Lecture 59

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Recall $\forall U \in \mathbb{R}^n$ open and $k \leq n \in \mathbb{N}$, $\Omega^k(U)$ is the space of $C^\infty k$-forms over U. Every element can then be described as

$$\omega = \sum_I f_I(x)dx_I, dx_I = dx_{i1} \wedge \cdots \wedge dx_{ik}, i1 < i2 < \cdots < ik$$

Recal we wanted to define $d : \Omega^k(U) \to \Omega^{k+1}(U)$ such that

1. For $k = 0, df = \sum_i (D_i f) dx_i$
2. $d(\omega \wedge \eta) = d\omega \wedge \eta + (-1)^k \omega \wedge d\eta$
3. $d(d\omega) = 0 \forall \omega$

We have shown that if such a d exists, it is unique. Defining ω as above, we have

$$d\omega = \sum_I df_I \wedge dx_I$$

Using this as the definition, it is tribial to show that this satisfies all properties.