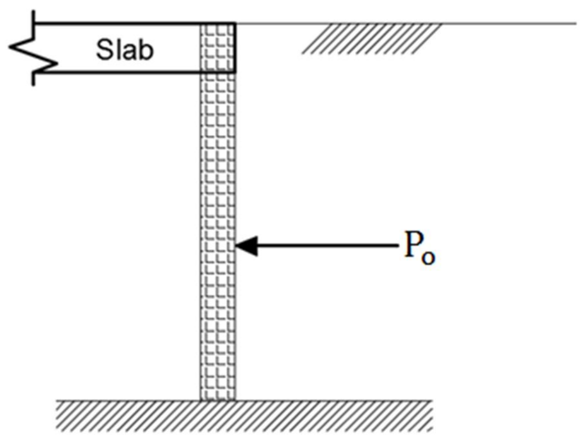
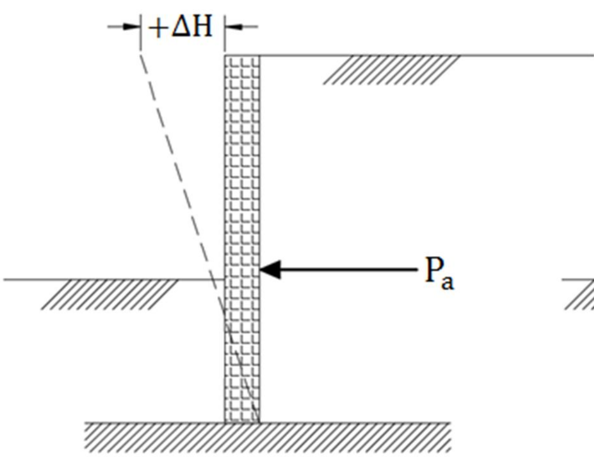
Chapter (7)
Lateral Earth Pressure

Introduction

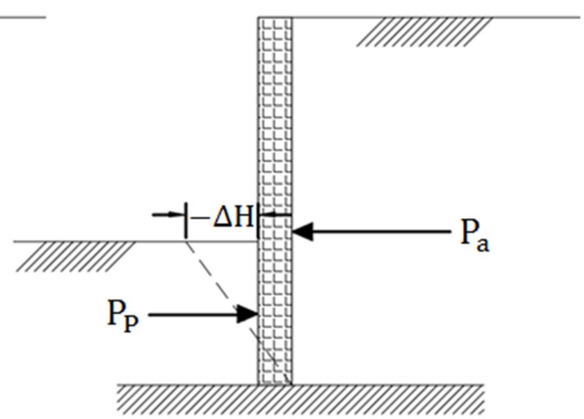
Vertical or near vertical slopes of soil are supported by retaining walls, cantilever sheet-pile walls, sheet-pile bulkheads, braced cuts, and other similar structures. The proper design of those structures required **estimation of lateral earth pressure**, which is a function of several factors, such as **(a)** type and amount of wall movement, **(b)** shear strength parameters of the soil, **(c)** unit weight of the soil, and **(d)** drainage conditions in the backfill. The following figures shows a retaining wall of height H. For similar types of backfill.



At rest LEP



Active LEP



Passive LEP

As shown in figure above, there are three types of Lateral Earth Pressure (LEP):

1. At Rest Lateral Earth Pressure:
The wall may be restrained from moving, for example; basement wall is restrained to move due to slab of the basement and the lateral earth force in this case can be termed as" P_o".

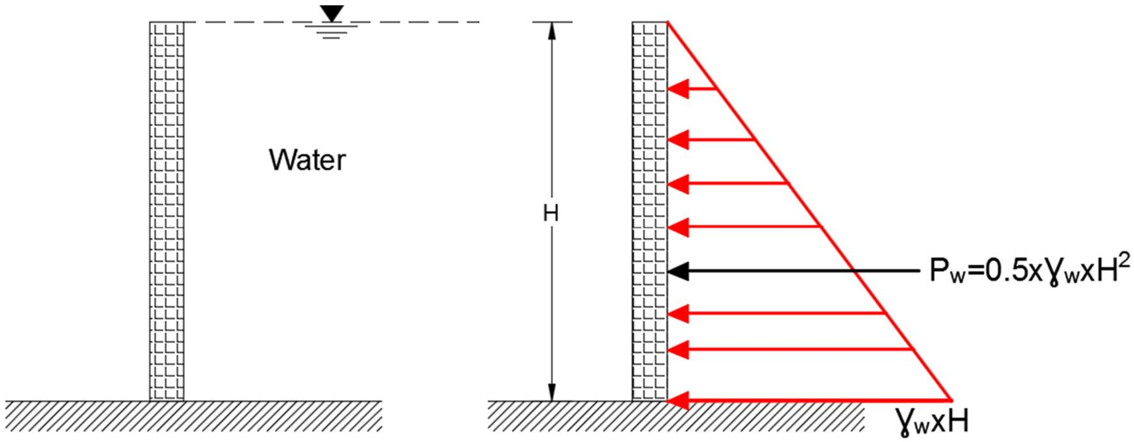
2. Active Lateral Earth Pressure:
In case of the wall is free from its upper edge (retaining wall), the wall may move away from the soil that is retained with distance " $+\Delta H$ " (i.e. the soil pushes the wall away) this means the soil is active and the force of this pushing is called active force and termed by " P_a".

3. Passive Lateral Earth Pressure:
For the wall shown above (retaining wall) in the left side there exist a soil with height less than the soil in the right and as mentioned above the right soil will pushes the wall away, so the wall will be **pushed into** the left soil (i.e. soil compresses the left soil) this means the soil has a passive effect and the force in this case is called passive force and termed by " P_P".

Now, we want to calculate the lateral pressure from water firstly and from earth (3 cases mentioned above) secondly.

