Lecture 23

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1 Plane Waves

Example 1.1 (Seawater)**.** In seawater, $\varepsilon_r = 80, \sigma = 4$). We are given that the magnetic field is

$$\vec{H}(0^+, t) = 100\cos(2\pi \times 10^3 t + 15^\circ)\hat{y}\text{mA m}^{-1}$$

We want to find $\vec{E}(z,t), \vec{H}(z,t)$, and the depth where $|\tilde{\vec{E}}(z)| = 0.01|\tilde{\vec{E}}(z = 0)|$.
The frequency from $\vec{H}$ is 1 kHz. Complex permittivity is

$$\varepsilon_c = \varepsilon - \frac{j\sigma}{\omega} = \frac{80}{36\pi} \times 10^{-9} - \frac{j4}{2000\pi}$$

so it is mostly imaginary. Here, seawater behaves as a good conductor, we we can use the approximation

$$\alpha = \beta = \sqrt{\pi f \mu \sigma} = 0.126$$

Then $\eta_c = (1 + j)\frac{\alpha}{\sigma} = 0.044e^{\frac{j\pi}{4}}$. The phasors are then

$$\tilde{\vec{E}} = |E_0|e^{j\phi_0}e^{-0.126z}e^{-j0.126z}$$

and

$$\tilde{\vec{H}} = \frac{|E_0|e^{j\phi_0}}{0.044e^{\frac{j\pi}{4}}}e^{-0.126z}e^{-j0.126z}$$

At $z = 0$, $\vec{H} = \Re\{\tilde{\vec{H}}(z = 0)e^{j\omega t}\}$. Then

$$\vec{H}(z = 0) = \frac{|E_0|}{0.044}\cos(\omega t + \phi_0 - 45^\circ)$$

Comparing with the given $\vec{H}$, we see that $\phi_0 = 60°$ and $|E_0| = 0.044 \times 100 = 4.4\text{mV}\,\text{m}^{-1}$. Substituting,

$$\vec{E}(z,t) = 4.44e^{-0.126z}\cos(2\pi \times 10^3 t - 0.126z + 60°)\hat{x}$$

$$\vec{H}(z,t) = 100e^{-0.126z}\cos(2\pi \times 10^3 t - 0.126z + 15°)\hat{y}$$

The required depth is $e^{-0.126d} = 0.01 \Rightarrow d = 37\text{m}$.