Lecture 31

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1 Waveguides

Electromagnetic waves attenuate exponentially in conductors. Therefore, ordinary cables are unsuitable for high frequency power transfer. The alternative is to use waveguides. Normally, we have waves that propagate along the z axis. We now want them to do so along the axis of the waveguide.

1.1 Parallel plates

Consider a wave travelling approximately along a pair of parallel plates. Due to imperfections, it travels at an angle and is reflected along each of the plates. Using the convention where x is normal to the plates,

$$\vec{k}_i = k\cos\theta_i\hat{x} + k\sin\theta_i\hat{z} = k_x\hat{x} + \beta\hat{z}$$

Then

$$\begin{aligned}\tilde{\vec{H}}_i &= H_0 e^{-jk_x x} e^{-j\beta z}\hat{y} \\ \tilde{\vec{H}}_r &= -\Gamma_\perp H_0 e^{jk_x x} e^{-j\beta z}\hat{y} \\ \Gamma_\perp &= \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}\end{aligned}$$

As $\eta_2 \to 0, \Gamma_\perp \to -1$, which gives

$$\tilde{\vec{H}} = H_0 e^{-j\beta z}\left(e^{-jk_x x} + e^{jk_x x}\right)\hat{y} = 2H_0\cos(k_x x)e^{-j\beta z}\hat{y}$$

To find the electric field more quickly, we can use

$$\nabla \times \tilde{\vec{H}} = j\omega\varepsilon\tilde{\vec{E}}$$

Then

$$\begin{aligned}
\tilde{\vec{E}} &= -\frac{j}{\omega\varepsilon}\nabla \times \tilde{\vec{H}} \\
&= -\frac{j}{\omega\varepsilon}\left(\hat{x}\frac{\partial}{\partial x} + \hat{z}\frac{\partial}{\partial z}\right) \times \tilde{H}_y\hat{y} \\
&= \frac{2H_0e^{-j\beta z}}{\omega\varepsilon}(\beta\cos(k_xx)\hat{x} + jk_x\sin(k_xx)\hat{z})
\end{aligned}$$

We know that $\vec{E}$ has no z component at the boundaries $x = 0, x = a$, since there it meets a conductor, and $\vec{E}$ fields only penetrate perpendicularly, These conditions give $k_x = \frac{n\pi}{a}$. There is another limit, since

$$\beta^2 + k_x^2 = k^2 = \omega^2\varepsilon\mu$$

Definition 1.1 (Cutoff Frequency)**.** The cutoff frequency of the nth mode is

$$\omega_{c,n} = \frac{n\pi}{a\sqrt{\varepsilon\mu}}$$

The nth mode cannot exist below this frequency.

When $\omega_{c,n} > \omega$, then

$$\beta = -jk\sqrt{\left(\frac{\omega_{c,n}}{\omega}\right)^2 - 1}$$

This is called the *evanescent* mode. It attenuates with constant

$$a_n = k\sqrt{\left(\frac{\omega_{c,n}}{\omega}\right)^2 - 1}$$

If not, we have the *propagating* mode. This gives a high-pass filter, since in the evanescent mode, β becomes imaginary, so $e^{-j\beta z}$ decays.

For the $n = 0$ mode, $k_x = 0$, so $\tilde{E}_z$ vanishes, and

$$\tilde{E}_x = \frac{\beta}{\varepsilon\omega}2H_0e^{-j\beta z} = 2\eta H_0e^{-j\beta z}$$

There is no cutoff. This means for frequencies below $\omega_{c,1}$, there is only one possible k_x.