Lecture 1

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1 Course Information

Office hours Fridays 14:00 - 17:00 MP408, including reading week. Problem set questions appear on quizzes.

2 Waves and Particles

$$E = h\omega \tag{1}$$

$$p = \frac{h}{\lambda} \tag{2}$$

The LHS of both equations are usually linked with *particles*, while the RHS are linked with *waves*.
Relativistically, we have

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{3}$$

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{4}$$

And the nonrelatavistic counterparts are trivial.

2.1 Electrons

Electrons are used to study surfaces, since they don't penetrate too deeply.

2.2 Wavefunctions

In 1 dimension, we have $\cos(kx - \omega t)$ or $\sin(kx - \omega t)$. We also know that these functions are complex in quantum, hence we use $e^{i(kx-\omega t)}$.
At $t = 0$, we form a wavepacket

$$\psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(k) e^{ikx} dk \tag{5}$$

A simpler expression is to sum over ks instead of integration.

Example 2.1. Let

$$g(k) = \begin{cases} 1 & i = k_0 \\ \frac{1}{2} & i = k_0 \pm \frac{\Delta}{2} \end{cases}$$

Then

$$\psi(x) = \frac{e^{ik_0 x}}{\sqrt{2\pi}} \left[1 + \cos\left(\frac{\Delta k_0 x}{2}\right)\right]$$

If we introduct time, e^{ikx} becomes $e^{i(kx-\omega(k)t)}$.
In a nonrelatavistic case,

$$E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m} \tag{6}$$

Assuming superposition,

$$\begin{aligned} \psi(x,t) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(k) e^{i(kx-\omega(k)t)} dk \\ &= \frac{1}{\sqrt{2\pi}} e^{i(k_0 x - \omega(k_0)t)} \int_{-\infty}^{\infty} g(k) e^{i(k-k_0)x} e^{-i(\omega(k)-\omega_0)t} \\ &\approx \frac{1}{\sqrt{2\pi}} e^{i(k_0 x - \omega_0 t)} \int_{-\infty}^{\infty} g(k) e^{i(k-k_0)(x-Vt)} \end{aligned}$$

where we define

$$V = \left(\frac{d\omega}{dk}\right)_{k_0} \tag{7}$$

Letting the final integral be $F(x,t)$, we see F moves at a phase velocity V.

For our nonrelatavistic particle,

$$\left(\frac{d\omega}{dk}\right)_{k_0} = \frac{\hbar k_0}{m} = \frac{p_0}{m} \tag{8}$$

For a relativistic particle,

$$\frac{\omega}{k} = \omega\lambda = \frac{E}{p} = \frac{c^2}{v} \tag{9}$$