Lateral Pressure from Water

As we learned previously in fluid mechanics course, the pressure of static fluid at a specific point is the same in all directions “Pascal’s Law”. So if there exist water in the soil (saturated soil) **we must** calculate the vertical stress for soil alone (effective stress) and calculate the vertical pressure for water alone (because the horizontal pressure of water is the same as vertical pressure), but for soil, each one –according soil parameters- having different transformation factor from vertical to horizontal pressure as we will discuss later. The following figure showing that the horizontal pressure of water against a wall is the same as vertical pressure:



At Rest Lateral Earth Pressure:

As stated above, the soil in this case is static and can't pushes the wall with any movement, the transformation factor of vertical pressure to horizontal pressure in this case is "K_o" and the lateral earth force is termed by "P_o"

Calculation of at rest lateral earth force "P_o" for different cases:

Firstly the value of K_o can be calculated as following:

$K_o = 1 - \sin\phi$

Always, (at rest) lateral earth pressure at any depth (z) may be calculated as following:

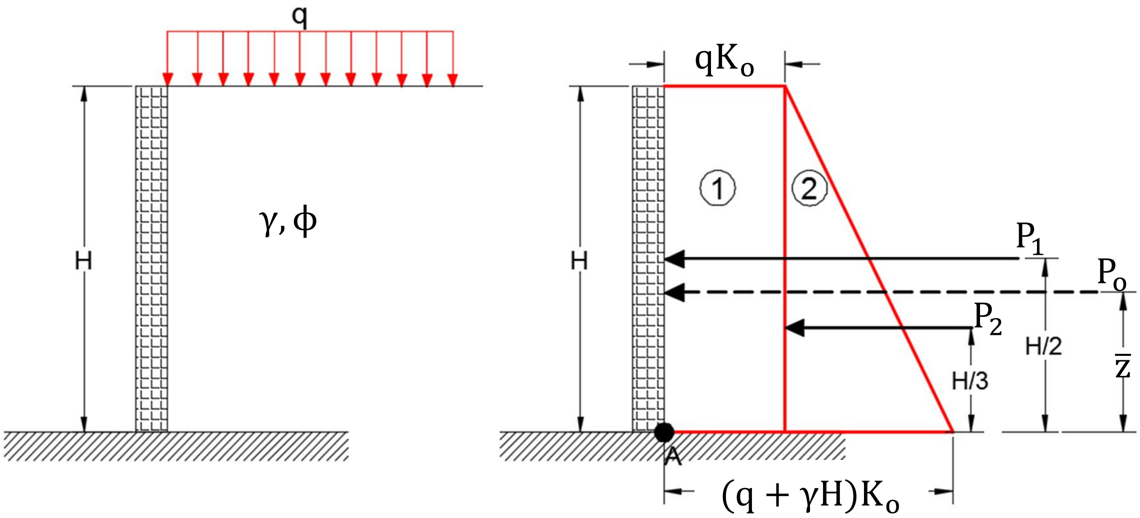
$\sigma_{h,o} =$ vertical effective pressure $(\sigma'_v) \times K_o +$ Pore water pressure (u)

$\sigma'_v = q + \gamma' \times z \quad , \quad u = \gamma_w \times z$

$\rightarrow \sigma_{h,o} = \sigma'_v \times K_o + u$

Note that the value of (u) doesn't multiplied by any factor since the horizontal pressure of water is the same as vertical pressure.

As shown there is no water table.



Firstly we calculate the vertical stress at each depth (each change):

At depth $z = 0.0$:

$\sigma'_v = q$

At depth $z = H$:

$\sigma'_v = q + \gamma \times H$

So, lateral at rest pressure at each depth now can be calculated:

At depth $z = 0.0$:

$\sigma_{h,o} = qK_o$

At depth $z = H$:

$\sigma_{h,o} = (q + \gamma \times H) \times K_o$

Now calculate the lateral forces P_1 and P_2 and then calculate P_o:

$P_1 = \text{Area of rectangle (1)} = qK_o \times H$ (per unit lenght)

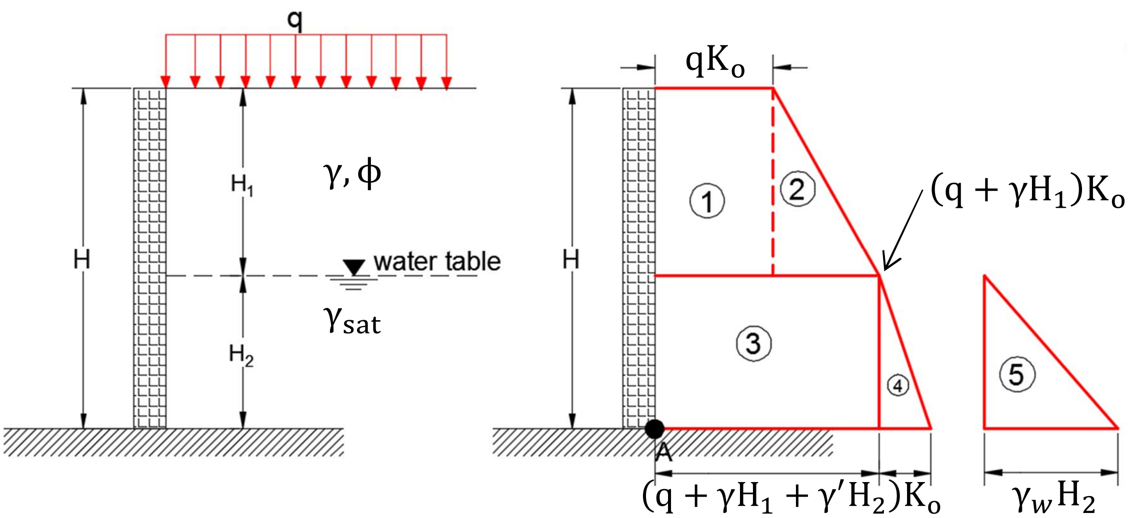
$P_2 = \text{Area of triangle (2)} = \frac{1}{2} \times (\gamma \times H \times K_o) \times H = \frac{1}{2} \times \gamma \times H^2 \times K_o$

