

# Lecture 3

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## 1 Definitions

We write down the definitions of some terms here for convenience.

Flux Density	$\Gamma = nv$
Reaction Rate	$r = n_b n_t \sigma v_b$
Collision Frequency	$\nu = v_b n_t \sigma$
Mean Free Path	$\lambda_b = \frac{1}{n_t \sigma}$
Beam Attenuation	$\Gamma = \Gamma_0 e^{-x/\lambda}$

## 2 Thermonuclear Fusion

The mean free path  $\lambda$  is on the order of  $10^7$  m. This means we need to make the particles go really fast. If velocity follows a Maxwellian Distribution. Defining  $dN$  to be the particles with energy between  $E$  and  $E + dE$ , the distribution is represented by  $f(E) = \frac{dN}{dE}$  which tends to  $E^{-1/2} e^{-E/kT}$ .

Recall

$$r = n_D n_T \overline{\sigma v} = n_D n_T \overline{\sigma(E) \sqrt{\frac{2E}{m}}} = n_D n_T \sqrt{\frac{2}{m}} \int f(E) \sigma(E) \sqrt{E} dE$$

Substituting  $f(E)$ , we eventually get

$$r = \frac{4}{\sqrt{\pi}} \left( \frac{M_r}{2kT} \right)^{3/2} \int_0^\infty \sigma(v) v^3 e^{-M_r v^2 / 2kT} dv$$

Where  $v$  is the relative velocity and  $M_r = \frac{m_1 m_2}{m_1 + m_2}$  is the reduced mass.

**Example 2.1.** For monogenergetic plasma  $E_D = E_T = 15\text{keV}$ , with mean approach energy around 15 keV, if we take the mean energy, we have  $\sigma$  around  $2 \times 10^{-30}$  and  $v$  around  $2.4 \times 10^6$ . Then their product is  $2.4 \times 10^{-24}$ .

If it's 10 keV, we get  $\sigma \bar{v} = 1.09 \times 10^{-22}$ .

### 3 Energy Distribution Among Fusion Products

With conservation of momentum, we get

$$\begin{aligned} \frac{E_1}{E_2} &= \frac{m_1 v_1^2}{m_2 v_2^2} \\ &= \frac{v_1}{v_2} \\ &= \frac{m_2}{m_1} \end{aligned}$$

where we use  $m_1 v_1 = m_2 v_2$  in the last equality.



The benefits of this are that

- There is no heat transfer problem for thermal electricity generation, since neutrons can travel for a long distance
- It also allows neutrons to reach Li for tritium breeding.

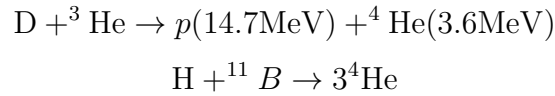
The disadvantages are that

- Neutron Radiation
  - There is material damage because of neutron bombardment

- There is induced radioactivity
- Transmutations produce hydrogen and helium that leads to embrittlement
- Thermal cycle for electricity generation (thermodynamics)
- Only 20% of the energy is available for self-heating plasma

**Definition 3.1** (Ignition). Ignition refers to self-sustaining plasma.

Exotic Fuels



## 4 The Need for Plasma Confinement

### 4.1 Energy Balance

If we have a confinement time a hundredth of fusion time, energy output is around 5-6 times the energy input. Then confinement time becomes

$$\tau_C = \frac{1}{100n\bar{\sigma}v} = \frac{1}{100n \times 10^{-22}}$$

Rearranging yields  $n\tau_C \approx 10^{20}$ . The Lawson Criteria is thus

$$n\tau_C \geq 10^{20}$$

### 4.2

Plasma pressure is

$$P_p = n_D k T_D + n_T k T_T + n_e k T_e$$

Plasma is quasi-neutral, meaning net charge is practically 0, or  $n_e = n_D + n_T$ . For  $T = 10\text{keV}$  and  $n_D = n_T = 10^{20}$ , pressure is 6.4 atm.

### 4.3 Electrostatics

Net force acting on each particle due to an electric field is  $eE$ . Combining this with pressure,

$$\frac{\Delta p}{\Delta x} = -enE$$

From Maxwell's Equations,

$$\nabla \cdot \vec{E} = \frac{ne}{\varepsilon_0} \Rightarrow \frac{dE}{dx} = \frac{ne}{\varepsilon_0}$$

Substituting,

$$\frac{dp}{dx} + E\varepsilon_0 \frac{dE}{dx} = 0 \Rightarrow \frac{d}{dx} \left( p + \frac{1}{2}\varepsilon_0 E^2 \right) = 0$$

Then the function in the derivative is a constant. Noting that the constant has to be greater than plasma pressure at some point ( $E \neq 0$ ), then in the exterior, we need the plasma pressure to be less than  $\frac{1}{2}\varepsilon_0 E_{\text{ext}}^2$ . The math shows that given the values of  $\varepsilon_0$  and plasma pressure, we need  $E \approx 10^9 \text{V/m}$ .

### 4.4 Magnetic Fields

$$P_m = \frac{B^2}{2\mu_0}$$

This gives  $B = 1.6\text{T}$ , which is feasible.