Quantitative Methods 2

Tutorial 3

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Assignment 1

1. Assignment tab on LMS
2. Group registration due by 5 pm, Friday 16/8
3. Assignment 1 due by 4 pm, Friday 23/8

Tutorial 3

1. Obtain and interpret descriptive statistics
2. Evaluate the (normal) distribution of a series
3. Hypothesis testing using
   – Parametric tests
   – Non-parametric test

Population vs. sample

- Population vs. sample quantities

Quantities	Population parameters	Sample statistics
Mean	μ	$\bar{x}$
Variance	σ^2	s^2
Standard deviation	σ	s

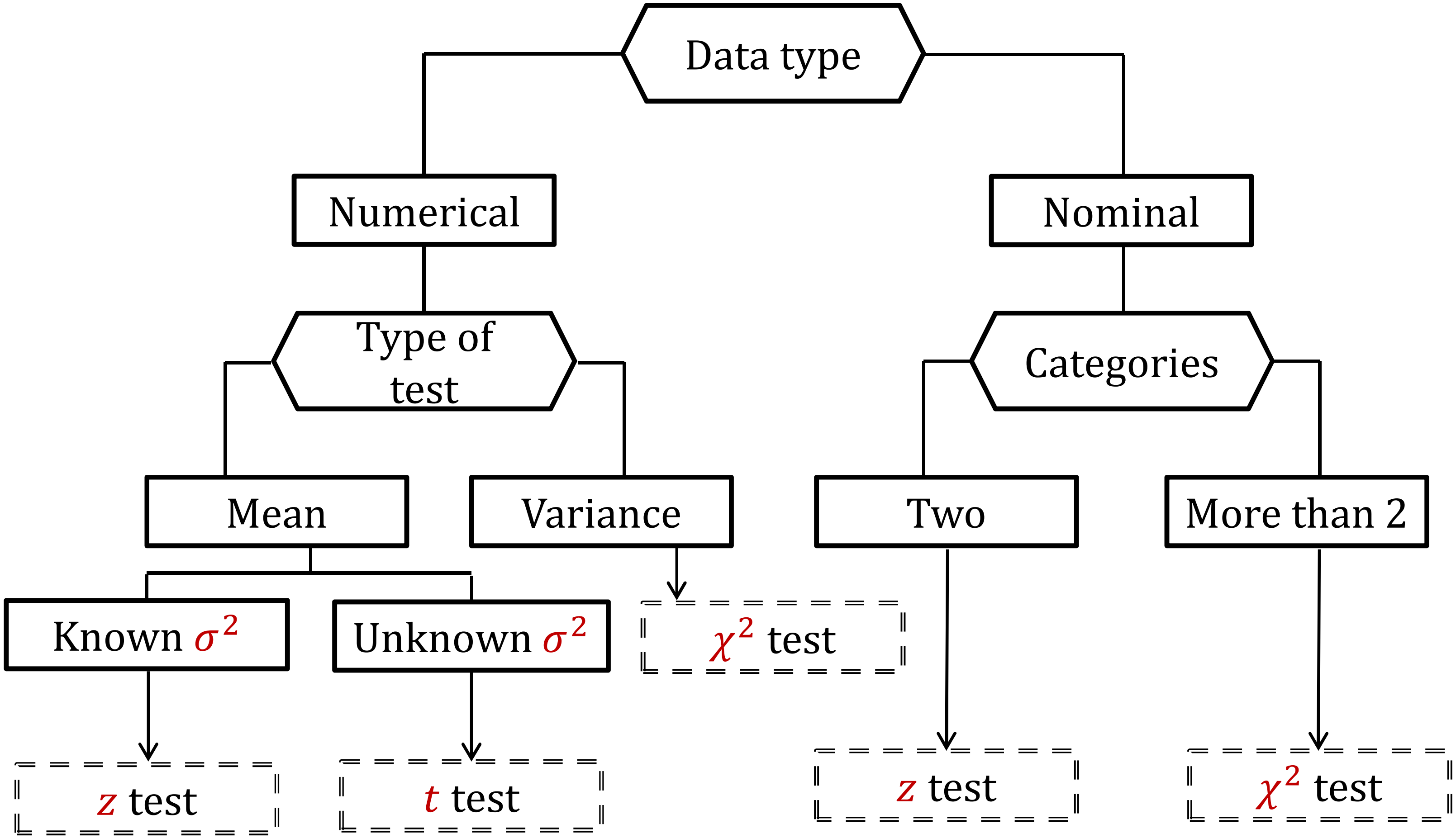
Hypothesis testing

- Always be about population parameters (<u>not</u> about sample statistics)

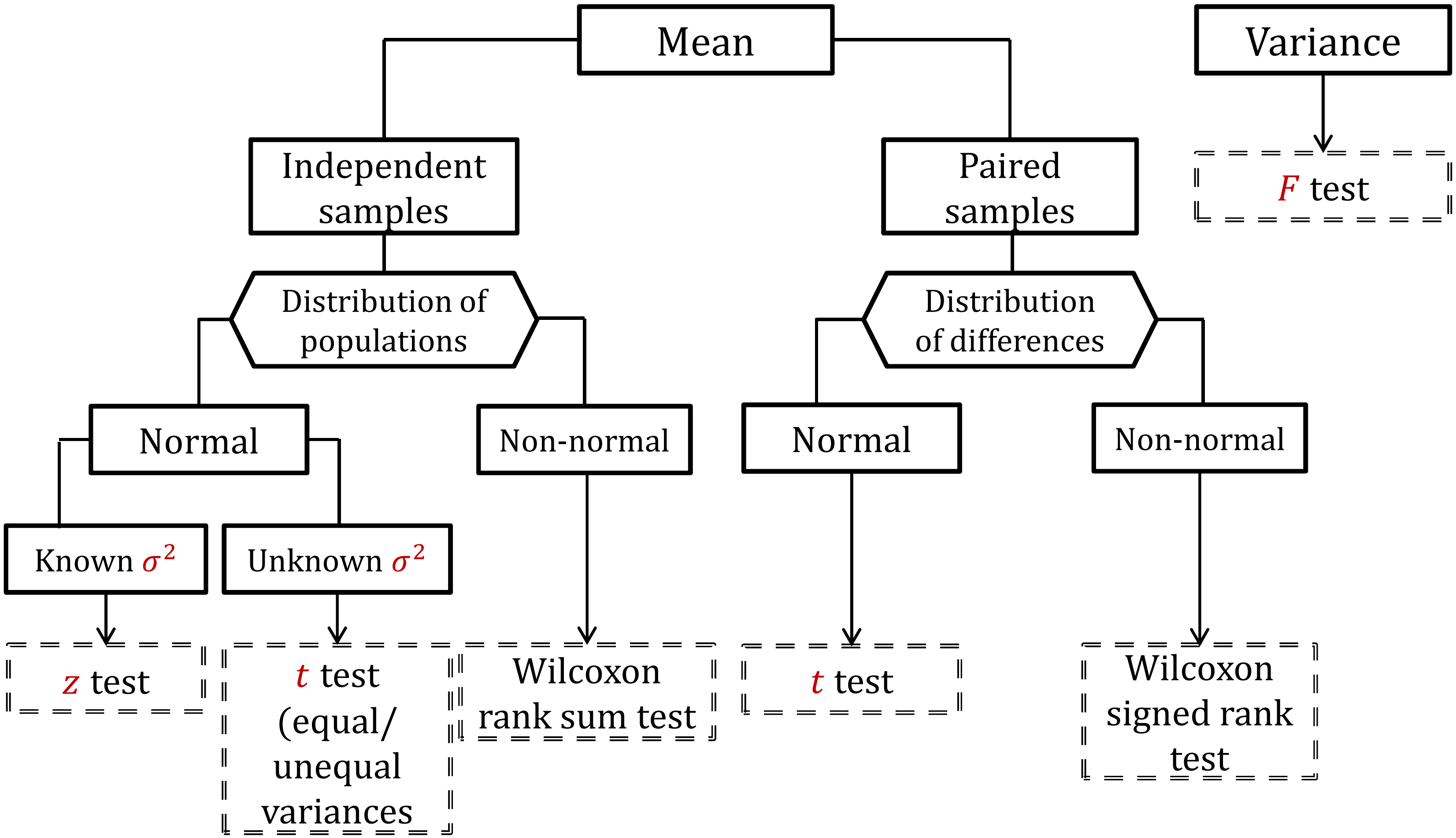
- Test of no difference (null hypothesis H_0) vs. difference (alternative hypothesis H_A)
  - H_A never contains the equal sign

