Chapter 3: Basics of Confinement

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1 General Need for Confinement

We need to achieve net energy output, so the Lawson Criteria has to be met, i.e. $n\tau \geq 10^{20}$. We also want to reduce plasma-wall interactions.

1.1 Wall Erosion

If there is no confinement,

$$\phi_{D^+} = \frac{1}{4}n\bar{c}, \bar{c} = \sqrt{\frac{8kT}{\pi m_e}}$$

For $T = 10\text{keV}, n = 10^{20}$, we have $\phi_{D^+} \to 10^{26}$ For carbon, with an erosional yield of around 1%, that's 10^{24} carbon molecules per metre squared. This number is too big.

1.2 Plasma Contamination

Example 1.1 (Carbon Walls). Carbon walls have $\frac{n_C}{n_D} \leq 1\%$. Carbon Influx is $\Gamma = y\phi_{D^+} \times 2\pi a \times 2\pi R$.

Definition 1.1 (Impurity Confinement Time).

$$\tau_{\text{imp}} = \frac{n_C V}{\Gamma_C}$$

where V stands for volume.

Assume $\tau_{\text{imp}} \approx \tau_E$. Then

$$y\phi_{D^+} \times 2\pi a \times 2\pi R = n_C \times \frac{2\pi R \times \pi a^2}{\tau_{\text{imp}}}$$

Using generic values $n_D \approx 10^{20}, y \approx 1\%, a \approx 1\text{m}, \tau \approx 1\text{s}$,

$$\phi_{D^+} \leq 10^{20}$$

This is a million times smaller than what we conventionally have.

2 Magnetic Confinement: General

2.1 Maxwell's Equations

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

In 1 dimensions, this simplifies to

$$\varepsilon_0 \frac{dE}{dx} = \rho = e|n_i - n_e|$$

We also know

$$\nabla \cdot \vec{B} = 0$$

meaning there are no magnetic "charges". The remaining laws are

$$\nabla \times \vec{E} = -\dot{\vec{B}}$$
$$\nabla \times \vec{B} = \mu_0 \left(\vec{j} + \varepsilon_0 \dot{\vec{E}} \right)$$

2.2 Diamagnetism

There are 3 types of magnetic materials. Diamagnetic materials (induced magnetic field in opposite direction), paramagnetic materials (induced magnetic field in same direction), ferromagnetic (no induced magnetic field). Here we ignore diamagnetic materials.

2.3 Quasineutrality

$$n_e \approx n_i$$

Since each negative and positive charge are generated in pairs. Now

$$\nabla \cdot \vec{E} = \frac{dE}{dx} = \frac{\rho}{\varepsilon_0} = \frac{en}{\varepsilon_0}$$

and

$$E = -\frac{dV}{dx} \Rightarrow V = \frac{nex^2}{2\varepsilon_0}$$

Choosing $V = 0, E = 0$ at $x = 0$,

$$V = \frac{10^{20} \times 1.6 \times 10^{-19} \times 1}{2 \times 8.85 \times 10^{-12}} = 10^{12}\text{V}$$

2.4 Debye Shielding

At the negative electrode, voltage is lower than the voltage of plasma, so $n_e \neq n_i$.

2.5 Sheath Thickness

$$\frac{d^2V}{dx^2} = -\frac{e(n_i - n_e)}{\varepsilon_0} \approx -\frac{en_i}{\varepsilon_0}$$

Then

$$V - V_0 = -\frac{enx^2}{2\varepsilon_0}$$

Definition 2.1 (Debye Length).

$$\frac{e\Delta V}{kT_e} = \frac{1}{2}\left(\frac{x}{\lambda_D}\right)^2$$

Where the Debye length λ_D is defined as above. Think of it as a scale length.

$$\lambda_D = 7430\left(\frac{T_e}{n}\right)^{1/2}$$

where T_e is in eV.

Example 2.1. Given $T = 10\text{keV}, n = 10^{20}$, we have $\lambda_D = 10^{-4}\text{m}$.

Rearranging, the sheath length is

$$x \approx \lambda_D \left(\frac{2e\Delta V}{kT}\right)^{1/2}$$

Example 2.2. If $V = 10^4\text{V}$, then $x \approx 10^{-4}\text{m}$.

2.6 Boltzmann Factor

Electric force is $F = qE = -enE$. Adding this to pressure gives

$$-(p_2 - p_1)A - enEA\Delta x = 0 \Rightarrow \frac{dP}{dx} = en\frac{dV}{dx}$$

From the ideal gas law, $P = n_e kT_e$, so

$$\frac{dn}{n} = \frac{edV}{kT_e} \Rightarrow n_e = n_\infty e^{eV/kT}$$

2.7 Shielding Again

Far from the electrodes, $n_e = n_i = n\infty, v = 0$. Ions are fixed in space. Say V goes from -10 to -1 $\frac{kT_e}{e}$. Then $x \approx 4\lambda_D$.

$$\begin{aligned}
\varepsilon_0 \frac{d^2V}{dx^2} &= -e(n_i - n_e) \\
&= -e\left(n_i - n_\infty e^{eV/dT}\right) \\
&= en\left(e^{eV/kT} - 1\right) \\
&\approx \frac{e^2 nV}{kT}
\end{aligned}$$

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