Chapter 2

STRESS AND STRAINS

Stress

Stress can be classified broadly in three types as described below:

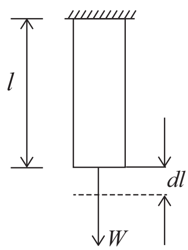


Figure 2.1

1. *Tensile stress*: It is illustrated in Fig. 2.1 where a tensile load W is applied to a uniform rod fixed at one end.

$$\text{Tensile stress, } \sigma = \frac{W}{\text{Cross-sectional area of rod}} = \frac{W}{A}$$

   unit is N/mm^2 or $\text{MN/m}^2, \sigma$ (Greek letter sigma).

2. *Compressive stress*: As shown in Fig. 2.2 when load W tends to compress a rod of cross-section area A, then compressive stress = $\dfrac{W}{A}$.

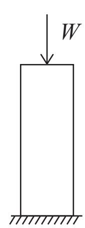


Figure 2.2

3. *Shear stress*: If two plates are joined together with rivet as shown in Fig. 2.3.

   The stress in rivet is known as shear stress, it is denoted by τ (Greek letter tau), shear stress in rivet, $\tau = \dfrac{F}{A}$.

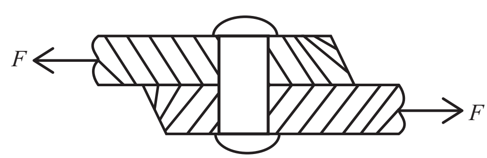


Figure 2.3

It may be noted that point A is the limit of proportionality and B is the elastic limit. Between point A and B it is a curve thus not linear relationship. Therefore, actually E is constant within the limit of proportionality, though in Hooke's law, we had mentioned, within 'elastic limit', because A and B are very close to each other. Let us name the important points on the graph:

A: Limit of proportionality
B: Elastic limit: It may be noted that on removal of load up to elastic limits, specimen comes back to its original dimension.
C: Higher yield point: This is the point where yielding of the material begins.
C′: Lower yield point: The stress associated with the lower yield point is known as yield strength.
D: Maximum stress: Here the stress is maximum because due to plastic behaviour of the material, area of cross section is very low.
E: Point of fracture: At this point 'waisting occurs' as shown in Fig. 2.5.

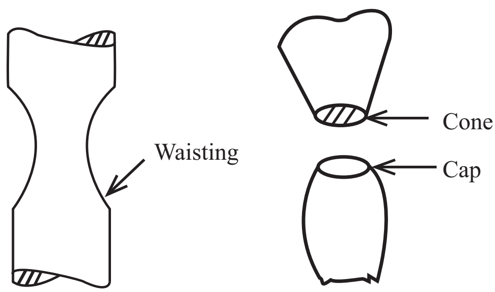


Figure 2.5

If the material is loaded beyond the elastic limit and then load is removed, a permanent extension remains, called *permanent set*.

Proof Stress: For engineering purposes it is desirable to know the stress to which a highly ductile material such as aluminium can be loaded safely before a permanent extension takes place.

This stress is known as the *proof stress or offset stress* and is defined as the stress at which a specified permanent extension has taken place in the tensile test. Proof stress is found from the stress-strain curve as given in Fig. 2.6. The extension specified is usually 0.1, 0.2 or 0.5 per cent of gauge length.

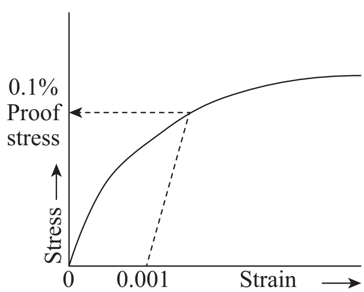


Figure 2.6

The proof stress here is found on the basis of 0.1 per cent strain.

Procedure: Draw a line parallel to the initial slope of the curve. The stress at the point where this line cuts the curve is the 0.1% *proof stress*. The 0.2 per cent proof stress is also found in the same manner.

Note: Though we define Hooke's law to be taken without elastic limit, but strictly speaking it is applicable up to the point of proportionality *B* in Fig. 2.4.

Brittle Materials: Fig. 2.6 shows that the stress-strain graph for brittle materials such as cast iron. The metal is almost elastic and up to fracture but does not obey Hooke's law. A material such as this which has little plasticity or ductility and does not neck down before fracture is termed 'brittle'. The modulus of elasticity for cast iron is not constant but depends on the portion of the curve from which it is calculated.

