Lecture 30

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1 Brewster Angle

It is the angle $\theta_i = \theta_B$ for which $\Gamma_t(\theta_B) = 0$. For the parallel case,

$$\sin^2\theta_B = \frac{1 - \frac{\mu_2\varepsilon_1}{\mu_1\varepsilon_2}}{1 - \left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2}$$

For interfaces between non-magnetic media, $\mu_1 = \mu_2 = \mu_0$, so this simplifies to

$$\theta_B = \arcsin\frac{1}{\sqrt{1 + \frac{\varepsilon_1}{\varepsilon_2}}}$$

For the perpendicular case,

$$\sin^2\theta_B = \frac{1 - \frac{\mu_1\varepsilon_2}{\mu_2\varepsilon_1}}{1 - \left(\frac{\mu_1}{\mu_2}\right)^2}$$

The brewster angle exists only for media with different μ.

Example 1.1. Consider a plane wave travelling from air to a medium with $\varepsilon_r = 4$. Both are non-magnetic. The incident electric field is

$$\tilde{\vec{E}}_i = (4\hat{x} + 5\hat{y} - 4\hat{z})e^{-j\sqrt{2}\pi(x+z)}$$

From the exponential term, $\sin\theta_i = \cos\theta_i$, so $\theta_i = \frac{\pi}{4}$. Since $n_1 = 1$ for air, we have $k_0 = 2\pi$. The wavelength is then 1m, and the frequency is 300MHz.

To find the reflected wave, we can use linear superposition. For the parallel component, $\theta_r = \theta_i$, and

$$\begin{aligned}\tilde{\vec{E}}_r &= \Gamma_{\parallel} E_0(\cos\theta_i \hat{x} + \sin\theta_i \hat{z})e^{-jk_0 n_1(x\sin\theta_i - z\cos\theta_i)} \\ &= \Gamma_{\parallel}(\hat{x} + \hat{z})e^{-j\sqrt{2}\pi(x-z)}\end{aligned}$$

For the perpendicular component,

$$\tilde{\vec{E}}_r = 5\Gamma_{\perp}\hat{y}e^{-j\sqrt{2}\pi(x-z)}$$

Now $\eta_1 = 120\pi, \eta_2 = 60\pi$. Snell's law provides $\theta = 20.705°$, which gives

$$\Gamma_{\parallel} = -0.2038, \Gamma_{\perp} = -0.4514$$

Substituting these yield the full result.