Lecture 32

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1 Parallel Plate Waveguide

Consider a waveguide composed of parallel plates $x = 0$ and $x = a$. The wave propogates in the z direction.

1.1 TE mode

$$\begin{aligned}
\tilde{H}_y &= H_0 \cos\left(\frac{n\pi x}{a}\right) e^{-j\beta_n z} \\
\tilde{E}_x &= \frac{\beta_n}{\omega\varepsilon} H_0 \cos\left(\frac{n\pi x}{a}\right) e^{-j\beta_n z} \\
\tilde{E}_z &= \frac{j}{\omega\varepsilon} \frac{n\pi}{a} H_0 \sin\left(\frac{n\pi x}{a}\right) e^{-j\beta_n z}
\end{aligned}$$

β_n is the propagation constant equal to $k\sqrt{1 - \left(\frac{f_{c,n}}{f}\right)^2}$. The cutoff frequency is

$$f_{c,n} = \frac{n}{2\sqrt{\varepsilon\mu}a}$$

Splitting the cosine term in $\tilde{H}_y$,

$$\tilde{H}_y = \frac{H_0}{2}\left(e^{j\frac{n\pi}{a}x} e^{-j\beta_n z} + e^{-j\frac{n\pi}{a}x} e^{-j\beta_n z}\right)$$

This can be split into an incident and reflected wave $\vec{k} = \pm\frac{n\pi}{a}\hat{x} + \beta_n\hat{z}$.

1.2 $\vec{k}_i$ as a a function of frequency

$f < f_{c,n} \Rightarrow \beta_n = -ja_n$ and we call this *evanescent*. Else, $\beta_n = k\sqrt{1 - \left(\frac{f_{c,n}}{f}\right)^2}$. Replacing f with ω and substituting $k = \omega\sqrt{\varepsilon\mu}$, we get

$$\beta_n = \sqrt{\varepsilon\mu}\sqrt{\omega^2 - \omega_{c,n}^2}$$

1.3 TE mode

$$\tilde{E}_y = E_0 \sin\left(\frac{n\pi x}{a}\right) e^{-j\beta_n z}$$
$$\tilde{H}_x = -\frac{\beta_n}{\omega\mu} E_0 \sin\left(\frac{n\pi x}{a}\right) e^{-j\beta_n z}$$
$$\tilde{H}_z = \frac{j}{\omega\mu}\frac{n\pi}{a} E_0 \cos\left(\frac{n\pi x}{a}\right) e^{-j\beta_n z}$$

Below $f_{c,1}$, there is no TE mode. Then $\beta_0 = \omega\sqrt{\varepsilon\mu}, v = \frac{\omega}{\beta} = \frac{1}{\sqrt{\varepsilon\mu}}$. At any higher modes, β is frequency dependent, and so is v, and one can show that

$$v_{p,n} = \frac{1}{\sqrt{\varepsilon\mu}} \times \frac{1}{\sqrt{1 - \left(\frac{f_{c,n}}{f}\right)^2}} > \frac{1}{\sqrt{\varepsilon\mu}}$$