Quantitative Methods 2

Tutorial 5

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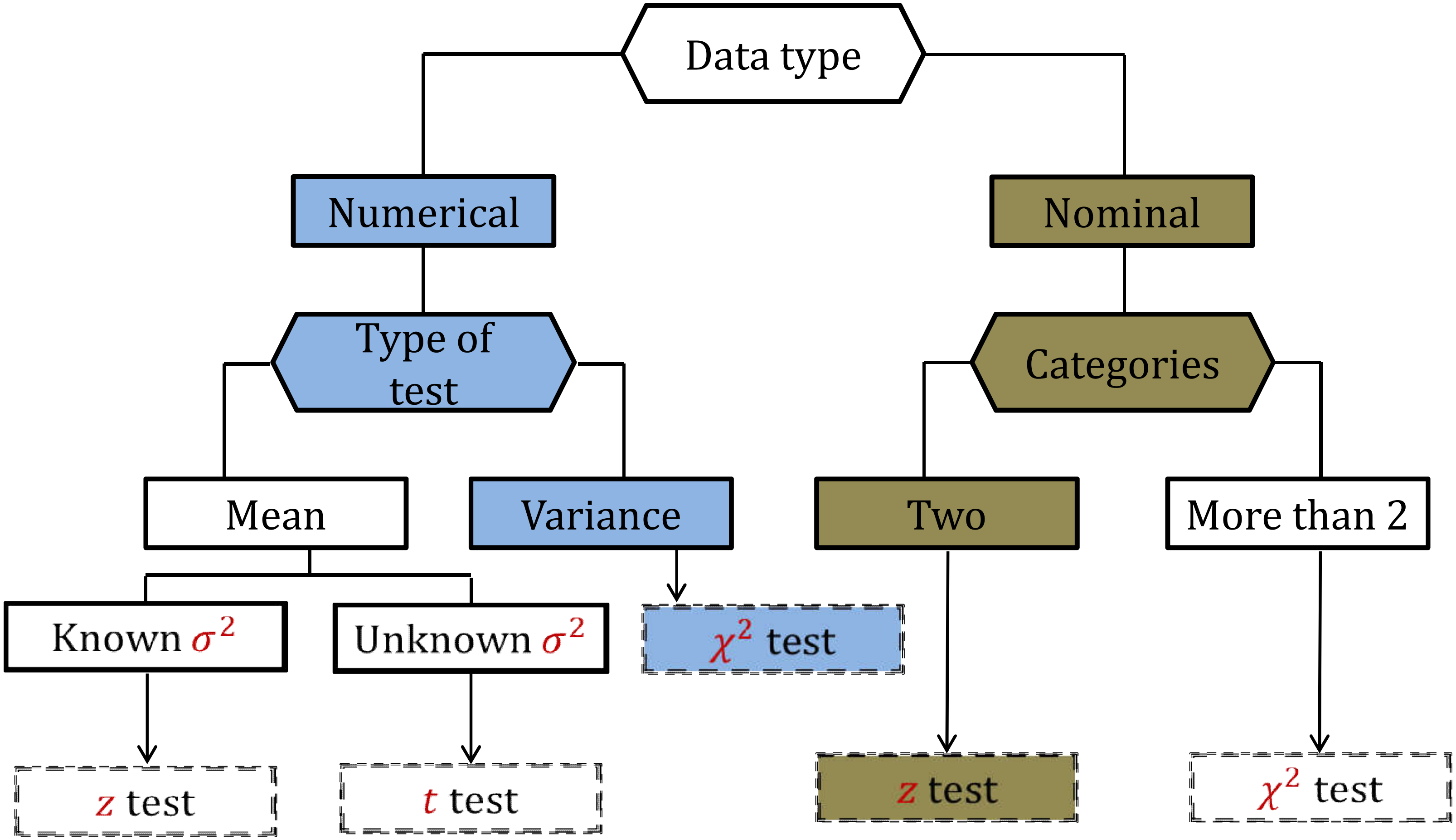
Last week