- Purpose: To see if the difference is real or due to randomness
  - Find the probability of obtaining a sample with the estimated statistic, assuming the population value stated in H_0 is true.
  - If such a probability is small enough then H_0 is false or rejected and vice versa.
  - How small is enough? It's determined in the significance level α

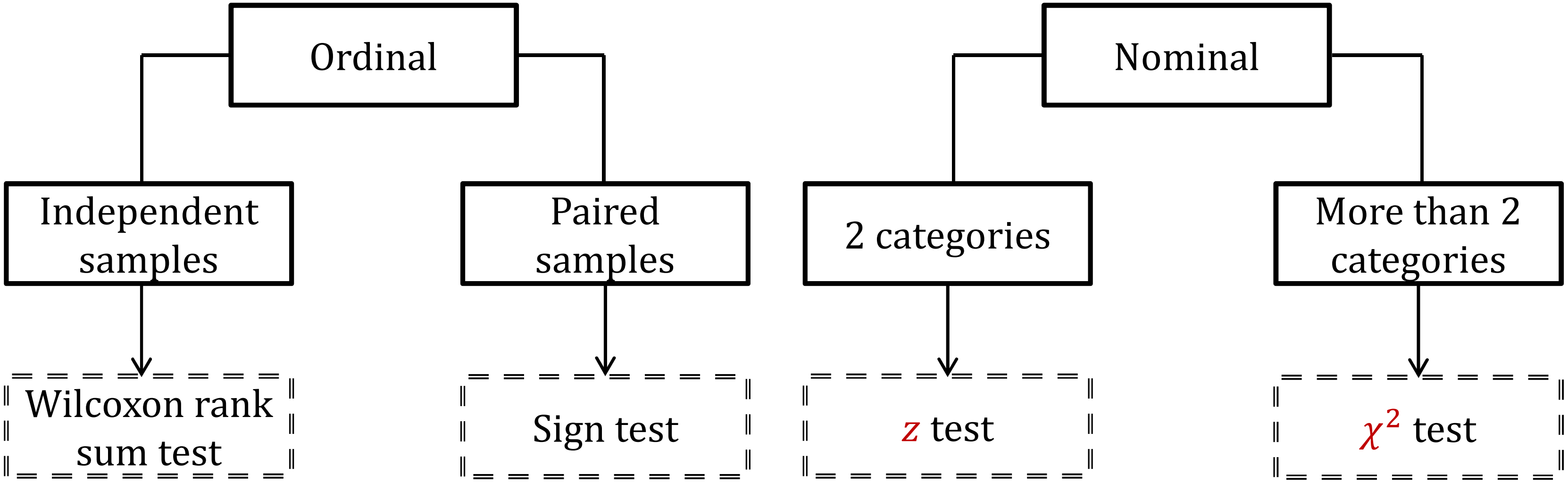
One population



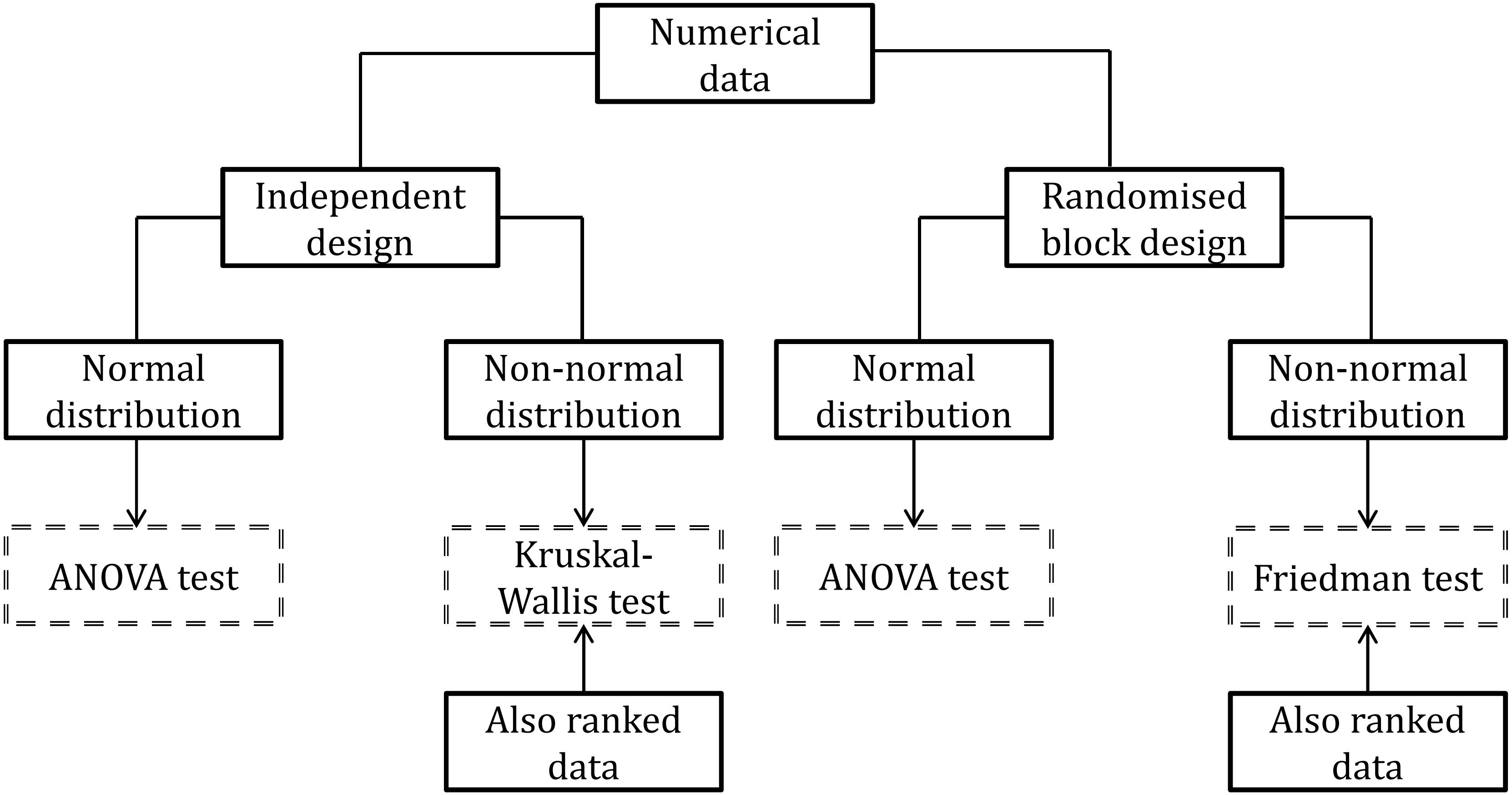
Two populations - Numerical



Two populations - Ordinal and Nominal



More than two populations



Hypothesis testing

- Choose the appropriate test based on:
  - Levels of measurement
  - Types of variable
  - Distribution of the underlying population
  - Relevant population parameter (e.g., σ^2) known/unknown