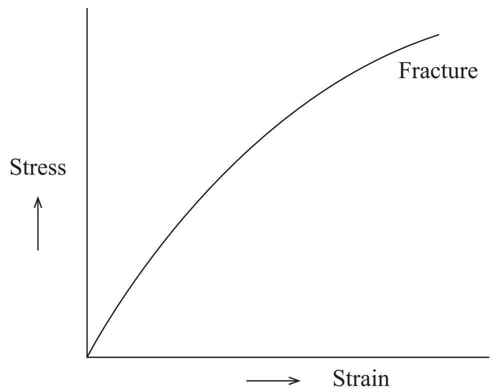


Figure 2.7 Cast iron in tension

The following table is very useful for mechanical properties:

Material	Percentage elongation	Yield stress MN/m^2	0.1% proof stress MN/m^2	Ultimate tensile stress MN/m^2
Copper annealed	60	–	60	220
Copper hard	4	–	320	400
Aluminium soft	35	–	30	90
Aluminium hard	5	–	140	150
Black mild steel	25–26			
Bright mild steel	14–17			
Structural steel	20	220–250	–	430–500
Cast iron	–			
Spheroidal				
Graphite				
Cast iron (annealed)	–	–	–	280–340
Stainless steel	60	230	–	600

Now $E = \dfrac{\text{stress}}{\text{strain}}$

Let F, L, A, dl be the force, length, area of cross section, and extension or contraction respectively,

then $E = \dfrac{F.L}{A.dl}$

EXAMPLE 2.1: A bar of mild steel has an overall length of 2.1 m. The diameter up to 700 mm length is 56 mm, the diameter of the remaining 1.4 m is 35 mm. Calculate the extension of the bar due to a tensile load of 55 kN.

$E = 200\ \text{GN/m}^2$.

SOLUTION:

$$\therefore \quad \text{Remember } 1\ \text{GN/m}^2 = 1\ \text{kN/mm}^2$$

$$\therefore \quad E = 200\ \text{kN/mm}^2$$

$$\text{We know } E = \frac{Fl}{Adl} \qquad \therefore \quad dl = \frac{F.l}{A.E}$$

Therefore, for portion of 700 mm,

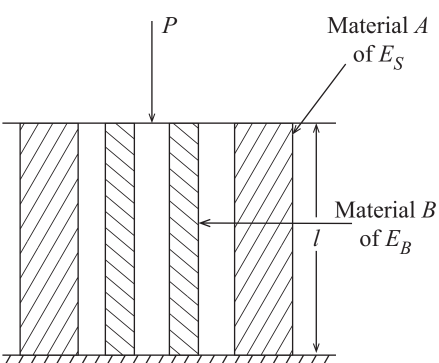
$$\text{the extension } dl_1 = \frac{55000 \times 700 \times 7 \times 4}{200000 \times 22 \times 56 \times 56} = \frac{5}{64}\ \text{mm} = 0.0178\ \text{mm}$$

Now dl_2 for 1400 mm length of 35 mm dia,

$$dl_2 = \frac{55000 \times 1400 \times 7 \times 4}{200000 \times 22 \times 35 \times 35} = 0.4\ \text{mm}$$

$$\text{Total extension} = dl_1 + dl_2 = 0.0178 + 0.4 = \mathit{0.4178}\ \text{mm}$$

Compound bars: When two or more materials (members) are rigidly fixed together so that they share the same load and extend or compress by same amount, the two members form compound bar. Let us say that in Fig. 2.7 we have to find stress in each material and amount of compression.



Let the outer tube of material A has outside dia as d_1 and inside dia as d_2 and inner tube of material B has outside dia as d_3 and inside dia as d_4. Both ends are joined rigidly to make compound bar of length l.

$$\text{Hence,} \quad d^2 = 5630 \text{ mm}^2$$
$$\text{or} \quad d = 75 \text{ mm}$$

EXAMPLE 2.3: A steel bar of 20 mm diameter and 400 mm long is placed concentrically inside a gunmetal tube (Fig. 2.9). The tube has inside diameter 22 mm and thickness 4 mm. The length of the tube exceeds the length of the steel bar by 0.12 mm. Rigid plates are placed on the compound assembly. Find: a) the load which will just make tube and bar of same length and b) the stresses in the steel and gunmetal when a load of 50 kN is applied. E for steel = 213 GN/m^2, E for gunmetal = 100 GN/m^2.

