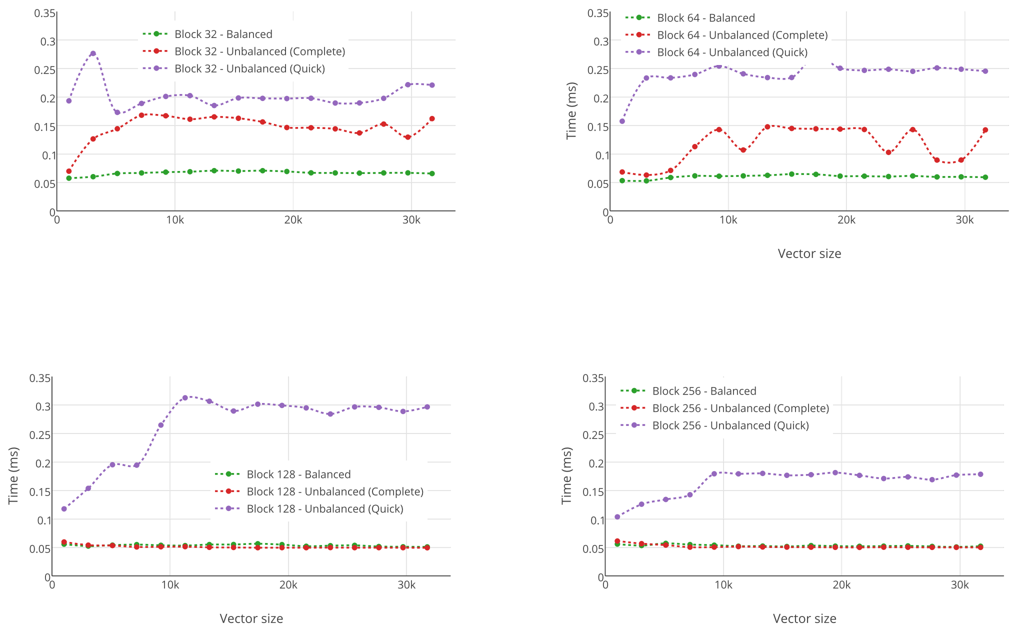


FIGURE 4.3: Time to execute 10k apply operations on sequential indices. Comparing performances for different block sizes and different implementation of the concatenation inner branch rebalancing (Complete/Quick).

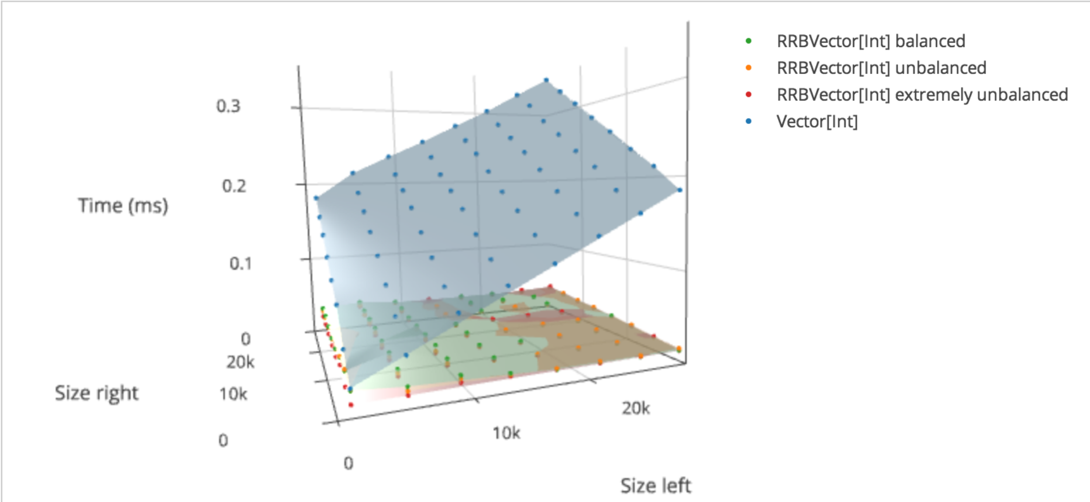


FIGURE 4.4: Execution time for a concatenation operation on two vectors. In theory (and in practice) Vector concatenation is $O(left + right)$ and the rrbVector concatenation operation is $O(log_{32}(left + right))$.

