Reinforced Concrete Design

Notation:

a = depth of the effective compression block in a concrete beam
A = name for area
A_g = gross area, equal to the total area ignoring any reinforcement
A_s = area of steel reinforcement in concrete beam design
A_s' = area of steel compression reinforcement in concrete beam design
A_{st} = area of steel reinforcement in concrete column design
A_v = area of concrete shear stirrup reinforcement
ACI = American Concrete Institute
b = width, often cross-sectional
b_E = effective width of the flange of a concrete T beam cross section
b_f = width of the flange
b_w = width of the stem (web) of a concrete T beam cross section
c = distance from the top to the neutral axis of a concrete beam *(see x)*
cc = shorthand for clear cover
C = name for centroid
= name for a compression force
C_c = compressive force in the compression steel in a doubly reinforced concrete beam
C_s = compressive force in the concrete of a doubly reinforced concrete beam
d = effective depth from the top of a reinforced concrete beam to the centroid of the tensile steel
d' = effective depth from the top of a reinforced concrete beam to the centroid of the compression steel
d_b = bar diameter of a reinforcing bar
D = shorthand for dead load
DL = shorthand for dead load
E = modulus of elasticity or Young's modulus
= shorthand for earthquake load
E_c = modulus of elasticity of concrete
E_s = modulus of elasticity of steel
f = symbol for stress
f_c = compressive stress
f_c' = concrete design compressive stress
f_{pu} = tensile strength of the prestressing reinforcement
f_s = stress in the steel reinforcement for concrete design
f_s' = compressive stress in the compression reinforcement for concrete beam design
f_y = yield stress or strength
f_{yt} = yield stress or strength of transverse reinforcement
F = shorthand for fluid load
F_y = yield strength
G = relative stiffness of columns to beams in a rigid connection, as is Ψ
h = cross-section depth
H = shorthand for lateral pressure load
h_f = depth of a flange in a T section
$I_{transformed}$ = moment of inertia of a multi-material section transformed to one material
k = effective length factor for columns
ℓ_b = length of beam in rigid joint
ℓ_c = length of column in rigid joint
l_d = development length for reinforcing steel
l_{dh} = development length for hooks
l_n = clear span from face of support to face of support in concrete design
L = name for length or span length, as is l
= shorthand for live load
L_r = shorthand for live roof load
LL = shorthand for live load
M_n = nominal flexure strength with the steel reinforcement at the yield stress and concrete at the concrete design strength for reinforced concrete beam design
M_u = maximum moment from factored loads for LRFD beam design

n = modulus of elasticity transformation coefficient for steel to concrete
$n.a.$ = shorthand for neutral axis (N.A.)
pH = chemical alkalinity
P = name for load or axial force vector
P_o = maximum axial force with no concurrent bending moment in a reinforced concrete column
P_n = nominal column load capacity in concrete design
P_u = factored column load calculated from load factors in concrete design
R = shorthand for rain or ice load
= radius of curvature in beam deflection relationships (*see* ρ)
R_n = concrete beam design ratio = M_u/bd^2
s = spacing of stirrups in reinforced concrete beams
S = shorthand for snow load
t = name for thickness
T = name for a tension force
= shorthand for thermal load
U = factored design value
V_c = shear force capacity in concrete
V_s = shear force capacity in steel shear stirrups
V_u = shear at a distance of d away from the face of support for reinforced concrete beam design
w_c = unit weight of concrete
w_{DL} = load per unit length on a beam from dead load
w_{LL} = load per unit length on a beam from live load
$w_{self\,wt}$ = name for distributed load from self weight of member
w_u = load per unit length on a beam from load factors
W = shorthand for wind load
x = horizontal distance
= distance from the top to the neutral axis of a concrete beam *(see c)*
y = vertical distance
β_1 = coefficient for determining stress block height, *a,* based on concrete strength, f'_c
Δ = elastic beam deflection
ε = strain
ε_t = strain in the steel
ε_y = strain at the yield stress
λ = modification factor for lightweight concrete
ϕ = resistance factor
ϕ_c = resistance factor for compression
γ = density or unit weight
ρ = radius of curvature in beam deflection relationships (see R)
= reinforcement ratio in concrete beam design = A_s/bd
$\rho_{balanced}$ = balanced reinforcement ratio in concrete beam design
υ_c = shear strength in concrete design

Reinforced Concrete Design

Structural design standards for reinforced concrete are established by the *Building Code and Commentary (ACI 318-14)* published by the American Concrete Institute International, and uses strength design (also known as *limit state* design).

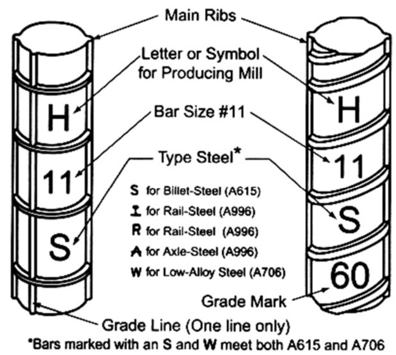
f'_c = concrete compressive design strength at 28 days (units of psi when used in equations)

Materials

Concrete is a mixture of cement, coarse aggregate, fine aggregate, and water. The cement hydrates with the water to form a binder. The result is a hardened mass with "filler" and pores. There are various types of cement for low heat, rapid set, and other properties. Other minerals or cementitious materials (like fly ash) may be added.

