Lecture 36

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1 Waveguide Examples

Example 1.1. Consider a rectangular waveguide with $a = 0.9\text{cm}, b = 0.8\text{cm}$ operating at $f = 10\text{GHz}$. It is filled with a dielectric $\varepsilon_r = 9$ for $z < 0$, and air for $z > 0$. $\text{TE}_{1,0}$ mode is incident from $z < 0$.
The cutoff frequencies are

$$\begin{aligned}
f_c &= \frac{1}{2\sqrt{\varepsilon\mu}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \\
&= \frac{1}{2\sqrt{\varepsilon\mu}a} \\
&= \frac{3 \times 10^8}{2\sqrt{\varepsilon_r}a}
\end{aligned}$$

For $z < 0$, $\varepsilon_r = 9$, so

$$f_c = \frac{3 \times 10^8}{2 \times 3 \times 0.009}$$

and for $z > 0$, $\varepsilon_r = 1$, so

$$f_c = \frac{3 \times 10^8}{2 \times 0.009}$$

Now

$$\beta_{1,0} = k\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

Plugging in the numbers, $\beta(z < 0) = 166.3\pi, \beta(z > 0) = -j88.9\pi$, which means this is an evanescent mode.

At $z < 0$, $\vec{E}$ only has a y component, by definition of TE.

$$\tilde{E}_y = E_0 \sin\frac{\pi x}{a} e^{-j\beta_{1,0}z} + \Gamma E_0 \sin\frac{\pi x}{a} e^{j\beta_{1,0}z}$$

As for the magnetic field,

$$Z_{TE} = \frac{\omega\mu}{\beta_{1,0}} \approx 48\pi\Omega$$

Then

$$\tilde{H}_x = -\frac{\tilde{E}_y}{Z_{TE}}$$

Note: this is not exactly correct. The sign of the reflected wave has to be flipped, like taking the conjugate.

For $z > 0$,

$$\tilde{E}_y(z > 0) = TE_0 \sin\frac{\pi x}{a} e^{-88.9\pi z}$$

$$Z_{TE} = \frac{\omega\mu}{\beta_{1,0}} = \frac{2\pi \times 10 \times 10^9 \times 4\pi \times 10^{-7}}{-j88.9\pi} = j90\pi\Omega$$

Then

$$\tilde{H}_x = -\frac{\tilde{E}_y}{j90\pi} = \frac{jT}{90\pi} E_0 \sin\frac{\pi x}{a} e^{-88.9\pi z}$$

Since $\tilde{E}_y$ is continuous at $z = 0$,

$$E_0 \sin\frac{\pi x}{a}(1 + \Gamma) = TE_0 \sin\frac{\pi x}{a} \Rightarrow 1 + \Gamma T$$

From continuity of $\tilde{H}_x$ at $z = 0$,

$$-\frac{E_0 \sin\frac{\pi x}{a}}{Z^d}(1 - \Gamma) = -\frac{TE_0 \sin\frac{\pi x}{a}}{Z^a} \Rightarrow \frac{1 - \Gamma}{Z^d} = \frac{T}{Z^a}$$

Solving,

$$\Gamma = \frac{Z^a - Z^d}{Z^a + Z^d} = e^{j56.1^\circ}, T = \frac{2Z^a}{Z^d = Z^a} = 1.557 + j0.83$$

It makes sense that $|\Gamma| = 1$, since all power is reflected; this can be verified with the Poynting vector.

$$\bar{S} = \frac{1}{2}\Re\{\hat{y}\tilde{E}_y \times [\hat{x}\tilde{H}_x^* + \hat{z}\tilde{H}_z^*]\}$$

If we look at the z component,

$$\tilde{E}_y \tilde{H}_x^* = \tilde{E}_y \times \frac{\tilde{E}_y^*}{Z^*} \in i\mathbb{R}$$

since Z_{TE} is imaginary. Then $\bar{S}$ has no z component. Power transmitted is $\int \vec{S} \cdot d\vec{A}$, but the normal is along z, so no power is transmitted.