

Lecture 27

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Example 0.1. A 10 GHz aircraft radar uses a narrow-beam scanning antenna mounted on a gimbal behind a dielectric radome. Even though the radome shape is far from planar, it is approximately planar over the narrow extent of the radar beam (couldn't finish copying)

At 10 GHz, the wavelength in air is 3 cm, and that in the material is

$$\lambda = \frac{\lambda_0}{\sqrt{\epsilon_r}} = \frac{3}{3} = 1\text{cm}$$

From transmission lines, there will be no reflection if thickness (total length) is an integer multiple of $\frac{\lambda}{2}$, so the radome will be stable for $d = 2.5\text{cm}$.

1 Incidence at an angle

Unlike in the normal case, we need to find the angles of reflection and transmission. All angles are measured with respect to the normal. The wavevectors are

$$\begin{aligned}\vec{k}_i &= k_0 n_1 (\sin \theta_i \hat{x} + \cos \theta_i \hat{z}) \\ \vec{k}_r &= k_0 n_1 (\sin \theta_r \hat{x} - \cos \theta_r \hat{z}) \\ \vec{k}_t &= k_0 n_2 (\sin \theta_t \hat{x} + \cos \theta_t \hat{z})\end{aligned}$$

We can split the \vec{E} field into parallel and perpendicular components.

$$\begin{aligned}\tilde{\vec{E}} &= E_0 (\cos \theta_i \hat{x} - \sin \theta_i \hat{z}) e^{-jk_0 n_1 \sin \theta_i x} e^{-jk_0 n_1 \cos \theta_i z} \\ \tilde{\vec{H}} &= \frac{1}{\eta_1} e^{-jk_0 n_1 \sin \theta_i x} e^{-jk_0 n_1 \cos \theta_i z} \hat{y}\end{aligned}$$

For the parallel components, the reflected waves are

$$\begin{aligned}\vec{\tilde{E}}_r &= \Gamma_{\parallel} (\cos \theta_i \hat{x} + \sin \theta_i \hat{z}) e^{-jk_0 n_1 \sin \theta_i x} e^{jk_0 n_1 \cos \theta_i z} \\ \vec{\tilde{H}}_r &= -\frac{\Gamma_{\parallel}}{\eta_1} e^{-jk_0 n_1 \sin \theta_i x} e^{jk_0 n_1 \cos \theta_i z} \hat{y}\end{aligned}$$

and the transmitted waves are

$$\begin{aligned}\vec{\tilde{E}}_t &= \tau_{\parallel} (\cos \theta_t \hat{x} - \sin \theta_t \hat{z}) e^{-jk_0 n_2 \sin \theta_t x} e^{-jk_0 n_2 \cos \theta_t z} \\ \vec{\tilde{H}}_t &= \frac{\tau_{\parallel}}{\eta_2} e^{-jk_0 n_2 \sin \theta_t x} e^{-jk_0 n_2 \cos \theta_t z}\end{aligned}$$

Applying boundary conditions,

$$\cos \theta_i e^{-jk_0 n_1 \sin \theta_i x} + \Gamma_{\parallel} \cos \theta_r e^{-jk_0 n_1 \sin \theta_i x} = \tau_{\parallel} \cos \theta_t e^{-jk_0 n_1 \sin \theta_i x}$$