Level of measurement	**Type of variable**	
	Quantitative	Qualitative
Nominal		
Ordinal		
Interval		
Ratio		

Hypothesis testing

- Significance level α
  - Pr(Type I Error): Probability of rejecting H_0 when it is true (an error). That is, the extent to which we can accept that error to happen.
  - Define the distance between the sample mean and the null hypothesis parameter
  - Conventional values of α: 0.1, 0.05, 0.01
- p-value (of a test statistic)
  - The smallest α that leads to a rejection of H_0
  - If $\alpha <$ p-value, we can't reject H_0, and vice versa
- The decision (i.e., to reject H_0 or not) is made based on the comparison
  - Significance level vs. p-value; OR
  - Critical value vs. test statistic

Hypothesis testing

1. Set up H_0 and H_A
2. Determine the appropriate test, test statistic and its sampling distribution
3. Specify α
4. Define the decision rule using <u>either</u>
   - Test statistic vs. critical values, <u>or</u>
   - p-value vs. α
   - Unless asked to use both
5. Calculate the test statistic
6. Make a decision and draw a conclusion

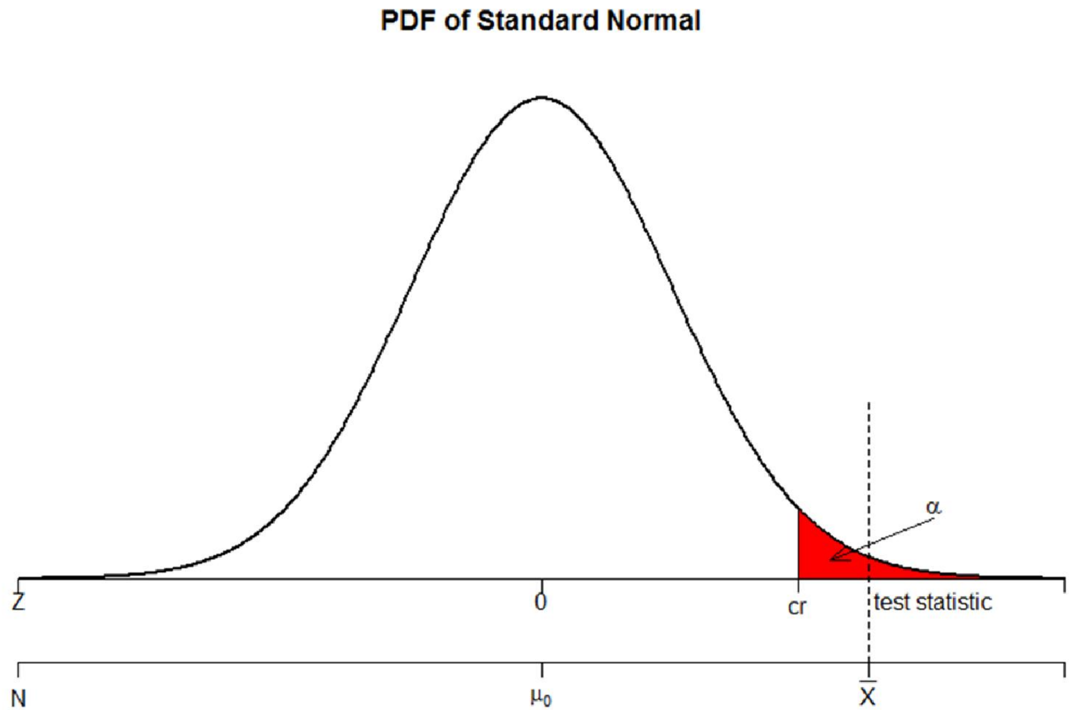
- Based on the comparison of <u>either</u>
  - Test statistic vs. critical values, <u>or</u>
  - p-value vs. α
  - Whatever use, make it consistent with step 4
- Decision: Reject or not able to reject H_0
- Conclusion: Statement in relation to the population values.

Normality test

- Visual assessment
- Descriptive measures: relative comparisons
  - Mean vs. median
  - Skewness
    - $\widehat{SK}$ vs. 0
    - $\left|\widehat{SK}\right|$ vs. $2s_{\widehat{SK}}$
  - Kurtosis
    - $\widehat{K}$vs. 3
    - $\left|\widehat{K}-3\right|$ vs. $2s_{\widehat{K}}$
- Hypothesis test: Jacque-Bera

