Lecture 5

niceguy

September 21, 2023

1 More Linear Algebra

For any ket $|\psi\rangle$, we can write it as a column vector

$$\begin{pmatrix} \langle v_1|\psi\rangle \\ \langle v_2|\psi\rangle \\ \vdots \end{pmatrix}$$

Similarly for any bra $\langle\phi|$,

$$\begin{pmatrix} \langle \phi|v_1\rangle \\ \langle \phi|v_2\rangle \\ \dots \end{pmatrix}$$

$$\begin{aligned} \langle\phi|A|\psi\rangle &= \langle\phi IAI|\psi\rangle \\ &= \sum_{ij} \langle\phi|u_i\rangle\langle u_i|A|u_j\rangle\langle u_j|\psi\rangle \end{aligned}$$

If we have $C = AB$, then the inner term becomes

$$\langle u_i|AB|u_j\rangle = \sum_k \langle u_i|A|u_k\rangle\langle u_k|B|u_j\rangle$$

2 Change of Variables

In different bases, we can express the same ket as

$$|\psi\rangle = \sum_i c_i |u_i\rangle = \sum_i |u_i\rangle\langle u_i|\psi\rangle$$

and

$$|\psi\rangle = \sum_k c'_k |t_k\rangle = \sum_k |t_k\rangle\langle t_k|\psi\rangle$$

Then

$$\begin{aligned}
\langle u_m|\psi\rangle &= \sum_k \langle u_m|t_k\rangle\langle t_k|\psi\rangle \\
&= \sum_k S_{mk}\langle t_k|\psi\rangle \\
c_m &= \sum_k S_{mk} c'_k
\end{aligned}$$

We can get the same relation in reverse by taking the adjoint of S. Applying this a few times, we can do this for a matrix too.

$$A_{kl} = \sum_{ij} S^*_{ki} A_{ij} S_{jl}$$

3 Eigenkets

Definition 3.1 (Degeneracy)**.** We say an eigenket is degenerate if the eigenspace has more than 1 dimension.