Lecture 4

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1 Bras and Kets

Not every bra has a corresponding ket. We avoid this by defining a set of generalised kets which is isomorphic with the bras. This comes at the cost of generalised kets not being normalisable.

1.1 Linear Operators

Let A and B be linear operators. Then we can define products AB where $AB|\psi\rangle = A\left(B|\psi\rangle\right)$ is also linear. Note that in general,

$$[A, B] = AB - BA \neq 0$$

Example 1.1. If $\langle\psi|\psi\rangle =$, we can define a linear operator

$$P_\psi = |\psi\rangle\langle\psi|$$

This is called a projection for obvious reasons. Note that it is equal to its square, which is how we define projections.

Example 1.2. Let

$$P_g = \sum_{i=1}^{g} |\phi_i\rangle\langle\phi_i|$$

Where ϕ_i is orthonormal. We can then show $P_g^2 = P_g$, so we can write

$$P_g|x\rangle = \sum_{i=1}^{g} |\phi_i\rangle\langle\phi_i|x\rangle$$

We can also define Hermitian conjugation, which is the adjoint, as in

$$\langle Tx, y\rangle = \langle x, T^* y\rangle$$

We say an operator is hermitian if it is equal to its adjoint. Projections are hermitian.

Definition 1.1 (Orthnormal Discrete Basis)**.** If we have a set $\{|u_i\rangle\}$ such that $\forall|\psi\rangle$,

$$|\psi\rangle = \sum_i c_i |u_i\rangle$$

and that

$$\langle u_i|u_j\rangle = \delta_{ij}$$

we say that $\{|u_i\rangle\}$ form an orthonormal discrete basis.

Definition 1.2 (Orthnormal Continuous Basis)**.** Likewise, for $\{|w_\alpha\rangle\}$ such that $\forall|\psi\rangle$,

$$|\psi\rangle = \int c(\alpha)|w_\alpha\rangle d\alpha$$

and that

$$\langle w_\alpha|w_{\alpha'}\rangle = \delta(\alpha - \alpha')$$

We say $\{|w_\alpha\rangle\}$ form an orthonormal continuous basis.

Using a discrete basis,

$$|\psi\rangle = \sum_i c_i|u_i\rangle$$
$$\langle u_j|\psi\rangle = \sum_i \langle u_j|u_i\rangle c_i = c_j$$

So

$$\begin{aligned} |\psi\rangle &= \sum_i |u_i\rangle\langle u_i|\psi\rangle \\ &= I|\psi\rangle \end{aligned}$$

where

$$I = \sum_i |u_i\rangle\langle u_i|$$

For a continuous basis, we similarly have

$$c(\alpha) = \langle w_\alpha|\psi\rangle$$

where

$$I = \int |w_\alpha\rangle\langle w_\alpha| d\alpha$$