Lecture 37

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Example 0.1. An air filled rectangular waveguide has cross-section $a = 5\text{cm}, b = 2\text{cm}$. A mode is

$$\tilde{H}_z = \cos(40\pi x)e^{-j\frac{40\pi\sqrt{7}}{3}z}$$

Since $\tilde{H}_z \neq 0$, it is TE. There is no attenuation along the z direction, so it is propagating. Since it has no y dependence, $n = 0$, and

$$\frac{m\pi x}{a} = 40\pi x \Rightarrow m = 2$$

This is the $\text{TE}_{2,0}$ mode. The frequency can be found by

$$\beta = k\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

Cutoff frequency is

$$f_c = \frac{1}{2\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = 20c_0 = 6\text{GHz}$$

Plugging into the formula above,

$$\frac{40\pi\sqrt{7}}{3} = \frac{2\pi f}{3\times 10^8}\sqrt{1 - \left(\frac{6\times 10^9}{f}\right)^2}$$
$$f = 8\text{GHz}$$

Find the range of frequencies where the guide supports a single mode.
For (m, n) pairs, the two lowest are $(1, 0)$ at 3GHz and $(2, 0)$ at 6GHz. Then this is the range of frequencies.