ASTM designations are

Type I:	Ordinary portland cement (OPC)
Type II:	Moderate heat of hydration and sulfate resistance
Type III:	High early strength (rapid hardening)
Type IV:	Low heat of hydration
Type V:	Sulfate resistant



GRADE 60

The proper proportions, by volume, of the mix constituents determine strength, which is related to the water to cement ratio (w/c). It also determines other properties, such as workability of fresh concrete. Admixtures, such as retardants, accelerators, or superplasticizers, which aid flow without adding more water, may be added. Vibration may also be used to get the mix to flow into forms and fill completely.

Slump is the measurement of the height loss from a compacted cone of fresh concrete. It can be an indicator of the workability.

Proper mix design is necessary for durability. The pH of fresh cement is enough to prevent reinforcing steel from oxidizing (rusting). If, however, cracks allow corrosive elements in water to penetrate to the steel, a corrosion cell will be created, the steel will rust, expand and cause further cracking. Adequate cover of the steel by the concrete is important.

Deformed reinforcing bars come in grades 40, 60 & 75 (for 40 ksi, 60 ksi and 75 ksi yield strengths). Sizes are given as # of 1/8" up to #8 bars. For #9 and larger, the number is a nominal size (while the actual size is larger).

Reinforced concrete is a composite material, and the average density is considered to be *150 lb/ft*^{3}. It has the properties that it will creep (deformation with long term load) and shrink (a result of hydration) that must be considered.

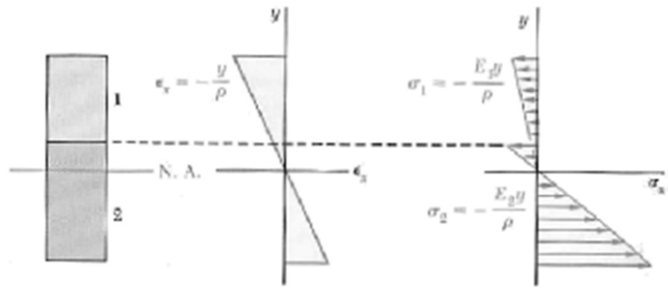
Construction

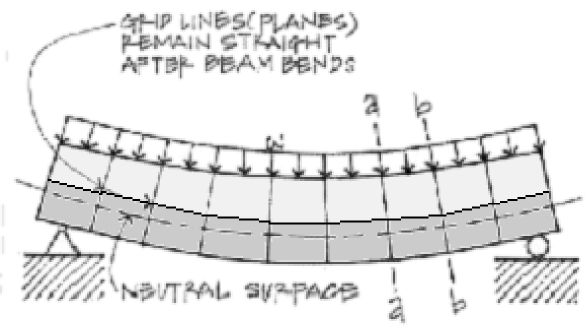
Because fresh concrete is a viscous suspension, it is cast or placed and *not poured*. Formwork must be able to withstand the hydraulic pressure. *Vibration* may be used to get the mix to flow around reinforcing bars or into tight locations, but excess vibration will cause segregation, honeycombing, and excessive *bleed* water which will reduce the water available for hydration and the strength, subsequently.

After casting, the surface must be worked. *Screeding* removes the excess from the top of the forms and gets a rough level. *Floating* is the process of working the aggregate under the surface and to "float" some paste to the surface. *Troweling* takes place when the mix has hydrated to the point of supporting weight and the surface is smoothed further and consolidated. *Curing* is allowing the hydration process to proceed with adequate moisture. Black tarps and curing compounds are commonly used. *Finishing* is the process of adding a texture, commonly by using a broom, after the concrete has begun to set.

Behavior

Plane sections of composite materials can still be assumed to be plane (strain is linear), *but* the stress distribution *is not* the same in both materials because the *modulus of elasticity* is different. (f=E·ε)





$$f_1 = E_1\varepsilon = -\frac{E_1 y}{R} \qquad f_2 = E_2\varepsilon = -\frac{E_2 y}{R}$$

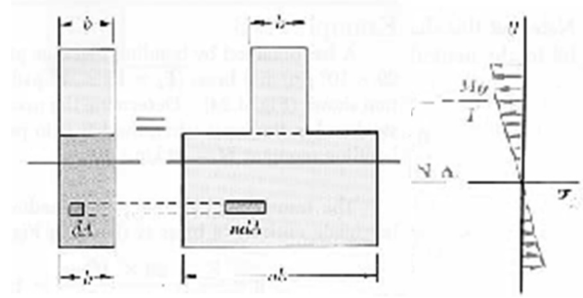
where R (or ρ) is the radius of curvature

In order to determine the stress, we can define n as the ratio of the elastic moduli: $n = \frac{E_2}{E_1}$

n is used to transform the width of the second material such that it sees the equivalent element stress.

Transformed Section y and I

In order to determine stresses in all types of material in the beam, we transform the materials into a single material, and calculate the location of the neutral axis and modulus of inertia for that material.



ex: When material 1 above is concrete and material 2 is steel

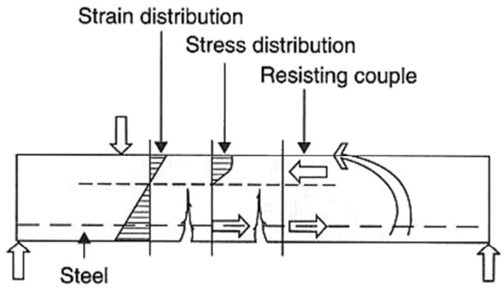
to transform steel into concrete $n = \frac{E_2}{E_1} = \frac{E_{steel}}{E_{concrete}}$

to find the neutral axis of the equivalent concrete member we transform the width of the steel by multiplying by n