$P_o = P_1 + P_2$

To find the location of P_o, take summation moments at point A (for ex.):

$P_1 \times \frac{H}{2} + P_2 \times \frac{H}{3} = P_o \times \bar{z} \rightarrow\rightarrow \bar{z} = \checkmark.$

In case of water table (if exist):



Firstly we calculate the vertical stress at each depth (each change):

At depth $z = 0.0$:

$\sigma'_v = q$

At depth $z = H_1$:

$\sigma'_v = q + \gamma \times H_1 \qquad u = 0.0$

At depth $z = H_2$:

$\sigma'_v = q + \gamma \times H_1 + \gamma' H_2 \quad (\gamma' = \gamma_{sat} - \gamma_w)$

$u = \gamma_w H_2$

Now calculate the lateral forces P_1, P_2, P_3, P_4 and P_5 then calculate P_o:

$P_1 = \text{area of recangle}(1) = qK_oH_1$

$P_2 = \text{area of triangle}(2) = \frac{1}{2}\gamma H_1^2 K_o$

$P_3 = \text{area of recangle}(3) = (q + \gamma H_1)K_o \times H_2$

$P_4 = \text{area of triangle}(4) = \frac{1}{2}\gamma' H_2^2 K_o$

$P_5 = \text{area of triangle}(5) = \frac{1}{2}\gamma_w H_2^2 \quad (\text{factor for water} = 1)$

$P_o = P_1 + P_2 + P_3 + P_4 + P_5$

To find the location of P_o, take summation moments at point A (for ex.):
(**Note**, P_5 must be included in moment equation)

(See example 7.1 Page 327)

Lateral Earth PressureTheories

Two theories are used to calculate lateral earth pressure (active and passive):
Rankine Earth Pressure theory and Coulomb's Earth Pressure theory.
Firstly we will learn rankine earth pressure theory (the most important) and then coulomb earth pressure theory.

Rankine Active Lateral Earth Pressure

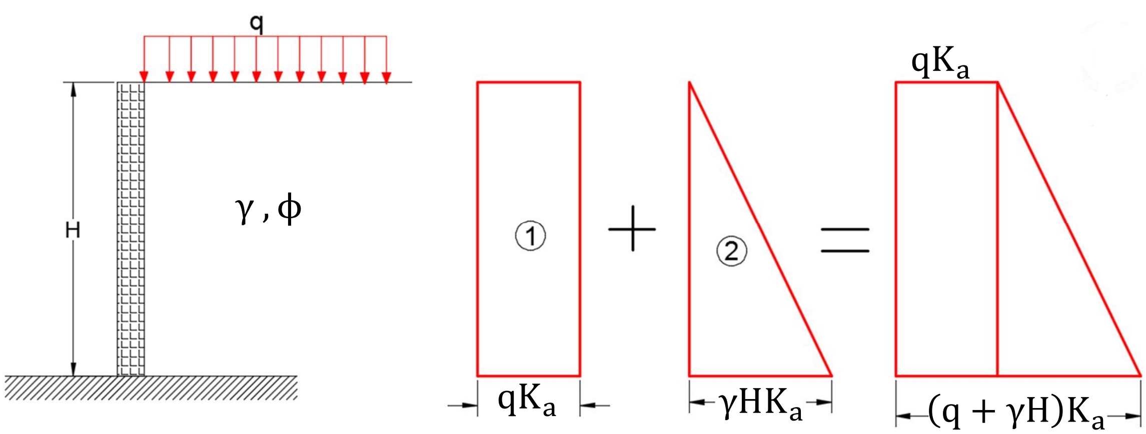
This theory is based mainly on the assumption of neglecting friction between the soil and the wall, so no shear forces are developed on soil particles.
As previously introduced, the soil in this case pushes the wall far away.
The transformation factor of vertical pressure to horizontal pressure in this case is "K_a" and the lateral earth force is termed by "P_a"

Firstly the value of K_a can be calculated as following:

$$K_a = \tan^2(45 - \frac{\phi}{2})$$

There are different cases:

In case of granular soil (pure sand):



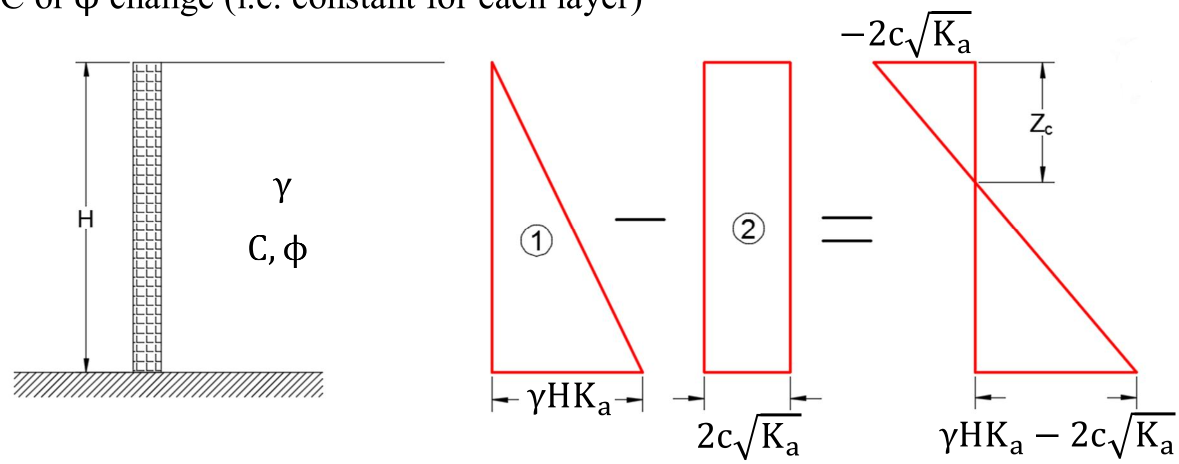
Exactly as the case of at rest LEP but here the transformation factor is K_a

$P_a = P_1 + P_2$

If the soil is $C - \phi$ soil:

The clay exerts a lateral earth pressure with value of $2c\sqrt{K}$ (in general).
In case of active earth pressure the value of K is K_a, and when the wall moves away from soil, the soil particles will disturbed and the cohesion of soil will decreased, so in case of active earth pressure we subtract the lateral earth pressure of clay because the cohesion of clay decreased.

