Quantitative Methods 2

Tutorial 6

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Last week

1. Test of variance
   - One population: χ^2 test
   - Two populations: F test
2. Test of proportion
   - One population: Z test
   - Two populations: Z test

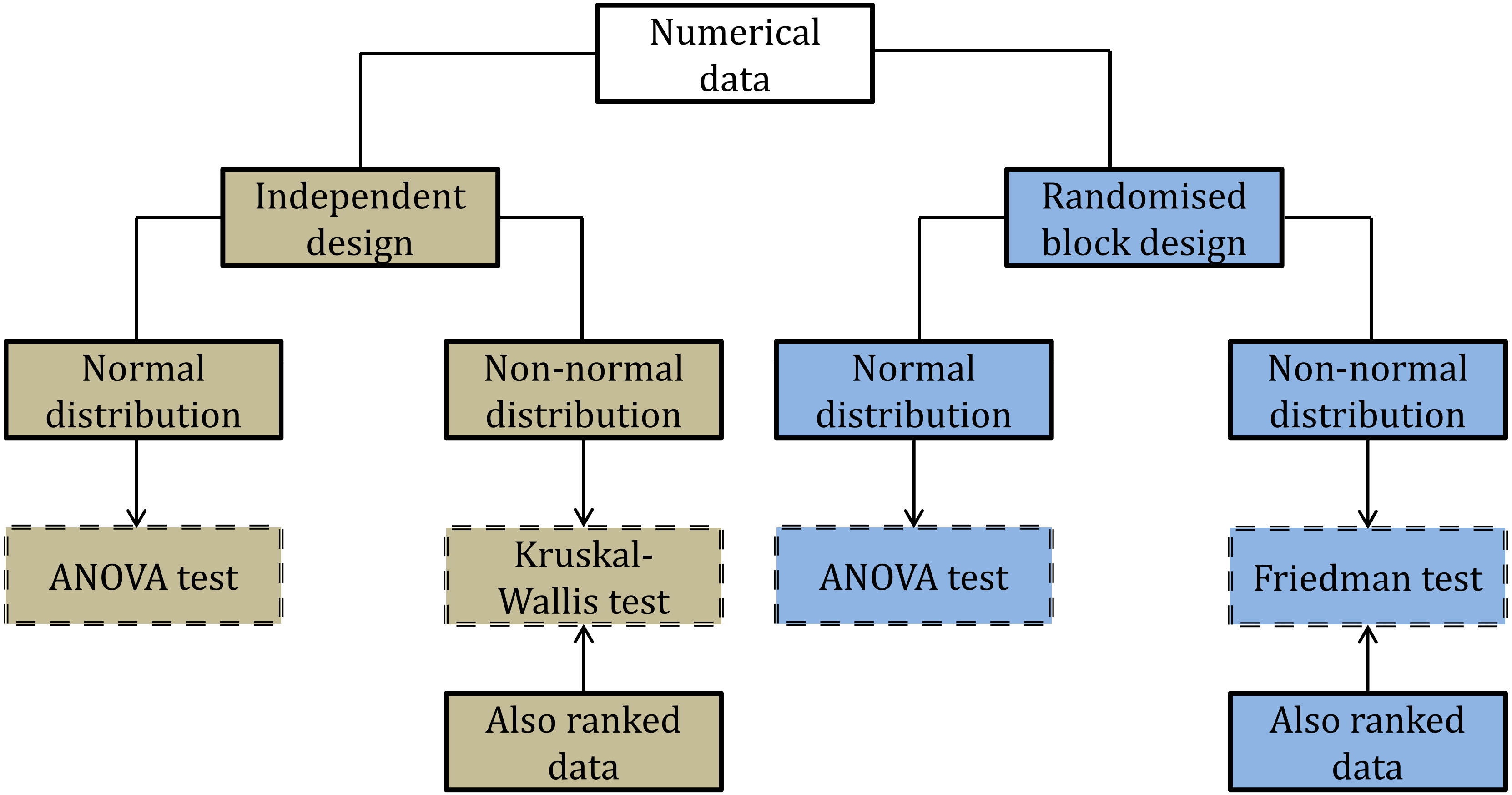
Tutorial 6

Test of population's central location for more than 2 populations

1. Independent measures design

2. Randomised block design

More than two populations



Independent measures design

Assumptions

Parametric (ANOVA F-test)	**Non-parametric (Kruskal-Wallis)**
Independent random samples	Independent random samples
Normal distribution	Populations differ at most with respect to their central locations
Equal variance	Continuous variables
	Measurement scale is at least ordinal

Independent measures design

Exercise 1

	Treatments (Groups)			
	Form_1	Form_2	Form_3	Form_4
Data	$X_{1,1}$	$X_{2,1}$	$X_{3,1}$	$X_{4,1}$
	$X_{1,2}$	$X_{2,2}$	$X_{3,2}$	$X_{4,2}$
	...	...	...	...
	$X_{1,30}$	$X_{2,30}$	$X_{3,30}$	$X_{4,30}$
Sample size	n_1	n_2	n_3	n_4
Means	$\bar{X}_1$	$\bar{X}_2$	$\bar{X}_3$	$\bar{X}_4$
Grand mean	$\bar{X}$			

