Tutorial 11

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1 Rolling Coin

Define θ to be the angle between the horizontal and the coin. Let ψ be the normal polar angle when treating the coin as a point mass, and ψ the angle of instantaneous rolling. Then

$$|\phi R| = |\psi a| \Rightarrow \psi = -\frac{R}{a}\psi$$

The negative sign comes from the fact that the rotation of the coin and the movement of the coin are in opposite direction. E.g. if a coin is rolling in the clockwise direction, its centre of mass travels in the anticlockwise direction.

It is easy to find translational kinetic energy.

$$T_{tr} = \frac{m}{2}((R - a\cos\theta)^2\dot{\phi}^2)$$

For cylinders, we have that

$$I_{11} = I_{22} = \frac{1}{4}ma^2, I_{33} = \frac{1}{2}ma^2$$

We have also derived in class that

$$\begin{aligned}
\Omega_1 &= \dot{\phi}\sin\theta\sin\psi + \dot{\theta}\cos\psi \\
\Omega_2 &= \dot{\phi}\sin\theta\cos\psi - \dot{\theta}\sin\psi \\
\Omega_3 &= \dot{\psi} + \dot{\phi}\cos\theta
\end{aligned}$$

We can simplify

$$\Omega_3 = \dot{\phi}\left(\cos\theta - \frac{R}{a}\right)$$

Then

$$T_r = \frac{1}{2}\Omega_i^2 I_{ii}$$