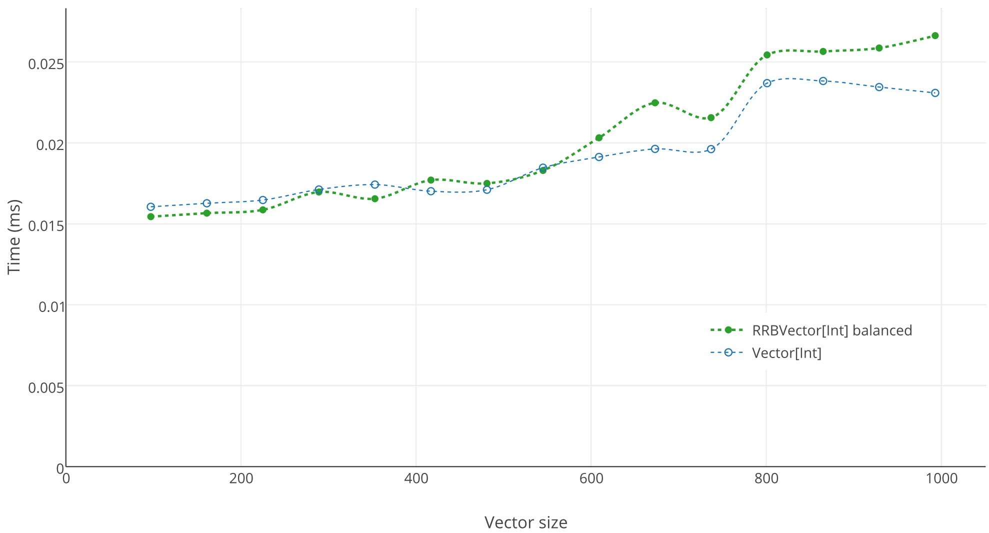


FIGURE 4.5: Time to execute 256 append operations. This shows the amortized cost of the append operation.

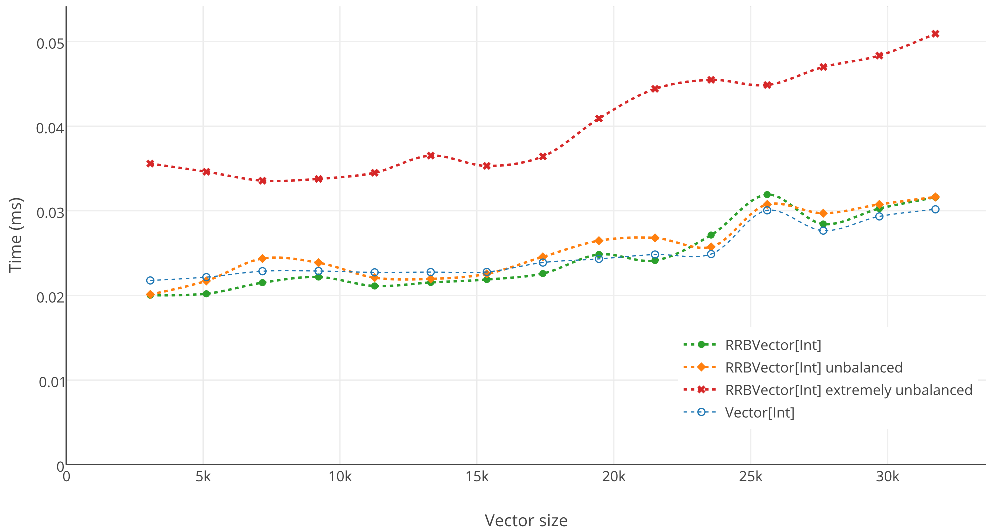


FIGURE 4.6: Time to execute 256 append operations. This shows the amortized cost of the append operation.

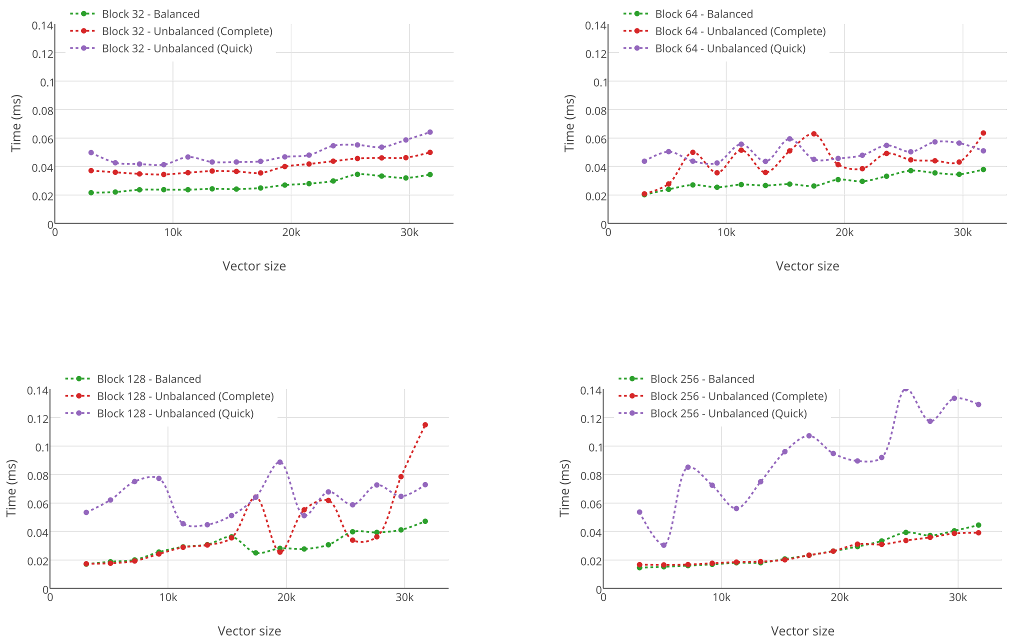


FIGURE 4.7: Time to execute 256 append operations. This shows the amortized cost of the append operation. Comparing performances for different block sizes and different implementation of the concatenation inner branch rebalancing (Complete/Quick).