The value of $2c\sqrt{K}$ is constant along the layer, and differ when the value of C or ϕ change (i.e. constant for each layer)



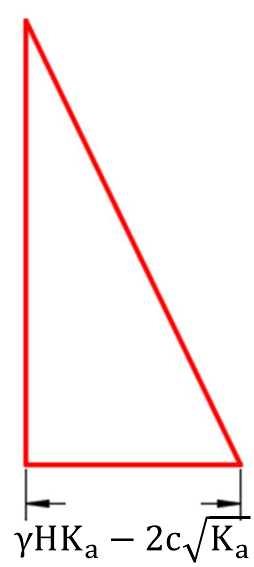
As we see, at depth z=0.0, the lateral earth pressure is $-2c\sqrt{K_a}$ this negative value (i.e. the soil will exerts a tensile stress on the wall) and this tensile stress will causes cracking on the wall from depth z=0.0 to depth = z_c.

Calculation of z_c:

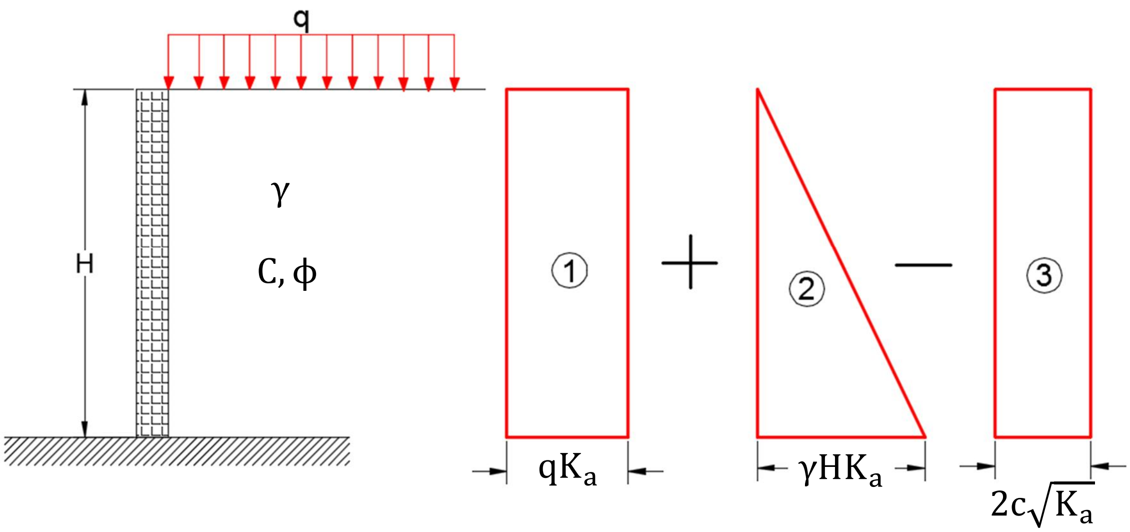
The lateral pressure will be zero at depth z_c:

$$\gamma\, z_c K_a - 2c\sqrt{K_a} = 0.0 \rightarrow z_c = \frac{2c\sqrt{K_a}}{\gamma K_a} = \frac{2c\sqrt{K_a}}{\gamma\sqrt{K_a}\times\sqrt{K_a}} = \frac{2c}{\gamma\sqrt{K_a}}$$

But, we know that soil can't develop any tension, so in design (in practice) we modify negative pressure to be zero and design for it (more safe because we enlarge lateral pressure) as shown:



If there exist surcharge:



So the final equation for active lateral earth pressure at and depth z can be calculated as following:

$\sigma_{h,a} = (q + \gamma H)K_a - 2c\sqrt{K_a}$

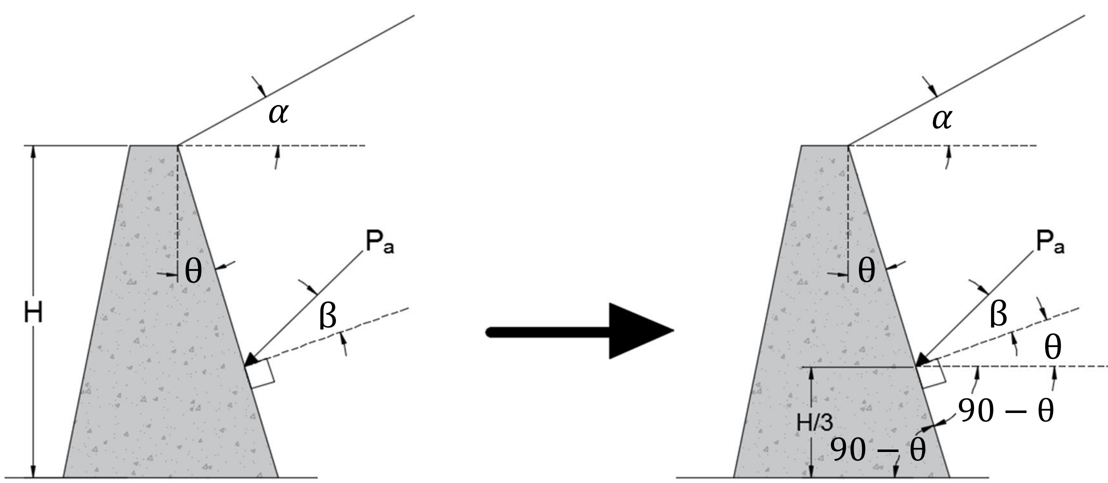
Note:

If there exist a water table, calculate the lateral force from water alone and then added it to the lateral force from soil to get total active force.

