Lecture 21

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April 21, 2024

1 Plane Wave Polarization

Definition 1.1 (Polarization)**.** The pattern traced by electric field vector as a function of time. This is *not* polarization in dielectrics.

Example 1.1. Plane wave propagating in the z direction.

$$\tilde{\vec{E}} = (E_x\hat{x} + E_y\hat{y})e^{-jkz} = E_x\left(\hat{x} + \frac{E_y}{E_x}\hat{y}\right)e^{-jkz}$$

Defining $E_x = |E_x|e^{j\delta_x}$ and similarly for y, we can simplify this to

$$\tilde{\vec{E}} = E_x e^{-jkz}\left(\hat{x} + Re^{j\delta}\right)$$

where R is the ratio of magnitudes, and $\delta_y - \delta_x = \delta$. Normalizing (ignoring constants), we have (at $z = 0$)

$$\tilde{\vec{E}} = \hat{x} + Re^{j\delta}\hat{y}$$

Going back to the time domain,

$$\vec{E} = \Re\{\tilde{\vec{E}}e^{j\omega t}\} = \cos(\omega t)\hat{x} + R\cos(\omega t + \delta)\hat{y}$$

If $\delta = 2n\pi$, the electric field moves along $y = Rx$ as time goes on. For $\delta = 2(n+1)\pi$, it oscillates along $y = -Rx$.
For $\delta = \frac{\pi}{2}$, this forms an ellipse. One can easily find that

$$E_x^2 + \left(\frac{E_y}{R}\right)^2 = 1$$

This is left handed; if you point your left thumb along z, your fingers trace $\vec{E}$ as they curl. Similarly, for $\delta = -\frac{\pi}{2}$, we have a right handed system. In general, a negative phase different implies righthandedness, and vice versa.