Lecture 26

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1 Plane Wave Incidence on Material Boundaries

For simple media, the wavenumber is in general

$$k = \omega\sqrt{\varepsilon\mu} = \omega\sqrt{\varepsilon_0\varepsilon_r\mu_0\mu_r} = \omega\sqrt{\varepsilon_0\mu_0}\sqrt{\varepsilon_r\mu_r} = k_0 n$$

where we define the **refractive index** to be

$$n = \sqrt{\varepsilon_r\mu_r}$$

If the incident waves are

$$\begin{aligned}\tilde{\vec{E}}_i &= E_i e^{-jk_0 n_1 z}\hat{x}\\ \tilde{\vec{H}}_i &= \frac{E_i}{\eta_1} e^{-jk_0 n_1 z}\hat{y}\end{aligned}$$

The reflected wave will be

$$\begin{aligned}\tilde{\vec{E}}_r &= \Gamma E_i e^{jk_0 n_1 z}\hat{x}\\ \tilde{\vec{H}}_r &= -\Gamma\frac{E_i}{\eta_1} e^{jk_0 n_1 z}\hat{y}\end{aligned}$$

where Γ is the reflection coefficient. The transmitted wave is

$$\begin{aligned}\tilde{\vec{E}}_t &= \tau E_i e^{-jk_0 n_2 z}\hat{x}\\ \tilde{\vec{H}}_t &= \tau\frac{E_i}{\eta_2} e^{-jk_0 n_2 z}\hat{y}\end{aligned}$$

By continuity of tangential $\vec{E}$ at $z = 0$, we have

$$\begin{aligned}
\vec{E}_r + \vec{E}_i &= \vec{E}_t \\
\Gamma E_i \hat{x} + E_i \hat{x} &= \tau E_i \hat{x} \\
1 + \Gamma &= \tau
\end{aligned}$$

If there is no surface current, the same applies to $\vec{H}$, so

$$\begin{aligned}
\frac{E_i}{\eta_1} - \Gamma \frac{E_i}{\eta_1} &= \tau \frac{E_i}{\eta_2} \\
\frac{1 - \Gamma}{\eta_1} &= \frac{\tau}{\eta_2}
\end{aligned}$$

Solving,

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}, \tau = \frac{2\eta_2}{\eta_1 + \eta_2}$$

In a perfect conductor, $\Gamma = -1, \tau = 0$. In the first medium,

$$\begin{aligned}
\tilde{\vec{E}}(z < 0) &= E_i e^{-jk_0 n_1 z} - E_i e^{jk_0 n_1 z} \hat{x} \\
&= -2jE_0 \sin(k_0 n_1 z) \hat{x}
\end{aligned}$$

and similarly

$$\tilde{\vec{H}}(z < 0) = \frac{E_i}{\eta_1} e^{-jk_0 n_1 z} + \frac{E_i}{\eta_1} e^{jk_0 n_1 z} = \frac{2E_i}{\eta_1} \cos(k_0 n_1 z) \hat{y}$$

At $z = 0$,

$$\begin{aligned}
\hat{n} \times (\vec{H}_2 - \vec{H}_1) &= \vec{J}_s \\
\hat{z} \times \left(0 - \frac{2E_i}{\eta_1} \hat{y} \right) &= \vec{J}_s \\
\vec{J}_s &= \frac{2E_i}{\eta_1} \hat{x}
\end{aligned}$$