1. Paired-sample test
   - Parametric: Z test or t test
   - Nonparametric: sign test or Wilcoxon signed rank test
2. Independent samples test
   - Parametric: Z test or t test
   - Nonparametric: Wilcoxon rank sum test

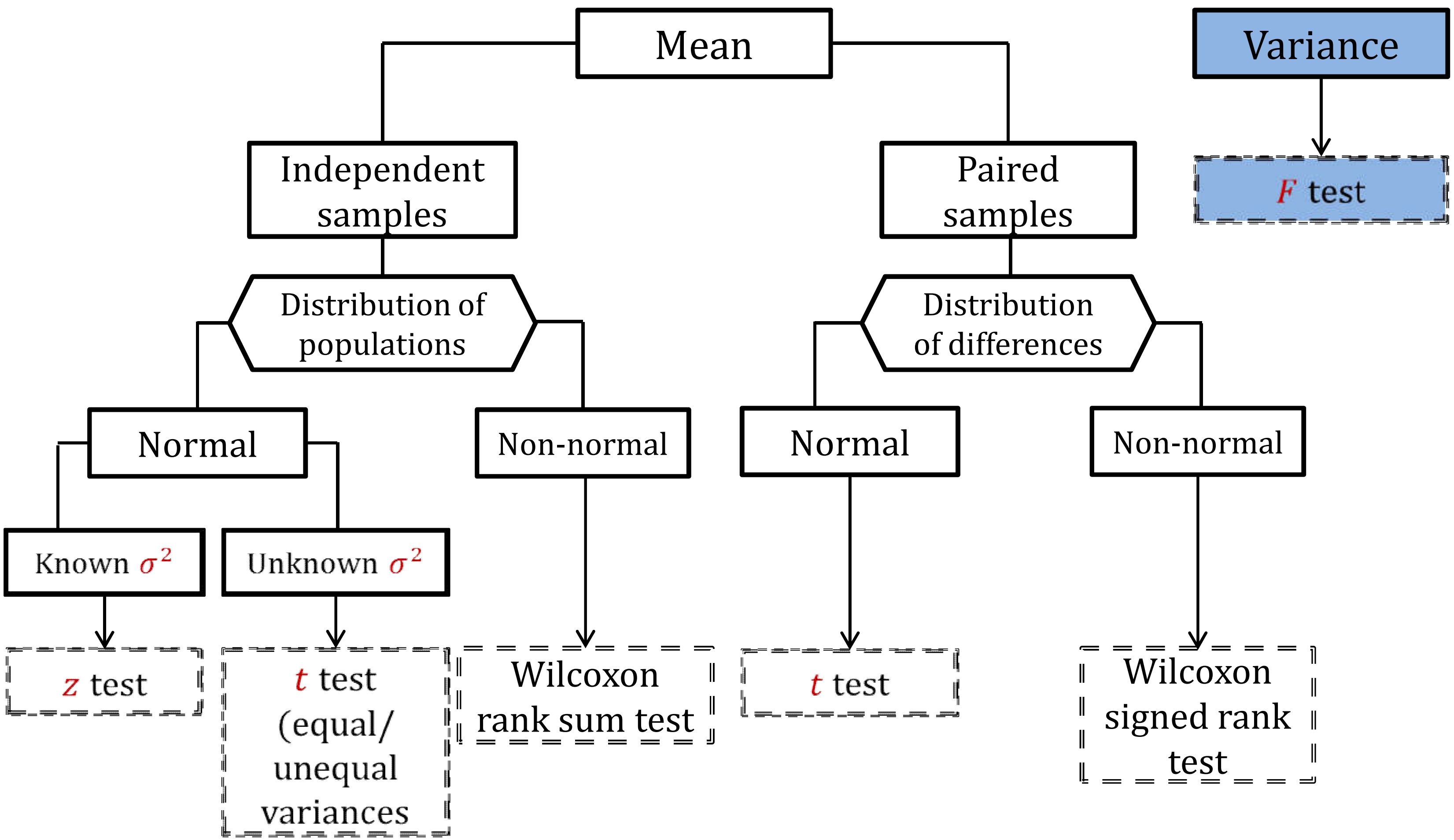
Tutorial 5

1. Test of variance
   - One population: χ^2 test
   - Two populations: F test
2. Test of proportion
   - One population: Z test
   - Two populations: Z test