SOLUTION:

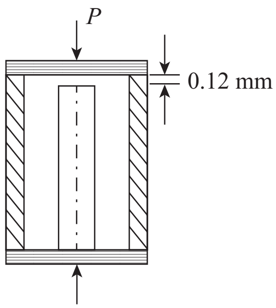


Figure 2.9

$$\text{Area of gunmetal tube, } A_g = \frac{\pi}{4}(0.03^2 - 0.022^2)$$
$$= 0.000327 \text{ m}^2$$
$$\text{Area of steel bar } A_s = \frac{\pi}{4}(0.02)^2 = 0.0003142 \text{ m}^2$$

a) For tube to compress 0.12 mm:

$$\text{strain} = \frac{0.12}{400} = 0.0003, \quad \text{Let } \sigma_1 \text{ be the stress in the tube}$$
$$\frac{\sigma_1}{E_g} = 0.0003, \frac{\sigma_1}{100} = 0.0003$$
$$\therefore \quad \sigma_1 = 0.0003 \times 100 = 0.03 \text{ GN/m}^2 = 30000 \text{ kN/m}^2$$
$$\text{Hence, load} = 30000 \times 0.000327 = 9.81 \text{ kN}$$

b) Load available to compress bar and tube as a compound bar is given by, let σ_2 be the additional stress produced in the gunmetal tube due to this load and σ_s be the corresponding stress in the steel bar, then

$$\text{Load on compound bar} = 50 - 9.81 = 40.19 \text{ kN}$$
$$P = \sigma_2 A_g + \sigma_s A_s$$
$$40.19 = \sigma_2 \times 0.000327 + \sigma_3 \times 0.0003142 \qquad \text{(i)}$$

Also

$$\frac{\sigma_2}{E_g} = \frac{\sigma_s}{E_s}, \quad \therefore \quad \sigma_2 = \frac{100}{2100}\sigma_s \tag{ii}$$

From Eqns. (i) and (ii)

$$\sigma_2 = 40,600 \text{ kN/m}^2 = 40.6 \text{ MN/m}^2$$

$$\sigma_s = 85300 \text{ kN/m}^2 = 85.3 \text{ MN/m}^2$$

$$\text{Final stress in gunmetal} = \sigma_1 + \sigma_2$$

$$= 40600 + 30000 = 70,600 \text{ kN/m}^2$$

$$= 70.6 \text{ MN/m}^2$$

Deformation of a Body Due to Self Weight

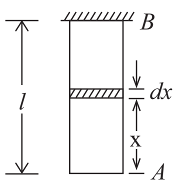


Figure 2.10

Let us consider a bar *AB* which is hanging freely under its own weight (see Fig. 2.10)

Let w = specific weight of the bar material

Now consider a small section dx at a distance x from A.

Weight of the bar for a length $= w\times$ volume

$= wAx$ (A is cross section of the bar)

Now elongation of the elementry length dx due to weight of the bar for length $x, (wAx)$

$$= \frac{pl}{A.E} = \frac{(wAx)\,dx}{A.E} = \frac{wx.dx}{E}$$

$$\text{Total elongation} = \int_0^l \frac{wxdx}{E} = \frac{w}{E}\int_0^l x.dx$$

$$= \frac{w}{E}\left[\frac{x^2}{2}\right]_0^l$$

$$\therefore \quad \boxed{\text{elongation, } dl = \frac{wl^2}{2E}}$$

Because total weight of bar, $W = wA.l$.

Now elongation dl can be written as $\dfrac{wAl.l}{2AE}$

Hence, $dl = \dfrac{Wl}{2AE}$

This result also proves that the extension due to own weight is half if same weight is applied at the end (of course neglecting extension due to self weight).

EXAMPLE 2.4: A steel bar *ABC* 18 m long is having cross-sectional area 4 mm^2 weighs 22.5 N (Refer Fig. 2.11). If modulus of elasticity of wire is 210 GN/m^2, find the deflections at *C* and *B*.

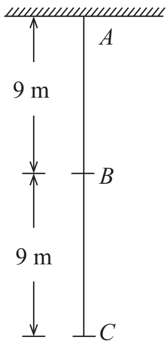


Figure 2.11

Deflection at C due to self weight of wire $AC = dl_c$

$$dl_c = \frac{Wl}{2AE} = \frac{22.5 \times 18000}{2 \times 4 \times 210000} = 0.241 \text{ mm}$$

Deflection at B:

Now deflection at B is due to two reasons: i) due to self weight of AB and ii) due to weight of BC.

$$dl_B = \frac{W/2 \times l/2}{2AE} + \frac{W/2 \times l/2}{A.E}$$

$$\therefore \quad dl_B = \frac{W/2 \times l/2}{AE}\left(\frac{1}{2} + 1\right)$$

$$= \frac{22.5 \times 9000}{2 \times 4 \times 210000}(1.5) = 0.181 \text{ mm}$$

Sometimes a machine member is a acted upon by a number of forces, some acting at outer edges while some are acting inside the body. In such cases in order to find out the total extension or contraction, the principle of superposition is applied. This has been very well made clear by the following examples:

EXAMPLE 2.5: A steel bar *ABC* of 400 mm length and 20 mm diameter is subjected to a point load as shown in Fig. 2.12. Determine the total change in the length of bar. Take $E = 200$ GPa.

