Lecture 4

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1 More on Lawson Criteria

Definition 1.1 (Disassembly Time)**.** We define disassembly time τ to be

$$\tau = \frac{r}{v_{\text{thermal}}}$$

At 10keV, $v_D \approx 10^6 \text{ms}^{-1}$. Substituting into the Lawson Criteria,

$$n\tau = 10^{20} \Rightarrow \frac{nr}{v_{\text{thermal}}} = 10^{20} \Rightarrow nr \approx 10^{26}$$

In air, we have $n_{\text{atm}} \approx 10^{25} \text{m}^{-3}$, which requires $r = 10\text{m}$, which is too big. Fusion energy is

$$E = \frac{4}{3}\pi r^3 n E_{DT} \approx 10^{17}\text{J}$$

which is 28 Megatons of TNT. This is all released in a time of $\tau = 10^{-5}\text{s}$, so it is an explosion.

If we try $n \approx 10^{31}\text{m}^{-3}$, then $r = 10^{-5}\text{m}$ and the energy of fusion of 10^5J, which is more reasonable. Time is $\tau \approx 10^{-11}\text{s}$, temperature 10keV, and pressure is to the order of 10^{12} atm. We need lasers to get to this point.

Here is a list of maximum values ever achieved.

- $\tau_E = 11\text{s}$
- $n_{\max} = 2 \times 10^{20}\text{m}^{-3}$
- $\overline{n_e}\tau_e \approx 10^{20}\text{m}^{-3}\,\text{s}$, where we have the line average of n_e

- $n_D(0)\tau_E T_i(0) = 1.53 \times 10^{21}\mathrm{keV\,s\,m^{-3}}$
- Pulse duration: 30 minutes
- $T_i(0) = 45\mathrm{keV}$
- $\langle\beta\rangle = 13\%$
- $Q = 0.68$ for actual DT
- $Q = 1.25$ (extrapolated)

2 Thermonuclear Reaction Power Density

Let's say we have a spherical reactor with energy flux Q. For it to be economically viable, $Q \geq 1\mathrm{MW\,m^{-2}}$. Material restrictions demand $Q < 10\mathrm{MW\,m^{-2}}$.

Example 2.1. Let's say we have a spherical plasma producing 1GW. Now

$$\frac{4}{3}\pi R^3 P_F = 10^9$$

By symmetry, the fusion energy density on the surface is the above divided by surface area, or $\frac{R}{3}P_\alpha$ where P_α (α particle makes up 20% of the fusion energy) is $\frac{1}{5}$ of P_F. Solving, $R \geq 1.25\mathrm{m}$. For it to be economically viable, we need $R \leq 9\mathrm{m}$. We have $P_f \approx 10^8\mathrm{W\,m^{-3}}$.

3 Radiation Losses

There is a minimum temperature beneath which power loss from radiation is greater than power generated from fusion. This is at a few keV for DT fusion, so it doesn't usually matter.

Blackbodies follow $A\sigma T^4$. Plugging the area to be around $100\mathrm{m}^2$ and temperature on the order of 10^8, energy produced is greater than the sun. Therefore, our plasma is not a blackbody. It doesn't absorb all of the wavelength it emits.

3.1 Bremsstrahlung

Breaking radiation. If an electron is deflected by a positive charge. $\lambda \approx$ 0.1nm, so this is in the form of xrays, whose mean free path is too long to be absorbed.

$$P \propto z^2\sqrt{T}$$

3.2 Line Radiation

This is from orbital electron transitions. This wouldn't be be an issue for smaller z because they are fully ionised.

3.3 Cyclotron Radiation

If an electron rotates about a magnetic field line.

$$P \propto n_e T_e B^2$$

The assumption is that if the reactor is big enough, most of this radiation will be reabsorbed.

4 Bremsstrahlung

Note: In this derivation, we're off by a factor of approximately 3 because of handwaviness.

$$P_{\text{rad}} = \frac{1}{4\pi\varepsilon_0} \times \frac{2}{3} \times \frac{e^2 a^2}{c^3} \tag{1}$$

This is the power radiated by an accelerating electrical charge (with $v << c$). We define b to be the minimum distance between the electron and the charge as it passes through. Now defining the electron to travel along the x axis, we have b lying on the y axis, and define $\theta = \arctan\left(\frac{b}{vt}\right)$. Then

$$F_y = F\sin\theta = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}\sin\theta = \frac{z_i e^2}{4\pi\varepsilon_0(b^2+v^2t^2)} \times \frac{b}{\sqrt{b^2+v^2t^2}}$$

Maximum force can be found by putting $t = 0$.

$$F_{\max} = \frac{z_i e^2}{4\pi\varepsilon_0 b} \Rightarrow a_{\max} = \frac{z_i e^2}{4\pi\varepsilon_0 b^2 m_e}$$

The impulse is equal to

$$I_Y = \int_{-\infty}^{\infty} F_y(t)dt = F_{\mathrm{y,max}}\Delta t$$

Now we have to find Δt. The integral is

$$I_y = \int_{-\infty}^{\infty} \frac{z_i e^2 b dt}{4\pi\varepsilon_0 (b^2 + v^2 t^2)^{3/2}} = \frac{2 z_i e^2}{4\pi\varepsilon_0 b v} \Rightarrow \Delta t = \frac{2b}{v}$$

Now the energy of radiation is the product of Δt and P_{rad}. Simplifying,

$$E_{\mathrm{rad}} \approx \frac{2b}{v} \times \frac{1}{4\pi\varepsilon} \times \frac{2}{3} \times \frac{e^2}{c^3} \left(\frac{z_i e^2}{4\pi\varepsilon_0 b^2 m_e}\right)^2$$

$\Gamma = n_e v_e \sigma n_i$ where $\sigma = 2\pi b db$. Since E is energy per collision and Γ is the unber of collisions about b, we have

$$dP = E\Gamma$$

Integrating,

$$P = \frac{8\pi e^6 n_e n_i z_i^2}{(4\pi\varepsilon_0)^3 \times 3 n_e^2 c^3} \int_{b_{\min}}^{\infty} \frac{db}{b^2}$$

$$\Delta y \Delta p \approx \hbar \tag{2}$$

where

$$\Delta p \approx m_e v_e$$

Substituting $\Delta y = b_{\min}$ and noting that

$$\frac{1}{2} m_e v_e^2 = \frac{3}{2} k T_e$$

we get a final result of

$$P = 5 \times 10^{-37} n_e n_i z_i^2 T_e^{1/2}$$

in reality. Note that T_e has units of keV.

5 Ignition

We can avoid line radiation by removing impurities, and cyclotron radiation by making the reactor big enough, but there is nothing we can really do about Bremsstrahlung.

Definition 5.1 (Ideal Ignition)**.**

$$P_\alpha = P_{\mathrm{br}}$$

For a DT reaction, this implies

$$\frac{\overline{\sigma v}(T_i)}{\sqrt{T_e}} \approx 3.6 \times 10^{-24}$$

In fact, sometimes we want radiation, or else the walls of the reactor would be hot enough to melt due to heat flux.