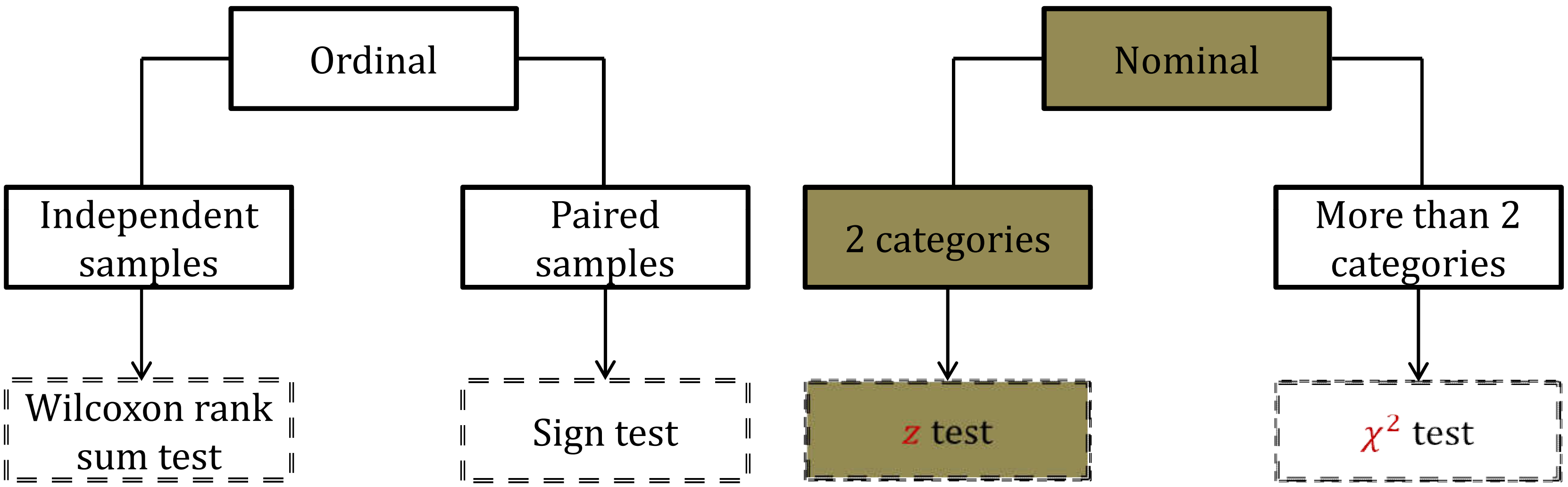
One population



Two populations - Numerical



Two populations - Ordinal and Nominal



Test of variance

1. One population

$$\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{\sigma^2}=\frac{(n-1)s^2}{\sigma^2}\sim\chi^2_{n-1}$$

$$CI=\left(\frac{(n-1)s^2}{\chi^2_{n-1,\alpha/2}},\frac{(n-1)s^2}{\chi^2_{n-1,1-\alpha/2}}\right)$$

- Lower bound value:

$$\frac{(n-1)s^2}{\sigma_l^2}=\chi_l^2=\chi^2_{n-1,\alpha/2}\Longrightarrow\sigma_l^2=\frac{(n-1)s^2}{\chi^2_{n-1,\alpha/2}}$$

- Upper bound value:

$$\frac{(n-1)s^2}{\sigma_u^2}=\chi_u^2=\chi^2_{n-1,1-\alpha/2}\Longrightarrow\sigma_u^2=\frac{(n-1)s^2}{\chi^2_{n-1,1-\alpha/2}}$$

