Plasma Flow to a Surface

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1 The Plasma Solution

For $\vec{B} = 0$, electron ionization is balanced by wall recombination. Initially, $T_e = 10000, T_i = 300$. There is a bigger difference because of both conservation of momentum and energy dictates the lighter particle gains more energy, or in the case of an applied voltage, electrons move faster, so they gain energy (traverse the field) faster. The time it takes for electrons to get to the walls is approximately $\frac{L}{\bar{c}_e}$, in the order of μs. After ion transit time, $\Gamma_e = \Gamma_i$. Only the most energetic electrons can reach the wall/overcome the potential barrier for recombination. This is equivalent to a cooling effect.

Consider particle flux in different directions. Some go along the field lines, i.e. towards the cathode or anode, but most go to the walls.

$$\Gamma_{\perp e} = \Gamma_{\perp i} \Rightarrow n_e v_{e\perp} = n_i v_{i\perp} \Rightarrow v_{e\perp} = v_{i\perp} \Rightarrow mn(v_e - v_i)\nu_{ei} = 0$$

There is ambipolar diffusion when $\lambda_{en}, \lambda_{in} < r$. There is no collison when the opposite occurs, $\lambda_{en}, \lambda_{in} > r$ and this is called freefall.

From the momentum equation,

$$\frac{\partial \vec{v}_e}{\partial t} + (\vec{v}_e \cdot \nabla)\vec{v}_e = \frac{q}{m_e}(\vec{E} + \vec{v}_e \times \vec{B}) - \frac{1}{\rho_0}\nabla p_e - \nu \vec{v}_e$$

If we assume $\overline{v}_e = 0$ (see Assignment 7), and $B = 0$, then we get the 1D equation

$$\frac{eE}{m_e} = -\frac{1}{\rho}\frac{dP_e}{dx}$$

If T_e is a constant, and we take E to be the negative derivative of V wrt x,

$$\frac{e}{m_e}\frac{dV}{dx} = \frac{kT}{\rho}\frac{dn}{dx}$$
$$\frac{dn}{n} = \frac{e}{kT_e}dV$$
$$n = n_0 e^{eV/kT_e}$$

We end with the **Boltzmann Relation**.

Defining S to be the ionization rate, in units of ions per area per time, then $\frac{d}{dx}(nv_i) = S$ and

$$v_i\frac{dv_i}{dx} = \frac{e}{m_i}E - \frac{1}{\rho_i}\frac{dp_i}{dx} - \nu v_i$$

Assume the last term doesn't exist. Momentum loss, approximating the new ions to have no momentum, is $m_i v_i S$. As always, T_i is taken to be constant, converting the pressure gradient to a density gradient. From the electron momentum equation, we can substituting m_e for m_i and get

$$\frac{eE}{m_e} = -\frac{kT_e}{m_e n_e}\frac{dn_e}{dx}$$
$$\frac{eE}{m_i} = -\frac{kT_e}{m_i n_e}\frac{dn_e}{dx}$$

This gives

$$v_i\frac{dv_i}{dx} = -\frac{kT_e}{m_i n_e}\frac{dn_e}{dx} - \frac{kT_i}{m_i n_i}\frac{dn_i}{dx} - \frac{v_2 S}{n_i}$$

If we further take $n_e = n_i$,

$$v_i\frac{dv_i}{dx} = -\frac{k(T_e+T_i)}{m_i n}\frac{dn}{dx} - \frac{vS}{n}$$
$$= -\frac{c_s^2}{n}\frac{dn}{dx} - \frac{vS}{n}$$

Where $c_s^2 = \frac{k(T_e+T_i)}{m_i}$ for low acoustic speed. Defining the Mach number as $M = \frac{v}{c_s}$, one can eventually derive

$$\frac{d}{dx}(nv_i) = S \Rightarrow n\frac{dv}{dx} + v\frac{dn}{dx} = S$$

and

$$\frac{dM}{dx} = \frac{S}{nc_s}\frac{1+M^2}{1-M^2}$$

At $M = 1$, the solution blows up, because its slope is infinite, and $\frac{dn}{dx}, E \to -\infty$. This is where the plasma ends, and we get to the wall sheath with thickness λ_D, where there is a layer of $n_e \neq n_i$.

