Quantitative Methods 2

Tutorial 9

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Last week

- Simple linear regression

$$E(Y) = \beta_0 + \beta_1 X$$

$$Y = \beta_0 + \beta_1 X + \varepsilon$$

- Multiple linear regression

$$E(Y) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_k X_k$$

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_k X_k + \varepsilon$$

Last week

$$SST = SSR + SSE \Leftrightarrow \sum_{i=1}^{n}(y_i - \bar{y})^2 = \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2 + \sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

- Coefficient of determination (model fit):

$$R^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST}$$

$$\bar{R}^2 = 1 - \frac{SSE/(n-k-1)}{SST/(n-1)} = 1 - \frac{n-1}{n-k-1}(1-R)^2$$

- Standard error of regression: $s_\varepsilon = \sqrt{\frac{SSE}{n-k-1}}$

Last week

- Hypothesis test statistic:
  - Single coefficient: $t = \frac{\widehat{\beta}_i - \beta_{0,i}}{s_{\widehat{\beta}_i}}$
  - Overall model significance/utility: $F = \frac{MSR}{MSE} = \frac{SSR/k}{SSE/(n-k-1)} = \frac{R^2/k}{(1-R^2)/(n-k-1)}$
    - H_0: $\beta_1 = \beta_2 = \cdots = \beta_k = 0$
    - H_A: At least one slope is not 0
- Interpretation of coefficients:
  - Positive or negative?
  - How much?
  - Significant (at what level)?
  - Multiple linear regression: "Holding other variables constant…"
- Confidence interval estimation: $\hat{\beta}_i \pm t_{\alpha/2,n-k-1} \times s_{\widehat{\beta}_i}$

Tutorial 9

- General F test
  - Compare unrestricted against restricted model
  - Test general linear restriction of parameters
  - General expression: $F = \frac{n-k-1}{m}\frac{SSE_r - SSE}{SSE}$
  - Same dependent variable: $F = \frac{n-k-1}{m}\frac{R^2 - R_r^2}{1-R^2} = \frac{(R^2 - R_r^2)/m}{(1-R^2)/(n-k-1)}$

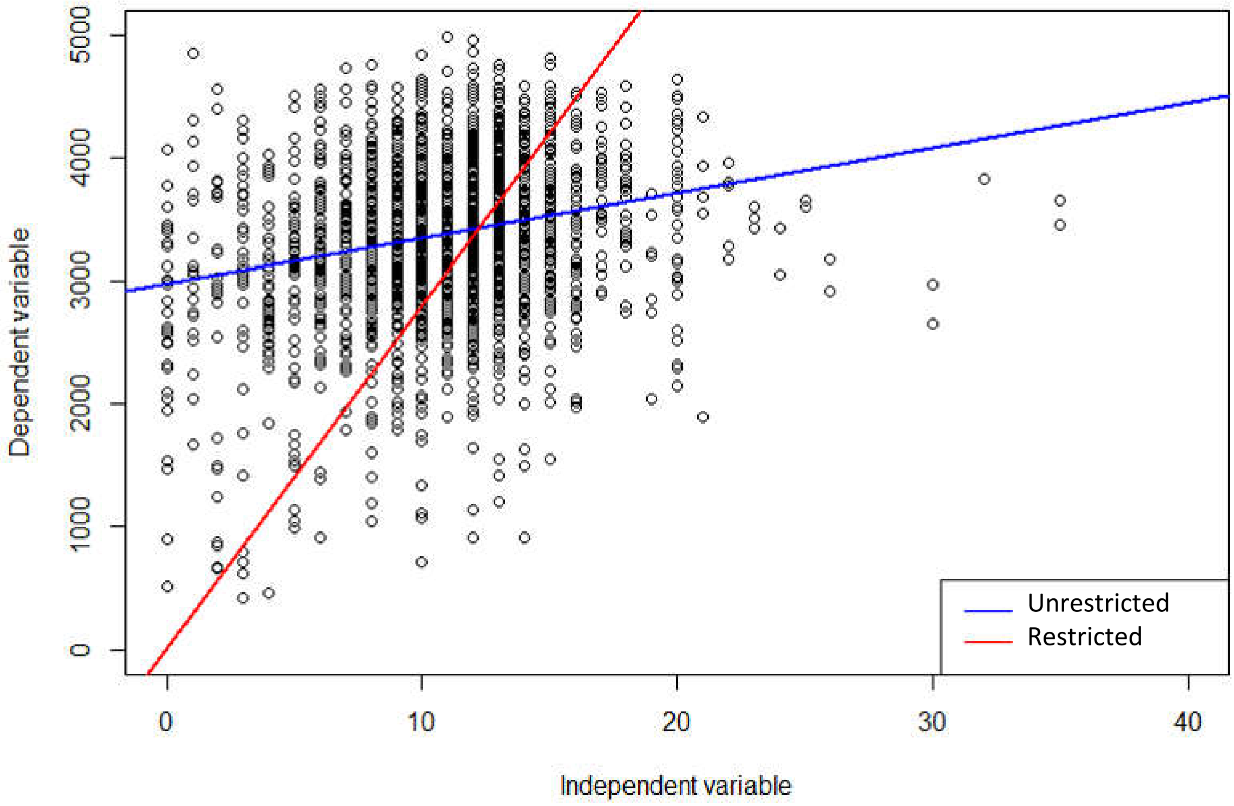
- Proof: $\frac{SSE_r - SSE}{SSE} = \frac{(SST_r - SSR_r) - (SST - SSR)}{SST - SSR}$