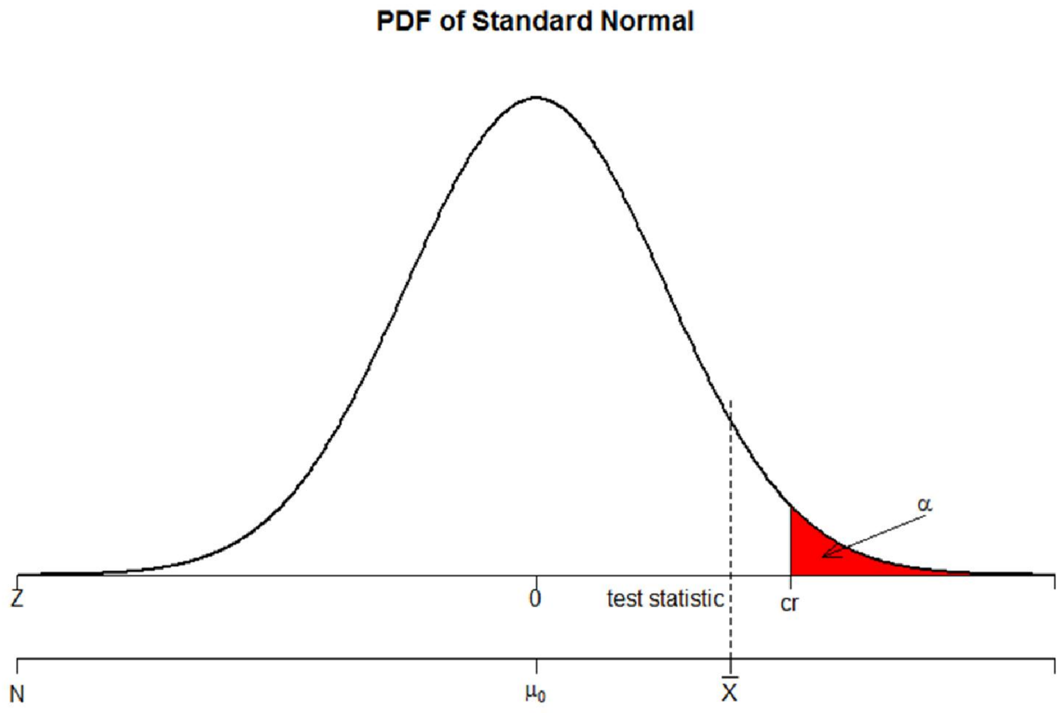
One-sided hypothesis

- H_0: $\mu = \mu_0$
- H_A: $\mu > \mu_0$
- Reject H_0?
  - Yes



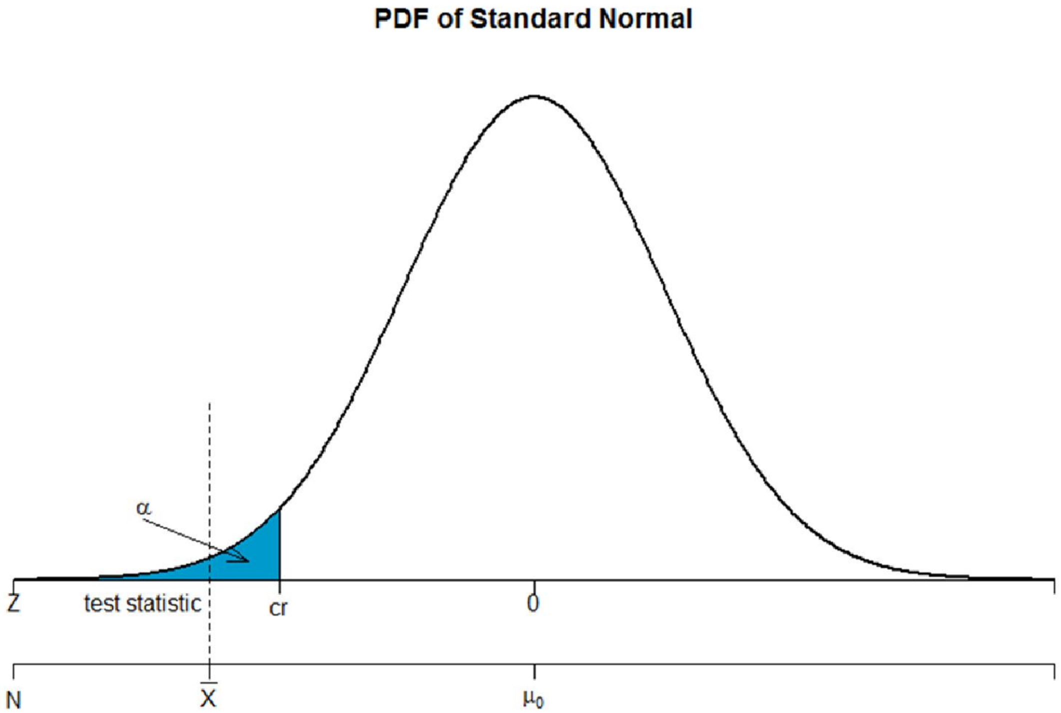
One-sided hypothesis

- $H_0\colon \mu = \mu_0$
- $H_A\colon \mu > \mu_0$
- Reject H_0?
  - No



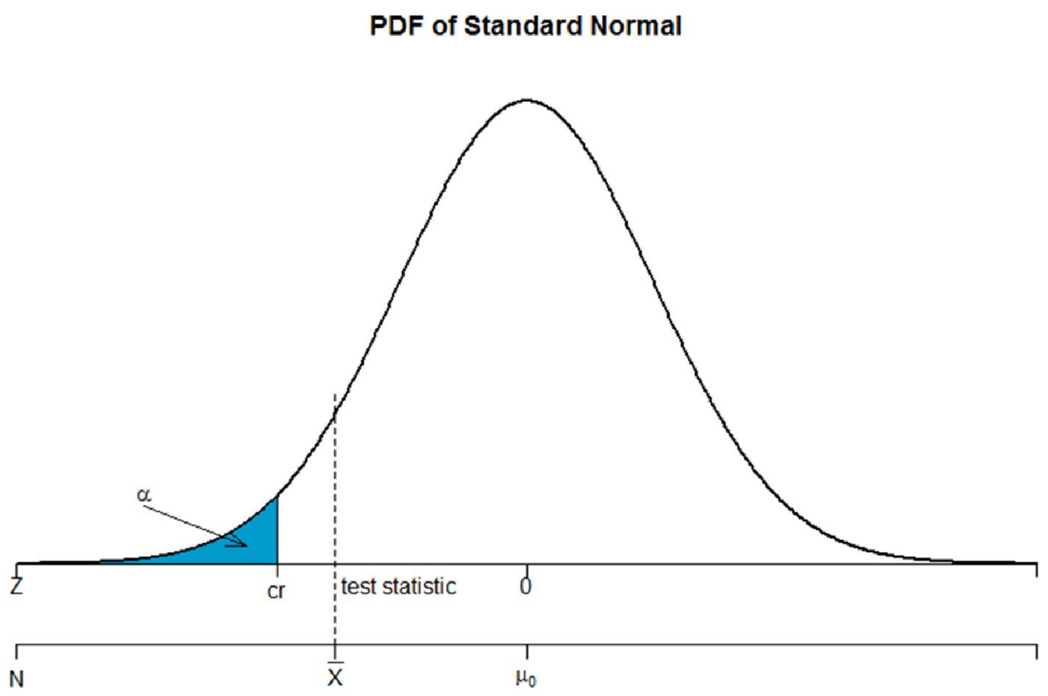
One-sided hypothesis

- H_0: $\mu = \mu_0$
- H_A: $\mu < \mu_0$
- Reject H_0?
  - Yes



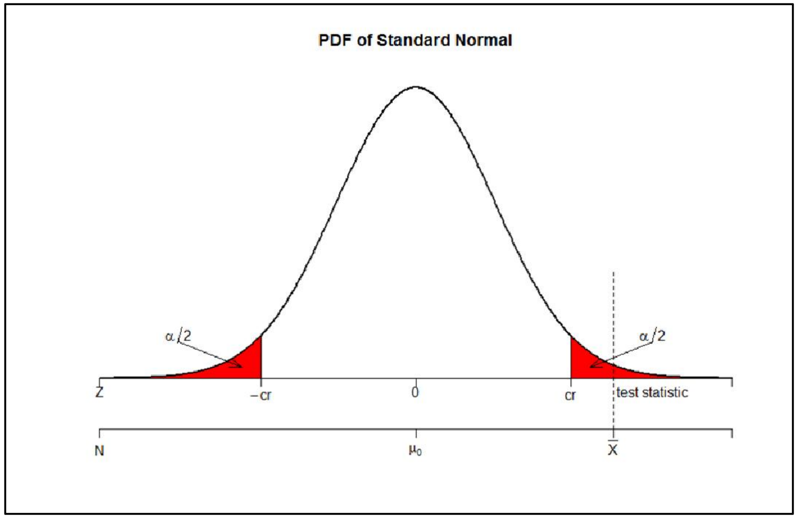
One-sided hypothesis

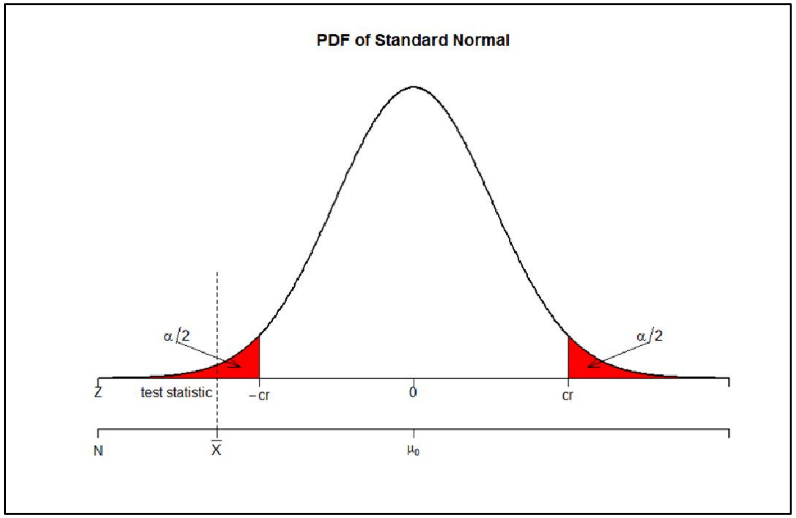
- H_0: $\mu = \mu_0$
- H_A: $\mu < \mu_0$
- Reject H_0?
  - No



Two-sided hypothesis

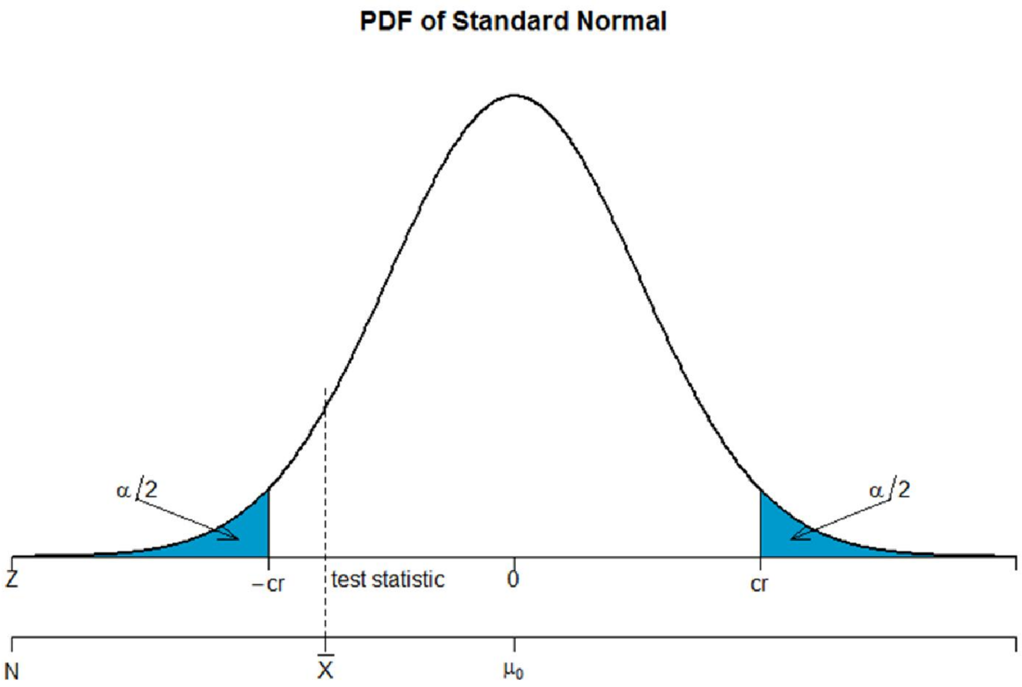
- H_0: $\mu = \mu_0$; H_A: $\mu \neq \mu_0$
- Reject H_0?
  - Yes





Two-sided hypothesis

- H_0: $\mu = \mu_0$
- H_A: $\mu \neq \mu_0$
- Reject H_0?
  - No



Critical values

1. Exercise 3: Two sided test:

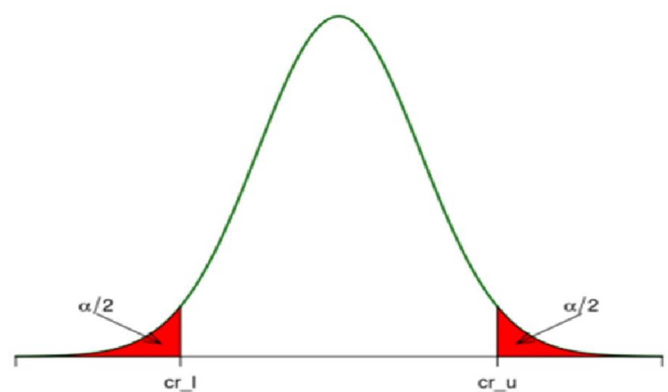
- $\alpha = 0.05$
- $H_0: \mu = 0.25$; $H_A: \mu \neq 0.25$

