Lecture 2

niceguy

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1 Basic Physics of Fusion

1.1 The Need for High Temperatures

We observe that fusion occurs where it is very hot (e.g. sun). This is because kinetic energy is needed (order of 100 KeV) to overcome the Coulomb barrier (around 10^{-14} metres). However, quantum tunneling can help reduce the energy needed.

For a monoatomic particle, energy is

$$E = \frac{3}{2}kT \tag{1}$$

which is on the order of 10 keV.

Reasons for Preferring Low z Fusion Fuels

- Coulomb repulsion is lowest
- Highest energy released per nucleon
- Radiation cooling is proportional to z^2
- Material erosion issues

1.2 Cross-Sections, etc.

Consider a box with particle density n_b. The length is defined such that particles traverse through it in 1 second. Cross-sectional area is 1 metre squared. Then the total flux is $n_b v_b$, where v_b is the velocity. Flux density

Γ_b is also $n_b v_b$, since area is 1. It has units of particles per metre squared per second.

Consider a cross section of the box with length dx, cross-sectional area 1 metre squared. Each target particle (with density n_t) has collision cross-section σ, and since dx is small enough, there is no overlap. Some particles are going to go through, but some will interact with target particles.

Definition 1.1. Reaction rate is denoted as r with units of events per volume per second.

$$r = n_b v_b n_t \sigma \tag{2}$$

Alternatively, one can see how r varies linearly with each of v_b, n_b, and n_t, and treat σ as a proportionality constant.

Definition 1.2. Collision frequency is denoted as ν_b, with units of collisions per second per beam particle.

Obviously,

$$\nu_b = v_b n_t \sigma \tag{3}$$

We have a similar equation for target particles

$$\nu_t = v_b n_b \sigma \tag{4}$$

Definition 1.3. We define collision time to be the mean time between collisions for a beam or target particle, i.

$$\tau = \frac{1}{\nu} \tag{5}$$

Definition 1.4. The mean free path is the distance a beam particle travels between collisions, i.e.

$$\lambda = v_b \tau_b \tag{6}$$

We consider also beam attenuation.

$$\text{Loss} = \Delta\Gamma = r dx$$

In other words,

$$\frac{d\Gamma}{dx} = -n_b v_b n_t \sigma = -\Gamma n_t \sigma$$

Rearranging yields,

$$\frac{d\Gamma}{dx} = -\frac{\Gamma}{\lambda}$$

whose solution is

$$\Gamma = \Gamma_0 e^{-\frac{x}{\lambda}} \tag{7}$$

We assume the target particles don't decay, or else it would be very sad. If the target is moving, we replace v_b with $|\vec{v_b} - \vec{v_t}|$ by magic of reference frames. We can even take the average of such if there is a distribution of velocities.

1.3 Target Particles with High Random Velocity

Assume the target particles are electrons. Compared to electrons, the beam is practically still, so we only really consider the mean speed of electrons. In the assignment, the value will be derived to be

$$\overline{c_e} = \sqrt{\frac{8kTe}{\pi m_e}}$$

We can then approximate

$$r = n_e n_b \sigma \sigma \overline{c_e} \tag{8}$$

1.4 Collision Cross-sectional Area

(Not) surprisingly, σ is a function of energy. It is shaped (really loosely) like a $\bigcap$. σ is small when energy is low, since the target particles occupy a small volume. It decreases when energy is high, because heuristically, particles need to "stay in the same place" long enough for events to occur.

$$\therefore r = n_t n_b \overline{\sigma(v)|\vec{v_t} - \vec{v_b}|}$$

Example 1.1. Rate could sometimes be expressed as

$$r = n_t n_b \overline{\sigma(E)\sqrt{\frac{2E}{m}}}$$

1.5 Cross-Sections from Experiments

$$\Gamma_{\text{out}} = \Gamma_{\text{in}} e^{-\frac{L}{\lambda}} \tag{9}$$

where L is the length of the box. This gives us λ.

- σ_{atom} is around 10^{-20}m^2
- σ_{nucl} is around 10^{-28}m^2, or one barn

1.6 Relative Velocity

Example 1.2. Given $\sigma_{D\to T}$ at $E_D = 100\text{eV}$. Since relative velocity is the same, both E_D and E_T can be found by $\frac{1}{2}mv^2$. The ratio of masses tell us $E_T = 150\text{keV}$.

2 Accelerater Fusion

100 keV in, 17 MeV out. But does not work because energy is lost through interactions with atoms instead of nucleons (8 orders of magnitude larger).

Remark. Energy lost is

$$dE = -n_t \mathcal{E}(E) dx$$

where $\mathcal{E}(E)$ is the stopping power.

Example 2.1. Witn $n_t = 10^{25}, \Delta x = 10^{-3}, E_{D^+} = 100\text{keV}, \mathcal{E} = 6 \times 10^{-22}$, we get

$$\Delta E = -10^{25} \times 6 \times 10^{-22} \times 10^{-3} = -6\text{keV}$$

Stopping distance can be derived by

$$\begin{aligned} L &= \int_0^L dx \\ &= \int_{E_{\text{in}}}^0 \frac{dE}{n_T \mathcal{E}(E)} \end{aligned}$$

We also say the range is equivalent to both

$$n_T L = \int_{E_{\text{in}}}^0 \frac{dE}{\mathcal{E}(E)}$$

Example 2.2. For 100 keV D^+ on T, and $n_T = 10^{19}\text{cm}^{-3}$, the range is

$$\frac{n_T L}{2} = 2\text{cm}$$

where the division by 2 is to convert the units to "per nucleon".

Fusion rate is

$$\begin{aligned} r dx &= n_b n_t \sigma v_b dx \\ &= n_D n_T \sigma v_b \frac{dE}{n_T \mathcal{E}(E)} \\ &= \Gamma_b \frac{\sigma(E)}{\mathcal{E}(E)} dE \end{aligned}$$

Reaction rate in units of reactions per area per second is

$$R = \int r dx$$

Yield is then

$$y = \frac{R}{\Gamma_b} = \int_{E_{\text{in}}}^{0} \frac{\sigma(E)}{\mathcal{E}(E)} dE$$

Example 2.3. For the same experiment as above, average $\sigma(E)$ is 1 barn, average $\mathcal{E}(E)$ is around 4×10^6 keV barns. The yield is then 2.5×10^{-5}. Unfortunately, energy efficiency is 0.4%.

If we use plasma, there wouldn't be parasitic reactions such as electron excitation. The major parasitic reaction, elastic collisions, have $\sigma_{\text{elastic}} \propto T^{-1.5}$, so we can get rid of that with $T \approx 10^8$K.