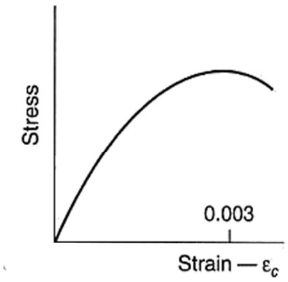
to find the moment of inertia of the equivalent concrete member, $I_{transformed}$, use the new geometry resulting from transforming the width of the steel

concrete stress: $f_{concrete} = -\frac{My}{I_{\text{transformed}}}$ steel stress: $f_{steel} = -\frac{Myn}{I_{transformed}}$

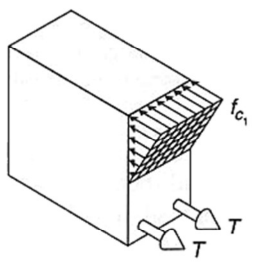
Reinforced Concrete Beam Members



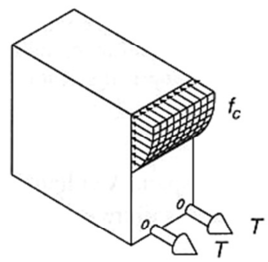
Stresses in the concrete above the neutral axis are compressive and nonlinearly distributed. In the tension zone below the neutral axis, the concrete is assumed to be cracked and the tensile force present to be taken up by reinforcing steel.



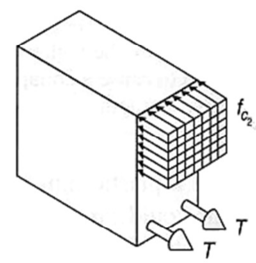
Typical stress-strain curve for concrete.



Working stress analysis. (Concrete stress distribution is assumed to be linear. Service loads are used in calculations.)



Actual stress distribution near ultimate strength (nonlinear).



Ultimate strength analysis. (A rectangular stress block is used to idealize the actual stress distribution. Calculations are based on ultimate loads and failure stresses.)

Strength Design for Beams

Strength design method is similar to LRFD. There is a *nominal* strength that is reduced by a factor ϕ which must exceed the factored design stress. For beams, the concrete only works in compression over a rectangular "stress" block above the n.a. from elastic calculation, and the steel is exposed and reaches the yield stress, F_y

For stress analysis in reinforced concrete beams

- the steel is transformed to concrete
- any concrete in tension is assumed to be cracked and to have <u>no strength</u>
- the steel can be in tension, and is placed in the bottom of a beam that has positive bending moment
-

The neutral axis is where there is no stress and no strain. The concrete above the n.a. is in compression. The concrete below the n.a. is considered ineffective. The steel below the n.a. is in tension.

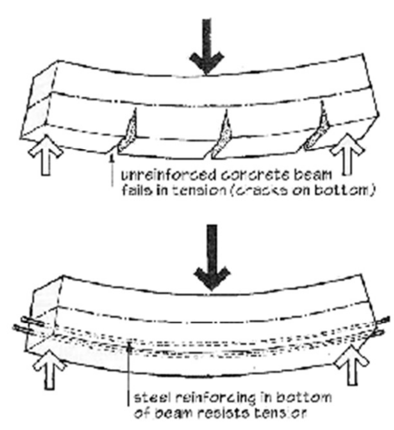
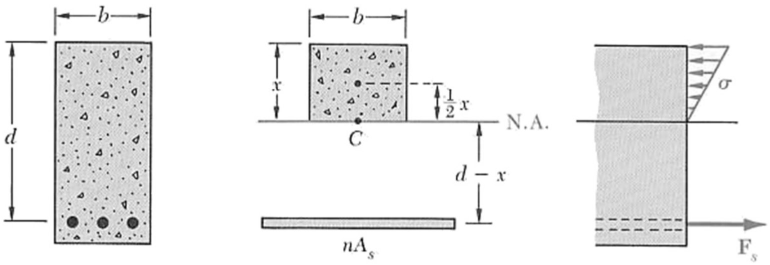


Figure 8.5: Bending in a concrete beam without and with steel reinforcing.

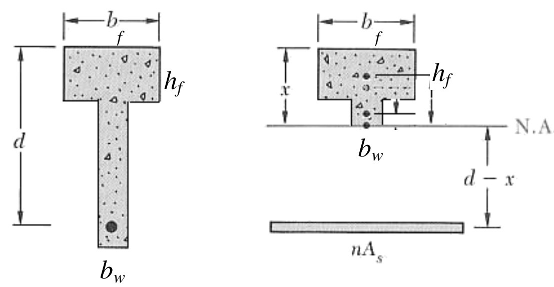
Because the n.a. is defined by the moment areas, we can solve for *x* (or *c*) knowing that d is the distance from the top of the concrete section to the centroid of the steel:

$$bx \cdot \frac{x}{2} - nA_s(\, d - x \,) = 0$$



x can be solved for when the equation is rearranged into the generic format with a, b & c in the binomial equation: $ax^2 + bx + c = 0$ by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

T-sections



If the n.a. is *above* the bottom of a flange in a T section, x is found as for a rectangular section.

