Lecture 2

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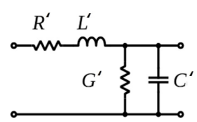
January 10, 2024

1 Parallel Plate Transmission Lines

We have shown that a parallel plate transmission line can be expressed as the following equivalent circuit.



Using KCL,

$$i(z,t) - i(z+\Delta z,t) = C'\Delta z\frac{\partial v(z+\Delta z,t)}{\partial t} + G'\Delta z v(z+\Delta z,t)$$

Dividing by Δz and taking the limit as it tends to zero,

$$-\frac{\partial i}{\partial t} = C'\frac{\partial v}{\partial t} + G'v$$

Similarly, for KVL,

$$-v(z,t) + i(z,t)R'\Delta z + L'\Delta z\frac{\partial i(z,t)}{\partial t} + v(z+\Delta z,t) = 0$$

and we get

$$\frac{\partial u}{\partial z} = -L'\frac{\partial i}{\partial t} - R'i$$

We solve this using phasors. Since we "inject" frequencies at the source, we can safely assume voltage and current are sinusoidal, with the same frequency. For

$$\begin{aligned} v(z,t) &= V_0 \cos(\omega t + \phi) \\ &= \Re\{V_0 e^{j(\omega t + \phi)}\} \\ &= \Re\{V_0 e^{j\phi} e^{j\omega t}\} \\ &= \Re\{\tilde{V}(z) e^{j\omega t}\} \end{aligned}$$

and similarly

$$i(z,t) = \Re\{\tilde{I}(z) e^{j\omega t}\}$$

We have now separated space- and time-dependent terms.

this eventually gives

$$\frac{\partial \tilde{I}}{\partial z} = -(G' + j\omega C')\tilde{V}$$

$$\frac{\partial \tilde{V}}{\partial z} = -(R' + jwL')\tilde{I}$$

Combining,

$$\begin{aligned} \frac{\partial^2 \tilde{V}}{\partial z^2} &= -(R' + jwL')\frac{\partial \tilde{I}}{\partial z} \\ &= (R' + jwL')(G' + jwC')\tilde{V} \\ &= \gamma^2 \tilde{V} \end{aligned}$$

where γ is as defined. It is a complex constant, with a real attenuation constant and imaginary phase constant.

Then the solution is

$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

and same for $\tilde{I}(z)$.

For a lossless transmission line, $R' = 0$, so

$$\gamma = j\omega\sqrt{L'C'}$$