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1 Dual Transformations

Recall for $T : V \to W, b \in \Omega^l(W)$, we define

$$T^*b(v_1, \ldots, v_l) = b(Tv_1, \ldots, Tv_l)$$

Similarly,

$$\alpha_*(x; v) = (\alpha(x), D\alpha \cdot v)$$

This gives the general transformation

$$\alpha^*\omega(x; v_1, \ldots, v_l) = \omega(\alpha(x); \alpha_*(x, v_1), \ldots, \alpha_*(x, v_l))$$

This preserves linear and wedge structure, as one can easily verify. Now we want to show that this commutes with d. By linearity, we need only show this for elementary dx_i or dx_I. For 1-forms,

$$\begin{aligned}
\alpha^*(dx_i)(x; v) &= dx_i(x)(\alpha_*(x; v)) \\
&= D\alpha(x)_i \cdot v \\
&= \sum_j D_j\alpha_i(x)v_j \\
&= \sum_j \frac{\partial \alpha_i}{\partial x_j} dx_j(v) \\
\alpha^*(dx_i) &= \sum_j \frac{\partial \alpha_i}{\partial x_j} dx_j \\
&= d\alpha_i
\end{aligned}$$