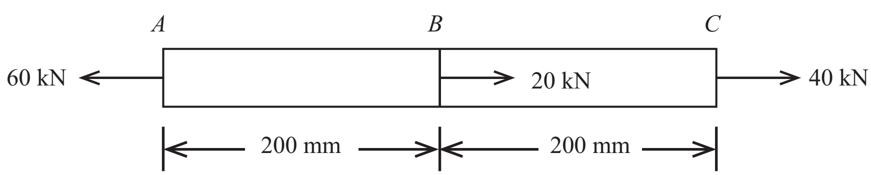
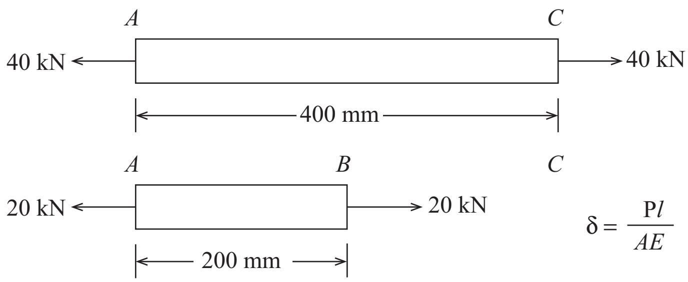


Figure 2.12

SOLUTION:
For simplification split it into two parts as under:



$$\delta = \frac{Pl}{AE}$$

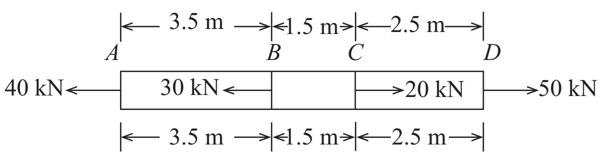
$$A = \frac{\pi}{4}(20)^2 = 314 \text{ mm}^2$$

$$\delta_{AC} = \frac{40 \times 10^3 \times 400}{314 \times 200000} = 0.255 \text{ mm}$$

$$\delta_{AB} = \frac{20 \times 10^3 \times 200}{314 \times 200000} = 0.064 \text{ mm}$$

Total $\quad \delta = 0.255 + 0.064 = 0.319$ mm **Ans.**

EXAMPLE 2.6: A copper rod *ABCD* of 800 mm^2 cross-sectional area and 7.5 m long is subjected to forces as shown in Fig. 2.13. Find the total elongation of the bar. Take $E = 100$ GPa



SOLUTION:
Splitting into three figures as shown below:

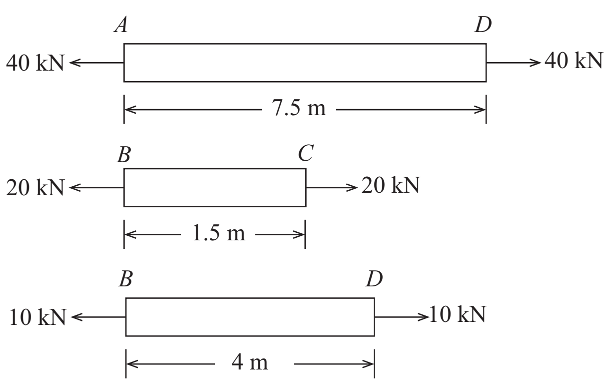


Figure 2.13

Hence, cross-sectional area at distance x from larger end $A' = \dfrac{\pi d'^2}{4} = \dfrac{\pi}{4}(d_1 - kx)^2$

$$\text{Stress at this section,} \quad \sigma' = \frac{P}{A'} = \frac{4P}{\pi(d_1 - kx)^2}$$

$$\therefore \quad \text{Strain} = \varepsilon' = \frac{\sigma'}{E} = \frac{4P}{\pi E(d_1 - kx)^2}$$

$$\text{Extension of elementary length} \quad dx = \varepsilon' dx = \frac{4P\,dx}{\pi E(d_1 - kx)^2}$$