If the n.a. is *below* the bottom of a flange in a T section, x is found by including the flange and the stem of the web (b_w) in the moment area calculation:

$$b_f h_f \left(x - \frac{h_f}{2} \right) + \left(x - h_f \right) b_w \frac{\left(x - h_f \right)}{2} - nA_s (d - x) = 0$$

Load Combinations (Alternative values are allowed)

$1.4D$

$1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$

$1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W)$

$1.2D + 1.0W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$

$1.2D + 1.0E + 1.0L + 0.2S$

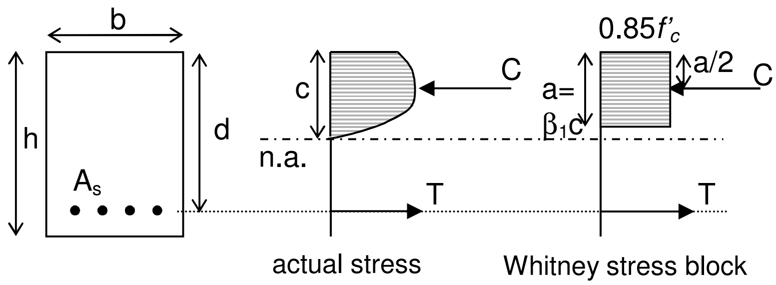
$0.9D + 1.0W$

$0.9D + 1.0E$

ASTM STANDARD REINFORCING BARS

Bar size, no.	Nominal diameter, in.	Nominal area, in.^{2}	Nominal weight, lb/ft
3	0.375	0.11	0.376
4	0.500	0.20	0.668
5	0.625	0.31	1.043
6	0.750	0.44	1.502
7	0.875	0.60	2.044
8	1.000	0.79	2.670
9	1.128	1.00	3.400
10	1.270	1.27	4.303
11	1.410	1.56	5.313
14	1.693	2.25	7.650
18	2.257	4.00	13.600

Internal Equilibrium



C = compression in concrete = stress x area = $0.85 f'_c ba$

T = tension in steel = stress x area = $A_s f_y$

$C = T$ and $M_n = T(d-a/2)$

where
- f'_c = concrete compression strength
- a = height of stress block
- β_1 = factor based on f'_c
- c = location to the neutral axis
- b = width of stress block
- f_y = steel yield strength
- A_s = area of steel reinforcement
- d = effective depth of section
  = depth to n.a. of reinforcement

$$\beta_1 = 0.85 - \left(\frac{f'_c - 4000}{1000}\right)(0.05) \geq 0.65$$

With C=T, $A_s f_y = 0.85 f'_c ba$ so a can be determined with $a = \dfrac{A_s f_y}{0.85 f'_c b} = \beta_1 c$

Criteria for Beam Design

For flexure design:

$M_u \leq \phi M_n$ $\phi = 0.9$ *for flexure (when the section is* <u>*tension*</u> *controlled)*

so for design, M_u *can be set to* $\phi M_n = \phi T(d-a/2) = \phi A_s f_y (d-a/2)$

Reinforcement Ratio

The amount of steel reinforcement is *limited*. Too much reinforcement, or **over-reinforcing** will not allow the steel to yield before the concrete crushes and there is a sudden failure. A beam with the proper amount of steel to allow it to yield at failure is said to be **under reinforced**.

The reinforcement ratio is just a fraction: $\rho = \dfrac{A_s}{bd}$ (or p). The amount of reinforcement is limited to that which results in a concrete strain of 0.003 and a minimum tensile strain of 0.004.

When the strain in the reinforcement is 0.005 or greater, the section is **tension controlled**. (For smaller strains the resistance factor reduces to 0.65 because the stress is less than the yield stress in the steel.) Previous codes limited the amount to $0.75\rho_{balanced}$ where $\rho_{balanced}$ was determined from the amount of steel that would make the concrete start to crush at the exact same time that the steel would yield based on strain (ε_y) of 0.002.

The strain in tension can be determined from $\varepsilon_t = \dfrac{d-c}{c}(0.003)$. At yield, $\varepsilon_y = \dfrac{f_y}{E_s}$.

The resistance factor expressions for **transition** and **compression controlled** sections are:

$$\phi = 0.75 + (\varepsilon_t - \varepsilon_y)\frac{0.15}{(0.005 - \varepsilon_y)} \textit{ for spiral members} \qquad \text{(not less than 0.75)}$$

$$\phi = 0.65 + (\varepsilon_t - \varepsilon_y)\frac{0.25}{(0.005 - \varepsilon_y)} \textit{ for other members} \qquad \text{(not less than 0.65)}$$

Maximum Reinforcement Ratio ρ for Singly Reinforced Rectangular Beams (tensile strain = 0.005) for which ϕ is permitted to be 0.9

f_y	f'_c = 3000 psi β_1 = 0.85	f'_c = 3500 psi β_1 = 0.85	f'_c = 4000 psi β_1 = 0.85	f'_c = 5000 psi β_1 = 0.80	f'_c = 6000 psi β_1 = 0.75
40,000 psi	0.0203	0.0237	0.0271	0.0319	0.0359
50,000 psi	0.0163	0.0190	0.0217	0.0255	0.0287
60,000 psi	0.0135	0.0158	0.0181	0.0213	0.0239
f_y	f'_c = 20 MPa β_1 = 0.85	f'_c = 25 MPa β_1 = 0.85	f'_c = 30 MPa β_1 = 0.85	f'_c = 35 MPa β_1 = 0.81	f'_c = 40 MPa β_1 = 0.77
300 MPa	0.0181	0.0226	0.0271	0.0301	0.0327
350 MPa	0.0155	0.0194	0.0232	0.0258	0.0281
400 MPa	0.0135	0.0169	0.0203	0.0226	0.0245
500 MPa	0.0108	0.0135	0.0163	0.0181	0.0196

Flexure Design of Reinforcement

One method is to "wisely" estimate a height of the stress block, a, and solve for A_s, and calculate a new value for a using M_u.