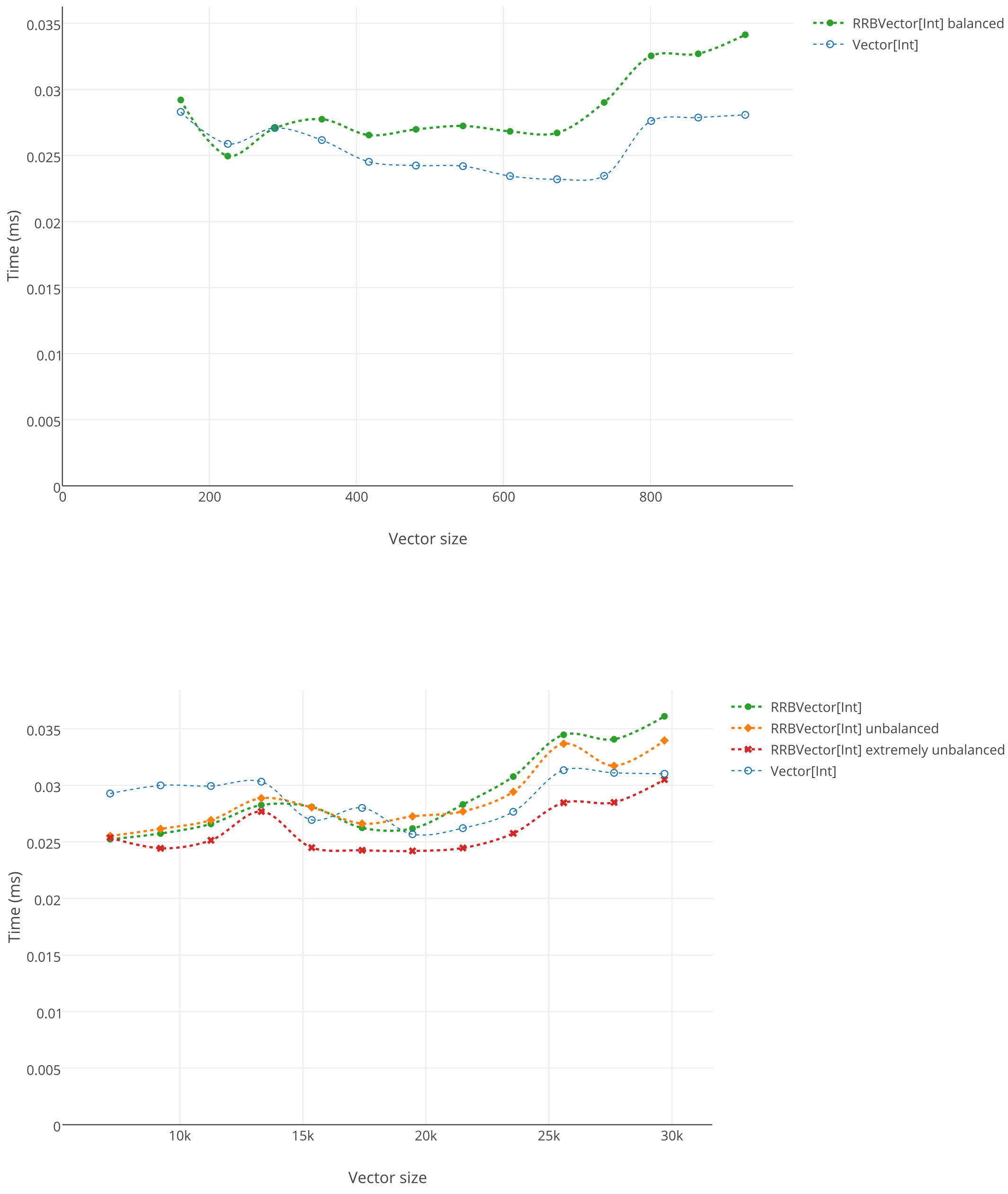


Figure 4.8: Time to execute 256 prepend operations. This shows the amortized cost of the prepend operation.

