Lecture 16

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1 Maxwell's Equations

Recall the following quantities.

Name	Symbol	Units
Electric field intensity	$\vec{E}$	$\mathrm{V\,m^{-1}}$
Electric field density	$\vec{D}$	$\mathrm{C\,m^{-2}}$
Magnetic field intensity	$\vec{H}$	$\mathrm{A\,m^{-1}}$
Magnetic flux density	$\vec{B}$	$\mathrm{T = Wb\,m^{-2}}$
Electric charge density	ρ_v	$\mathrm{C\,m^{-3}}$
Volume current density	$\vec{J}$	$\mathrm{A\,m^{-2}}$

There are 16 unknowns in total, with 5 vectors (15 unknowns) and 1 scalar.

1.1 Gauss Law

$$\oiint_S \vec{D} \cdot d\vec{s} = \int_V \rho_v dV = Q_{\text{enclosed}}$$

$$\vec{\nabla} \cdot \vec{D} = \rho_v$$

The sign convention is that positive charges are electric flux sources, and negative charges are sinks correspondingly.

1.2 Gauss Law for Magnetism

$$\oiint_S \vec{B} \cdot d\vec{s} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

1.3 Faraday Law

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{S}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

We can then define

$$V_{\text{emf}} = -\frac{d}{dt} \Phi_m$$

where Φ_m is the integral of magnetic flux over the surface. We call V_{emf} the electromotive force. We differentiate it from voltage, because we usually use them in time-independent scenarios.
When $\vec{E} = \vec{E}_0 \cos(\omega t + \phi_E), \vec{B} = \vec{B}_0 \cos(\omega t + \phi_B)$, we get phasors $\tilde{\vec{E}}, \tilde{\vec{B}}$. Substituting,

$$\vec{\nabla} \times \tilde{\vec{E}} = -j\omega \tilde{\vec{B}}$$

Example 1.1. Consider a closed circuit with 2 resistors $R_1 = 100\Omega$ and $R_2 = 200\Omega$ in series. Consider a magnetic flux pointing out of the plane with

$$\vec{B} = 10^{-3} \cos(2\pi \times 1000t)\hat{z}$$

Assuming the area of the loop is 1cm^2, find the voltages across each resistor.

$$\begin{aligned} V_{\text{emf}} &= -\frac{d}{dt} \int 10^{-3} \cos(2\pi \times 1000t) dx dy \\ &= 10^{-3} \times 10^{-4} \times 2000\pi \sin(2000\pi t) \\ &= 2\pi \times 10^{-4} \sin(2\pi \times 1000t) \end{aligned}$$

Assume this V_{emf} is a voltage source. Using the right-hand rule, the current/$\vec{E}$ field (for a positive V) goes counterclockwise, for $d\vec{s}$ to point in the $\hat{z}$ direction, as implicitly assumed in the integral. Then voltage division suffices.