2 The Self-Sustaining Plasma

The source term is a reaction rate, so it can be written as

$$S = n_e n_g \langle \sigma v \rangle \approx n_e n_g \sigma_{\mathrm{iz}} \overline{c}_e$$

where we assume the cross-section is constant. For $T_e = 10 - 100\text{eV}, \sigma \approx 10^{-20}\text{m}^2$. See Figure 3D1. Plugging this into the derivative of the Mach number gives

$$\frac{dM}{dx} = \frac{n_g \sigma_{\mathrm{iz}} \overline{c}_e}{c_s}\frac{1+M^2}{1-M^2}$$

Integrating, assumine $M = 0$ at $x = 0$,

$$\int_0^M \frac{1-M^2}{1+M^2} dM = \frac{n_g \sigma_{iz} \overline{c}_e}{c_s} \int_0^x dx \frac{n_g \sigma_{iz} \overline{c}_e}{c_s} x = 2(\arctan M) - M$$

$M = 1$ at $x = L$. The right hand side under this condition is $\frac{\pi}{2} - 1 \approx 0.57$, so

$$\frac{\sigma_{iz} \overline{c}_e}{c_s} = \frac{0.57}{n_g L}$$

For $T_i << T_e$, we can simplify c_S^2 and get

$$\frac{\sigma_{iz} \overline{c}_e}{c_S} = \left(8\pi \frac{m_i}{m_e}\right)^{1/2} \sigma_{iz}$$

T_e is a function of $n_g L$.

3 Plasma Density Variations

From the momentum and continuity equations,

$$\frac{dM}{dx} = \frac{S}{nc_s}\frac{1+M^2}{1-M^2}$$

$$v\frac{dn}{dx} + n\frac{dv}{dx} = S \Rightarrow \frac{dn}{dx} = \frac{1}{v}\left(S - n\frac{dv}{dx}\right)$$

Now

$$\frac{dM}{dn}\left(S - n\frac{dv}{dx}\right) = \frac{Sv}{nc_s}\frac{1+M^2}{1-M^2}$$

which implies

$$\frac{dv}{dx} = c_s\frac{dM}{dx} = \frac{S}{n}\frac{1+M^2}{1-M^2}$$

so

$$\frac{dM}{dn}S\left(1 - \frac{1+M^2}{1-M^2}\right) = \frac{SM}{n}\frac{1+M^2}{1-M^2}$$

and

$$\frac{dn}{n} = -\frac{2MdM}{1+M^2} \Rightarrow \frac{n}{n_0} = \frac{1}{1+M^2}$$

at the boundary, $n = \frac{n_0}{2}$. Plugging into the Boltzmann relation, the voltage difference is

$$\Delta v = -\ln 2\frac{kT_e}{e} \approx -0.69\frac{kT_e}{e}$$

This is the **Pre-sheath voltage drop**.

4 The Sheath Voltage Drop

$$\Gamma_e = \frac{1}{4}n_e\bar{c}_e = \frac{1}{4}n_0\bar{c}_e e^{eV/kT_e}$$

In steady state, $\Gamma_e = \Gamma_i$, or else there will be charge build up (i.e. not steady state). Then Γ_i at the edge is equal to Γ_i at the wall, or $\frac{1}{2}n_0c_s$.

Substituting,

$$\begin{aligned}\frac{eV}{kT_e} &= \ln\frac{2c_s}{\bar{c}_e} \\ &= \ln\frac{2\sqrt{k(T_e+T_i)/m_i}}{\sqrt{8kT_e/\pi m_e}} \\ &= \frac{1}{2}\ln\left(\frac{\pi}{2}\frac{m_e}{m_i}\left(1+\frac{T_i}{T_e}\right)\right)\end{aligned}$$

so if we exclude the pre-sheath drop,

$$\frac{e\Delta V_{\text{sheath}}}{kT_e} = \frac{1}{2}\ln\left[\frac{\pi}{2}\frac{m_e}{m_i}\left(1+\frac{T_i}{T_e}\right)\right] + 0.69$$

For hydrogenic plasmas,

$$\Delta V_{\text{sheath}} \approx \Delta V_{\text{wall}} \approx \frac{3kT_e}{e}$$

The slope at the sheath ($\frac{dV}{dx}$) is the above value divided by Debye length.

