Lecture 5

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1 Cyclotron Radiation

The Lorentz Force is $F = 2e\vec{v} \times \vec{B} = 2ev^{\perp}B$. The centrifugal force is $\frac{m\left(v^{\perp}\right)^2}{R}$, and the gyro radius is $R = \frac{mv^{\perp}}{zeB}$. Time taken per cycle is $\tau = \frac{2\pi R}{v^{\perp}}$. The gyrofrequency is $\Omega = \frac{2\pi}{\tau} = \frac{zeB}{m}$.

Example 1.1. For $B = 5\text{T}$ and an electron, we get

$$\Omega = \frac{1.6 \times 10^{-19} \times 5}{9.1 \times 10^{-31}} = 10^{12}\text{rad}\,\text{s}^{-1}$$

The mean free path is then

$$\lambda_{cy} = \frac{2\pi c}{\Omega} \approx 2\text{mm}$$

Similarly, we get $\Omega = 2.4 \times 10^8\text{rad}\,\text{s}^{-1}$ for D^+. Its mean free path is approximately 8 m.

Power is

$$P_{\text{rad}} = \frac{1}{4\pi\varepsilon_0}\frac{2}{3}\frac{e^2}{c^3}a^2 n$$

where

$$a = \frac{\left(v^{\perp}\right)^2}{R} = \frac{ev^{\perp}B}{m}$$

Plugging $\left(v^{\perp}\right)^2 \approx \frac{2kT}{m}$, we get

$$P_{\text{cy}} = \frac{1}{3\pi\varepsilon_0}\frac{e^4 B^2 kT}{c^2 m^3} n$$

Or

$$P_{\text{cy}} = 6.21 \times 10^{-17} n_e T_e B^2$$

where T_e is in keV and n is in m^{-3}. Another way to derive it is

$$\beta = \frac{P_P}{P_M} = \frac{n_e k T_e + n_i k T_i}{B/2\mu_0} = \frac{4nkT\mu_0}{B^2}$$

Then

$$P_{\text{cy}} = \frac{4}{3} \frac{n_e^2 (kT)^2 e^4 \mu_0}{\pi \varepsilon_0 m_e^3 c^2 \beta}$$

2 Lawson Criteria

2.1 Including Conversion Efficiency

We define

$$\begin{aligned} \tau &= \frac{\text{total heat content}}{\text{total heating rate}} \\ &= \frac{\int \frac{3k}{2} \left(n_e T_e + n_D T_D + n_T T_T \right) dV}{P_{\text{in}}} \end{aligned}$$

Example 2.1. For $n_D = n_T = \frac{1}{2} n_i = \frac{1}{2} n_e$, $T_e = T_D = T_T$, we have

$$\tau = \frac{3 n_i k T}{P_{\text{in}}}$$

Breakeven requires $\epsilon(P_{\text{in}} + P_{\text{fusion}} = P_{\text{fusion}}$, where ϵ is the efficiency of converting heat to electricity. Continuing from the previous example,

$$\begin{aligned} (1-\epsilon) P_{\text{in}} &= \epsilon P_{\text{fusion}} \\ (1-\epsilon) \frac{3 n_i k T}{\tau} &= \epsilon \frac{n_i}{2} \frac{n_i}{2} \overline{\sigma v} E_{DT} \\ n_i \tau &= \frac{12(1-\epsilon) k T}{\epsilon \overline{\sigma v} E_{DT}} \\ n_i \tau &= \frac{24 k T}{\overline{\sigma v} E_{DT}} \end{aligned}$$

Where we take $\epsilon = \frac{1}{3}$.

2.2 Specific Energy Loss Mechanisms

Ignoring radiative effects, there is only conduction and convection. Similarly, we define

$$\tau_{CC} = \frac{3n_i kT}{P_{CC}}$$

Considering sum of power,

$$P_{\text{in}} = P_{\text{rad}} + P_{\text{cc}}$$

And we get

$$n_i\tau_{CC} = \frac{12(1-\epsilon)kT}{\epsilon\overline{\sigma v}E_{DT} - 4(1-\epsilon)C_1\sqrt{T} - 4(1-\epsilon)c_2T^2}$$

where

$$P_{\text{rad}} = P_{\text{br}} + P_{\text{cy}} = c_1 n_D^2\sqrt{T} + c_2 n_D^2 T^2$$

Recall from thermodynamic that

$$Q = K\frac{\Delta T}{\Delta x}$$

where K is the thermal conductivity. For a torus with radii a and R (bigger one in bigger letters), we have

$$\frac{\Delta T}{\Delta x} \approx \frac{T_0}{a}$$

Total conducted loss is then

$$Q_{TOT} = 2\pi a \times 2\pi R \times Q$$

Loss per volume is

$$P_C = \frac{Q_{TOT}}{2\pi R\pi a^2} = \frac{2Q}{a} = \frac{2KT_0}{a^2}$$

Recall

$$K = nk\chi$$

where k is the Boltzmann constant, and χ is thermal diffusivity. Experimentally, χ is inversely proportional with B, and is around $1\text{m}^2\,\text{s}^{-1}$ at $B = 3\text{T}$.

$$P_C = \frac{2n\chi kT}{a^2}$$

and

$$\tau_{CC} \approx \frac{3nkT}{P_C} = \frac{3nkT}{2nkT\chi/a^2}$$

so

$$\tau_{\text{non-rad}} \approx \frac{a^2}{\chi}$$

For τ_{nr} at 1 second, we have a around 1 metre and χ around 1 metre squared per second.

3 Impurities Can Prevent Ignition

$P_z \mathrm{W\,m^{-3}}$

- Bremsstrahlung proprtional to z^2
- Line Radiation (see tables)

$$P_z \approx n_e n_z L_(T)$$

where the last term is tabulated.

Ignition requires $P_z < P_\alpha$. Letting $n_e = n_D + n_T + zn_z$,

$$\frac{n_D}{n_e} = \frac{1}{2}(1 - zf_z)$$

where

$$f_z = \frac{n_z}{n_e}$$

Plugging into the inequality,

$$\begin{aligned} n_e n_z L_z(T) &< n_D n_T \overline{\sigma v}(T) E_\alpha \\ f_z L_z(T) &< \frac{1}{4}(1 - zf_z)^2 \overline{\sigma v}(T) E_\alpha \\ \frac{f_z}{(1 - zf_z)^2} &< \frac{\overline{\sigma v}(T) E_\alpha}{4L_z(T)} \end{aligned}$$

Note that we can allow for more impurities as temperature goes up. Factors include more ionisation.

3.1 Impurity Production Mechanisms

- Sputtering
- Chemical Erosion
- Evaporation
- Melting