Lecture 17

niceguy

April 19, 2024

1 Ampere Maxwell

$$\oint_C \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{s} + \frac{d}{dt} \int_S \vec{D} \cdot d\vec{s}$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

The first term on the right hand side is the conduction current I_c, and the second is the displacement current I_d. The term on the left is the magnetomotive force V_{mmf}.

In phasors, the differential form becomes

$$\vec{\nabla} \times \tilde{\vec{H}} = \tilde{\vec{J}} + j\omega \tilde{\vec{D}}$$

Note: from Faraday's Law,

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{\nabla} \times \vec{E} = -\vec{\nabla} \cdot \frac{\partial \vec{B}}{\partial t}$$

$$0 = \frac{\partial}{\partial t} \left(\vec{\nabla} \cdot \vec{B} \right)$$

Where the constant has to vanish. This means we actually only have 7 relations; 2 vector equations from Gauss and Faraday, then one from Ampere. There are $16 - 7 = 9$ equations left, where $\vec{D}, \vec{B}, \vec{J}$ are all functions of $\vec{E}, \vec{H}$.

2 Simple Media

In these media, $\vec{D} = \varepsilon\vec{E}, \vec{B} = \mu\vec{H}, \vec{J} = c\vec{E}$. $\varepsilon = \varepsilon_0\varepsilon_r, \mu = \mu_0\mu_r$. Then

$$\vec{\nabla} \times \vec{E} = -\mu\frac{\partial\vec{H}}{\partial t}$$

and

$$\vec{\nabla} \times \vec{H} = \sigma\vec{E} + \varepsilon\frac{\partial\vec{E}}{\partial t}$$