Tokamaks

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1 Simple Pinches

Recall how simple pinches work. By the Biot-Savart Law, we obtain

$$I^2 = \frac{8\pi}{\mu_0} AnkT$$

Which is the basic pinch relation with $p = nkT$ and A being the cross-section area.

Example 1.1. With $n = 10^{20}, T = 10^4, A = 1$, we get $I = 10^6$A.

- end losses through electrodes
- inherently unstable (Magnetohydrodynamic instabilities)
- Operate quickly (high n, low τ)
- Feedback control of B (technically infeasible)
- Stiffen $\vec{B}$ by adding $B_{\text{axial}} > B_\theta$

2 Toroidal Pinches

2.1 Safety Factor q

Adding a plasma current I gives a poloidal B. A stellarator uses complex magnetic coils to give a twist to the magnetic field. Too much twist will lead to MHD instability, but some twist is needed for there to be confinement (cf self pinch).

2.2 Plasma Stability Determined by Exact Amount of Twist

Definition 2.1 (Safety Factor)**.**

$$q = \frac{rB_T}{RB_p}$$

Ideally, we want $q \approx 3$. q can also be thought of the number of toroidal loops needed to complete one poloidal loop.

Example 2.1. For a constant $j(r)$, then

$$B_\theta = \frac{\mu_0}{2\pi r} \int_0^r j dA = \frac{\mu_0 r j}{2}$$

which is proportional to r. Then q is a constant.

There is usually a sawteeth pattern found in the T_e/t graph, since as electron temperature goes up, MHD instability is more likely, so temperature falls.

2.3 First Stability Limit

The Kruskal-Shafranov Stability Limit (experimental) requires

$$Q(a) \geq 2.5$$

$$\begin{aligned} B_p(a) &= \frac{\mu_0 I_p}{2\pi a} \\ q(a) &= \frac{2\pi B_T a^2}{\mu_0 R I_p} \\ I_p &= \frac{2\pi B_T a^2}{\mu_0 R q(a)} \\ &\leq \frac{2\pi B_T a^2}{\mu_0 R (2.5)} \end{aligned}$$

Example 2.2. For JET, where $a = 1.5, R = 3, B_T = 3.5$, we require $I_p \leq 5 \times 10^6$. For ITER, where $a = 3, R = 6.2, B_T = 5.3$, we want $I_p \leq 7 \times 10^6$, actually 15MA.

If we make an ellipse-shaped donut, this allows us to use a greater current as above.

2.4 Second Stability Limit

Davis doesn't understand this, so it's fine if we don't either.
Recall

$$\beta = \frac{P_p}{P_m} = \frac{\sum nkT}{B^2/2\mu_0}$$

We similarly define

$$\beta_p = \frac{P_p}{B_\theta^2(a)/2\mu_0} = \frac{nk(T_e + T_i)}{B_\theta^2(a)/2\mu_0}$$

Observe that at a high plasma pressure, the poloidal field lines are distorted. The second stability limit is given by

$$\frac{B_\theta^2(a)}{2\mu_0} > \frac{a}{R_0} P_p$$

or

$$\beta_P < \frac{R_0}{a}$$

For $\frac{a}{R} = 0$, $B_\theta = 0$. For $q \geq 2.5$,

$$\begin{aligned} \frac{a^2 B_T^2}{2\mu_0} &\geq (2.5)^2 \frac{R^2 B_\theta^2}{2\mu_0} \\ &\geq (2.5)^2 R^2 \frac{a}{R} P_p \\ &\geq 6aRP_p \\ \beta &\leq \frac{a}{6R} \end{aligned}$$

For $\frac{a}{R} \approx \frac{1}{3}$, $\beta \leq 5\%$.

3 Tokamak Density Limits

The Greenwald limit gives

$$\overline{n}_e \leq n_G = \frac{I}{\pi a^2} \times 10^{20}$$

before MHD instabilities occur.

Example 3.1. For DIII-D, where $I = 1.5\text{MA}, a = 0.7, n_G = 1 \times 10^{20}$. For JET, where $I = 5\text{MA}, a = 1.25, n_G = 1 \times 10^{20}$. For ITER, $I = 15\text{MA}, a = 2, n_G = 1.2 \times 10^{20}$.