1. guess a (less than n.a.)
2. $A_s = \dfrac{0.85 f'_c ba}{f_y}$
3. solve for a from

   setting $M_u = \phi A_s f_y (d - a/2)$:

   $$a = 2\left(d - \frac{M_u}{\phi A_s f_y}\right)$$
4. repeat from 2. until a found from step 3 matches a used in step 2.

Design Chart Method:

1. calculate $R_n = \dfrac{M_n}{bd^2}$ $\left(R_n = \dfrac{M_u}{\phi bd^2}\right)$
2. find curve for f'_c and f_y to get ρ
3. calculate A_s and a, where:

   $$A_s = \rho bd \textit{ and } a = \frac{A_s f_y}{0.85 f'_c b}$$

Any method can simplify the size of d using h = 1.1d

Maximum Reinforcement

Based on the limiting strain of 0.005 in the steel, $c = 0.375d$ so

$a = \beta_1(0.375d)$ to find $A_{s\text{-max}}$

(β_1 is shown in the table above)

from Reinforced Concrete, 7th, Wang, Salmon, Pincheira, Wiley & Sons, 2007

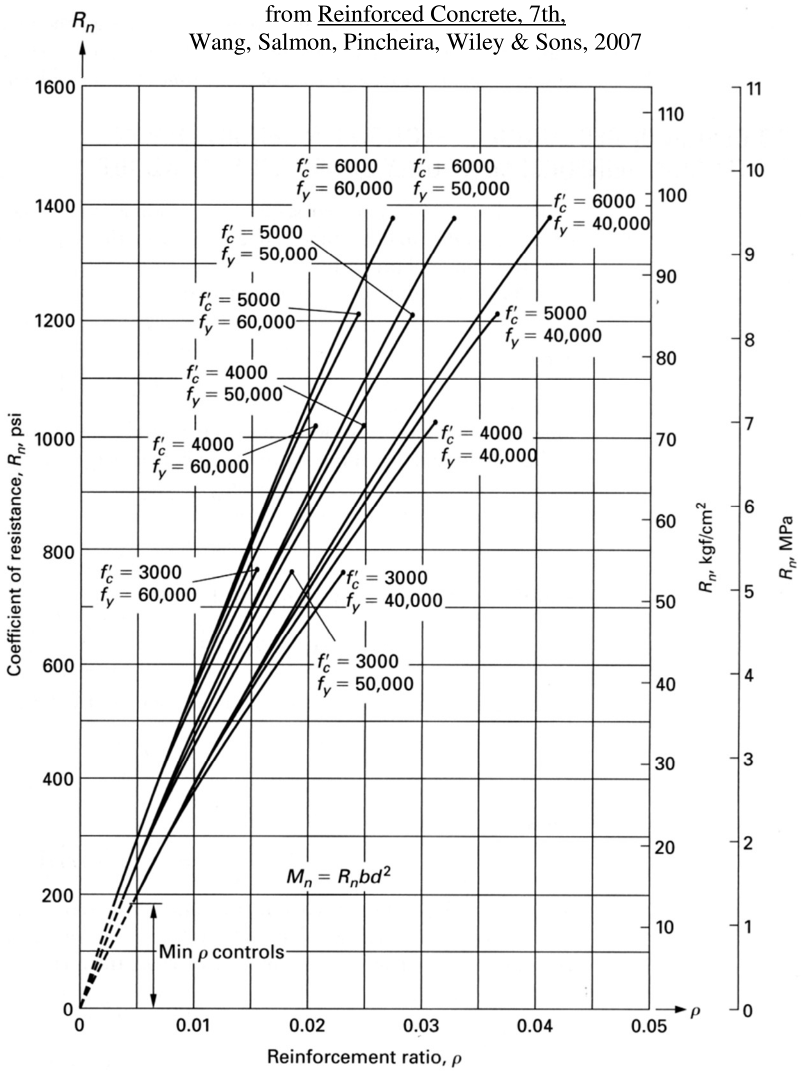


Figure 3.8.1 Strength curves (R_n vs ρ) for singly reinforced rectangular sections. Upper limit of curves is at ρ_{max}. (tensile strain of 0.004)

Minimum Reinforcement

Minimum reinforcement is provided even if the concrete can resist the tension. This is a means to control cracking.

Minimum required: $A_s = \frac{3\sqrt{f'_c}}{f_y}(b_w d)$ but not less than: $A_s = \frac{200}{f_y}(b_w d)$

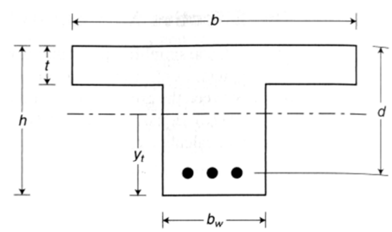
where f'_c is in psi, and λ is for material:

This can be translated to $\rho_{min} = \frac{3\sqrt{f'_c}}{f_y}$ but not less than $\frac{200}{f_y}$

Lightweight Concrete

Lightweight concrete has strength properties that are different from normalweight concretes, and a modification factor, λ, must be multiplied to the strength value of $\sqrt{f'_c}$. for concrete for some specifications (ex. shear). Depending on the aggregate and the lightweight concrete, the value of λ ranges from 075 to 0.85, 0.85 or 0.85 to 1.0. λ is 1.0 for normalweight concrete.

