Lecture 5

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1 Summary

Note: These are all from the previous lecture

$$\begin{aligned}
\tilde{V}(z) &= V_0^+ e^{-\gamma d} + V_0^- e^{\gamma d} \\
\tilde{I}(z) &= \frac{V_0^+}{Z_0} e^{-\gamma d} - \frac{V_0^-}{Z_0} e^{\gamma d}
\end{aligned}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{V_0^-}{V_0^+}$$

2 Standing Waves

Replacing z with $-d$, and using Γ to reduce unknowns,

$$\begin{aligned}
\tilde{V}(d) &= V_0^+ e^{j\beta d} \left(1 + \Gamma e^{-j2\beta d}\right) \\
&= \frac{V_0^+}{Z_0} e^{j\beta d} \left(1 - \gamma e^{-j2\beta d}\right)
\end{aligned}$$

The standing wave pattern is the "envelope", or $|\tilde{V}(d)|$, which can be expressed as

$$|\tilde{V}(d)| = |V_0^+||1 + \Gamma e^{-j2\beta d}|$$

Definition 2.1 (Distance Dependent Reflection Coefficient).

$$\Gamma_d = \Gamma e^{-j2\beta d}$$

The maxima is obviously when Γ_d is real. The distance between two successive maxima is then $\frac{\pi}{\beta} = \frac{\lambda}{2}$. This explains why the standing wave pattern is constant when there is no reflection; $\Gamma = 0$. One can also determine, since Γ has a negative sign for current, that the standing wave for current is out of phase (by exactly π), so it forms a destructive interference pattern with that of voltage.

3 Impedance Transformation

$$\begin{aligned} Z(d) &= \frac{\tilde{V}(d)}{\tilde{I}(d)} \\ &= Z_0 \frac{1+\Gamma_d}{1-\Gamma d} \end{aligned}$$

To be continued...