Lecture 3

niceguy

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1 Potential Barriers

Let's say we have a long wave chain $\psi(x, 0)$ that is approaching a potential barrier $V(x)$. We start by approximating ψ to have only one frequency. Then we can write

$$\psi(x, t) = \phi(x)e^{-i\frac{E}{\hbar}t} \tag{1}$$

Substituting into Schrödinger's Equation, we get (see complement H_1 in textbook)

$$\frac{d^2}{dx^2}\phi(x) + \frac{2m}{\hbar^2}(E - V(x))\phi(x) = 0 \tag{2}$$

Now if $E > V$, we put

$$E - V = \frac{\hbar^2 k^2}{2m} \tag{3}$$

This substituted into 2 gives

$$\phi(x) = Ae^{ix} + A'e^{-ikx}, |A| = |A'| \tag{4}$$

In quantum mechanics, in fact there is a nonzero probability that the wave is reflected.

$$\phi(x) = A_2 e^{ik_2 x} \tag{5}$$

If $E < V$, then we put

$$V - E = \frac{\hbar^2 \rho^2}{2m} \tag{6}$$

Plugging into 2 again, we get

$$\phi(x) = Be^{\rho x} + B' e^{-\rho x} \tag{7}$$

Classically, there is total reflection. However, in quantum mechanics, for $x > 0$ we have

$$\phi(x) = B' e^{-px} \tag{8}$$

2 Math facts of Quantum

Note: This corresponds to the second chapter of the textbook.

In section A of the textbook, we discuss the space of a 1 particle wave functions, where the integral of norm squared is 1. We define a vector space $\mathcal{F}$ with elements being functions that are square integrable and satisfies our physical assumptions (boundary conditions). This is an infinite-dimension vector space.

In sections B - F, we talk about bracket notation. We link $\psi(z) \in \mathcal{F}$ with $|\psi\rangle \in \mathcal{E}_{\mathcal{F}}$. We will look at more general kets later.

For kets $|\phi\rangle \neq |\psi\rangle$ in $\mathcal{E}$, introduce a scalar product that is a complex number. We define in $E_{\vec{r}}$

$$(|\phi\rangle, |\psi\rangle) = \int \phi^*(\vec{r}) \psi(\vec{r}) d\vec{r} \tag{9}$$

This is an inner product.

We also introduce a dual space $\mathcal{E}^*$. Recall a linear functional χ maps from a ket to a complex number. The dual space is the set of all χs. In fact, we can prove that all χ can be written uniquely as

$$\chi\left(|\psi\rangle\right) = \langle \chi | \psi \rangle \tag{10}$$