$$\text{Total extention of the bar} = \delta = \frac{4P}{\pi E}\int_0^l \frac{dx}{(d_1 - kx)^2} = \frac{4P}{\pi E}\left[\frac{(d_l - kx)^{-1}}{-1 \times -k}\right]_o^l$$

$$= \frac{4P}{\pi Ek}\left[\frac{1}{d_1 - kx}\right]_0^l$$

$$= \frac{4P}{\pi Ek}\left\{\frac{1}{d_1 - kl} - \frac{1}{d_1}\right\} \quad \text{but} \quad k = \frac{d_1 - d_2}{l}$$

$$\therefore \quad = \frac{4P}{\pi E(d_1 - d_2)}\left[\frac{1}{d_1 - d_1 + d_2} - \frac{1}{d_1}\right]$$

$$= \frac{4Pl}{\pi E(d_1 - d_2)}\left(\frac{1}{d_2} - \frac{1}{d_1}\right)$$

$$= \frac{4Pl}{\pi E(d_1 - d_2)} \cdot \frac{d_1 - d_2}{d_1 d_2}$$

$$\therefore \quad \boxed{\delta = \frac{4Pl}{\pi E d_1 d_2}}$$

If both the diameters are equal to d.

$$\text{Then} \quad \delta = \frac{4Pl}{\pi E d^2}$$

EXAMPLE 2.7: A round steel rod of different cross-sections is loaded as shown in Fig. 2.15. Find the maximum stress induced in the rod and its deformations. Take $E = 210$ GPa.

Extension of Tapered Rectangular Strip

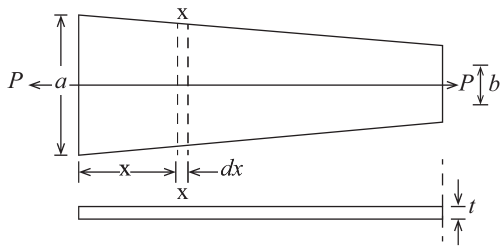


Figure 2.16

Consider any section $x-x$ distant x from the bigger end

$$\text{Width of the section} = t$$

$$\therefore \quad \text{Area of the section} = \frac{P}{t(a-kx)}$$

$$\therefore \quad \text{Extension of an elemental length } dx = \frac{Pdx}{t(a-kx)E}$$

$$\therefore \quad \text{Total extension of the rod} = \delta = \frac{P}{tE}\int_0^l \frac{dx}{a-kx}$$

$$= -\frac{P}{tE}\cdot\frac{1}{k} - \log_e[(a-kx)]_0^l$$

$$= -\frac{P}{tE}[\log_e(a-kl) - \log_e a]$$

$$= \frac{P}{tkE}\left[\frac{a}{a-k}\right]$$

$$\text{But} \quad k = \frac{a-b}{l}$$

$$\delta = \frac{P.l}{Et(a-b)}\log e\frac{a}{b}$$

EXAMPLE 2.8: A straight bar of steel rectangular in section is 3 m long and of thickness of 12 mm. The width of rod varies uniformly from 110 mm or one end to 35 mm at the other end. If the rod is subjected to an axial load (tensile) of 25 kN, find the extension of the rod. Take $E = 200000\ \text{N/mm}^2$.

$$\text{Extension of the rod, } \delta = \frac{Pl}{Et(a-b)} \log e \frac{a}{b}$$

$$P = 25000\ \text{N},\ l = 3000\ \text{mm},\ t = 12\ \text{mm}$$

$$a = 110\ \text{mm},\ ab = 35\ \text{mm}\ \&\ E = 200000\ \text{N/mm}^2$$

$$\therefore \quad \delta = \frac{25000 \times 3000}{2 \times 10^5 \times 12(110-35)} \log_e \frac{110}{35}$$

$$= \frac{25000 \times 3000}{2 \times 10^5 \times 12 \times 75} \times 1.1452$$

$$= 0.477\ \text{mm} \quad \textbf{Ans}$$

EXAMPLE 2.9: A rigid bar *AB* is attached to two vertical rods as shown in Fig. 2.17 is horizontal before the load is applied. Determine the vertical movement of *P* if it is of magnitude 60 kN.

