

Lecture 10

niceguy

April 18, 2024

1 Power Flow in AC

$$\begin{aligned}\tilde{V}(d) &= V_0^+ e^{j\beta d} + \Gamma V_0^+ e^{-j\beta d} \\ \tilde{I}(d) &= \frac{V_0^+}{Z_0} e^{j\beta d} - \frac{\Gamma V_0^+}{Z_0} e^{-j\beta d}\end{aligned}$$

Recall voltage is energy per unit charge (energy can have units eV). Then

$$p = \frac{dW}{dt} = v \frac{d(qv)}{dt}$$

Using our sinusoidal expressions for voltage and current,

Definition 1.1 (Instantaneous Power). It is defined as

$$p(t) = V_0 I_0 \cos(\omega t + \phi_v) \cos(\omega t + \phi_i)$$

We are more interested in the average value of power. Using the product-to-sum formula,

$$\begin{aligned}p(t) &= \frac{V_0 I_0}{2} [\cos(\phi_v - \phi_i) + \cos(2\omega t + \phi_v + \phi_i)] \\ \bar{p}(t) &= \frac{1}{T} \int_0^T p(t) dt \\ &= \frac{V_0 I_0}{2} \cos(\phi_v - \phi_i)\end{aligned}$$

the sinusoidal term averages out to 0. This agrees with the intuition that power is maximised when phase difference is minimised. If we use phasors, where $\tilde{V} = V_0 e^{j\phi_v}$, $\tilde{I} = I_0 e^{j\phi_i}$, then

$$\frac{1}{2} \tilde{V} \tilde{I}^* = \frac{1}{2} V_0 I_0 e^{j(\phi_v - \phi_i)}$$

whose real part yields average power.

2 Power Flow in Transmission Lines

Converting our equations \tilde{V}, \tilde{I} into the time domain, $V_0^+ = |V_0^+| e^{j\phi_+}$, $\Gamma = |\Gamma| e^{j\theta_\Gamma}$. Expanding,

$$v(d) = |V_0^+| \cos(\omega t + \beta d + \phi_+) + |\Gamma V_0^+| \cos(\omega t - \beta d + \phi_+ + \theta_\Gamma)$$

$$i(d) = \frac{|V_0^+|}{Z_0} \cos(\omega t + \beta d + \phi_+) - \frac{|\Gamma V_0^+|}{Z_0} \cos(\omega t - \beta d + \phi_+ + \theta_\Gamma)$$

Power is

$$p(d) = \frac{|V_0^+|^2}{Z_0} \cos^2(\omega t + \beta d + \phi_+) - \frac{|\Gamma V_0^+|^2}{Z_0} \cos^2(\omega t - \beta d + \phi_+ + \theta_\Gamma)$$

Note: the cross terms cancel out.

We call the first term **incident** power and the second **reflected** power. Taking the time average,

$$\bar{p}(d) = \frac{1}{2} \frac{|V_0^+|^2}{Z_0} - \frac{1}{2} \frac{|\Gamma V_0^+|^2}{Z_0} = \frac{|V_0^+|^2}{2Z_0} (1 - |\Gamma|^2)$$

Example 2.1. For $R_g = 10\Omega$, $l = 0.75\lambda$, $v_g = 1\text{V}$, $Z_0 = 50\Omega$, $v_p = 3 \times 10^8 \text{m s}^{-1}$, $R_L = 25\Omega$. This is a quarter-wave transformer, where we know

$$Z_{\text{in}} = \frac{Z_0^2}{Z_L} = 100\Omega$$

The input voltage is then

$$v_i = v_g \times \frac{100}{100 + 10} \approx 0.909\text{V}$$

and current is 100 times smaller than that. Input power (time averaged) is half of the product, or

$$p = \frac{v_i^2}{2Z_{\text{in}}} = \frac{0.909^2}{2 \times 100} = 4.1322\text{mW}$$

To find V_0^+ , $\Gamma = \frac{25-50}{25+50} = -\frac{1}{3}$.

$$v_{\text{in}} = 0.909 = V_0^+ e^{j\beta l} + \Gamma V_0^+ e^{-j\beta l}$$

Solving yields $V_0^+ = j0.68\Omega$.