Lecture 28

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1 Plane Wave Incidence

1.1 Incident Wave

$$\tilde{\vec{E}}_i = (\cos\theta_i \hat{x} - \sin\theta_i \hat{z}) e^{-jk_0 n_1 \sin\theta_i x} e^{-jk_0 n_1 \cos\theta_i z}$$

$$\tilde{\vec{H}}_1 = \frac{1}{\eta_1} e^{-jk_0 n_1 \sin\theta_i x} e^{-jk_0 n_1 \cos\theta_i z} \hat{y}$$

$$n_1 = \sqrt{\varepsilon_{r_1} \mu_{r_1}}, \eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}$$

1.2 Reflected Wave

$$\tilde{\vec{E}}_r = \Gamma_\parallel (\cos\theta_r \hat{x} + \sin\theta_r \hat{z}) e^{-jk_0 n_1 \sin\theta_r x} e^{jk_0 n_1 \cos\theta_r z}$$

$$\tilde{\vec{H}}_r = -\frac{\Gamma_\parallel}{\eta_1} e^{-jk_0 n_1 \sin\theta_r x} e^{jk_0 n_1 \sin\theta_r z} \hat{y}$$

1.3 Transmitted Wave

$$\tilde{\vec{E}}_t = \tau_\parallel (\cos\theta_t \hat{x} - \sin\theta_t \hat{z}) e^{-jk_0 n_2 \sin\theta_t} e^{-jk_0 n_2 \cos\theta_t z}$$

$$\tilde{\vec{H}}_t = \frac{\tau_\parallel}{\eta_2} e^{-jk_0 n_2 \sin\theta_t x} e^{-jk_0 n_2 \cos\theta_t z} \hat{y}$$

1.4 Boundary Conditions

$$E_{x,i}(z=0) + E_{x,r}(z=0) = E_{x,t}(z=0)$$
$$H_{y,i}(z=0) + H_{y,r}(z=0) = H_{y,t}(z=0)$$

In order to satisfy this $\forall x$, all exponentials have to agree by identity. This gives the law of reflection $\theta_i = \theta_r$, and also Snell's law $n_1 \sin\theta_i = n_2 \sin\theta_t$. Cancelling out the exponentials, we obtain

$$\cos\theta_i(1 + \Gamma_{\parallel}) = \tau_{\parallel} \cos\theta_t$$

For the magnetic field, we similarly have

$$\frac{1}{\eta_1}(1 - \Gamma_{\parallel}) = \frac{\tau_{\parallel}}{\eta_2}$$

Solving the pair of equations, we can find

$$\Gamma_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$
$$\tau_{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

For normal incidence, $\theta_i = \theta_r = \theta_t = 0$, so $\Gamma_{\parallel} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}, \tau_{\parallel} = \frac{2\eta_2}{\eta_1 + \eta_2}$.

Note that

$$\eta_1 \cos\theta_i = \frac{\tilde{E}_{x,i}}{\tilde{H}_{y,i}}$$

and

$$\eta_2 \cos\theta_t = \frac{\tilde{E}_{x,t}}{\tilde{H}_{y,t}}$$

From Snell's law, $\exists \theta_i = \theta_c$ at which $\sin\theta_t = 1$. Above this angle, the transmitted wave propagagtes along the x direction. Since $\sin^2\theta_t > 1$, we have $\cos\theta_t = \sqrt{1 - \sin^2\theta_t} = \pm j\alpha$ where α is real. It makes no physical sense to have a growing solution, so the only solution is

$$\tilde{E}_t, \tilde{H}_t \; e^{-jk_0 n_2 (\pm j\alpha) z} = e^{k_0 n_2 \alpha z}$$

The exponention can be split into propogation and attenuation terms, i.e.

$$e^{-jk_0 n_1 \sin\theta_i x} e^{-k_0 n_2 \alpha z}$$

The transmitted wave has no power, so all power is transferred to the reflected wave. We call this **total internal reflection**.