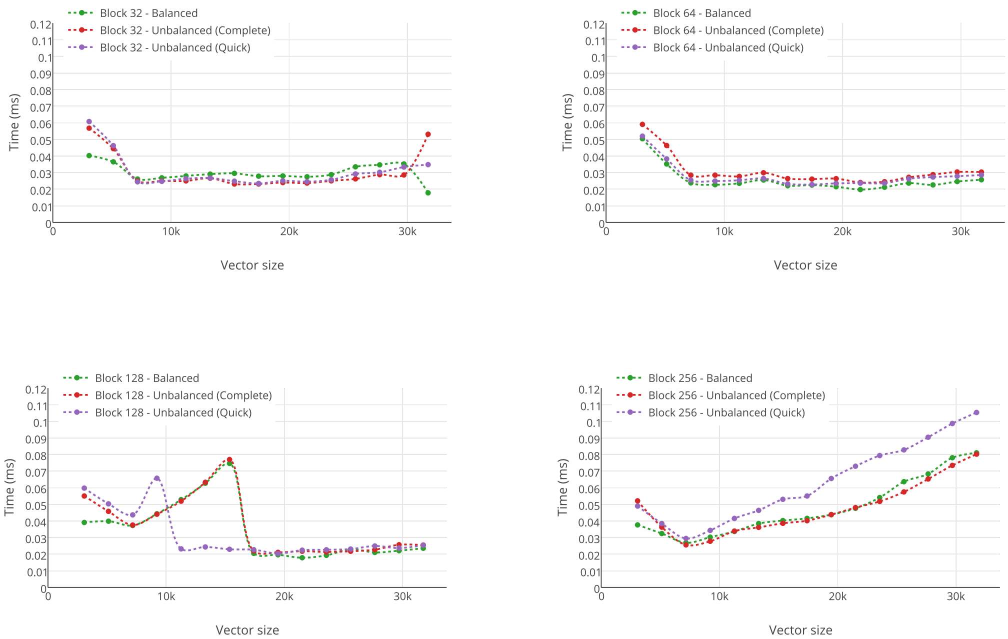
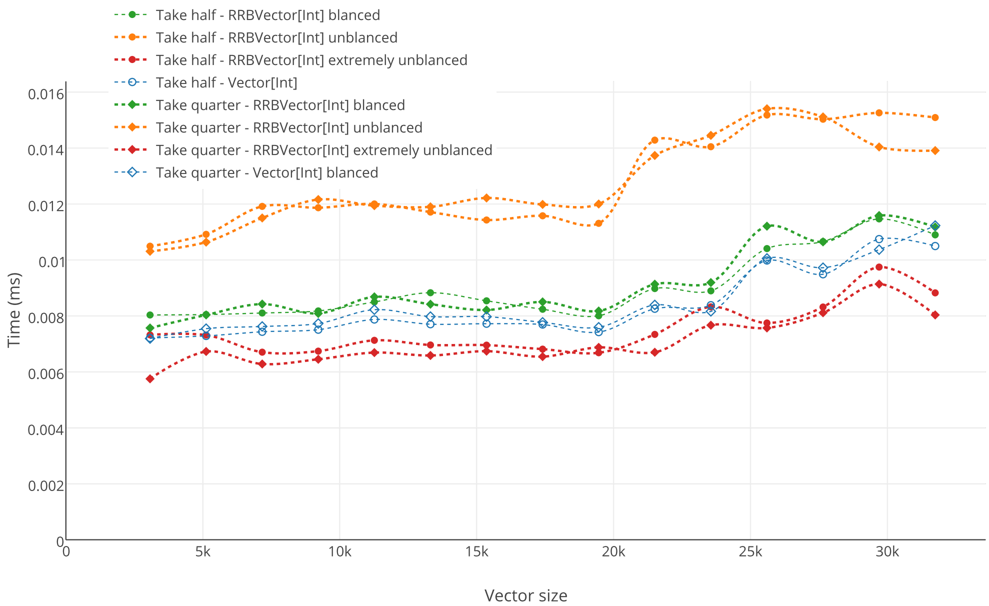


FIGURE 4.9: Time to execute 256 prepend operations. This shows the amortized cost of the append operation. Comparing performances for different block sizes and different implementation of the concatenation inner branch rebalancing (Complete/Quick).



