Lecture 4

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1 Propagating Standing Waves in a Transmission Line

1.1 Summary up till Now

$$\tilde{V}(z) = V_0^+ e^{-\gamma d} + V_0^- e^{\gamma d}$$
$$\tilde{I}(z) = \frac{V_0^+}{Z_0} e^{-\gamma d} - \frac{V_0^-}{Z_0} e^{\gamma d}$$

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')} = \alpha + j\beta$$

The phase velocity is $v_p = \frac{\omega}{\beta}$, and the wavelength is $\lambda = \frac{2\pi}{\beta}$.

1.2 Transmission Line Circuit

Consider a lossless TL, with $Z_0 = \sqrt{\frac{L'}{C'}}, \gamma = j\beta$. At $z = 0$,

$$\tilde{V}(z = 0) = V_L = V_0^+ + V_0^-$$

$$\tilde{I}(z = 0) = \tilde{I}_L = \frac{V_0^+ - v_0^-}{Z_0}$$

and

$$Z_L = Z_0 \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}$$

Definition 1.1 (Reflection Coefficient)**.**

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{V_0^-}{V_0^+}$$

Note that $Z_L = Z_0 \Rightarrow \Gamma = 0 \Rightarrow V_0^- = 0$. This means that there is no reflection, and there is only the plus wave. The following relation is instantly satisfied

$$Z_0 = \frac{\tilde{V}}{\tilde{I}}$$

In an open circuit, $Z_L = \infty, \Gamma = 1, V_0^+ = V_0^-$, and so

$$\tilde{V}(z) = V_0^+(e^{-j\beta z} + e^{j\beta z}) = 2V_0^+ \cos(\beta z)$$

and

$$\tilde{I}(z) = \frac{V_0^+}{Z_0}[e^{-j\beta z} - e^{j\beta z}] = -\frac{2jV_0^+}{Z_0} \sin(\beta z)$$

Definition 1.2 (Standing Wave Ratio)**.**

$$S = \frac{|\tilde{V}|_{\max}}{|\tilde{V}|_{\min}}$$

For a matched line, $|\tilde{V}(z)| = |V_0^+|$, so $S = 1$. For an open circuit, $|\tilde{V}(z)| = 2|V_0^+||\cos(\beta z)|$, so $S = \infty$.