Lecture 29

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April 5, 2024

1 Recap

For a parallel case,

1.1 Incident

$$\tilde{\vec{E}}_i = (\cos\theta_i\hat{x} - \sin\theta_i\hat{z})e^{-jk_0n_1(x\sin\theta_i + z\cos\theta_i)}$$
$$\tilde{\vec{H}}_i = \frac{1}{\eta_1}e^{-jk_0n_1(x\sin\theta_i + z\cos\theta_i)}\hat{y}$$

1.2 Reflected

$$\tilde{\vec{E}}_r = \Gamma_\parallel(\cos\theta_r\hat{x} + \sin\theta_r\hat{z})e^{-jk_0n_1(x\sin\theta_r - z\cos\theta_r)}$$
$$\tilde{\vec{H}}_r = \frac{\Gamma_\parallel}{\eta_1}e^{-jk_0n_1(x\sin\theta_r - z\cos\theta_r)}\hat{y}$$

1.3 Transmitted

$$\tilde{\vec{E}}_t = \tau_\parallel(\cos\theta_t\hat{x} - \sin\theta_t\hat{z})e^{-jk_0n_2(x\sin\theta_t + z\cos\theta_t)}$$
$$\tilde{\vec{H}}_t = \frac{\tau_\parallel}{\eta_2}e^{-jk_0n_2(x\sin\theta_t + z\cos\theta_t)}\hat{y}$$

2 Total Internal Reflection

For $n_1 > n_2$, there is $\theta_i = \theta_c$ where $\sin\theta_t = 1$. At an angle greater than that, $\sin\theta_t > 1$, and

$$\cos\theta_t = \sqrt{1-\sin^2\theta_t} = \pm j\sqrt{\left(\frac{n_1}{n_2}\sin\theta_i\right)^2 - 1}$$

This is multiplied by $-jk_0n_2$ in the exponent. We cannot admit a solution that grows to infinity, so the only accepted solution is the negative j solution. The reflection coefficient becomes complex and has a magnitude of 1, since

$$\Gamma_\parallel = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} = \frac{jx-y}{jx+y}$$

3 Perpendicular Case

The electric field is perpendicular to the plane spanned by the wavevector and the normal.

3.1 Incident

$$\begin{aligned}\tilde{\vec{H}}_i &= \frac{1}{\eta_1}(-\cos\theta_i\hat{x} + \sin\theta_i\hat{z})e^{-jk_0n_1(x\sin\theta_i + z\cos\theta_i)}\\ \tilde{\vec{E}}_i &= e^{-jk_0n_1(x\sin\theta_i + z\cos\theta_i)}\hat{y}\end{aligned}$$

3.2 Reflected

$$\begin{aligned}\tilde{\vec{H}}_r &= \frac{\Gamma_\perp}{\eta_1}(\cos\theta_r\hat{x} + \sin\theta_r\hat{z})e^{-jk_0n_1(x\sin\theta_r - z\cos\theta_r)}\\ \tilde{\vec{E}}_r &= \Gamma_\perp e^{-jk_0n_1(x\sin\theta_r - z\cos\theta_r)}\hat{y}\end{aligned}$$

3.3 Transmitted

$$\begin{aligned}\tilde{\vec{H}}_t &= \frac{\tau_\perp}{\eta_2}(-\cos\theta_t\hat{x} + \sin\theta_t\hat{z})e^{-jk_0n_2(x\sin\theta_t + z\cos\theta_t)}\\ \tilde{\vec{E}}_t &= \tau_\perp e^{-jk_0n_2(x\sin\theta_t + z\cos\theta_t)}\hat{y}\end{aligned}$$

Similar to the parallel case, we can use boundary conditions to find the coefficients

$$\Gamma_{\perp} = \frac{\frac{\eta_2}{\cos\theta_t} - \frac{\eta_1}{\cos\theta_i}}{\frac{\eta_2}{\cos\theta_t} + \frac{\eta_1}{\cos\theta_i}}$$

and

$$\tau_{\perp} = \frac{\frac{2\eta_2}{\cos\theta_t}}{\frac{\eta_2}{\cos\theta_2} + \frac{\eta_1}{\cos\theta_i}}$$