$t_{cr\_u} = t_{\alpha/2,n-1} = t_{0.005,39}$ [EViews: scalar tcr_u=@qtdist(0.995,39)]

$t_{cr\_l} = t_{1-\alpha/2,n-1} = t_{0.995,39}$ [EViews: scalar tcr_l=@qtdist(0.005,39)]

Use the t distribution table: Find t_{cr_u} and apply the symmetry: $t_{cr\_l} = -t_{cr\_u}$

Degrees of freedom	$t_{0.100}$	$t_{0.050}$	$t_{0.025}$	$t_{0.010}$	$t_{0.005}$
29	1.311	1.699	2.045	2.462	[illegible]
30	1.310	1.697	2.042	2.457	[illegible]
35	1.306	1.690	2.030	2.438	[illegible]
40	[illegible]	[illegible]	[illegible]	[illegible]	2.704
45	1.301	1.679	2.014	2.412	2.690



Critical values

2. Exercise 3: One-sided test

- Right tail test: $H_0\colon \mu = 0.25$; $H_A\colon \mu > 0.25$

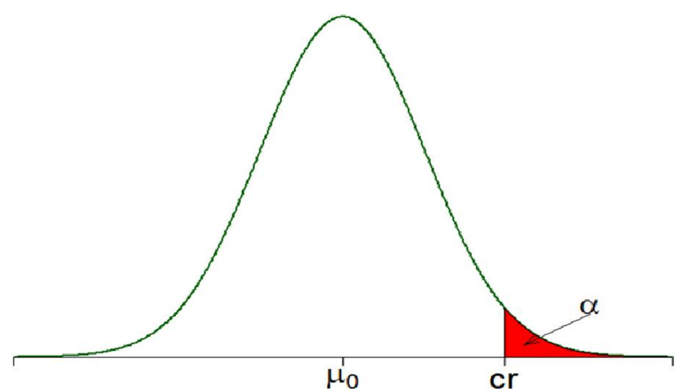
  $t_{cr\_rt} = t_{\alpha,n-1} = t_{0.01,39}$ [EViews: scalar tcr_rt=@qtdist(0.99,39)]

- Left tail test: $H_0\colon \mu = 0.25$; $H_A\colon \mu < 0.25$

  $t_{cr\_lt} = t_{1-\alpha,n-1} = t_{0.99,39}$ [EViews: scalar tcr_lt=@qtdist(0.01,39)]