5 Basic Consequences of Plasma Sheath

- Sputtering Increased

$$E_i \approx 2kT_i + 3kT_e$$

$$\Gamma_i^{\text{no sheath}} = \frac{1}{4}n_0\sqrt{\frac{8kT_i}{\pi m_i}}$$

$$\Gamma_i^{\text{sheath}} = \frac{1}{2}n_0c_s = \frac{1}{2}n_0\sqrt{\frac{k(T_e+T_i)}{m_i}}$$

- Heat flux reduced
  No sheath: $P_e = 2kT_e \times 1/4n\bar{c}_e, P_i = 2kT_i \times \frac{1}{4}n\bar{c}_i$
  Sheath: $P_e = 1kT_e\frac{1}{4}e^{-3}n_0\bar{c}_e, P_i = (2kT_i + 3kT_e)\frac{1}{2}n_0\bar{c}_i (e \approx 2.72)$
  Power of electrons is reduced by around 20 times, and power of ions is up by 2-3 times. Total heat flux is reduced by around 10 times.

- The sheath cools electrons

- Sets boundary conditions for n, T

6 Plasma Heat Flux to a Wall

6.1 Tokamak Geometry

- Limiters
- Divertors
  - Double-Null poloidal divertor

In the divertor, a separatrix is where the open and closed field lines meet.

$$\Gamma = D_\perp \frac{dn}{dr}\bigg|_{r=a} = D_\perp \frac{n}{lambda}$$

where λ is the width of the scrape-off layer.

$$\Gamma^{\text{total}} \approx 2LD_\perp \frac{n}{\lambda}$$

$$\Gamma^{\text{limiters}} \approx \lambda n_0 c_s \Rightarrow \lambda \approx \sqrt{\frac{2D_\perp L}{c_s}}$$

Example 6.1 (JET)**.** For $L = 10, T_i = 100\text{eV}, T_e = 50\text{eV}$, we have

$$D_\perp \approx 1\text{m}^2\,\text{s}^{-1}, c_s = \sqrt{\frac{k(T_e+T_i)}{m_i}} \approx 10^5\text{m}^2\,\text{s}^{-1}, \lambda \approx \sqrt{\frac{2\times 1\times 10}{10^5}} \approx 1\text{cm}$$

Example 6.2 (Reactor)**.** Consider a reactor with 2.5GW thermal power. 20% of it, 500 mW, is transferred to the walls. Taking 8 limiter surfaces, $A_L = 2\pi r\lambda$, and total A_L is $8 \times 2\pi \times 1 \times 0.01 = 0.5\text{m}^2$. Heat load is

$$q = \frac{5\times 10^8}{0.5} \approx 10^9\text{W}\,\text{m}^{-2}$$

This is a small problem, since there is no material that can take this q.

- Put surfaces at steep angles to B field
- Sweep strike point: change the current temporally, so the strike point changes over time
- Add impurities to induce radiation (so more power is radiated instead)
- Spread out B field in divertor

6.2 Sheath Energy Transmission Factor

$$Q_e^{\text{conv}} = 2kT_e\Gamma_e = 2kT_e\frac{1}{4}n\bar{c}_e = 2kT_e \times \frac{1}{4}n_0\bar{c}_e e^{eV_w/kT_e}$$

where V_w is wall potential.

$$Q_i^{\text{conv}} = 2kT_i\Gamma_i + e(-V_w)\Gamma_i$$

so

$$Q^{\text{TOT}} = (2kT_e + 2kT_i - eV_w)\Gamma = (2kT_e + 2kT_i - eV_w) \times \frac{1}{2}n_0c_s$$

Definition 6.1. The sheath energy transmission factor is defined as

$$\gamma_s = \frac{2kT_e + 2kT_i - eV_w}{kT_e}$$

for $T_e = T_i, eV_w \approx -3kT_e$ which implies

$$\gamma_s = 7 \Rightarrow Q_w = 7kT_e\Gamma_w$$

and

$$\gamma_s = 5 \Rightarrow Q_w = 5kT_e\Gamma_w$$

7 How Do Edge Conditions Get Established?

Edge conditions include $n(a), T(a)$.

$$P_{\text{conduction, convection}} = P_{\text{in}} - P_{\text{rad}}$$

The equation holds assuming all power in is emitted in one way or another. Ignore fusion power.