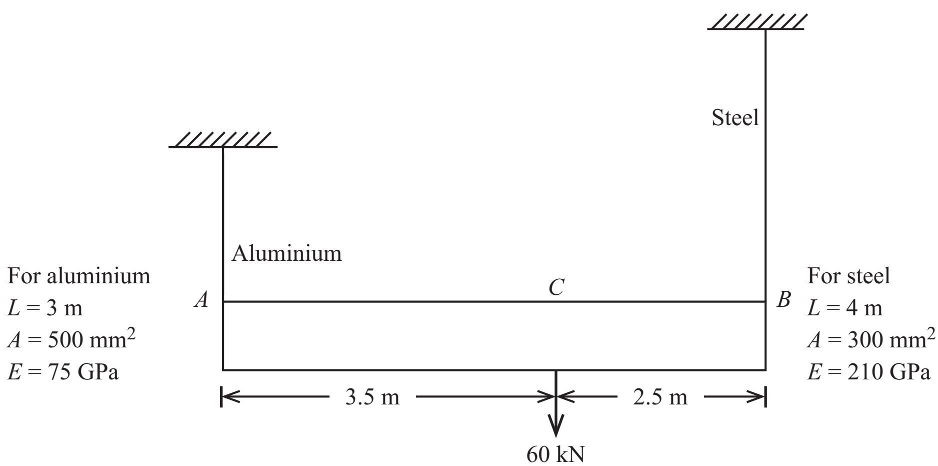


Figure 2.17

SOLUTION:

For Al, $\sum M_B = 0,\ 6P_{Al} = 2.5 \times 60$

$$\therefore \quad P_A = \frac{2.5 \times 60}{6} = 25 \text{ kN} = 25000 \text{ N}$$

$$\delta_{Al} = \frac{PL}{A.E} = \frac{25000 \times 3000}{500 \times 75000} = 2 \text{ mm}$$

For steel $\sum M_A = 0$ gives

$$P_{st} = 3.5 \times 60$$

$$P_{st} = \frac{3.5 \times 60}{6} = 35 \text{ kN}$$

$$\sigma_{ST} = \frac{35000 \times 4 \times 1000}{300 \times 200000} = 2.33 \text{ mm}$$

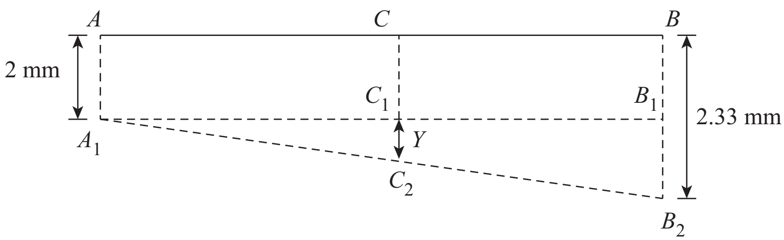


Figure 2.18

Now from similar triangles $A_1\ C_1\ C_2$ and A_1, B_1, B_2

$$\frac{Y}{A_1C_1} = \frac{B_1B_2}{A_1B_1}; \qquad \frac{Y}{3.5} = \frac{2.33 - 2}{6}$$

$$\therefore \quad Y = 0.1925 \text{ mm}$$

Now vertical movement of

$$\begin{aligned} P &= CC_2 \\ &= CC_1 + Y \\ &= 2 + 0.1925 = 2.1995 \text{ mm} \quad \textbf{Ans} \end{aligned}$$

Bar of Uniform Strength

As we have seen earlier that the stress due to self weight is not constant. It increases with the increase of distance from the lower end.

We wish to find the shape of the bar of which the self weight is considered and is having uniform stress on all sections when subjected to an axial P. Figure 2.19 shows such a bar of uniform stress in which the area increases from the lower end to the upper end.

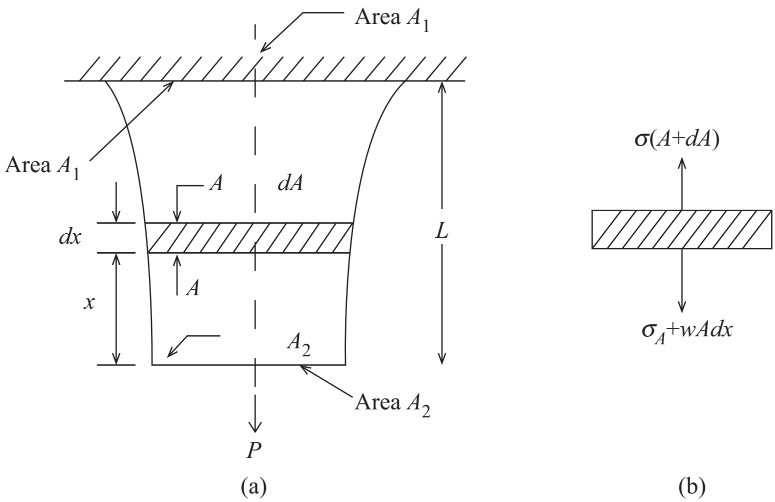


Figure 2.19

Let L be the length of bar, having area A_1, and area A_2 be cross-sectional areas of the bar at top and bottom, respectively.

