Lecture 9

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1 Special cases of Transmission Lines

$$Z_{\text{in}} = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$$

1. Matched Line: $Z_L = Z_0$.
   This implies $Z_{\text{in}} = Z_0$, and there is no impedance tranformation, no standing wave.

2. $l = \frac{n\lambda}{2}, n \in \mathbb{Z}^+$
   Then $\beta l = \frac{2\pi}{\lambda} \frac{n\lambda}{2} = n\pi$. Then its tangent vanishes, and $Z_{\text{in}} = Z_L$. This is then a half-wave line.

3. $l = (n + \frac{1}{2})\frac{\lambda}{2}, n \in \mathbb{N}$
   The tangent term becomes $\tan(\beta l) = \tan\left[\frac{2\pi}{\lambda}\frac{\lambda}{2}\left(n + \frac{1}{2}\right)\right] = \tan\left(n\pi + \frac{\pi}{2}\right) = \pm\infty$. This implies $Z_{\text{in}} = \frac{Z_0^2}{Z_L}$. This is the quarter-wave transformer, or impedance inverter.

Definition 1.1 (Normalised Impedance)**.**

$$z = \frac{Z}{Z_0}$$

The quarter-wave transformer "inverts impedance" because

$$z_{\text{in}} = \frac{Z_0}{Z_L} = \frac{1}{z_L}$$

Example 1.1. Use a quarter wave transformer to *match* a 5Ω load to a $Z_0 = 50\Omega$ transmission line.
We add a transmission line between the existing one and the load, with Z_0'. Then

$$\begin{aligned}\frac{Z_0}{Z_0'} &= \frac{Z_0'}{R_L}\\ \frac{50}{Z_0'} &= \frac{Z_0'}{5}\\ Z_0' &= 5\sqrt{10}\\ &= 158\Omega\end{aligned}$$

where the equation comes from the results of a quarter-wave transformer.

2 Open-Circuited Transmission Line

As $Z_L \to \infty$, we have

$$Z_{\text{in}} = \frac{1}{jY_0 \tan(\beta l)}, Y_0 = \frac{1}{Z_0}$$

which is the characteristic admittance (recall admittance is purely imaginary). Also

$$Y_{\text{in}} = jY_0 \tan(\beta l)$$

Since the sign of $\tan(\beta l)$ changes, it can act like an inductor or capacitor depending on its arguments. Susceptance against βl becomes a tangent curve, so it starts out as a capacitor and alternates.

3 Short-Circuited Transmission Line

Obviously $Z_{\text{in}} = jZ_0 \tan(\beta l)$. A similar graph can be sketched for reactance, but it starts out acting as an inductor.

Example 3.1. Use a short-circuited transmission line ($Z_0 = 50\Omega$) to implement $C = 4\text{pF}$ at $f = 2.25\text{GHz}$. Phase velocity of the line is $0.75c \approx 2.25 \times 10^8 \text{m s}^{-1}$.
The impedance of the capacitor is

$$Z = \frac{1}{j\omega C} = \frac{1}{j2\pi f C} = \frac{1}{j2\pi \times 2.25 \times 10^9 \times 4 \times 10^{-12}} = -j17.684\Omega$$

We have

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{v} = \frac{4.5 \times 10^9 \pi}{2.25 \times 10^8} = 20\pi$$

Equating the impedance with $Z_0 \tan(\beta l)$,

$$\begin{aligned}
Z_0 \tan(\beta l) &= -17.684 \\
50 \tan(20\pi l) &= -17.684 \\
\tan(20\pi l) &= -0.354 \\
l &= (4.46 + 5n)\mathrm{cm}
\end{aligned}$$