Tokamaks

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1 Simple Pinches

Recall how simple pinches work. By the Biot-Savart Law, we obtain

$$I^2 = \frac{8\pi}{\mu_0} AnkT$$

Which is the basic pinch relation with $p = nkT$ and A being the cross-section area.

Example 1.1. With $n = 10^{20}, T = 10^4, A = 1$, we get $I = 10^6$A.

- end losses through electrodes
- inherently unstable (Magnetohydrodynamic instabilities)
- Operate quickly (high n, low τ)
- Feedback control of B (technically infeasible)
- Stiffen $\vec{B}$ by adding $B_{\text{axial}} > B_\theta$

2 Toroidal Pinches

2.1 Safety Factor q

Adding a plasma current I gives a poloidal B. A stellarator uses complex magnetic coils to give a twist to the magnetic field. Too much twist will lead to MHD instability, but some twist is needed for there to be confinement (cf self pinch).

2.2 Plasma Stability Determined by Exact Amount of Twist

Definition 2.1 (Safety Factor)**.**

$$q = \frac{rB_T}{RB_p}$$

Ideally, we want $q \approx 3$. q can also be thought of the number of toroidal loops needed to complete one poloidal loop.

Example 2.1. For a constant $j(r)$, then

$$B_\theta = \frac{\mu_0}{2\pi r} \int_0^r j dA = \frac{\mu_0 r j}{2}$$

which is proportional to r. Then q is a constant.

There is usually a sawteeth pattern found in the T_e/t graph, since as electron temperature goes up, MHD instability is more likely, so temperature falls.

2.3 First Stability Limit

The Kruskal-Shafranov Stability Limit (experimental) requires

$$Q(a) \geq 2.5$$

$$\begin{aligned} B_p(a) &= \frac{\mu_0 I_p}{2\pi a} \\ q(a) &= \frac{2\pi B_T a^2}{\mu_0 R I_p} \\ I_p &= \frac{2\pi B_T a^2}{\mu_0 R q(a)} \\ &\leq \frac{2\pi B_T a^2}{\mu_0 R(2.5)} \end{aligned}$$

Example 2.2. For JET, where $a = 1.5, R = 3, B_T = 3.5$, we require $I_p \leq 5 \times 10^6$. For ITER, where $a = 3, R = 6.2, B_T = 5.3$, we want $I_p \leq 7 \times 10^6$, actually 15MA.

If we make an ellipse-shaped donut, this allows us to use a greater current as above.

2.4 Second Stability Limit

Davis doesn't understand this, so it's fine if we don't either.
Recall

$$\beta = \frac{P_p}{P_m} = \frac{\sum nkT}{B^2/2\mu_0}$$

We similarly define

$$\beta_p = \frac{P_p}{B_\theta^2(a)/2\mu_0} = \frac{nk(T_e + T_i)}{B_\theta^2(a)/2\mu_0}$$

Observe that at a high plasma pressure, the poloidal field lines are distorted. The second stability limit is given by

$$\frac{B_\theta^2(a)}{2\mu_0} > \frac{a}{R_0} P_p$$

or

$$\beta_P < \frac{R_0}{a}$$

For $\frac{a}{R} = 0$, $B_\theta = 0$. For $q \geq 2.5$,

$$\begin{aligned} \frac{a^2 B_T^2}{2\mu_0} &\geq (2.5)^2 \frac{R^2 B_\theta^2}{2\mu_0} \\ &\geq (2.5)^2 R^2 \frac{a}{R} P_p \\ &\geq 6aRP_p \\ \beta &\leq \frac{a}{6R} \end{aligned}$$

For $\frac{a}{R} \approx \frac{1}{3}$, $\beta \leq 5\%$.

3 Tokamak Density Limits

The Greenwald limit gives

$$\overline{n}_e \leq n_G = \frac{I}{\pi a^2} \times 10^{20}$$

before MHD instabilities occur. I has units of MA and a has units of metres.

Example 3.1. For DIII-D, where $I = 1.5\text{MA}, a = 0.7, n_G = 1 \times 10^{20}$. For JET, where $I = 5\text{MA}, a = 1.25, n_G = 1 \times 10^{20}$. For ITER, $I = 15\text{MA}, a = 2, n_G = 1.2 \times 10^{20}$.

4 Low and High Confinement Modes

L mode is when there is low confinement, same for H mode. τ_E in H mode is approximately twice of that of low mode.

5 Emperical Scaling for τ_E

$$\tau^E_{\text{non-rad}} = \frac{a^I 2}{\chi_\perp}$$

Classically,

$$\chi_\perp = D_\perp = D\frac{\nu^2}{\Omega^2}, D = \frac{kT}{m\nu}\Omega = \frac{eB}{m}$$

Then

$$\tau^E_{\text{non-rad}} \propto \sqrt{T} B^2 a^2 n^{-1}$$

It makes sense that $\tau \propto \sqrt{T}$, since it is harder to confine with more energy. $\tau \propto B^2$ also makes sense; a greater magnetic field can confine better. Having it inversely proportional to n means confinement cannot be improved by increasing density; the effects cancel each other out. In practice,

$$\tau_L = 0.048\frac{I^{0.85} R_0^{1.2} a^{0.3} \kappa^{0.5} \overline{n}^{0.1} B_0^{0.2} A^{0.5}}{P^{0.5}}$$
$$\tau_H = 0.145\frac{I^{0.93} R_0^{1.39} a^{0.58} \kappa^{0.78} \overline{n}^{0.41} B_0^{0.15} A^{0.19}}{P^{0.69}}$$

where I is the plasma current in MA, κ is elongation, $\overline{n}$ is density in 10^{20}m^{-3}, A is the atomic mass of plasma ions, R_0 is the major radius, and P is input power in MW.

$\tau \propto R_0 a$, similar to a^2. It is not so good that $\tau \propto T^0$. There is a weak dependence on magnetic field, which is bad, but the (positive) dependence on number density is better than in theory.

Example 5.1 (ITER)**.** For $I = 15, R_0 = 6.2, a = 2, \kappa = 1.7, \overline{n} = 0.91, A = 2, P = 70, B_0 = 5.3$, we have $\tau_H = 3.9$.

Lawson Parameter (ITER):
For $N_G = 1.2 \times 10^{20}, \tau_H = 3.9$, so $n\tau = 4.2 \times 10^{20}$.