Let w be the specific weight of the bar material (1.2. weight per unit volume of the bar).

The forces acting on the elementary stripe are:

i) Weight of the strip acting downward and is equal to $w\times$ volume of strip.
ii) Force on section AB due to uniform stress is equal to $\sigma \times A$. This is acting downward. A is area of elementary stripe.
iii) Force on section CD due to uniform (σ) is equal to $\sigma(A+dA)$. This is acting upwards.

$$\text{Total force acting upwards} = \text{Total force acting downwards}$$

$$\sigma(A+dA) = \sigma \times A + wA\,dx$$

$$\sigma_A + \sigma dA = \sigma A + wA.dx$$

$$\text{or} \quad \frac{dA}{A} = \frac{w}{\sigma}dx$$

Using equation,

$$A_1 = A_2 e^{\frac{wL}{\sigma}}$$

$$A_1 = 450\, e^{\frac{0.000075 \times 22000}{1777.8}} = 450 e^{9.28 \times 10^{-4}}$$

$$A_1 = 450.4 \text{ mm}^2 \quad \textbf{Ans}$$

EXAMPLE 2.11: A steel rod of 25 mm dia passes centrally through a copper tube of 30 mm inside diameter and 40 mm outside diameter. Copper tube is 850 mm long and is closed by rigid washers of negligible thickness, which are fastened by nut threaded on the rod as shown in Fig. 2.20. The nuts are tightened till the load on the assembly is 20 kN. Calculate: i) the initial stresses on the copper tube and steel rod and ii) also calculate increase in the stresses, when one nut is tightened by one-quarter of a turn relative to the other. Take pitch of the thread as 1.5 mm. E for copper = 100 GPa, E for steel = 100 GPa

SOLUTION:

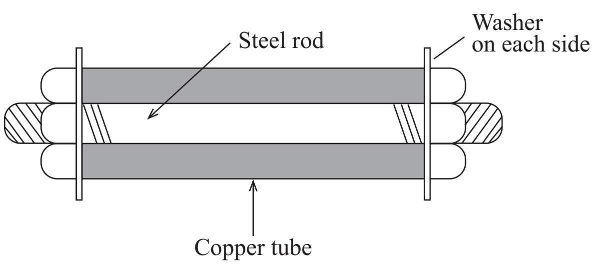


Figure 2.20

$$\text{Let} \quad \sigma_s = \text{Stress in steel rod}$$

$$\sigma_c = \text{Stress in copper rod}$$

i)

$$A_s = \frac{\pi}{4}(D_s)^2 = \frac{\pi}{4}(25)^2 = 156.25\pi \text{ mm}^2$$

$$A_c = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(40^2 - 30^2) = 175\pi \text{ mm}^2$$

Tensile rod on steel = Compressive load on copper tube

$$\sigma_s = \frac{A_c}{A_s} \times \sigma_c = \frac{175\pi}{156.25\pi} \times \sigma_c$$

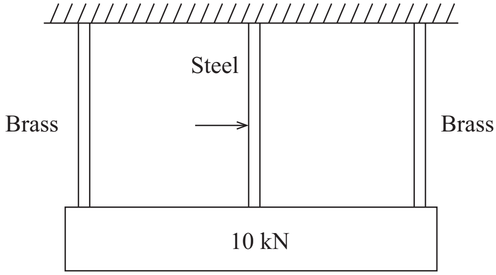


Figure 2.21

SOLUTION:

$$\sigma_s \times 100 + 100\sigma_b + 100\sigma_b \qquad \text{(i)}$$

$$100\sigma_s + 200\sigma_b = 10000$$

$$\sigma_s + 2\sigma_b = 100 \text{ N/mm}^2 \qquad \text{(ii)}$$

$$\frac{\sigma_s}{200 \times 10^3} = \frac{\sigma_b}{100 \times 10^3}$$

$$\therefore \quad \sigma_s = 2\sigma_b \qquad \text{(iii)}$$

substituting for σ_s in (ii)

$$2\sigma_b + 2\sigma_b = 100;$$

$$\sigma_b = \frac{100}{4} = 25 \text{ MPa} \quad \textbf{Ans.}$$

$$\sigma_s = 2 \times 25 = 50 \text{ MPa} \quad \textbf{Ans.}$$

EXAMPLE 2.13: Two steel rods and one copper rod each of 20 mm diameter together support a load of 50 kN as shown in Fig. 2.22. Find the stress in each rod. Take $E_s = 200$ GPa, $E_b = 100$ GPa

