Lecture 2

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1 Superpositions of Waves

$$\psi(x,t) = \int_{-\infty}^{\infty} \frac{g(k)}{\sqrt{2\pi}} \exp(i(kx - \omega(k)t)) \tag{1}$$

We showed that for a nonrelatavistic particle, the group velocity is equal to that of a classical particle. The phase velocity is $\frac{v}{2}$ for a nonrelatavistic particle, and $\frac{c^2}{v}$ otherwise.

Definition 1.1 (Group Velocity Dispersion)**.** The group velocity dispersion is defined as

$$\left(\frac{d^2\omega}{dk^2}\right)_{k_0}$$

In 3D, we know the wavefunction satisfies

$$i\hbar\frac{\partial\psi(\vec{r},t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi(\vec{r},t) \tag{2}$$

More generally,

$$i\hbar\frac{\partial\psi(\vec{r},t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi(\vec{r},t) + v(\vec{r})\psi(\vec{r},t) \tag{3}$$

We have stationary solutions of the form

$$\psi(\vec{r},t) = \phi(\vec{r})\chi(t)$$

$$\begin{aligned}
\phi(\vec{r})\frac{d\chi(t)}{dt} &= \left[-\frac{\hbar^2}{2m}\nabla^2\phi(\vec{r}) + v(\vec{r})\phi(\vec{r})\right]\chi(t) \\
\frac{d\chi(t)}{dt}\frac{1}{\chi(t)} &= \left[-\frac{\hbar^2}{2m}\nabla^2\phi(\vec{r}) + v(\vec{r})\phi(\vec{r})\right]\frac{1}{\phi(\vec{r})}
\end{aligned}$$

Since both sides are functions of t and $\vec{r}$ respectively, they must be constants. Calling that constant E and integrating,

$$\chi(t) = A\exp\left(-\frac{iEt}{\hbar}\right)$$

Note that E is also an eigenvalue of the operator H, where

$$H = -\frac{\hbar^2}{2m}\nabla^2 + v(\vec{r}) \tag{4}$$

This is obtained by letting RHS be equal to E.

In some problems, Es are discrete, and we have eigenfunctions

$$H\phi_n(\vec{r}) = E_n\phi_n(\vec{r}) \tag{5}$$

Writing $\omega_n = \frac{E_n}{\hbar}$, we get

$$\phi_n(\vec{r}, t) = \phi_n(\vec{r})\exp(-i\omega_n t) \tag{6}$$

and the general wavefunction becomes

$$\psi(\vec{r}, t) = \sum_n c_n\phi_n(\vec{r})e^{-i\omega_n t} \tag{7}$$

2 The Born Rule

For a normalised wavefunction,

$$d\mathcal{P}(\vec{r}, t) = |\psi(\vec{r}, t)|^2 d\vec{r} \tag{8}$$

This is the probability that a position **measurement** at time t will find the particle within $d\vec{r}$ of $\vec{r}$.