Use the *t* distribution table: $t_{cr\_lt} = -t_{cr\_rt}$

Degrees of freedom	$t_{0.100}$	$t_{0.050}$	$t_{0.025}$	$\boxed{t_{0.010}}$	$t_{0.005}$
29	1.311	1.699	2.045	[illegible].462	2.756
30	1.310	1.697	2.042	[illegible].457	2.750
35	1.306	1.690	2.030	[illegible].438	2.724
$\boxed{40}$	[illegible]	[illegible]	[illegible]	2.423	2.704
45	1.301	1.679	2.014	2.412	2.690

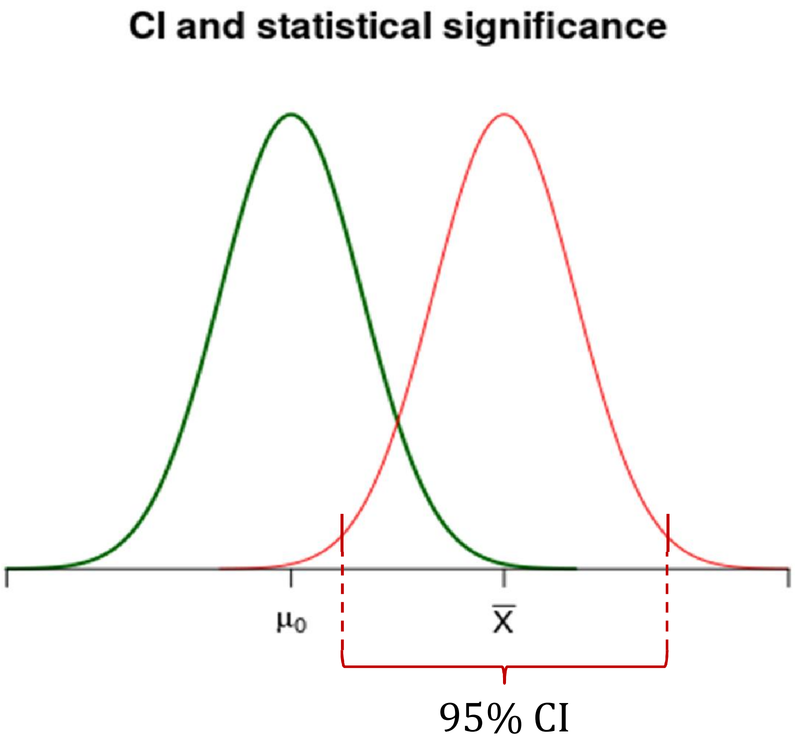


Confidence interval

- Not a probability
- Always agree with the p-value of the statistic with regard to statistical significance
- Recall significance level defines the distance between the sample mean and the null hypothesis parameter
- The confidence level defines the distance for how close the confidence limits are to sample mean

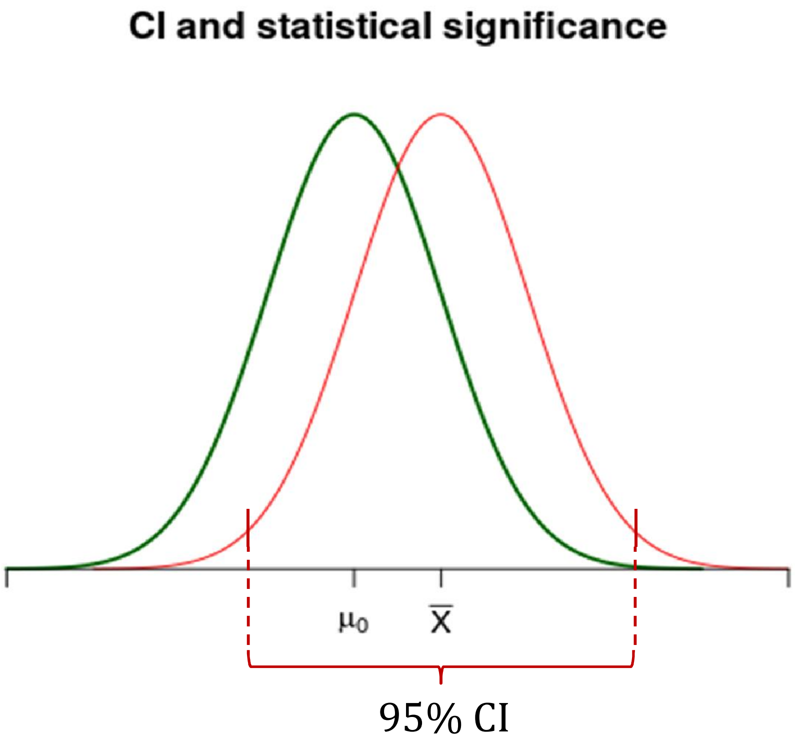
Confidence interval

- Reject H_0 at 5% sig. level
- 95% CI doesn't contain μ_0



Confidence interval

- Not to reject H_0 at 5% sig. level
- 95% CI contains μ_0



EViews command

- Create a text object to save and modify commands.

  `Object → New Object → Text`

- Examples of finding critical values associated with significance levels, and p-values associated with test statistics
  - Standard normal distribution (Z test)
  - t distribution (t test)
  - Chi-squared distribution

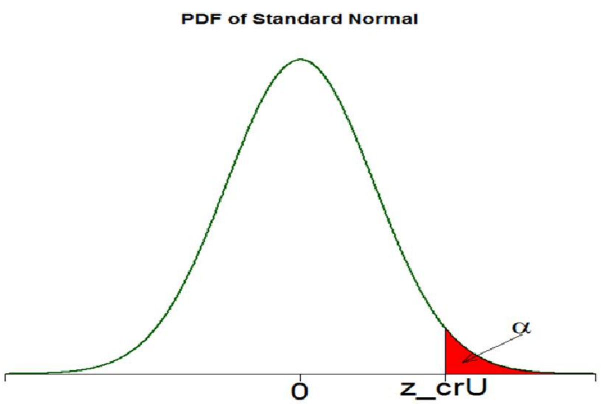
Standard normal distribution

1. Critical value

a/ One-sided, upper test

H_0: $\mu = \mu_0$; H_A: $\mu > \mu_0$

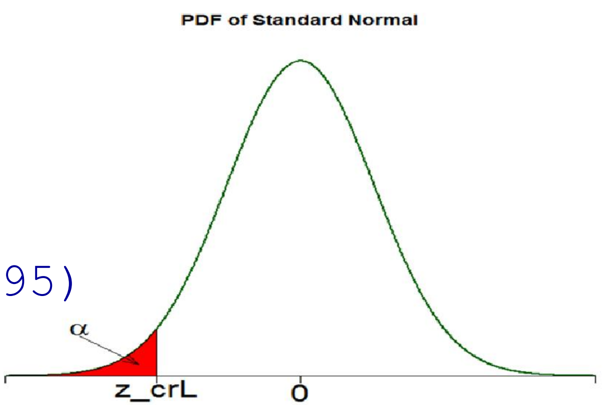
- $\alpha = 0.05$
- Reject H_0 if z statistic $> z_{\text{critical}}$
- Find z_{critical}: `scalar z_crU=@qnorm(0.95)`



b/ One-sided, lower test

H_0: $\mu = \mu_0$; H_A: $\mu < \mu_0$

- $\alpha = 0.05$
- Reject H_0 if z statistic $< z_{\text{critical}}$
- Find z_{critical}: `scalar z_crL = -@qnorm(0.95)`