Generalized Case for Rankine Active Pressure:

In previous section, we calculate rankine active lateral pressure for the **vertical wall and horizontal backfill**, but here we will calculate the lateral earth pressure for general case (inclined wall and inclined backfill):

This General case for Granular soil only (pure sand)



$\alpha =$ inclination of backfill with horizontal
$\theta =$ inclination of wall with vertical
$\beta =$ inclination of P_a with the normal to the wall

From trigonometry, the angle between the normal to the wall and horizontal is θ.

Calculation of P_a:

$P_a =$ Vertical force $\times K_a$

Vertical force $=$ area of vertical pressure digram $= \frac{1}{2}\gamma H^2$

The value of K_a in this case is calculated from the following equation:

$$K_a = \frac{\cos(\alpha - \theta)\sqrt{1 + \sin^2\phi - 2\sin\phi\cos\psi_a}}{\cos^2\theta\left(\cos\alpha + \sqrt{\sin^2\phi - \sin^2\alpha}\right)}$$

$$\psi_a = \sin^{-1}\left(\frac{\sin\alpha}{\sin\phi}\right) - \alpha + 2\theta$$

The angle β is:

$$\beta = \tan^{-1}\left(\frac{\sin\phi\sin\psi_a}{1 - \sin\phi\cos\psi_a}\right)$$

$$P_a = \frac{1}{2}\gamma H^2 \times K_a$$

The location of P_a is $\frac{H}{3}$ from base as shown above

If we need the horizontal and vertical components of P_a:

$P_{a,h} = P_a\cos(\beta + \theta)$

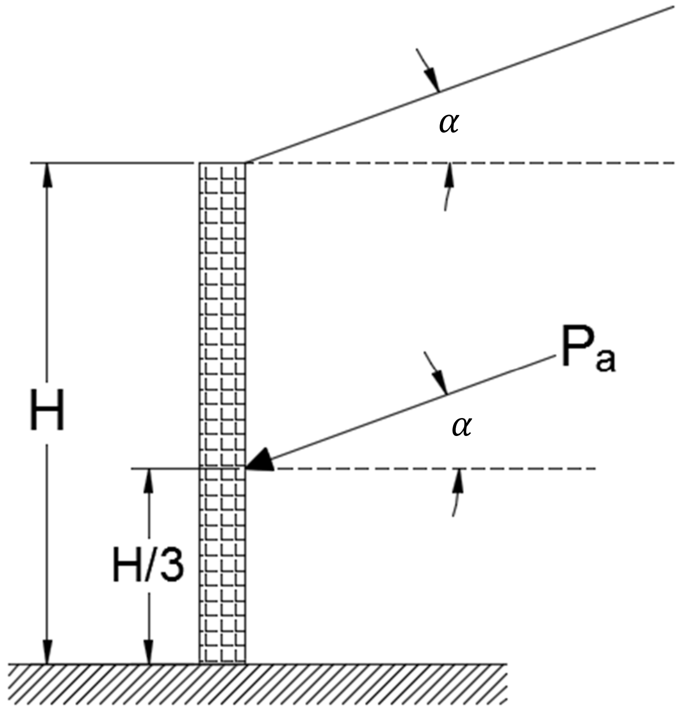
$P_{a,v} = P_a\sin(\beta + \theta)$

Case of vertical wall and inclined backfill:

Two cases:

1. **For pure sand**
2. **For $C - \phi$ soil**

For Pure sand:



Here P_a is inclined with angle α with horizontal.

Effective vertical pressure $= \gamma H$

$$P_a = \frac{1}{2}\gamma H^2 K_a$$

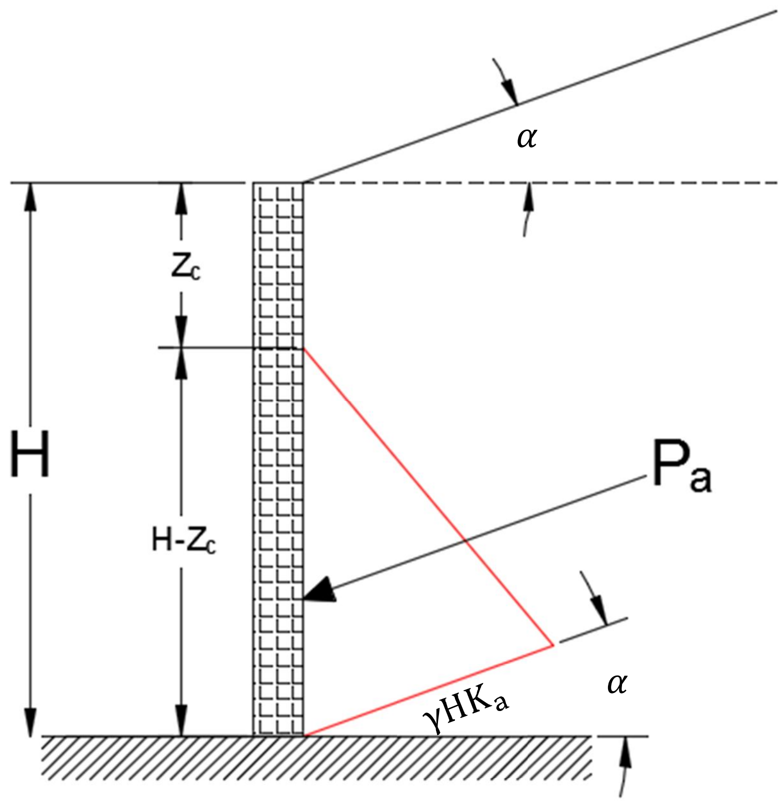
K_a in this case is calculated from the following equation:

$$K_a = \cos\alpha \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi}}$$

Or, by using (**Table 7.1Page 337)** easier than the equation above.