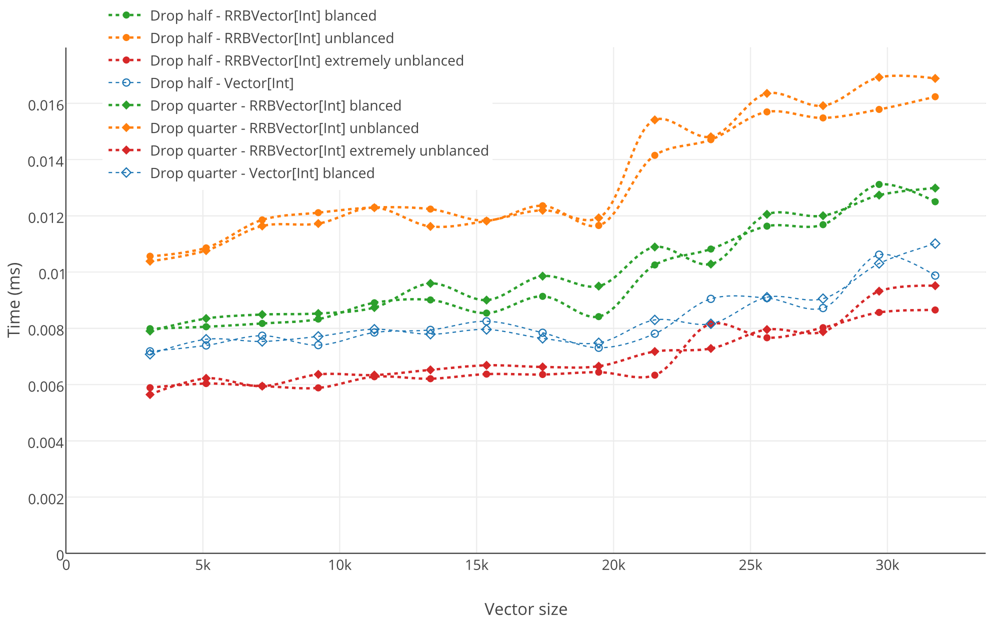
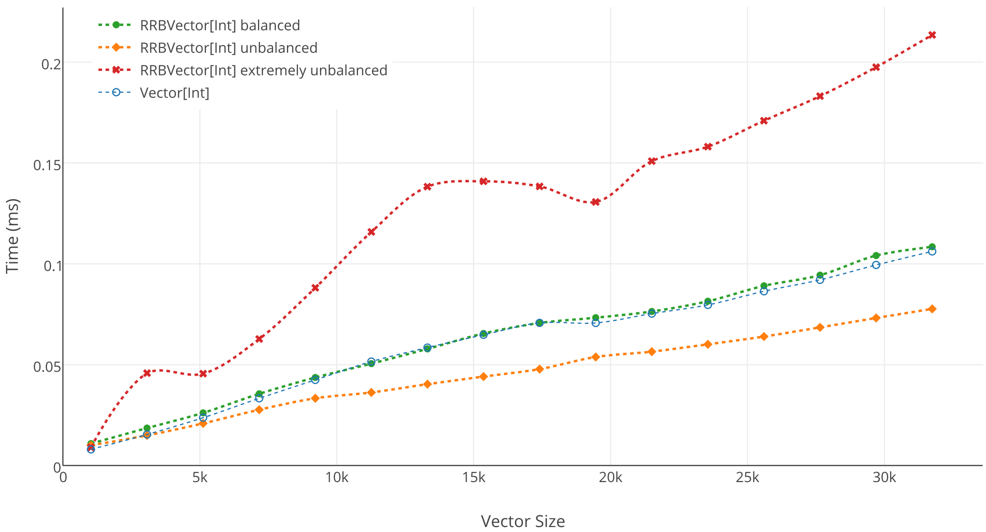


FIGURE 4.10: Execution time of take and drop.



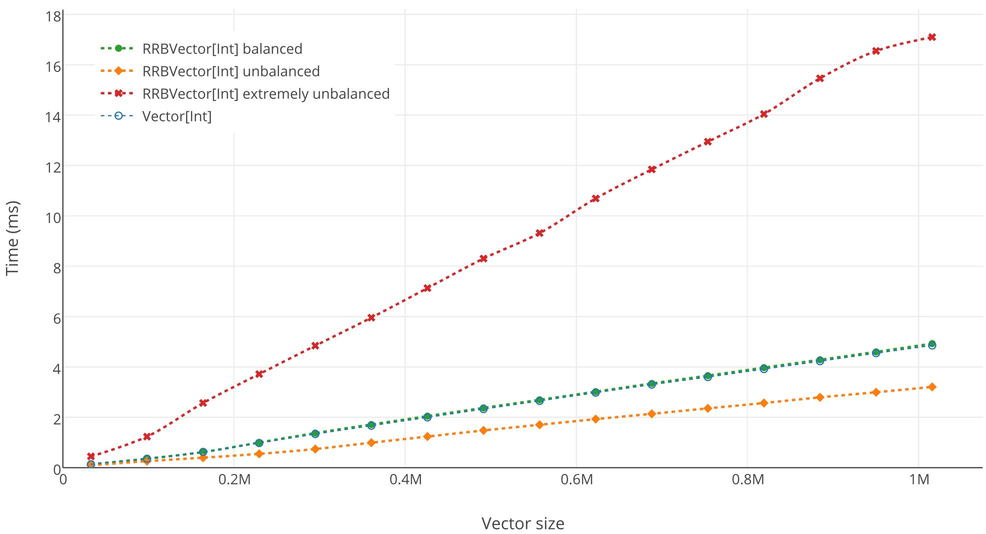


FIGURE 4.11: Excecution time to iterate through all the elements of the vector.