Standard normal distribution

1. Critical value (cont.)

c/ Two-sided test

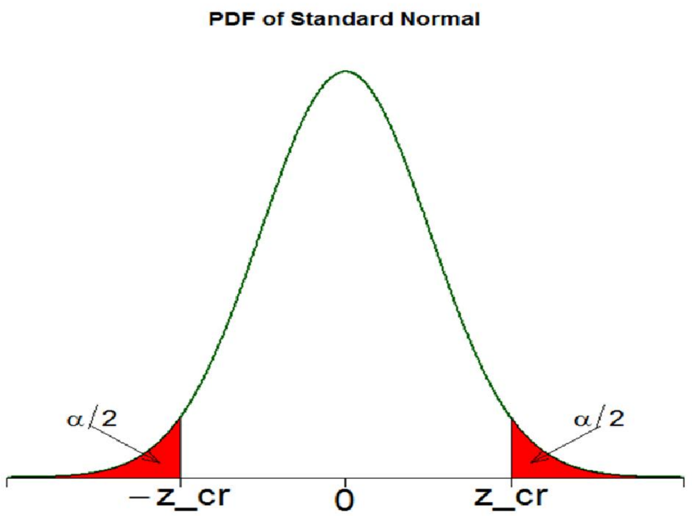
H_0: $\mu = \mu_0$; H_A: $\mu \neq \mu_0$

- $\alpha = 0.05$
- Reject H_0 if: z statistic $> z_{\text{critical}}$

  OR z statistic $< -z_{\text{critical}}$
- Find z_{critical}:

```
scalar z_cr = @qnorm(0.975)
```

- Your turn:
  - Find z_{critical} for $\alpha = 0.1$ and $\alpha = 0.01$ for one- and two-sided tests
  - Compare values between EViews and standard normal distribution table (they should be equal)



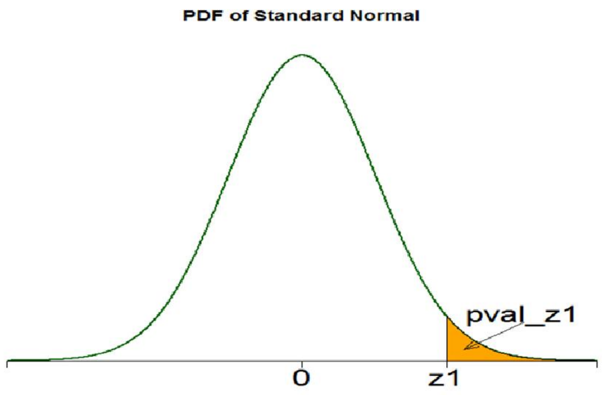
Standard normal distribution

2. p-value of *z* statistic

a/ One-sided test

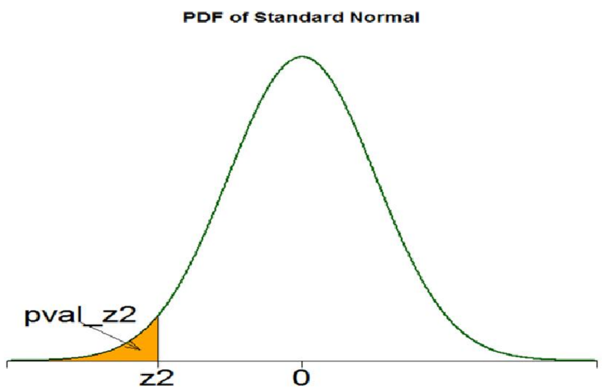
- Suppose z statistic $= 1.68$

```
scalar pval_z1 = 1-@cnorm(1.68)
```



- Suppose z statistic $= -1.68$

```
scalar pval_z2 = @cnorm(-1.68)
```



- In general:

```
p-value(z) = 1 - p-value(-z)
```

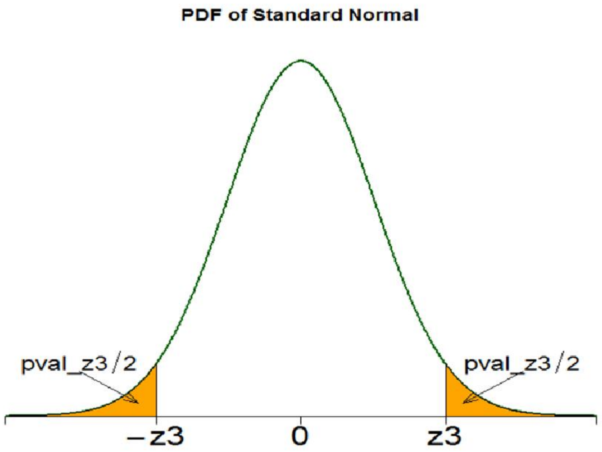
Standard normal distribution

2. p-value of *z* statistic (cont.)

b/ Two-sided test

- Suppose *z* statistic = 1.68

```
scalar pval_z3=2*(1-@cnorm(1.68))
```



- Suppose *z* statistic = −1.68

```
scalar pval_z3 = 2*@cnorm(-1.68)
```

- Your turn:
  - Find these p-values.
  - Compare EViews and standard normal distribution table outcomes
  - Also, compare one-sided and two-sided p-values

t distribution

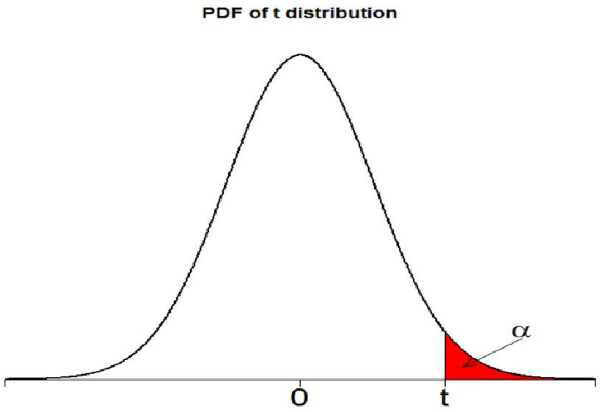
1. Critical value:

a/ One-sided, upper test

H_0: $\mu = \mu_0$; H_A: $\mu > \mu_0$

- $\alpha = 0.05$, suppose degrees of freedom (d.f) = 49
- Reject H_0 if t statistic > t_{critical}
- Find t_{critical}:

```
scalar t_crU = @qtdist(0.95,49)
```

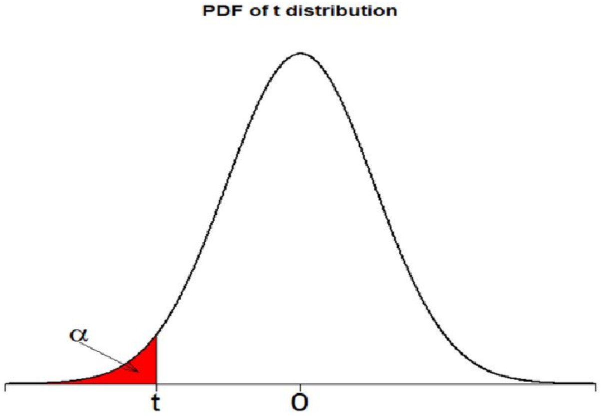


b/ One-sided, lower test

H_0: $\mu = \mu_0$; H_A: $\mu < \mu_0$

- $\alpha = 0.05$, d.f = 49
- Reject H_0 if t statistic < t_{critical}
- Find t_{critical}:

```
scalar t_crL = -@qtdist(0.95,49)
```



