Lecture 22

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1 Plane Waves in Lossy Media

For a lossy media where $G' \neq 0$, we consider two cases. For low G', the plane wave decays in amplitude, but everything else stays the same. For higher G', the waves become out of phase, and amplitude decays as expected.

Example 1.1. Consider the field $\tilde{\vec{E}} = \tilde{E}_x(z)\hat{x}, \tilde{\vec{H}} = \tilde{H}_y(z)\hat{y}$ in a medium $(\varepsilon, \mu, \sigma \neq 0)$.

Lossless	Lossy
$\tilde{E}_x(z) = E_0 e^{-jkz}$	$\tilde{E}_x(z) = E_0 e^{-jk_c z}$
$\tilde{\vec{H}} = \frac{\hat{k} \times \tilde{\vec{E}}}{\eta}$	$\tilde{\vec{H}} = \frac{\hat{k}_c \times \tilde{\vec{E}}}{\eta_c}$

Where $\varepsilon_c = \varepsilon - \frac{j\sigma}{\omega}, k_c = \omega\sqrt{\varepsilon_c \mu}, \eta_c = \sqrt{\frac{\mu}{\varepsilon_c}}$. Depending on $\{\sigma, \omega\}$, ε_c can be almost real or even almost imaginary.

Example 1.2. For water, $\varepsilon_c = 81\varepsilon_0 - \frac{j4}{2\pi f}$. At 60Hz, $\varepsilon_c = 81\varepsilon_0(1 - j1.48 \times 10^7)$, which is almost imaginary. At 100MHz, $\varepsilon_c = 81\varepsilon_0(1 - j8.9)$, so it is simply complex.

For a good conductor, $\sigma >> \omega\varepsilon$, then $\varepsilon \approx -\frac{j\sigma}{\omega}$. Substituting,

$$\begin{aligned}
k_c &= \omega\sqrt{-\frac{j\sigma}{\omega}\mu} \\
&= \sqrt{-j}\sqrt{\omega\mu\sigma} \\
&= \frac{1-j}{\sqrt{2}}\sqrt{\omega\mu\sigma} \\
&= (1-j)\sqrt{\pi f \mu\sigma}
\end{aligned}$$

Then $\tilde{\vec{E}}, \tilde{\vec{H}}$ are proportional to

$$e^{-j(1-j)\sqrt{\pi f\mu\sigma}z} = e^{-j\sqrt{\pi f\mu\sigma}z}e^{-\sqrt{\pi f\mu\sigma}z}$$

Definition 1.1 (Skin Depth)**.** The skin depth of a medium is

$$\delta_s = \frac{1}{\sqrt{\pi f\mu\sigma}}$$

Example 1.3. Skin depth of Copper at $f = 10\text{GHz}$.
Copper has conductivity $5.8 \times 10^7 \mathrm{S\,m^{-1}}$. At this frequency,

$$\delta_s = \frac{1}{\sqrt{\pi \times 10^{10} \times 4\pi \times 10^{-7} \times 58 \times 10^7}} = 0.66\mu\text{m}$$

These can be used as RF shields.

Example 1.4 (Microwave Ovens)**.** How thick should microwave ovens be to ensure that microwaves do not leak? We typically use $3 - 5\delta_s$ at frequency of operation.

To quantify losses, consider the surface current density $\vec{J}_s$.

$$\begin{aligned} J &= \frac{J_s}{\delta_s} \\ \sigma E &= \frac{J_s}{\delta_s} \\ E &= \frac{J_s}{\sigma\delta_s} \end{aligned}$$

2 Complex Impedance

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} \approx \sqrt{\frac{\mu}{-\frac{j\sigma}{\omega}}} = \frac{1+j}{\sqrt{2}}\sqrt{\frac{\omega\mu}{\sigma}} = \frac{1+j}{\sigma}\sqrt{\pi f\mu\sigma} = \frac{1+j}{\sigma\delta_s}$$

3 "Good" Dielectric

$$\varepsilon >> \frac{\sigma}{\omega}$$

Therefore

$$\varepsilon_c = \varepsilon \left(1 - \frac{j\sigma}{\omega\varepsilon}\right)$$

and

$$k_c = \omega\sqrt{\varepsilon_c \mu} = \omega\sqrt{\varepsilon\mu}\sqrt{1 - \frac{j\sigma}{\omega\varepsilon}} \approx k\left(1 - \frac{j\sigma}{2\omega\varepsilon}\right)$$

Frequency cancels out, so we can also write

$$k_c \approx \omega\sqrt{\varepsilon\mu} - \frac{j\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

and

$$\tilde{\vec{E}}, \tilde{\vec{H}} \propto e^{-j\omega\sqrt{\varepsilon\mu}z} e^{-\frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}z}$$