For $C-\phi$ soil

Here the force P_a is inclined with angle α with horizontal, but the calculation of P_a will differ because there exist clay and there is a tensile stress from clay at depth Z_c as shown:



The value of Z_c in this case is calculated as following:

$$Z_c=\frac{2c}{\gamma}\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}$$

Calculation of P_a:

Vertical pressure$=\gamma H$

Horizontal pressure $=\gamma HK_a$

$K_a=K'_a\cos\alpha \rightarrow$ Horizontal pressure $=\gamma HK'_a\cos\alpha$

$$P_a=\frac{1}{2}\times(\gamma HK'_a\cos\alpha)\times(H-Z_c)$$

K'_a can be calculated from (**Table 7.2 Page 338**)

Note

The calculated value of P_a is inclined by angle α with horizontal, so:

$P_{a,h} = P_a \cos(\alpha)$
$P_{a,v} = P_a \sin(\alpha)$

Rankine Passive Lateral Earth Pressure

As previously introduced, the wall in this case pushed into the soil.
The transformation factor of vertical pressure to horizontal pressure in this case is "K_P" and the lateral earth force is termed by "P_P"

Firstly the value of K_P can be calculated as following:

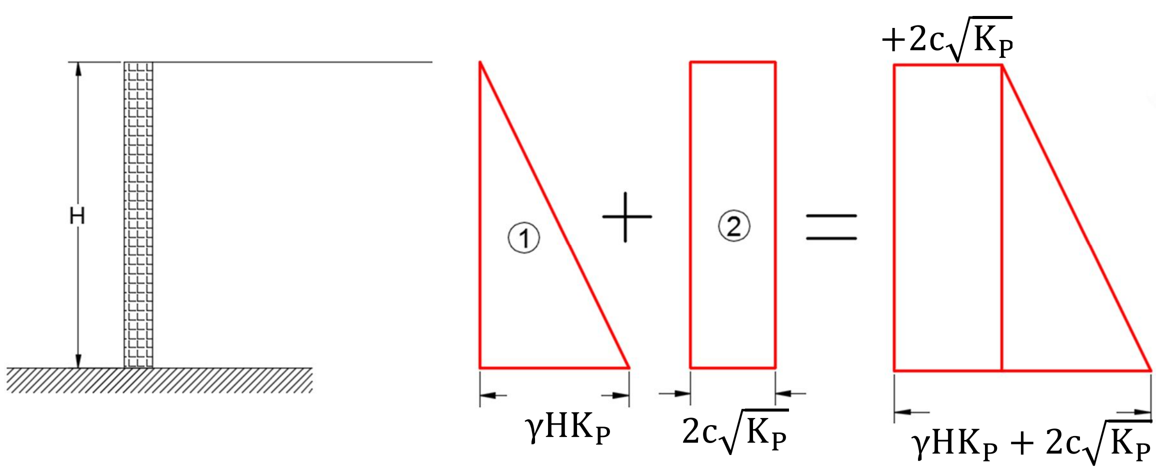
$$K_P = \tan^2(45 + \frac{\phi}{2})$$

The only difference between passive and active is in the formula of calculating K.

If the soil is $C - \phi$ soil:

In case of passive earth pressure the value of K is K_P, and when the wall moves into the soil, the soil particles will converges and the cohesion of soil will increased, so in case of passive earth pressure we **add** the lateral earth pressure of clay because the cohesion of clay increased.

The value of $2c\sqrt{K}$ is constant along the layer, and differ when the value of C or ϕ change (i.e. constant for each layer)



If there exist surcharge, will be added to vertical pressure and the final form for passive rankine pressure will be:

$$\sigma_{h,P} = (q + \gamma H)K_P + 2c\sqrt{K_P}$$

Rankine Passive Pressure (Vertical wall and inclined backfill):

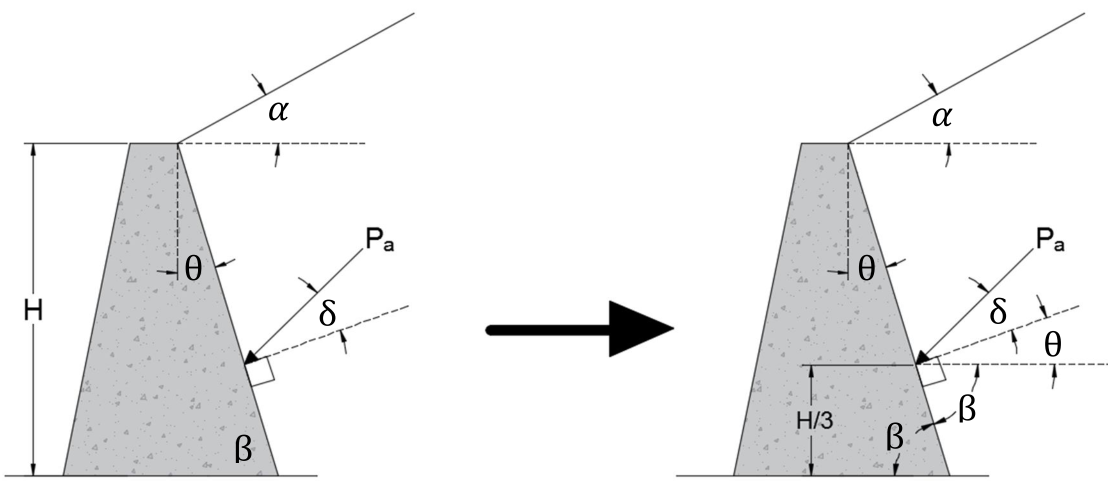
The same as rankine active pressure for this case, the only difference is in the equation of calculating K_P (negative sign in active transformed to positive sign in passive). **(Page 363).**

Coulomb's Lateral Earth Pressure Theory

The main assumption of this theory is considering the friction between the wall and the soil, this friction angle between the soil and the wall is (δ).
So there exist shear stresses on the soil particles and the equations for calculating passive lateral earth coefficient will differ from equations of active lateral earth coefficient.
This theory deal only with granular soil (pure sand).