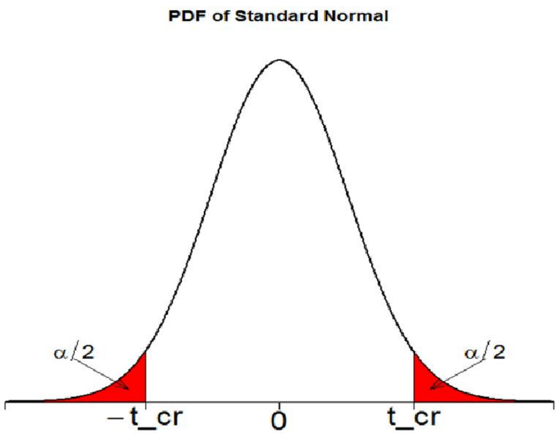
t distribution

1. Critical value (cont.)

c/ Two-sided test

H_0: $\mu = \mu_0$; H_A: $\mu \neq \mu_0$

- $\alpha = 0.05$, d.f=49
- Reject H_0 if: t statistic $> t_{\text{critical}}$
  OR t statistic $< -t_{\text{critical}}$
- Find t_{critical}: `scalar t_cr = @qtdist(0.975,49)`
- Your turn:
  - Find t_{critical} for $\alpha = 0.1$ and $\alpha = 0.01$ for one- and two-sided tests. Also, vary d.f and observe the difference.
  - Compare values between EViews and t distribution table (they should be equal)



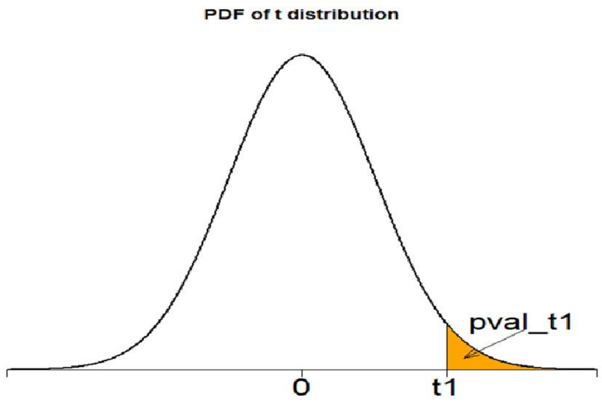
t distribution

2. p-value of t statistic (still, d.f=49)

a/ One-sided test

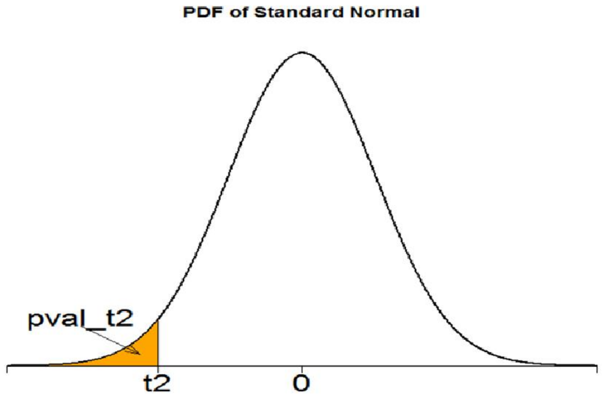
- Suppose t statistic $= 1.68$

```
scalar pval_t1 = 1-@ctdist(1.68,49)
```



- Suppose t statistic $= -1.68$

```
scalar pval_t2 = @ctdist(-1.68,49)
```



- In general:

```
p-value(t) = 1 - p-value(-t)
```

t distribution

2. p-value of *t* statistic (cont.)

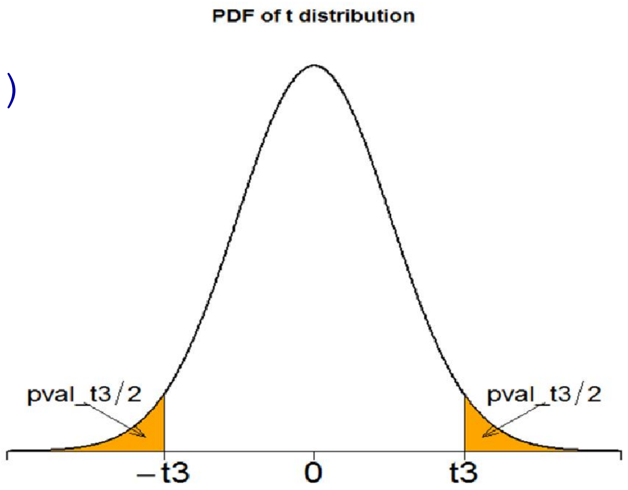
b/ Two-sided test

- Suppose *t* statistic = 1.68

```
scalar pval_t3=2*(1-@ctdist(1.68,49))
```

- Suppose *t* statistic = −1.68

```
scalar pval_t3 = 2*@ctdist(-1.68,49)
```



- Your turn:
  - Find these p-values.
  - Compare EViews and t distribution table outcomes
  - Also, compare one-sided and two-sided p-values

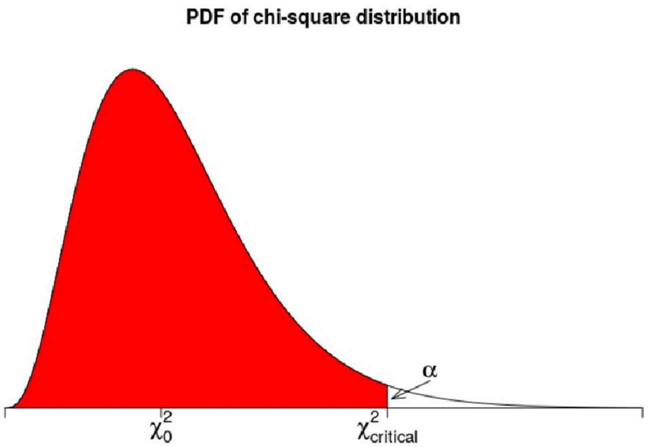
Chi-squared distribution

1. Critical value

- χ^2 is always positive (by definition)

  $H_0: \chi^2 = \chi_0^2; H_A: \chi^2 > \chi_0^2$
- $\alpha = 0.05$, degrees of freedom (d.f) = 8
- Reject H_0 if $\chi^2 > \chi^2_{\text{critical}}$
- Find χ^2_{critical}:

```
scalar cr_chi = @qchisq(0.95,8)
```



- Your turn:
  - Find the critical value. Vary α and d.f and observe the difference
  - What if $H_A: \chi^2 < \chi_0^2$? Reject H_0 if $\chi^2 < \chi^2_{\text{critical}}$
  - Compare EViews and chi-squared distribution table outcomes

Chi-squared distribution

1. p-value of chi-squared statistic

- Suppose χ^2 statistic $= 20, \mathrm{d.f} = 8$
- Find p-value χ^2_{critical}:

```
scalar pval_chi = 1-@cchisq(20,8)
```

- Notice the difference of command

compared with z and t distributions

- Your turn:
  - Find the p value. Vary χ^2 and d.f and observe the difference
  - Compare EViews and chi-squared distribution table outcomes