Lecture 10

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1 Power Flow in AC

$$\tilde{V}(d) = V_0^+ e^{j\beta d} + \Gamma V_0^+ e^{-j\beta d}$$
$$\tilde{I}(d) = \frac{V_0^+}{Z_0} e^{j\beta d} - \frac{\Gamma V_0^+}{Z_0} e^{-j\beta d}$$

Recall voltage is energy per unit charge (energy can have units eV). Then

$$p = \frac{dW}{dt} = v\frac{d(qv)}{dt}$$

Using our sinusoidal expressions for voltage and current,

Definition 1.1 (Instantaneous Power)**.** It is defined as

$$p(t) = V_0 I_0 \cos(\omega t + \phi_v)\cos(\omega t + \phi_i)$$

We are more interested in the average value of power. Using the product-to-sum formula,

$$\begin{aligned}
p(t) &= \frac{V_0 I_0}{2}\left[\cos(\phi_v - \phi_i) + \cos(2\omega t + \phi_v + \phi_i)\right] \\
\overline{p}(t) &= \frac{1}{T}\int_0^T p(t)dt \\
&= \frac{V_0 I_0}{2}\cos(\phi_v - \phi_i)
\end{aligned}$$

the sinusoidal term averages out to 0. This agrees with the intuition that power is maximised when phase difference is minimised.
If we use phasors, where $\tilde{V} = V_0 e^{j\phi_v}, \tilde{I} = I_0 e^{j\phi_i}$, then

$$\frac{1}{2}\tilde{V}\tilde{I}^* = \frac{1}{2}V_0 I_0 e^{j(\phi_v - \phi_i)}$$

whose real part yields average power.

2 Power Flow in Transmission Lines

Converting our equations $\tilde{V}, \tilde{I}$ into the time domain, $V_0^+ = |V_0^+|e^{j\phi_+}, \Gamma = |\Gamma|e^{j\theta_\Gamma}$. Expanding,

$$v(d) = |V_0^+|\cos(\omega t + \beta d + \phi_+) + |\Gamma V_0^+|\cos(\omega t - \beta d + \phi_+ + \theta_\Gamma)$$
$$i(d) = \frac{|V_0^+|}{Z_0}\cos(\omega t + \beta ad + \phi_i) - \frac{|\Gamma V_0^+|}{Z_0}\cos(\omega t - \beta d + \phi_+ + \theta_\Gamma)$$

Power is

$$p(d) = \frac{|V_0^+|^2}{Z_0}\cos^2(\omega t + \beta d + \phi_+) - \frac{|\Gamma V_0^+|^2}{Z_0}\cos^2(\omega t - \beta d + \phi_+ + \theta_\Gamma)$$

Note: the cross terms cancel out.
We call the first term **incident** power and the second **reflected** power. Taking the time average,

$$\overline{p}(d) = \frac{1}{2}\frac{|V_0^+|^2}{Z_0} - \frac{1}{2}\frac{|\Gamma V_0^+|^2}{Z_0} = \frac{|V_0^+|^2}{2Z_0}(1 - |\Gamma|^2)$$

Example 2.1. For $R_g = 10\Omega, l = 0.75\lambda, v_g = 1\text{V}, Z_0 = 50\Omega, v_p = 3 \times 10^8 \text{m s}^{-1}, R_L = 25\Omega$. This is a quarter-wave transformer, where we know

$$Z_{\text{in}} = \frac{Z_0^2}{Z_L} = 100\Omega$$

The input voltage is then

$$v_i = v_g \times \frac{100}{100 + 10} \approx 0.909\text{V}$$

and current is 100 times smaller than that. Input power (time averaged) is half of the product, or

$$p = \frac{v_i^2}{2Z_{\text{in}}} = \frac{0.909^2}{100} = 4.1322\text{mW}$$