Flexural Analysis of Reinforced Concrete Beams

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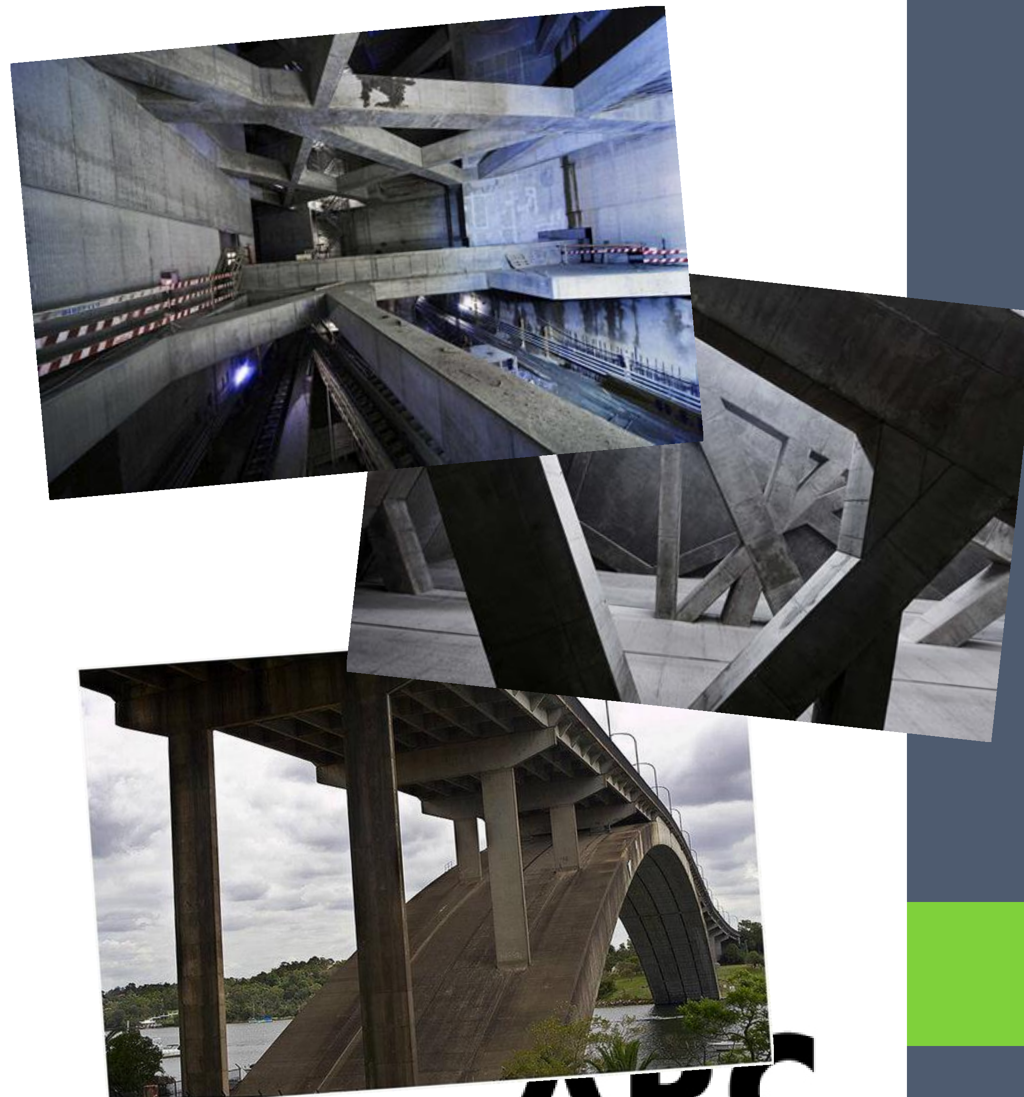
Structural Concrete

- It’s everywhere
- Beams are one of the most common structural components
- Parking ramps, high rises, bridges…