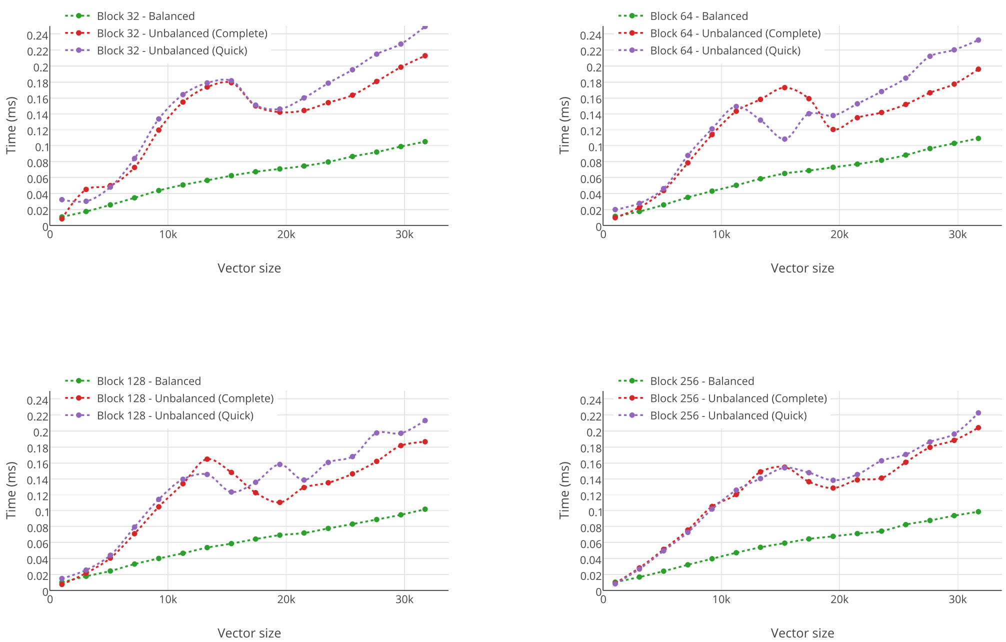
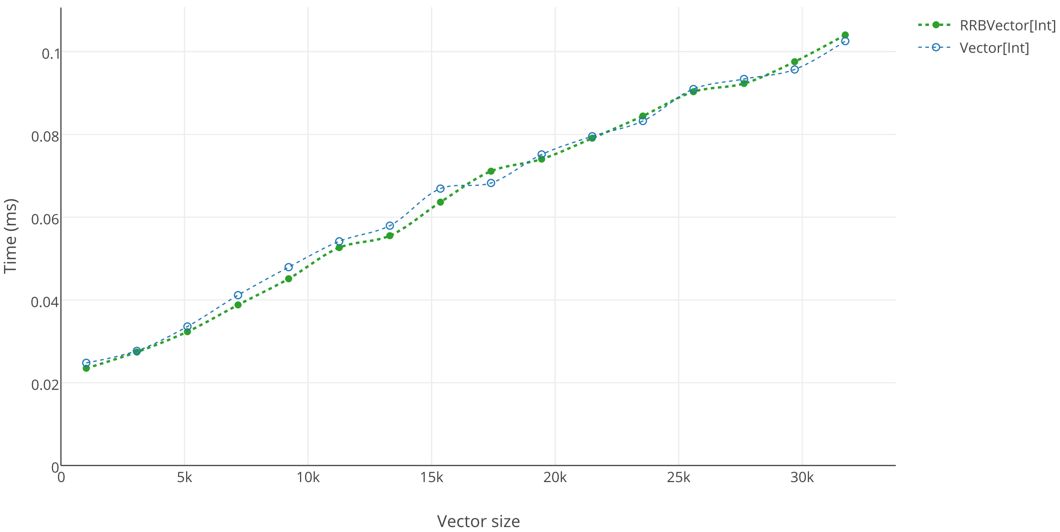


FIGURE 4.12: Excecution time to iterate through all the elements of the vector. Comparing performances for different block sizes and different implementation of the concatenation inner branch rebalancing (Complete/Quick).



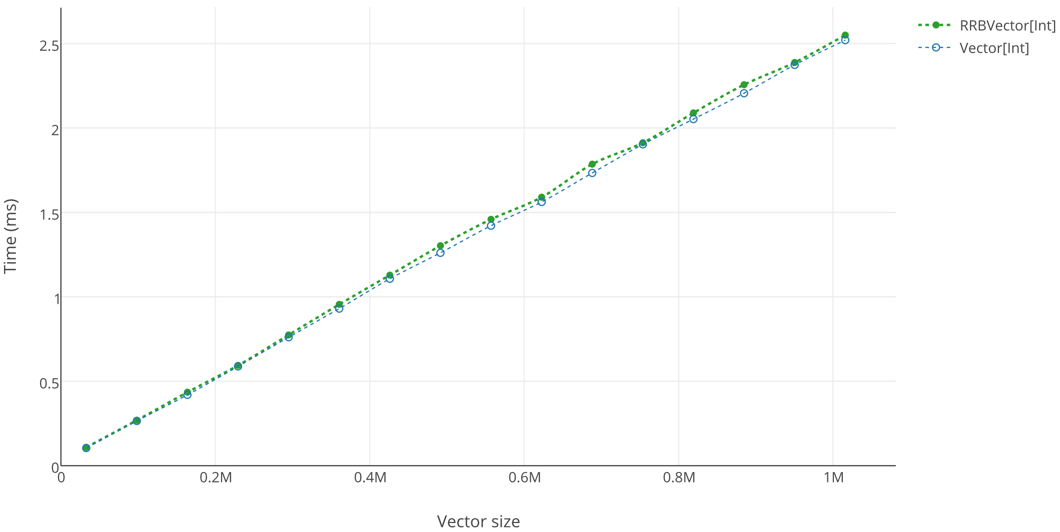


FIGURE 4.13: Execution time to build a vector of a given size.