For models with the same dependent variable: $SST_r = SST$

Hence: $\frac{SST_r - SSR_r - (SST - SSR)}{SST - SSE} = \frac{SSR - SSR_r}{SST - SSR} = \frac{(SSR - SSR_r)/SST}{(SST - SSR)/SST} = \frac{R^2 - R_r^2}{1 - R^2}$

Tutorial 9

- General F test



Tutorial 9

- General F test
  - Multiple restrictions of coefficients

$$\beta_1 = 0, \beta_2 = 0$$

  - Restriction of multiple coefficients

$$\beta_1 = 2\beta_2$$

  - Both

$$\beta_1 = 3, \beta_2 = 1.5\beta_3$$

Exercise 2c

- Model (restricted)

$$time = \beta_0 + \beta_1 depart + 2reds + \beta_3 trains + \varepsilon$$

- Estimation

$$time - 2reds = \beta_0 + \beta_1 depart + \beta_3 trains + \varepsilon$$

EViews: (time – 2*reds) c depart trains

- Can't do:

EViews: time c depart 2*reds trains

$$time = \beta_0 + \beta_1 depart + \beta_2 2reds + \beta_3 trains + \varepsilon$$

Exercise 2d

- Unrestricted model

$$time = \beta_0 + \beta_1 depart + \beta_2 reds + \beta_3 trains + \varepsilon$$

C(1) C(2) C(3) C(4)

- Linear restriction test
  - $H_0: \beta_3 = 3\beta_2$; $H_A: \beta_3 \neq 3\beta_2$
  - Using Eviews on the unrestricted model: C(4) = 3*C(3)
  - Using general F test: Estimate the restricted model

$$time = \beta_0 + \beta_1 depart + \beta_2 (reds + 3trains) + \varepsilon$$

Exercise 2e

- Unrestricted model

$$time = \beta_0 + \beta_1 depart + \beta_2 reds + \beta_3 trains + \varepsilon$$

- Restricted model

$$time - 1.8 reds - 3.2 trains = \beta_0 + \beta_1 depart + \varepsilon$$

- $F = \frac{n-k-1}{m} \frac{SSE_r - SSE}{SSE} = 6.76$

EViews p-values

Parameter	Population	Test	EViews default reported p-values
Variance	One population	χ^2 test	One-sided, smaller value (see Week 4 lecture slides) Note: it's more straightforward to rely on the comparison between the test statistic χ^2_{obs} and the critical value χ^2_{cr}
	Two populations	F-test	Two-sided (see Week 4 lecture slides, especially for one-sided test hypotheses and statistic formation)
Proportion	One population	Z-test	Two-sided
	Two populations	t-test (to produce approximated results)	Two-sided

EViews p-values

Parameter	Population	Test	EViews default reported p-values
Correlation and regression	Pearson correlation coefficient	t-test	Two-sided
	Coefficient (slopes or intercept)	t-test	Two-sided
	Model significance/utility	F-test	One-sided
	Linear restriction of coefficient	One conjecture ($<$, $>$ or $=$): t-test or F-test Note: the conjecture can involve more than one parameter	t-test: Two-sided F-test: One-sided
		More than one conjecture: F-test	One-sided
	Heteroskedasticity White test	F-test	One-sided