1 Waves in Transmission Lines

Note: variables with a tilde denote phasors.

Recall we have

$$-\frac{\partial v}{\partial z} = R'i + L'\frac{\partial i}{\partial t} \tag{1}$$

$$-\frac{\partial i}{\partial z} = G'v + C'\frac{\partial v}{\partial t} \tag{2}$$

Using phasors and assuming

$$v(z,t) = \Re\{V(z)e^{j\omega t}\}$$

and similarly for i, we can solve for $\tilde{V}$ and $\tilde{I}$. Defining

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')} = \alpha + j\beta \tag{3}$$

Definition 1.1 (Characteristic Impedance)**.** The characteristic impedance is defined to be

$$Z_0 = \frac{R' + j\omega L'}{\gamma} \tag{4}$$

which is the quotient of V_0^+ and I_0^+.

Note for lossless lines, $Z_0 = \sqrt{\frac{L'}{C'}}$, and it is real. However, a lossy line can still have a real Z_0.

2 Physical Meaning of v, i on a Transmission Line

Recall

$$\tilde{V}^+ = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

Focusing on the first term,

$$\tilde{V}^+ = |V_0^+| \exp(j\phi_+ - \alpha z - j\beta z)$$

Going back to the time domain,

$$
\begin{aligned}
v^+(z,t) &= \Re\{|V_0^+| \exp(j\phi_+ - \alpha z - j\beta z + j\omega t)\} \\
&= |V_0^+| e^{-\alpha z} \cos(\omega t - \beta z + \phi_+)
\end{aligned}
$$

This is periodic in time and space. Period in time is $T = \frac{2\pi}{\omega}$, and wavelength is $\lambda = \frac{2\pi}{\beta}$. Without loss of generality, let $\phi_+ = 0$. Phase velocity is $\frac{\lambda}{T}$, or $v = f\lambda$.