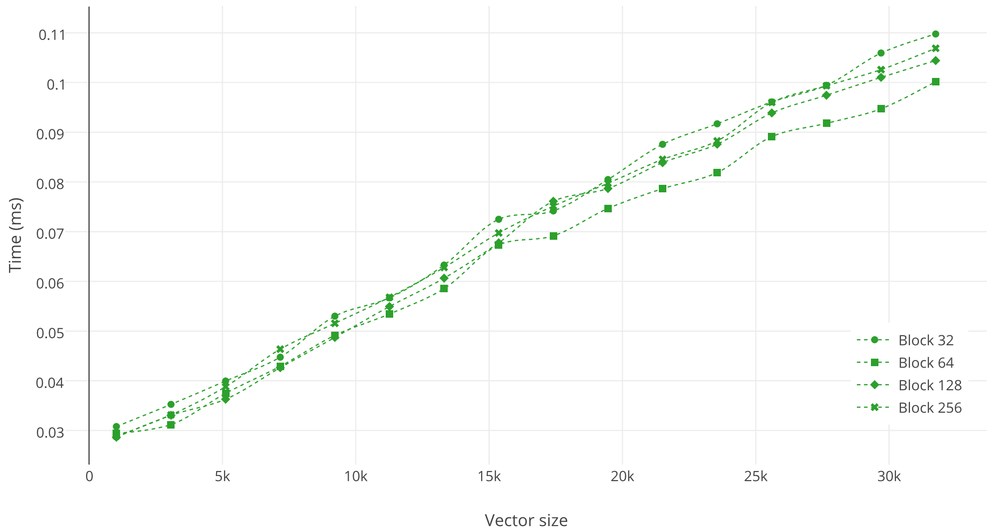


FIGURE 4.14: Execution time to build a vector of a given size. Comparing performances for different block sizes.

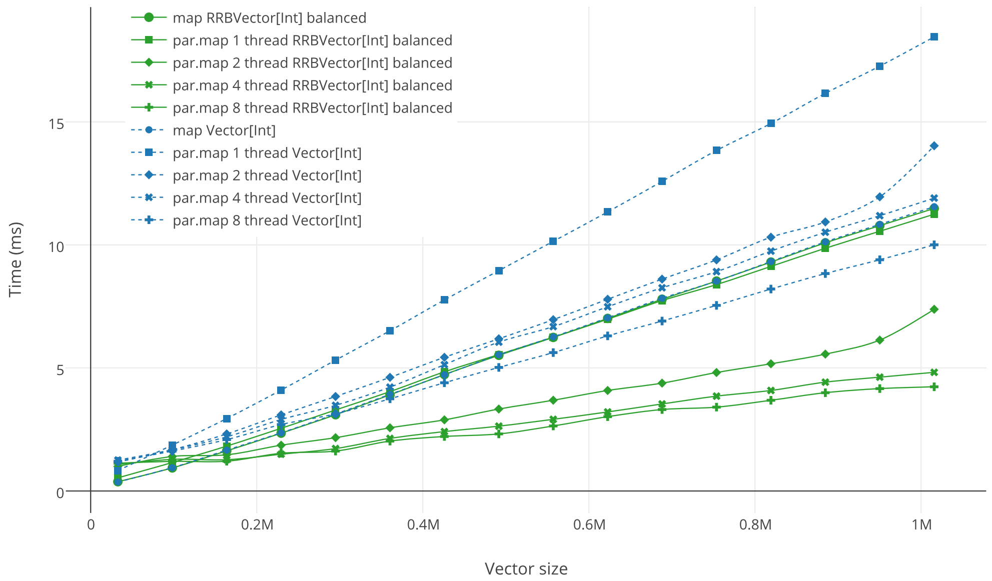


FIGURE 4.15: Benchmark on map and parallel map using the function (x=>x) to show the difference time used in the framework. This time represents the time spent in the splitters and combiners of the parallel collection (iterator and builder for the sequential version).

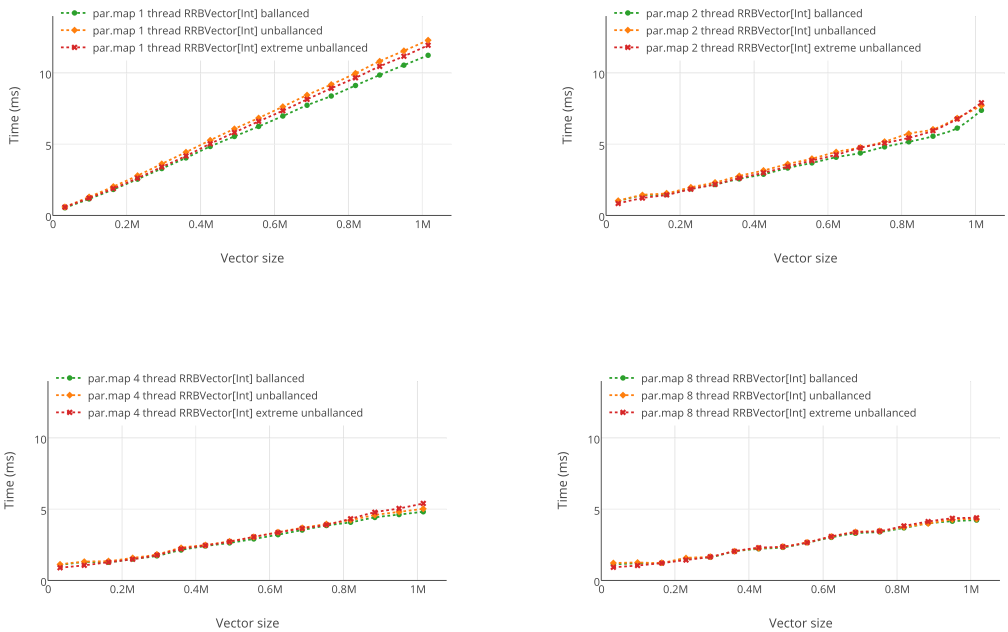


FIGURE 4.16: Benchmark on map and parallel map using the function (x=>x) to show the difference time used in the framework. This time represents the time spent in the splitters and combiners of the parallel collection.