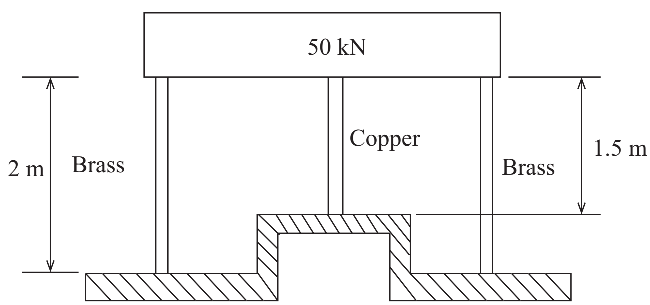


Figure 2.22

$$A_c = A_s = \frac{\pi}{4}(20)^2 = 314 \text{ mm}^2$$

$$\text{Total area of steel } A's + 314 \times 2 = 628 \text{ mm}^2$$

$$\sigma_s A'_s + \sigma_c A_c = 50000$$

$$628\sigma_s + 314\sigma_c = 50000$$

$$2\sigma_s + \sigma_c = 159.24 \tag{i}$$

$$\frac{\sigma_s l_s}{E_s} = \frac{\sigma_c l_c}{E_c};$$

$$\frac{\sigma_s \times 2000}{200000} = \frac{\sigma_c \times 1500}{100000}$$

$$\sigma_s = 1.5\sigma_c \tag{ii}$$

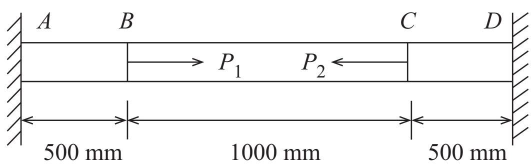
Substituting for σ_s from Eqn. (ii) in Eqn. (i)

$$2 \times 1.5\sigma_c + \sigma_c = 159.24$$

$$\therefore \quad \sigma_c = 39.81 \text{ MPa} \quad \textbf{Ans.}$$

$$\sigma_s = 1.5 \times 39.81 = 59.7 \text{ MPa} \quad \textbf{Ans.}$$

EXAMPLE 2.14: A uniform bar *ABCD* has built-in ends *A&D*. It is subjected to two point loads P_1 and P_2 equal to 80 kN and 40 kN at *B* and *C* as shown in Fig. 2.23. Find values of reactions at *A* and *D*.



SOLUTION:

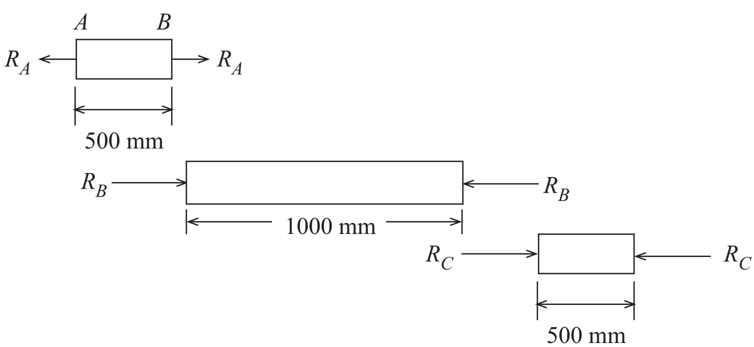


Figure 2.23

2.10 Figure 2.26 shows a rigid bar *ABC* hinged at *A* and suspended at two points *B* and *C* by two bars *BD* and *CE*, made of aluminium and steel, respectively. The bar carries a load of 20 kN midway between *B* and *C*. The cross-sectional area of aluminium bar *BD* is 3 mm^2 and that of steel bar *CE* is 2 mm^2. Determine the loads taken by the two bars *BD* and *CE*.

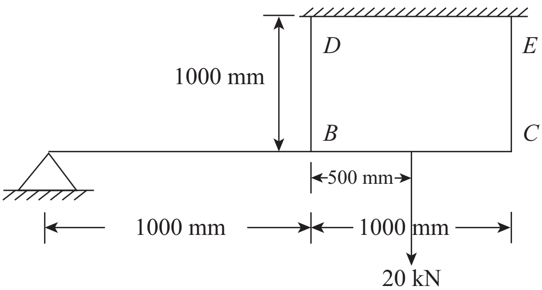


Figure 2.26

[**Ans** $P_a = 3.481$ kN, $P_s = 13.26$ kN]