Cover for Reinforcement



Cover of concrete over/under the reinforcement must be provided to protect the steel from corrosion. For indoor exposure, 1.5 inch is typical for beams and columns, 0.75 inch is typical for slabs, and for concrete cast against soil, 3 inch minimum is required.

Bar Spacing

Minimum bar spacings are specified to allow proper consolidation of concrete around the reinforcement. The minimum spacing is the maximum of 1 in, a bar diameter, or 1.33 times the maximum aggregate size.

T-beams and T-sections (pan joists)

Beams cast with slabs have an effective width, b_E, that sees compression stress in a wide flange beam or joist in a slab system with positive bending.

For *interior* T-sections, b_E is the smallest of

L/4, b_w + 16t, or center to center of beams

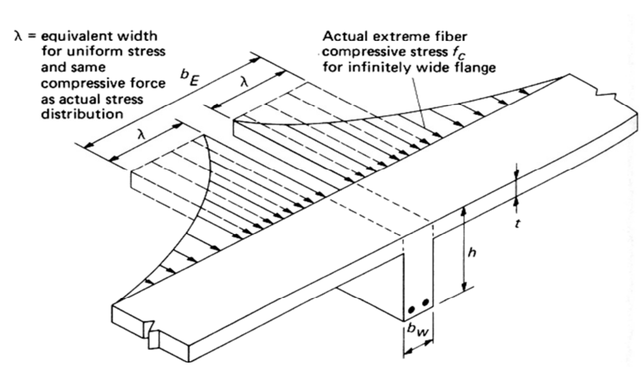


Figure 9.3.1 Actual and equivalent stress distribution over flange width.

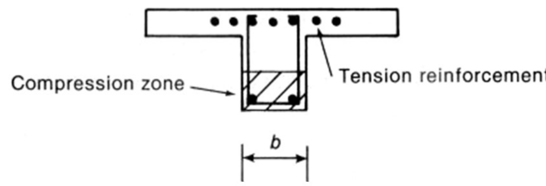
For *exterior* T-sections, b_E is the smallest of

b_w + L/12, b_w + 6t, or b_w + ½(clear distance to next beam)

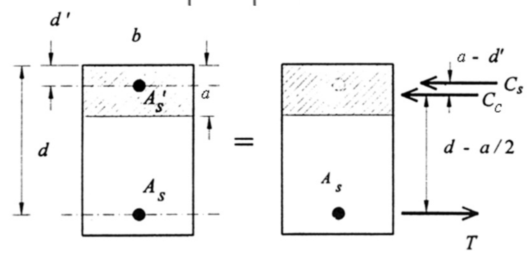
When the **web** is in tension the minimum reinforcement required is the same as for rectangular sections with the web width (b_w) in place of b. $M_n = C_w(d\text{-}a/2)+C_f(d\text{-}h_f/2)$ (h_f is height of flange or t)

When the **flange** is in tension (negative bending), the minimum reinforcement required is the greater value of $A_s = \dfrac{6\sqrt{f'_c}}{f_y}(b_w d)$ or $A_s = \dfrac{3\sqrt{f'_c}}{f_y}(b_f d)$

where f'_c is in psi, b_w is the beam width, and b_f is the effective flange width



Compression Reinforcement



If a section is *doubly reinforced*, it means there is steel in the beam seeing compression. The force in the compression steel that *may not be yielding* is

$C_s = A_s'(f'_s - 0.85f'_c)$

The total compression that balances the tension is now:

$T = C_c + C_s$.

And the moment taken about the centroid of the compression stress is $M_n = T(d\text{-}a/2)+C_s(a\text{-}d')$

where A_s' is the area of compression reinforcement, and d' is the effective depth to the centroid of the compression reinforcement

Because the compression steel may not be yielding, the neutral axis x must be found from the force equilibrium relationships, and the stress can be found based on strain to see if it has yielded.

Slabs

One way slabs can be designed as "one unit"-wide beams. Because they are thin, control of deflections is important, and minimum depths are specified, as is minimum reinforcement for shrinkage and crack control when not in flexure. Reinforcement is commonly small diameter bars and welded wire fabric.

Maximum spacing between bars is also specified for shrinkage and crack control as five times the slab thickness not exceeding 18".

For required flexure reinforcement the spacing limit is three times the slab thickness not exceeding 18".

Table 7.3.1.1—Minimum thickness of solid nonprestressed one-way slabs

Support condition	Minimum $h^{[1]}$
Simply supported	$\ell/20$
One end continuous	$\ell/24$
Both ends continuous	$\ell/28$
Cantilever	$\ell/10$

[1]Expression applicable for normalweight concrete and f_y = 60,000 psi. For other cases, minimum h shall be modified in accordance with 7.3.1.1.1 through 7.3.1.1.3, as appropriate.

7.3.1.1.1 For f_y other than 60,000 psi, the expressions in Table 7.3.1.1 shall be multiplied by **(0.4 + f_y/100,000)**.

