Lecture 3

niceguy

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1 Definitions

We write down the definitions of some terms here for convenience.

Flux Density	$\Gamma = nv$
Reaction Rate	$r = n_b n_t \sigma v_b$
Collision Frequency	$\nu = v_b n_t \sigma$
Mean Free Path	$\lambda_b = \frac{1}{n_t \sigma}$
Beam Attenuation	$\Gamma = \Gamma_0 e^{-x/\lambda}$

2 Thermonuclear Fusion

The mean free path λ is on the order of 10^7m. This means we need to make the particles go really fast. If velocity follows a Maxwellian Distribution. Defining dN to be the particles with energy between E and $E + dE$, the distribution is represented by $f(E) = \frac{dN}{dE}$ which tends to $E^{-1/2}e^{-E/kT}$.

Recall

$$r = n_D n_T \overline{\sigma v} = n_D n_T \overline{\sigma(E)\sqrt{\frac{2E}{m}}} = n_D n_T \sqrt{\frac{2}{m}} \int f(E)\sigma(E)\sqrt{E} dE$$

Substituting $f(E)$, we eventually get

$$r = \frac{4}{\sqrt{\pi}} \left(\frac{M_r}{2kT} \right)^{3/2} \int_0^\infty \sigma(v) v^3 e^{-M_r v^2/2kT} dv$$

Where v is the relative velocity and $M_r = \frac{m_1 m_2}{m_1 + m_2}$ is the reduced mass.

Example 2.1. For monogenergetic plasma $E_D = E_T = 15\text{keV}$, with mean approach energy around 15 keV, if we take the mean energy, we have σ around 2×10^{-30} and v around 2.4×10^6. Then their product is 2.4×10^{-24}.

If it's 10 keV, we get $\overline{\sigma v} = 1.09 \times 10^{-22}$.

3 Energy Distribution Among Fusion Products

With conservation of momentum, we get

$$\begin{aligned} \frac{E_1}{E_2} &= \frac{m_1 v_1^2}{m_2 v_2^2} \\ &= \frac{v_1}{v_2} \\ &= \frac{m_2}{m_1} \end{aligned}$$

where we use $m_1 v_1 = m_2 v_2$ in the last equality.

$$\mathrm{D} + \mathrm{T} \to \mathrm{He}(3.5\text{MeV}) + n(14.1\text{MeV})$$

The benefits of this are that

- There is no heat transfer problem for thermal electricity generation, since neutrons can travel for a long distance
- It also allows neutrons to reach Li for tritium breeding.

The disadvantages are that

- Neutron Radiation
  - There is material damage because of neutron bombardment

  - There is induced radioactivity
  - Transmutations produce hydrogen and helium that leads to embrittlement

- Thermal cycle for electricity generation (thermodynamics)
- Only 20% of the energy is available for self-heating plasma

Definition 3.1 (Ignition)**.** Ignition refers to self-sustaining plasma.

Exotic Fuels

$$\mathrm{D} +^3 \mathrm{He} \to p(14.7\mathrm{MeV}) +^4 \mathrm{He}(3.6\mathrm{MeV})$$
$$\mathrm{H} +^{11} B \to 3^4\mathrm{He}$$

4 The Need for Plasma Confinement

4.1 Energy Balance

If we have a confinement time a hundredth of fusion time, energy output is around 5-6 times the energy input. Then confinement time becomes

$$\tau_C = \frac{1}{100n\overline{\sigma v}} = \frac{1}{100n \times 10^{-22}}$$

Rearranging yields $n\tau_C \approx 10^{20}$. The Lawson Criteria is thus

$$n\tau_C \geq 10^{20}$$

4.2

Plasma pressure is

$$P_p = n_D kT_D + n_T kT_T + n_e kT_e$$

Plasma is quasi-neutral, meaning net charge is practically 0, or $n_e = n_D + n_T$. For $T = 10\mathrm{keV}$ and $n_D = n_T = 10^{20}$, pressure is 6.4 atm.

4.3 Electrostatics

Net force acting on each particle due to an electric field is eE. Combining this with pressure,

$$\frac{\Delta p}{\Delta x} = -enE$$

From Maxwell's Equations,

$$\nabla \cdot \vec{E} = \frac{ne}{\varepsilon_0} \Rightarrow \frac{dE}{dx} = \frac{ne}{\varepsilon_0}$$

Substituting,

$$\frac{dp}{dx} + E\varepsilon_0 \frac{dE}{dx} = 0 \Rightarrow \frac{d}{dx}\left(p + \frac{1}{2}\varepsilon_0 E^2\right) = 0$$

Then the function in the derivative is a constant. Noting that the constant has to be greater than plasma pressure at some point ($E \neq 0$), then in the exterior, we need the plasma pressure to be less than $\frac{1}{2}\varepsilon_0 E_{\text{ext}}^2$. The math shows that given the values of ε_0 and plasma pressure, we need $E \approx 10^9$V/m.

4.4 Magnetic Fields

$$P_m = \frac{B^2}{2\mu_0}$$

This gives $B = 1.6$T, which is feasible.