Lecture 19

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1 Plane Waves

E&M phasors satisfy

$$(\nabla^2 + k^2)\tilde{\vec{E}} = 0$$
$$(\nabla^2 + k^2)\tilde{\vec{H}} = 0$$

where
$$k^2 = \omega^2 \varepsilon_c \mu, \varepsilon_c = \varepsilon_0 \varepsilon_r + \frac{\sigma}{j\omega}$$

For a lossless medium $\sigma = 0$,

$$\tilde{\vec{E}} = \tilde{E}_x(z)\hat{z} \Rightarrow \tilde{E}_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

Plugging into Faraday's Law, we can solve for $\tilde{\vec{H}}$:

$$\tilde{\vec{H}} = \left(\frac{k}{\omega\mu} E_x^+ e^{-jkz} - \frac{k}{\omega\mu} E_x^- e^{jkz}\right)\hat{y}$$

Definition 1.1 (Intrinsic Impedance)**.**

$$\eta = \frac{\omega\mu}{k} = \sqrt{\frac{\mu}{\varepsilon}}$$

In free space, $\eta = \eta_0 = 120\pi\Omega \approx 377\Omega$. Then substituting,

$$\tilde{\vec{H}} = \left(\frac{E_x^+}{\eta} e^{-jkz} - \frac{E_x^-}{\eta} e^{jkz}\right)$$

If we only look at the + components, we can similarly find wave propogation speed, which is

$$v_p = \frac{\omega}{k} = \frac{1}{\sqrt{\varepsilon\mu}}$$

Definition 1.2 (Wavevector)**.** The wavevector $\vec{k}$ is a vector with magnitude k (as how we define wavenumbers) in the direction of wave propogation.

2 General Plane Waves

We shall solve the Helmholz equations for lossless media in general. We first solve it for an arbitrary component $\tilde{E}_x$. The solution is (by observation)

$$\tilde{E}_x = E_x e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

and $k_x^2 + k_y^2 + k_z^2 = k^2$.