Lecture 7

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1 Transients in Tranmission Lines

For a DC source, $\beta = \frac{2\pi}{\lambda} = 0$, so

$$\begin{aligned} Z_{\text{in}} &= Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \\ &= Z_0 \frac{Z_L}{Z_0} \\ &= Z_L \end{aligned}$$

Before steady state, we have "waves" of voltage "correction" that look like travelling step functions. This is because no information can travel faster than the speed of light; it takes time for the current to "know" that the generator voltage has changed. Defining R_0, R_g, R_L to be the resistances of the transmission line, generator, and load respectively, the first wave is

$$v_1^+ = v_g \frac{R_0}{R_g + R_0}$$

When the wave passes, a reflected wave comes back and lowers the voltage

$$v_1^- = \Gamma_L v_1^+$$

where we define

$$\Gamma_L = \frac{R_L - R_0}{R_L + R_0}$$

The second positive wave is then

$$v_2^+ = \Gamma_g v_1^-$$

where

$$\Gamma_g = \frac{R_g - R_0}{R_g + R_0}$$

and so on. One can prove that the final voltage, or the sum, is

$$V = \frac{v_1^+(1 + \Gamma_L)}{1 - \Gamma_g \Gamma_L}$$

This is equivalent to our initial findings at steady state.