Coulomb's Active Lateral Earth Pressure

General case (inclined wall and inclined backfill):



$\alpha =$ inclination of backfill with horizontal
$\theta =$ inclination of wall with vertical
$\beta =$ inclination of wall with the horizontal
$\delta =$ friction angle between soil and wall
From trigonometry, the angle between the normal to the wall and horizontal is θ.

Calculation of P_a:

$P_a = \text{Vertical force} \times K_a$

$\text{Vertical force} = \text{area of vertical pressure digram} = \frac{1}{2}\gamma H^2$

The value of K_a in this case is calculated from **equation 7.26 Page 342.**
The value of δ is less than ϕ since δ is friction angle between wall and soil, but ϕ is friction angle between soil itself.

In general, the value of $\delta = \left(\frac{1}{2} \to \frac{2}{3}\right)\phi$ so there are tables for calculating K_a for $\delta = \frac{2}{3}\phi$ and for $\delta = \frac{1}{2}\phi$

(Table 7.4 Page 343) for $\delta = \frac{2}{3}\phi \to$

K_a is obtained according the following angles:
α, β and ϕ

(Table 7.5 Page 344) for $\delta = \frac{1}{2}\phi$

Now, we can easily calculate the value of P_a:

$$P_a = \frac{1}{2}\gamma H^2 \times K_a$$

$$P_{a,h} = P_a \cos(\delta + \theta) \qquad P_{a,v} = P_a \sin(\delta + \theta)$$

Special case (when the wall is vertical "$\beta = 90$" and the backfill is horizontal "$\alpha = 0$") .

$$P_a = \frac{1}{2}\gamma H^2 \times K_a \quad \text{(Inclined by angle } \delta \text{ with horizontal)}$$

K_a can be calculated from(**Table 7.3 P342**)according the values of δ and ϕ, so for this special case:

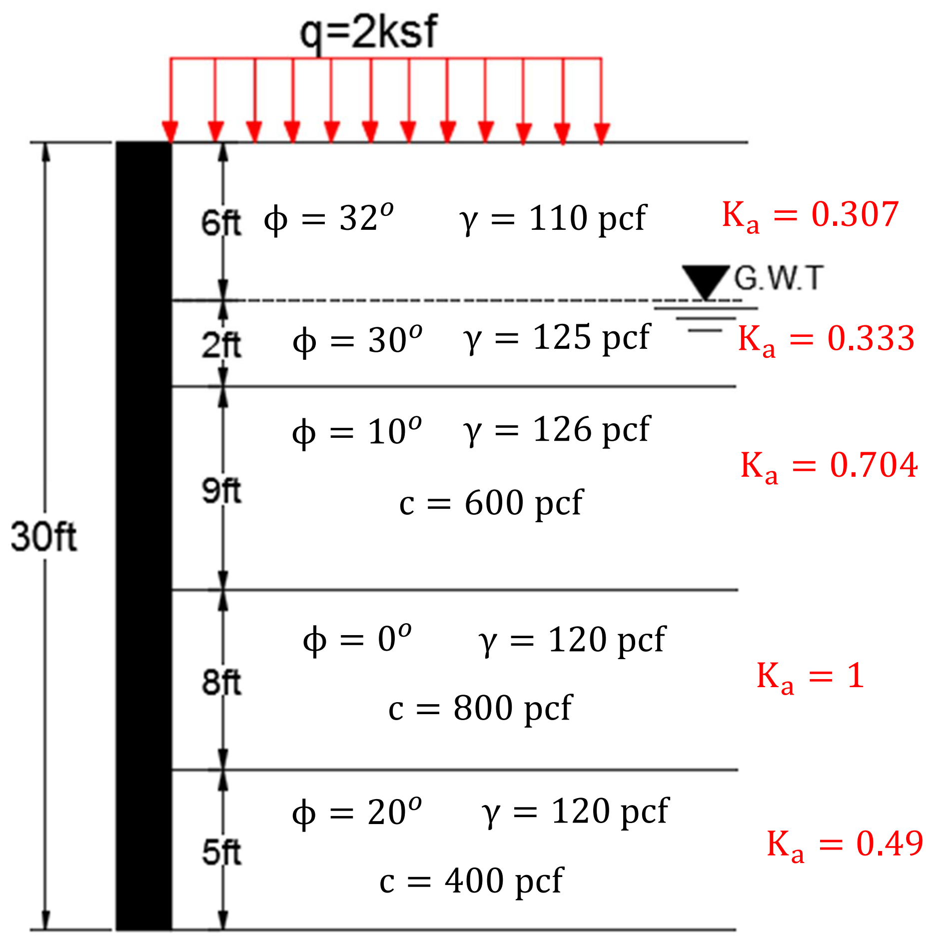
$$P_{a,h} = P_a \cos\delta \qquad P_{a,v} = P_a \sin\delta$$

Coulomb's Passive Lateral Earth Pressure

The same as coulomb active pressure for this case, the only difference is in the equation of calculating K_P (negative sign in active transformed to positive sign in passive). **(Page 365).**

Problems:

For the shown figure below. Plot the pressure diagram and find the resultant force F and its location under **active conditions.**



Solution

Note that the value of ϕ is differ for each layer, so the value of K_a will differ for each layer, thus the first step is to calculate the value of K_a for each layer.

$K_a = \tan^2\left(45 - \frac{\phi}{2}\right)$

For $\phi = 32 \rightarrow K_a = \tan^2\left(45 - \frac{32}{2}\right) = 0.307$

Calculate K_a for each layer by the same way.

The values of K_a are written on each layer on the figure above.

Now, calculate the lateral earth pressure at each depth (each change) from soil alone (i.e. vertical effective pressure x K_a), then the water will considered alone.

Before calculating the pressure, at each change we calculate the lateral earth pressure **just before** and **just after** the layer because the value of K_a is differ before and after the layer.

