# Lecture 12

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## 1 Smith Chart Basic Operations

### 1.1 Impedance Transformation

For $Z_0 = 50\Omega, Z_L = 130 + j90$, we can find the normalised load $z_L = \frac{Z_L}{Z_0} = 2.6 + j1.8$. The reflection coefficient is

$$\Gamma(d = 0) = \Gamma_L = \frac{z_L - 1}{z_L + 1} = 0.6e^{j21.8^\circ} \Rightarrow \Gamma_d = 0.6e^{j(21.8^\circ - 2\beta d)}$$

The standing wave ratio is $\frac{1+|\Gamma|}{1-|\Gamma|}$. Recall also

$$z = \frac{1 + \Gamma_d}{1 - \Gamma_d} = \frac{1 + |\Gamma|e^{j(\theta_\Gamma - 2\beta d)}}{1 - |\Gamma|e^{\theta_\Gamma - 2\beta d}}$$

When $\theta_\Gamma - 2\beta d = 2n\pi$, $z$ becomes the standing wave ratio. We can read the SWR value as the value of $r$ at the crossing of the positive real axis.

$$2\beta d = 2 \times \frac{2\pi}{\lambda} d = \frac{4\pi d}{\lambda}$$

For $d = 0.3\lambda$, we can rotate (clockwise) the complex $\Gamma$ along the outer scale (from 0.25) by 0.3, and this gives $z_{\text{in}} = 0.26 + j0.12$. Denormalising, $Z_{\text{in}} = 12.7 + j5.8\Omega$. The distance to the maxima is $(0.25 - 0.22)\lambda = 0.03\lambda$, where $0.25\lambda$ is known to be the maximum, and $0.22\lambda$ is the angle of $\Gamma$ according to $z$. The minimum is a quarter wavelength after that, at $0.28\lambda$.

*Fun fact: a reflection about the origin/rotation by $\pi$ gives the admittance, since*

$$
\begin{aligned}
y &= \frac{1}{z} \\
&= \frac{1-\Gamma_d}{1+\Gamma_d} \\
&= \frac{1+(-\Gamma_d)}{1-(-\Gamma_d)}
\end{aligned}
$$