Randomised block design

Assumptions

Parametric (ANOVA F-test)	Non-parametric (Friedman)
X_{ij} are independent random samples (size $= 1$) from $k \times b$ (sub-) populations	Randomised block design
Each (sub-) population is normally distributed	Sampled populations have similar spreads and shapes
Equal variance	
Block and treatment effects are additive (no interactions)	

Randomised block design

Exercise 3

Block	Treatments(Groups)				Block means
	Lec_1	Lec_2	Lec_3	Lec_4	
1	[illegible]	[illegible]	$X_{1,3}$	$X_{1,4}$	$\bar{X}_{B,1}$
2	[illegible]	[illegible]	$X_{2,3}$	$X_{2,4}$	$\bar{X}_{B,2}$
…	…	…	…	…	…
8	[illegible]	[illegible]	$X_{8,3}$	$X_{8,4}$	$\bar{X}_{B,8}$
Treatment means	[illegible]	[illegible]	$\bar{X}_{T,3}$	$\bar{X}_{T,4}$	

Parametric tests

Independent measures design	Randomised block design
$SS = SST + SSE$	$SS = SST + SSB + SSE$
$SS = \sum_{j=1}^{k} \sum_{i=1}^{n_j} (x_{ij} - \bar{x})^2$	$SS = \sum_{j=1}^{k} \sum_{i=1}^{n_j} (x_{ij} - \bar{x})^2$
$SST = \sum_{j=1}^{k} n_j (\bar{x}_j - \bar{x})^2$	$SST = b \sum_{j=1}^{k} (\bar{x}_{T,j} - \bar{x})^2$
	$SSB = k \sum_{i=1}^{b} (\bar{x}_{B,i} - \bar{x})^2$
$SSE = \sum_{j=1}^{k} \sum_{i=1}^{n_j} (x_{ij} - \bar{x}_j)^2 = SS - SST$	$SSE = \sum_{j=1}^{k} \sum_{i=1}^{n_j} (x_{ij} - \bar{x}_{T,j} - \bar{x}_{B,i} - \bar{x}_j)^2 = SS - SST - SSB$
$F = \frac{s^2}{s_p^2} = \frac{MST}{MSE} = \frac{SST/k-1}{SSE/n-k} \sim F_{k-1,n-k}$ • Always one-sided test	$F_T = \frac{MST}{MSE} = \frac{SST/k-1}{SSE/n-k-b+1} \sim F_{k-1,n-k-b+1}$ $F_B = \frac{MSB}{MSE} = \frac{SSB/b-1}{SSE/n-k-b+1} \sim F_{b-1,n-k-b+1}$

Non-parametric tests

- Independent design: Kruskal-Wallis test

$$H = \frac{12}{n(n+1)} \sum_{j=1}^{k} \frac{T_j^2}{n_j} - 3(n+1) \sim \chi^2_{k-1}$$

  - T_j^2: sum of ranks of each sample across the pooled sample (all observations)

- Randomised block design: Friedman test

$$F_r = \frac{12}{b \times k \times (k+1)} \sum_{j=1}^{k} T_j^2 - 3b(k+1) \sim \chi^2_{k-1}$$

  - T_j^2: sum of ranks of each treatment across blocks

Hypotheses

- Parametric test

$H_0: \mu_1 = \mu_2 = \mu_3$

H_A: Not all population means are equal

  - $H_A: \mu_1 = \mu_2 \neq \mu_3$
  - $H_A: \mu_1 \neq \mu_2 = \mu_3$
  - $H_A: \mu_1 \neq \mu_2 \neq \mu_3$

- Non-parametric test

$H_0: \eta_1 = \eta_2 = \eta_3$

H_A: Not all population medians are equal

  - $H_A: \eta_1 = \eta_2 \neq \eta_3$
  - $H_A: \eta_1 \neq \eta_2 = \eta_3$
  - $H_A: \eta_1 \neq \eta_2 \neq \eta_3$

General repeated measure design

- 2 populations: Paired-samples
  - One sample
  - Data often collected over time, though not the only case
- More than 2 populations: Randomised block
  - More than 2 samples: identical in every other way, except for the characteristic/series being tested
  - Data collected across treatments
- Combined design. E.g., biomedical trials, panel surveys
  - More than one sample: Placebo, Current treatment, New treatment
  - Identical in every other way, made through a randomisation
  - Data collected across treatments <u>and/or</u> over time

Exercise 3

- Calculate grand mean using group/treatment statistics

scalar gmean = (@mean(lecturer_1)+@mean(lecturer_2)+@mean(lecturer_3)+@mean(lecturer_4))/4

Or more generally:

scalar gmean = (@sum(lecturer_1)+@sum(lecturer_2)+@sum(lecturer_3)+@sum(lecturer_4))/(@obs(lecturer_1)+@obs(lecturer_2)+@obs(lecturer_3)+@obs(lecturer_4))