Definition 7.1 (Recycling)**.** Recycling refers to particles returning to the plasma after neutralizing at the wall.

Assume

$$\Gamma_{\text{wall}} \approx \Gamma_{\text{in}} >> \Gamma_{\text{recycle}}$$

Γ_{in} can be due to gas injection, pellet injection, or neutral beams. Then in the toroidal direction,

$$\Gamma_{\text{wall}} = \frac{1}{2}n_ac_s\lambda(2\pi R \times 2)$$

For $T_e = T_i = t_a$,

$$Q_c = 7kT_a\Gamma_{\text{Wall}}$$

Example 7.1 (JET)**.** For $R = 3\text{cm}, P_{\text{conduction}} = P_\Omega - P_{\text{rad}} \approx 1\text{MW}, \Gamma_{\text{in}} = 10^{22}\text{s}^{-1}, \lambda \approx 1\text{cm}$,

$$\begin{aligned}
&\Rightarrow kT_a = \frac{P_{\text{cond}}}{7\Gamma_{\text{in}}} = \frac{10^6}{7 \times 10^{22}} = 1.4 \times 10^{-17}\text{J} \approx 100\text{eV} \\
&\Rightarrow c_s \approx 10^5 \text{m}\,\text{s}^{-1} \\
&\Rightarrow n_a \frac{\Gamma_{\text{in}}}{2\pi R c_s \lambda} = \frac{10^{22}}{2\pi 3 \times 10^5 \times 10^{-2}} \approx 5 \times 10^{17}\text{m}^{-3}
\end{aligned}$$

Example 7.2 (ITER)**.** For $R = 6\text{m}, P_{\text{cond}} \approx \frac{1}{2}P_{\text{in}} \approx 30\text{MW}, \Gamma_{\text{in}} \approx 10^{24}\text{s}, \lambda_{\text{SOL}} \approx 1\text{cm}$. Then

$$\begin{aligned}
&\Rightarrow kT_a = \frac{P_{\text{cond}}}{7\Gamma_c} = \frac{3 \times 10^7}{7 \times 10^{24}} = \frac{3 \times 10^7}{7 \times 10^{24}} = 4 \times 10^{-18}\text{J} \approx 30\text{eV} \\
&\Rightarrow c_s \approx 5 \times 10^4 \text{m}\,\text{s}^{-1} \\
&\Rightarrow n_a = \frac{\Gamma_c}{2\pi R c_s A} = \frac{10^{24}}{2\pi \times 6 \times 5 \times 10^4 \times 0.1} = 5 \times 10^{18}\text{m}^{-3}
\end{aligned}$$

8 The Non-Self Sustaining Plasma

Consider the Tokamak Scrape-off plasma.

$$\lambda_{\text{nfp}}^{\text{neutral}} = \frac{v\_\text{neutral}}{n_e \sigma_{iz} \overline{c}_E}$$

Consider $n_e = 10^{18}\text{m}^{-3}, \tau_{iz} = 10^{-20}\text{m}^2, \overline{c}_e \approx 10^6 \text{m}\,\text{s}^{-1}$, for $T_e = 10\text{eV}$.
Case 1: thermal D_2 (300K) implies $v \approx 10^3 \text{m}\,\text{s}^{-1}$. Then

$$\lambda = \frac{10^3}{10^{18} \times 10^{-20} \times 10^6} \approx 0.1\text{m}$$

and in case 2, where 100 eV backscattered D^0 implies $v \approx 10^5 \text{m}\,\text{s}^{-1}$, so $\lambda = 10\text{m}$

$$\begin{aligned}
\Gamma_{\text{out}} &= \overline{n}_e \times \frac{2\pi R \pi a^2}{\tau_p} \\
&\quad + S_0 2\pi R \times 2\pi a \times \lambda_{\text{edge}} \\
S_0 &= \frac{\overline{n}_e a}{2\tau_p \lambda_{\text{edge}}}
\end{aligned}$$