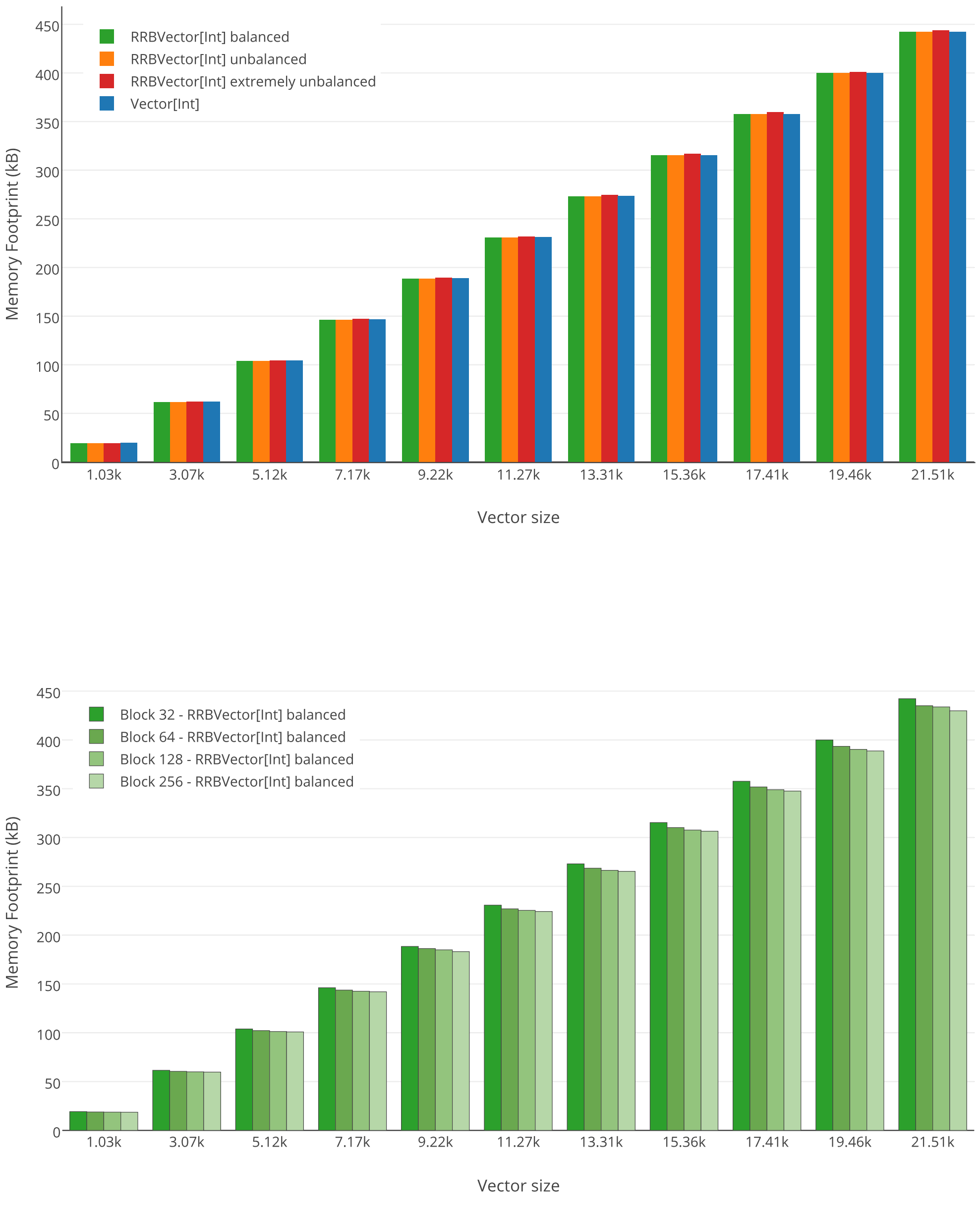


Figure 4.17: Memory Footprint

Chapter 5

Testing

5.1 Teststing correctness

5.1.1 Unit tests

5.1.2 Invariant Assertions

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Chapter 6

Related Work

6.1 RRB-Vectors in Clojure

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Chapter 7

Conclusions

Bibliography

[1] GitHub - Scala 2.11 - ParVector.scala. https://github.com/scala/scala/blob/f4267ccd96a9143c910c66a5b0436aaa64b7c9dc/src/library/scala/collection/parallel/immutable/ParVector.scala.