7.3.1.1.2 For nonprestressed slabs made of lightweight concrete having w_c in the range of 90 to 115 lb/ft^3, the expressions in Table 7.3.1.1 shall be multiplied by the greater of (a) and (b):

(a) **1.65 – 0.005w_c**
(b) **1.09**

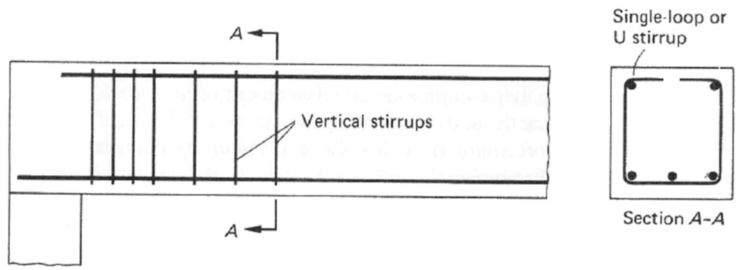
Shrinkage and temperature reinforcement (and minimum for flexure reinforcement):

Minimum for slabs with grade 40 or 50 bars: $\rho = \frac{A_s}{bt} = 0.002 \;\; or \;\; A_{s\text{-}min} = 0.002bt$

Minimum for slabs with grade 60 bars: $\rho = \frac{A_s}{bt} = 0.0018 \; or \; A_{s\text{-}min} = 0.0018bt$

Shear Behavior

Horizontal shear stresses occur along with bending stresses to cause tensile stresses where the concrete cracks. Vertical reinforcement is required to bridge the cracks which are called **shear stirrups** (or **stirrups**).



The maximum shear for design, V_u is the value at a distance of d from the face of the support.

Nominal Shear Strength

The shear force that can be resisted is the shear stress × cross section area: $V_c = \upsilon_c \times b_w d$

The shear stress for beams (one way) $\upsilon_c = 2\lambda\sqrt{f'_c}$ so $\phi V_c = \phi 2\lambda\sqrt{f'_c}\, b_w d$

where b_w = the beam width or the minimum width of the stem.
ϕ = 0.75 for shear
λ = modification factor for lightweight concrete

One-way joists are allowed an increase to $1.1 \times V_c$ if the joists are closely spaced.

Stirrups are necessary for strength (as well as crack control): $V_s = \frac{A_v f_{yt} d}{s} \le 8\sqrt{f'_c}\, b_w d \; (max)$

where A_v = area of all vertical legs of stirrup
s = spacing of stirrups
d = effective depth

For shear design:

$$V_u \le \phi V_c + \phi V_s \qquad \phi = 0.75 \text{ for shear}$$

Spacing Requirements

Stirrups are required when V_u is greater than $\frac{\phi V_c}{2}$. A minimum is required because shear failure of a beam without stirrups is sudden and brittle and because the loads can vary with respect to the design values.

Table 3-8 ACI Provisions for Shear Design*

		$V_u \le \frac{\phi V_c}{2}$	$\phi V_c \ge V_u > \frac{\phi V_c}{2}$	$V_u > \phi V_c$	
Required area of stirrups, A_v**		none	greater of $\frac{50 b_w s}{f_{yt}}$ and $\frac{0.75\sqrt{f'_c}\, b_w s}{f_{yt}}$		$\frac{(V_u - \phi V_c)s}{\phi f_{yt} d}$
Stirrup spacing, s	Required	—	smaller of $\frac{A_v f_{yt}}{50 b_w}$ and $\frac{A_v f_{yt}}{0.75\sqrt{f'_c}\, b_w}$		$\frac{\phi A_v f_{yt} d}{V_u - \phi V_c}$
	Recommended Minimum†	—	—	4 in.	
	Maximum†† (ACI 11.5.4)	—	$\frac{d}{2}$ or 24 in.	$\frac{d}{2}$ or 24 in. for $(V_u - \phi V_c) \le \phi 4\sqrt{f'_c}\, b_w d$	
				$\frac{d}{4}$ or 12 in. for $(V_u - \phi V_c) > \phi 4\sqrt{f'_c}\, b_w d$	

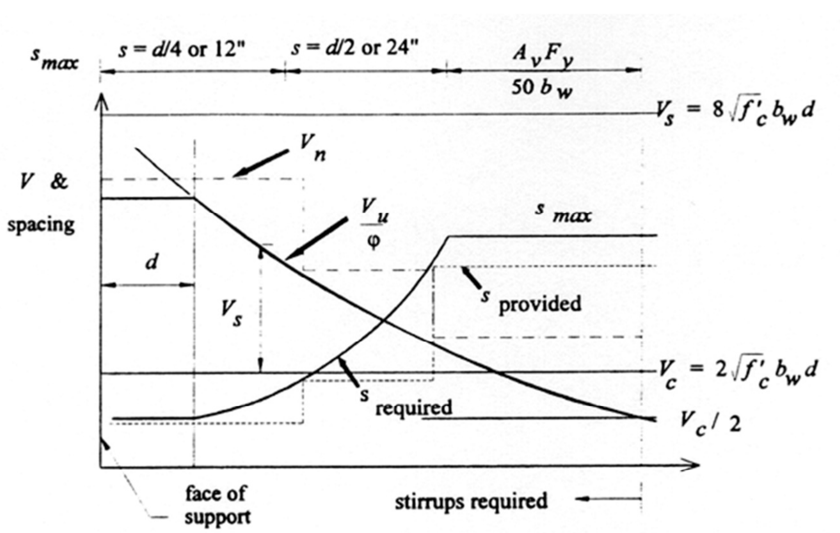
*Members subjected to shear and flexure only; $\phi V_c = \phi 2\lambda\sqrt{f'_c} b_w d$ $\phi = 0.75$ (ACI 11.3.1.1)
**$A_v = 2 \times A_b$ for U stirrups; $f_y \le 60$ ksi (ACI 11.5.2)
†A practical limit for minimum spacing is d/4
††Maximum spacing based on minimum shear reinforcement (= $A_v f_y / 50 b_w$) must also be considered (ACI 11.5.5.3).