Example 8.1 (JET)**.** For $\overline{n}_e \approx 3 \times 10^9 \mathrm{m}^{-3}, a = 1\mathrm{m}, \tau_p = 1\mathrm{s}, \lambda_{\mathrm{wdge}} \approx 1\mathrm{cm}$. Then $S_0 = 1.5 \times 10^{21} \mathrm{m}^{-3}\,\mathrm{s}^{-1}$. From the momentum equation in the first section,

$$
\begin{aligned}
\frac{dM}{dx} &= \frac{S_0}{nc_s} \frac{1+M^2}{1-M^2} \\
\frac{n}{n_0} &= \frac{1}{1+M^2} \\
\frac{dM}{dx} &= \frac{S_0}{c_s} \frac{1_M^2}{n_0} \frac{1+M^2}{1-M^2} \\
\frac{S_0 x}{c_s n_0} &= \frac{M}{1+M^2}
\end{aligned}
$$

At $x = L, M = 1, \frac{S_0 L}{n_0 c_s} = \frac{1}{2} \Rightarrow \frac{1}{2} n_0 c_s = \Gamma = S_0 L$.

$$
\begin{aligned}
P_{\mathrm{div}} &= \gamma_S k T_{\mathrm{edge}} \Gamma_w - 2\lambda_{\mathrm{edge}} \times 2\pi R \\
&= \gamma_S k T_{\mathrm{edge}} S_0 L 2\lambda_{\mathrm{edge}} \times 2\pi R \\
&= \gamma_S k T_{\mathrm{edge}} \times \frac{\overline{n}_e a}{2\tau_p \lambda_{\mathrm{edge}}} L \times 2\lambda_{\mathrm{edge}} \times 2\pi R \\
k T_{\mathrm{edge}} &= \frac{P_{\mathrm{dev}} \tau_p}{\gamma_S 2\pi R a L \overline{n}_e}
\end{aligned}
$$

Example 8.2 (ITER)**.** $P_{\text{conduction, convection}} = 100\mathrm{MW}, R = 6\mathrm{m}, a = 2, \gamma_S = 7, \overline{n}_e \approx 10^{20}, \tau_p \approx 1\mathrm{s}, L = 20\mathrm{m}, D_\perp \approx 0.3\mathrm{m}^2\,\mathrm{s}^{-1}$. Then

$$
\begin{aligned}
k T_{\mathrm{edge}} &= \frac{10^8 \times 1}{2 \times 2\pi \times 6 \times 2 \times 20 \times 10^{20}} = 9.6 \times 10^{-17} \mathrm{J} \approx 600\mathrm{eV} \\
c_s &= \sqrt{\frac{2.95 \times 10^{-17}}{3.34 \times 10^{-22}}} = 2.4 \times 10^5 \mathrm{m}\,\mathrm{s}^{-1} \\
\lambda_{\mathrm{edge}} &= \sqrt{\frac{2D_\perp L}{c_s}} = \sqrt{\frac{2 \times 0.3 \times 20}{2.4 \times 10^5}} = 7\mathrm{mm} \\
S_0 &= \frac{10^{20} \times 2}{2 \times 1 \times 0.007} = \frac{\overline{n}_e a}{2\tau_p \lambda_{\mathrm{edge}}} = 1.4 \times 10^{22} \\
n_0 &= \frac{2 S_0 L}{c_s} = \frac{2 \times 1.4 \times 10^{22} \times 20}{2.4 \times 10^5} = 2.3 \times 10^{18} \mathrm{m}^{-3}
\end{aligned}
$$

9 Plasma-Surface Interactions

1. Material Erosion

2. Hydrogen Retention

9.1 Material Erosion

1. Physical Sputtering Think of this as billiard ball collisions. Yield increases with angle (up to 90°) for smooth surfaces, but it peaks before that for rough surfaces

2. Chemical Sputtering/Erosion The graph of yield against temperature looks like $\bigcap$, or $y = -x^2$

3. Melting If the surface melts, it becomes rough. B fields are usually directed such that they are close to parallel to the surface, to spread the power distribution. With increased roughness, B fields are more likely to hit the surface perpendicularly, which greatly increases power density.

4. Enhanced Bombardment Induced Erosion at high temperature This applies for any combinations, but for convenience, we use He^+ on C, so chemical effects can be ignored. Yield is constant until around 1000 K, when it grows exponentially.