240A Winter 2022: Computing Assignment IV

DUE: 6pm on 3/3. Upload to Canvas an RMD file and either a PDF or html file.

Your compiled file should contain slides that you will present during discussion (if your group is chosen).

One selected group will present problem 1 and the other will present problem 2

1. Power of a t-test

You will generate data from a regression model with one right-hand-side variable. The model is:

$$y_i = \beta_0 + x_i\beta_1 + e_i,$$

where $\beta_0 = 0$, $x_i \sim iidN(0,1)$, and $e_i = \sum_{j=1}^{J} z_{ji}^2 - J$, where $z_{ji} \sim iidN(0,1)$. The error, e_i, is distributed as $\chi^2_{(J)}$ minus its mean. I chose this structure because it implies that the finite sample distribution of estimators and test-statistics is not normal.

You are interested in the power of a two-sided t-test of $H_0 : \beta_1 = 0$.

1. Simulate the power curve for the test for $J = 1$ and $n \in \{25, 100\}$. Plot the power against β_1 for each of the two values of n. Include on your plot the asymptotic power curve (which we derived in class).
2. Simulate the power curve for the test for $J \in \{1, 5\}$ and $n = 100$. Plot the power against β_1 for each of the two values of J. Include on your plot the asymptotic power curve (which we derived in class).
3. Simulate the power curve for the test for $J = 1$ and $\beta_1 \in \{2/n^{0.4}, 2/n^{0.5}, 2/n^{0.6}\}$. Plot the power against n for each of the three sequences of β_1.

2. Testing a hypothesis about the ratio of coefficients

Consider the model:

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + e_i$$

You want to test $H_0 : \beta_1/\beta_2 = \theta_0$. Your test will be based on a Wald statistic using either the bootstrap or asympototic theory.

Suppose the data generating process is $\beta_0 = 0$, $\beta_1 = 1$, $\beta_2 \in \{0.1, 0.5, 1\}$, $x_{ji} \sim iidN(0,1)$, and $e_i \sim iidN(0,3)$. Hansen (2021) shows this example in Section 9.17 of his book.

Questions

1. Derive the Wald statistic and apply the delta method to obtain its asymptotic null distribution. Call this statistic W_0.
2. It is possible to re-write the null hypothesis as $H_0 : \beta_1 - \theta_0\beta_2 = 0$. Derive the Wald statistic and apply the delta method to obtain its asymptotic null distribution. Call this statistic W_1.
3. Compare the size of four test procedures: (i) Reject if W_0 exceeds the 0.9 quantile of its asymptotic null distribution, (ii) Reject if W_1 exceeds the 0.9 quantile of its asymptotic null distribution, (iii) Reject if W_0 exceeds the 0.9 quantile of its bootstrap distribution under the null, (iv) Reject if W_1 exceeds the 0.9 quantile of its bootstrap distribution under the null.

Simulation Procedure

1. Draw a sample of size $n = 25$ from the DGP. Regress y_i on x_{1i} and x_{2i} to obtain the OLS estimates $\hat{\beta}_1$ and $\hat{\beta}_2$.
2. Compute W_0 and W_1. We are simulating the size of the testing procedures, which means that the null hypothesis is true. That means, when $\beta_2 = 0.1$, we test $H_0 : \beta_1/\beta_2 = 10$, when $\beta_2 = 0.5$, we test $H_0 : \beta_1/\beta_2 = 2$, and when $\beta_2 = 1$, we test $H_0 : \beta_1/\beta_2 = 1$. More simply, use $\theta_0 = 1/\beta_2$
3. Use a *pairs bootstrap* to draw 200 bootstrap samples. For each bootstrap resample, compute W_0^* and W_1^*. Remember that you need to re-center the statistic so that the null hypothesis is true in the bootstrap world.
4. Repeat step 1-3 a large number of times. For each run, you will return the result of the four tests.
5. Report the rejection rate and interpret the results.