Lecture 6

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1 Observables

Consider a Hermitian (self-adjoint) operator A with an orthonormal system of eigenvectors that for a basis

$$A|\psi_n^i\rangle = a_n|\psi_n^i\rangle$$

and we can write

$$I = \sum_{n,i} |\psi_n^i\rangle\langle\psi_n^i|$$

1.1 Complete Set of Commuting Observables

We have a set of $A, B, C, \ldots$ that all commute. The eigenvalues of all operators determine a unique common eigenvector, up to a phase.

Example 1.1 (Spinless Particle)**.**

$$\psi(\vec{r}) \in \mathcal{F}$$

Consider 2 sets of functions that can be used to expand ψ. Defining

$$\xi_{\vec{r}}(\vec{r}') = \delta(\vec{r} - \vec{r}')$$

Then

$$\psi(\vec{r}) = \int \xi_{\vec{r}}(\vec{r}')\psi(\vec{r}')d\vec{r}'$$

Since

$$\langle \phi | \psi \rangle = \int \phi^* \psi d\vec{r}$$

we get

$$\begin{aligned}\langle \vec{r}' | \vec{r} \rangle &= \int_\delta (\vec{r}' - \vec{r}'') \delta(\vec{r} - \vec{r}'') d\vec{r}'' \\ &= \delta(\vec{r} - \vec{r}')\end{aligned}$$

and

$$\begin{aligned}\langle \vec{p}' | \vec{p} \rangle &= \int \frac{1}{(2\pi\hbar)^3} \exp\left(\frac{i}{\hbar}(\vec{p} - \vec{p}') \cdot \vec{r}\right) d\vec{r} \\ &= \delta(\vec{p} - \vec{p}')\end{aligned}$$

Similarly, we can identify

$$\langle \vec{r} | \psi \rangle = \psi(\vec{r})$$

and

$$\begin{aligned}\langle \vec{p} | \psi \rangle &= \int \frac{\exp\left(-\frac{i}{\hbar}\vec{p} \cdot \vec{r}'\right)}{(2\pi\hbar)^{3/2}} \psi(\vec{r}') d\vec{r}' \\ &= \psi(\vec{p})\end{aligned}$$