Lecture 44

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1 General Definition of a Manifold

Recall that for a general manifold, we need only a chart from an open set in $\mathbb{R}^k_+$ to a neighbourhood of $P \in M^k$. A set is open in $\mathbb{R}^k_+$ when it is the intersection between an open set in $\mathbb{R}^k$ and the set R^k_+ itself.

Lemma 1.1. *Let $f : S \subseteq \mathbb{R}^m \to \mathbb{R}^k$, and suppose*

Definition 1.1 (Transition Functions)**.** Consider two coordinate patches α_0, α_1, where each map from U_i to V_i respectively, and $W = V_1 \cap V_0 \neq \emptyset$. Letting $W_i = \alpha_i^{-1}(W)$, we define $\alpha_1^{-1}\alpha_0$ to be the transition function.

Proof. Assume it admits an extension to an open set in $\mathbb{R}^k$. This is U itself if it is already open in $\mathbb{R}^k$. We will only (possibly) use the extension at a boundary point, where the derivative is determined (see last lecture). Therefore there is no confusion in labelling it (still) as α_i. Let $\alpha^{-1}(p_0) = z_0$. $D\alpha(x_0)$ has rank k, and this has to be true for an open neighbourhood of x_0. This is because we can find k column vectors that are linearly independent; the determinant, a continuous function, has to be locally nonzero, so the vectors are still linearly independent. Let $\pi : \mathbb{R}^n \to \mathbb{R}^k$ be the map that preserves exactly these k coordinates. Then $g = \pi \circ \alpha$ has a nonsingular derivative. By the inverse function theorem, g, g^{-1} are C^r, Then take a small open neighbourhood of A of $\mathbb{R}^n$ containing p, and define $\square$