The general formula for active lateral earth pressure at any depth is:

$\sigma_{h,a} = (q + \gamma H)K_a - 2c\sqrt{K_a} \qquad \gamma_w = 62.4\text{pcf}$

$q = 2\text{ksf} = 2000\text{psf}$

$\mathbf{@z = 0.0}$:

$\sigma_{h,a} = (2000 + 0) \times 0.307 - 0 = 614 \text{ psf}$

$\mathbf{@z = 6ft}$:

Just before ($K_a = 0.307, c = 0$):

$\sigma_{h,a} = (2000 + 110 \times 6) \times 0.307 - 0 = 816.62 \text{ psf}$

Just after ($K_a = 0.333 , c = 0$):

$\sigma_{h,a} = (2000 + 110 \times 6) \times 0.333 - 0 = 885.8 \text{ psf}$

$\mathbf{@z = 8ft}$:

Just before ($K_a = 0.333, c = 0$):

$\sigma_{h,a} = (2000 + 110 \times 6 + (125 - 62.4) \times 2) \times 0.333 - 0 = 927.5 \text{ psf}$

Just after ($K_a = 0.704 , c = 600$):

$$\sigma_{h,a} = (2000 + 110 \times 6 + (125 - 62.4) \times 2) \times 0.704 - 2 \times 600 \times \sqrt{0.704} = 953.9 \text{ psf}$$

$\mathbf{@z = 17ft}$:

Just before ($K_a = 0.704, c = 600$):

$$\sigma_{h,a} = (2000 + 110 \times 6 + (125 - 62.4) \times 2 + (126 - 62.4) \times 9) \times 0.704 - 2 \times 600 \times \sqrt{0.704} = 1356.9 \text{ psf}$$

Just after ($K_a = 1 , c = 800$):

$$\sigma_{h,a} = (2000 + 110 \times 6 + (125 - 62.4) \times 2 + (126 - 62.4) \times 9) \times 1 - 2 \times 800 \times \sqrt{1} = 1757.6 \text{ psf}$$

$@\mathbf{z = 25ft}$:

Just before $(K_a = 1, c = 800)$:

$$\sigma_{h,a} = \begin{pmatrix} 2000 + 110 \times 6 + (125 - 62.4) \times 2 + (126 - 62.4) \times 9 \\ +(120 - 62.4) \times 8 \end{pmatrix} \times 1$$

$$-2 \times 800 \times \sqrt{1} = 2218.4 \text{ psf}$$

Just before $(K_a = 0.49, c = 400)$:

$$\sigma_{h,a} = \begin{pmatrix} 2000 + 110 \times 6 + (125 - 62.4) \times 2 + (126 - 62.4) \times 9 \\ +(120 - 62.4) \times 8 \end{pmatrix} \times 0.49$$

$$-2 \times 400 \times \sqrt{0.49} = 1311 \text{ psf}$$

$@\mathbf{z = 30ft}$:

$$\sigma_{h,a} = \begin{pmatrix} 2000 + 110 \times 6 + (125 - 62.4) \times 2 + (126 - 62.4) \times 9 \\ +(120 - 62.4) \times 8 + (120 - 62.4) \times 5 \end{pmatrix} \times 0.49$$

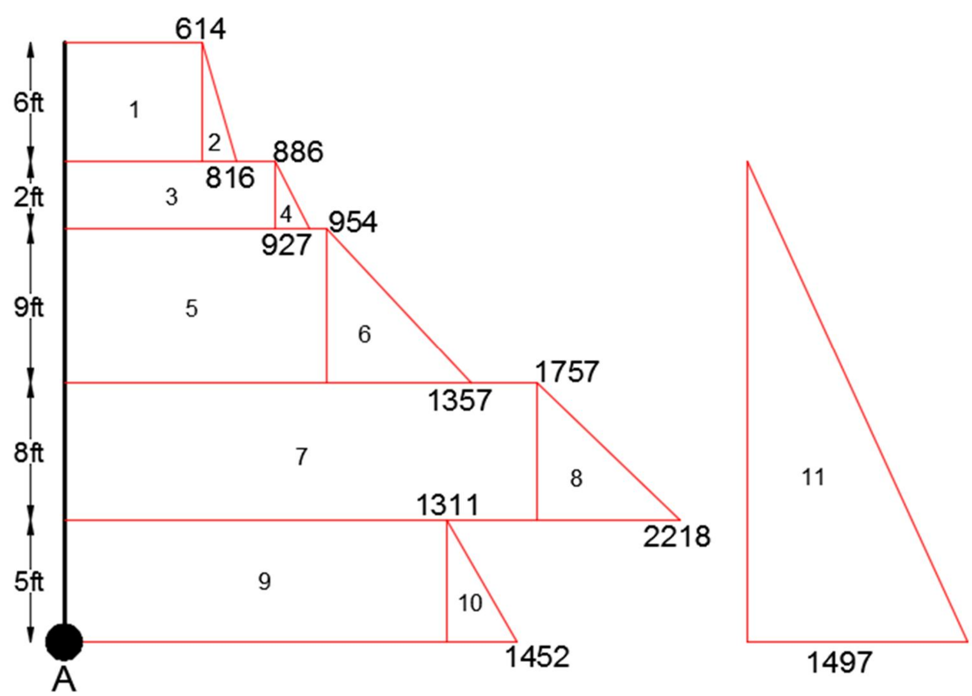
$$-2 \times 400 \times \sqrt{0.49} = 1452 \text{ psf}$$

Now we calculate the pore water pressure:

Water starts at depth 6ft to the end (i.e. depth of water is 24 ft)

$u = 62.4 \times 24 = 1497.6 \text{ psf}$

Now we draw the pressure diagram as following:



Now calculate the force for each shape (1 to 11) i.e. area of each shape and then sum all of these forces to get total active lateral force P_a.

$P_a \cong 57214 \text{ Ib/ft}'$ ✓.

To calculate the location of P_a, take summation moments at point A (Include the moment from water force "don't forget it")

The location of P_a is ≅ 10.7 ft (above point A) ✓.