

Lecture 22

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March 4, 2024

1 Plane Waves in Lossy Media

For a lossy media where $G' \neq 0$, we consider two cases. For low G' , the plane wave decays in amplitude, but everything else stays the same. For higher G' , the waves become out of phase, and amplitude decays as expected.

Example 1.1. Consider the field $\vec{E} = \tilde{E}_x(z)\hat{x}, \vec{H} = \tilde{H}_y(z)\hat{y}$ in a medium $(\varepsilon, \mu, \sigma \neq 0)$.

Lossless	Lossy
$\tilde{E}_x(z) = E_0 e^{-jkz}$	$\tilde{E}_x(z) = E_0 e^{-jk_c z}$
$\vec{H} = \frac{\hat{k} \times \vec{E}}{\eta}$	$\vec{H} = \frac{\hat{k}_c \times \vec{E}}{\eta_c}$

Where $\varepsilon_c = \varepsilon - \frac{j\sigma}{\omega}$, $k_c = \omega \sqrt{\varepsilon_c \mu}$, $\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}}$. Depending on $\{\sigma, \omega\}$, ε_c can be almost real or even almost imaginary.

Example 1.2. For water, $\varepsilon_c = 81\varepsilon_0 - \frac{j4}{2\pi f}$. At 60Hz, $\varepsilon_c = 81\varepsilon_0(1 - j1.48 \times 10^7)$, which is almost imaginary. At 100MHz, $\varepsilon_c = 81\varepsilon_0(1 - j8.9)$, so it is simply complex.

For a good conductor, $\sigma \gg \omega\varepsilon$, then $\varepsilon \approx -\frac{j\sigma}{\omega}$. Substituting,

$$\begin{aligned} k_c &= \omega \sqrt{-\frac{j\sigma}{\omega} \mu} \\ &= \sqrt{-j} \sqrt{\omega \mu \sigma} \\ &= \frac{1-j}{\sqrt{2}} \sqrt{\omega \mu \sigma} \\ &= (1-j) \sqrt{\pi f \mu \sigma} \end{aligned}$$

Then $\tilde{\vec{E}}, \tilde{\vec{H}}$ are proportional to

$$e^{-j(1-j)\sqrt{\pi f \mu \sigma} z} = e^{-j\sqrt{\pi f \mu \sigma} z} e^{-\sqrt{\pi f \mu \sigma} z}$$

Definition 1.1 (Skin Depth). The skin depth of a medium is

$$\delta_s = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

Example 1.3. Skin depth of Copper at $f = 10\text{GHz}$.

Copper has conductivity $5.8 \times 10^7 \text{S m}^{-1}$. At this frequency,

$$\delta_s = \frac{1}{\sqrt{\pi \times 10^{10} \times 4\pi \times 10^{-7} \times 58 \times 10^7}} = 0.66 \mu\text{m}$$

These can be used as RF shields.

Example 1.4 (Microwave Ovens). How thick should microwave ovens be to ensure that microwaves do not leak? We typically use $3 - 5\delta_s$ at frequency of operation.

To quantify losses, consider the surface current density \vec{J}_s .

$$\begin{aligned} J &= \frac{J_s}{\delta_s} \\ \sigma E &= \frac{J_s}{\delta_s} \\ E &= \frac{J_s}{\sigma \delta_s} \end{aligned}$$

2 Complex Impedance

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} \approx \sqrt{\frac{\mu}{-\frac{j\sigma}{\omega}}} = \frac{1+j}{\sqrt{2}} \sqrt{\frac{\omega\mu}{\sigma}} = \frac{1+j}{\sigma} \sqrt{\pi f \mu \sigma} = \frac{1+j}{\sigma \delta_s}$$

3 "Good" Dielectric

$$\varepsilon \gg \frac{\sigma}{\omega}$$

Therefore

$$\varepsilon_c = \varepsilon \left(1 - \frac{j}{\omega \varepsilon} \right)$$

and

$$k_c = \omega \sqrt{\varepsilon_c \mu} = \omega \sqrt{\varepsilon \mu} \sqrt{1 - \frac{j\sigma}{\omega \varepsilon}} \approx k \left(1 - \frac{j\sigma}{2\omega \varepsilon} \right)$$

Frequency cancels out, so we can also write

$$k_c \approx \omega \varepsilon \mu - \frac{j\sigma}{2} \sqrt{\frac{\mu}{\varepsilon}}$$

and

$$\vec{\tilde{E}}, \vec{\tilde{H}} \propto e^{-j\omega \sqrt{\varepsilon \mu} z} e^{-\frac{\sigma}{2} \sqrt{\frac{\mu}{\varepsilon}} z}$$