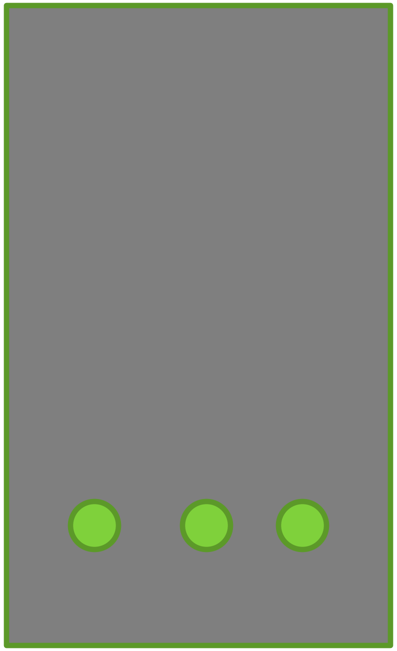
Analysis versus Design

- Analysis:
  - Determining the strength of a beam with given dimensions and reinforcement.

- Design:
  - Creating a beam that will carry a specified load or combination of loads.

Analysis

- <u>Ex</u>: Determine the flexural strength of the following member:



- b = 10in
- h = 25in
- d = 23in
- Steel: Three No. 8 bars (A_s = 2.37in^2)

Step 1

- Start with the reinforcement ratio:

$$\rho = \frac{A_s}{bd}$$

- Compare to the balance nt ratio to check if steel will yield:

$$\rho_b = \frac{\alpha f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_y}$$

Obtain "a":

- "a" represents the stress block caused by concrete compression.

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

- Insert into nominal str

$$M_n = A_s f_y \left(d - \frac{a}{2} \right)$$

- This yields the stren

Where does all this come from?

$$M_n = \rho f_y b d^2 \left(1 - \frac{\beta f_y \rho}{\alpha f'_c}\right)$$

$$M_n = \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f'_c}\right)$$

$$R = \rho f_y \left(1 - 0.59 \frac{\rho f_y}{f'_c}\right)$$

$$M_n = R b d^2$$

Concrete Dimensions to Resist a Given Area (Beam Design)

- Find cross section of concrete and area of steel required for a simply supported rectangular beam
  - Span = 15ft
  - Dead Load = 1.27 kips/ft
  - Live Load = 2.15 kips/ft
  - f'c = 4000 psi
  - fy = 60,000 psi

Step 1

- Load factoring:
  - Dead Load: Factor = 1.2
  - Live Load: Factor = 1.6
  - Ultimate Load:

$$wu = 1.2DL + 1.6LL$$

  - Ultimate Moment:

$$Mu = 1/8 * wu * L^2$$

Step 2:

- Maximum reinforcement ratio:

$$\rho = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_t}$$

Note: For $\phi = 0.90$, use $\epsilon_t = 0.005$.

- Lim

$$\rho_{max} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004}$$

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Step 3:

- Substitute known values into nominal strength equation:

$$M_n = \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f'_c}\right)$$

- bd^2 will be left c
- Use relations be to solve
  - If all else fails, guess

Step 4:

- Using b and d, determine required steel area:

$$A_s = \rho bd$$

- Select steel reinforcement and obtain dimensions for effective depth and cover

Step 5:

- Calculate the new reinforcement ratio based on the designed dimensions
  - Assume a reasonable value of “a” and solve for A_s in the nominal strength equation:

$$M_n = A_s f_y \left(d - \frac{a}{2} \right)$$

  - Check the assumed “a … ion:

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

Step 6:

- Use judgment to determine if the new calculated “a” value is reasonably close to the “a” value guessed.

- If so, the design of the beam is complete.

Similar Applications

- **Doubly Reinforced Beams**
- **T Beams**

References

- Nilson, Arthur H, David Darwin, and Charles W Dolan. *Design of Concrete Structures*. 13th ed. N.p.: McGraw Hill India, 2003. N. pag. Print.

- Images courtesy of Google Inc.