Test of variance

2. Two populations

$$\frac{(n_1 - 1)s_1^2}{\sigma_1^2} \sim \chi^2_{n_1-1}; \frac{(n_2 - 1)s_2^2}{\sigma_1^2} \sim \chi^2_{n_2-1};$$

$$\frac{\chi^2_{n_1-1}/(n_1 - 1)}{\chi^2_{n_2-1}/(n_2 - 1)} = \frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2} \sim F_{n_1-1,n_2-1}$$

$$CI = \left(\frac{s_1^2/s_2^2}{F_{n_1-1,n_2-1,\alpha/2}}, \frac{s_1^2/s_2^2}{F_{n_1-1,n_2-1,1-\alpha/2}}\right)$$

Test of variance

Note the reversion of notation compared with t or Z distribution

	Lower quantile	Upper quantile
χ^2	$\chi^2_{n-1,1-\alpha/2}$	$\chi^2_{n-1,\alpha/2}$
F	$F_{n_1-1,n_2-1,1-\alpha/2}$	$F_{n_1-1,n_2-1,\alpha/2}$
t	$t_{\alpha/2,n-1}$	$t_{1-\alpha/2,n-1}$
Z	$Z_{\alpha/2}$	$Z_{1-\alpha/2}$
EViews commands $(n = 10, \alpha = 0.1)$ $(n_1 = 10, n_2 = 20)$	scalar chi_l=@qchisq(0.05,9) scalar f_l=@qfdist(0.05,9,19) scalart_l=@qtdist(0.05,9) scalar z_l=@qnorm(0.05)	scalar chi_u=@qchisq(0.95,9) scalar f_u=@qfdist(0.95,9,19) scalart_u=@qtdist(0.95,9) scalar z_u=@qnorm(0.95)

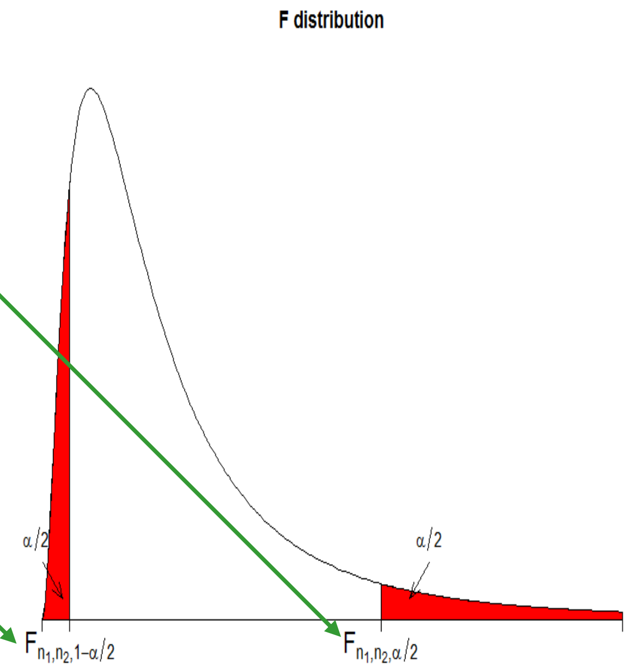
The F distribution

- Non-negative, asymmetric distribution
- Paper F distribution table

usually only reports the right hand side critical values ($F_{n_1,n_2,\alpha/2}$)

- To derive $F_{n_1,n_2,1-\alpha/2}$, use:

$$F_{n_1,n_2,1-\alpha/2} = \frac{1}{F_{n_2,n_1,\alpha/2}}$$



Exercise 2b

- What if:

$$H_0: \sigma_1^2 = 2\sigma_2^2; H_A: \sigma_1^2 \neq 2\sigma_2^2$$

$$\Rightarrow H_0: \frac{\sigma_1^2}{2\sigma_2^2} = 1 \,; H_A: \frac{\sigma_1^2}{2\sigma_2^2} \neq 1$$

Test statistic:

$$F_{obs} = \frac{s_1^2}{2s_2^2}$$

- For convenience, e.g., when using a paper F distribution table, choose the population with the greater sample variance as the numerator

$$s_1^2 > s_2^2 \Longrightarrow \frac{s_1^2}{s_2^2} > 1 \Longrightarrow \text{right side test}$$