Lecture 20

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April 21, 2024

1 Plane Waves

We can simplify the general solution as such

$$\tilde{\vec{E}} = \vec{E}_0 e^{-jk_x x} e^{-jk_y y} e^{-jk_z z} = \vec{E}_0 e^{-j\vec{k}\cdot\vec{R}}$$

Gauss Law for plane-waves gives (with no charge density)

$$\begin{aligned}
\vec{\nabla} \cdot \tilde{\vec{D}} &= \rho_v \\
&= 0 \\
\vec{\nabla}\left(\varepsilon \tilde{\vec{E}}\right) &= 0 \\
\vec{\nabla}\left(\vec{E}_0 e^{-j\vec{k}\cdot\vec{R}}\right) &= 0 \\
-j\vec{k} \cdot \vec{E}_0 e^{-j\vec{k}\cdot\vec{R}} &= 0
\end{aligned}$$

so $\vec{k}, \vec{E}$ are perpendicular. Since $\vec{H}$ is proportional to $\vec{k} \times \vec{E}$, these vectors form a right-handed triplet.

Example 1.1. Plane wave propogates on the $x - z$ plane at an angle of $\varphi = 30^\circ$ from the x axis, and $|\vec{E}| = 1\,\mathrm{V\,m^{-1}}, E_y = 0$. What is $\tilde{\vec{E}}, \tilde{\vec{H}}$? Frequency is $f = 3\mathrm{GHz}, \varepsilon_r = 2, \mu_r = 1$.
It is easy to find $\tilde{\vec{E}}$ given the direction.

$$\tilde{\vec{E}} = \left(-\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{z}\right) e^{-jk_x x - jk_z z}$$

Phase velocity is

$$\frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{\sqrt{\varepsilon_r \mu_r}} = 1.5 \times 10^8$$

Wavelength is thus

$$\lambda = \frac{v_p}{f} = \frac{1.5 \times 10^8}{3 \times 10^9} = 0.05\text{m} = 5\text{cm}$$

The magnitude of k is $\frac{2\pi}{\lambda} = 40\pi\text{m}$. Substituting,

$$\tilde{\vec{E}} = \left(-\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{z}\right) e^{-j40\pi(\sqrt{3}x/2 + z/2)}$$

Now

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = 120\pi \times \sqrt{\frac{1}{4}} = 60\pi\Omega$$

Combining, we have

$$\begin{aligned}
\tilde{\vec{H}} &= \frac{\hat{k} \times \tilde{\vec{E}}}{\eta} \\
&= \frac{1}{60\pi}\left(\frac{\sqrt{3}}{2}\hat{x} + \frac{1}{2}\hat{z}\right) \times (\tilde{E}_x\hat{x} + \tilde{E}_z\hat{z}) \\
&= \frac{\hat{i}}{60\pi}\left(-\sqrt{3}\frac{1}{2}\tilde{E}_z + \frac{1}{2}\tilde{E}_x\right)
\end{aligned}$$