NOTE: section numbers are pre ACI 318-14

Economical spacing of stirrups is considered to be greater than d/4. Common spacings of d/4, d/3 and d/2 are used to determine the values of ϕV_s at which the spacings can be increased.

$$\phi V_s = \frac{\phi A_v f_{yt} d}{s}$$

This figure shows that the size of V_n provided by $V_c + V_s$ (long dashes) exceeds V_u/ϕ in a step-wise function, while the spacing provided (short dashes) is at or less than the required s (limited by the maximum allowed). *(Note that the maximum shear permitted from the stirrups is $8\sqrt{f'_c}\, b_w d$)*



The minimum recommended spacing for the first stirrup is 2 inches from the face of the support.

Torsional Shear Reinforcement

On occasion beam members will see twist along the axis caused by an eccentric shape supporting a load, like on an L-shaped spandrel (edge) beam. The torsion results in shearing stresses, and closed stirrups may be needed to resist the stress that the concrete cannot resist.

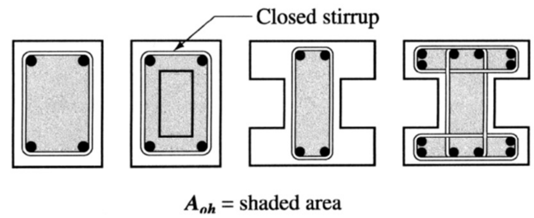


Fig. R11.6.3.6(b)—Definition of $\mathbf{A_{oh}}$

Development Length for Reinforcement

Because the design is based on the reinforcement attaining the yield stress, the reinforcement needs to be properly bonded to the concrete for a finite length (*both sides*) so it won't slip. This is referred to as the development length, l_d. Providing sufficient length to anchor bars that need to reach the yield stress near the end of connections are also specified by hook lengths. *Detailing reinforcement is a tedious job.* The equations for development length must be modified if the bar is epoxy coated or is cast with more than 12 in. of fresh concrete below it. Splices are also necessary to extend the length of reinforcement that come in standard lengths. The equations for splices are not provided here.

<u>Development Length in Tension</u>

With the proper bar to bar spacing and cover, the common development length equations are:

#6 bars and smaller: $l_d = \dfrac{d_b f_y}{25\lambda\sqrt{f'_c}}$ or 12 in. minimum

#7 bars and larger: $l_d = \dfrac{d_b f_y}{20\lambda\sqrt{f'_c}}$ or 12 in. minimum

<u>Development Length in Compression</u>

$$l_d = \frac{d_b f_y}{50\lambda\sqrt{f'_c}} \le 0.0003 f_y d_b \text{ or 8 in. minimum}$$

<u>Hook Bends and Extensions</u>

The minimum hook length is $l_{dh} = \dfrac{d_b f_y}{50\lambda\sqrt{f'_c}}$ but not less than the larger of $8d_b$ and 6 in.

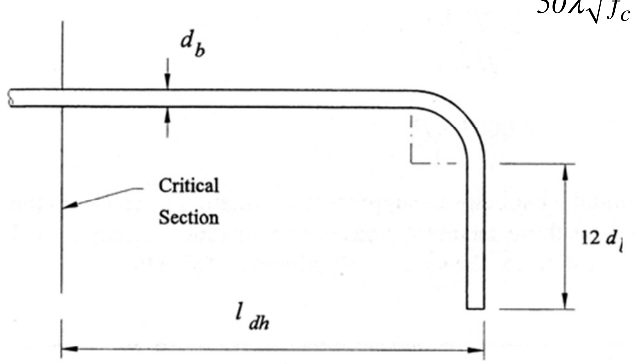


Figure 9-17: Minimum requirements for 90°bar hooks.

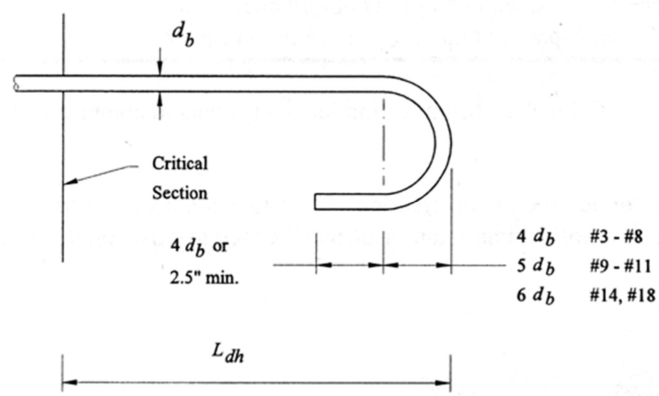


Figure 9-18: Minimum requirements for 180° bar hooks.

Modulus of Elasticity & Deflection

E_c for deflection calculations can be used with the transformed section modulus in the elastic range. After that, the cracked section modulus is calculated and E_c is adjusted.

Code values:

$$E_c = 57{,}000\sqrt{f'_c}\ \textit{(normal weight)} \quad E_c = w_c^{1.5}33\sqrt{f'_c},\ w_c = 90\ lb/ft^3 - 160\ lb/ft^3$$

Deflections of beams and one-way slabs need not be computed if the overall member thickness meets the minimum specified by the code, and are shown in Table 7.3.1.1 (see *Slabs*). The span lengths for continuous beams or slabs is taken as the clear span, l_n.

Criteria for Flat Slab & Plate System Design

Systems with slabs and supporting beams, joists or columns typically have multiple bays. The horizontal elements can act as one-way or two-way systems. Most often the flexure resisting elements are continuous, having positive and negative bending moments. These moment and shear values can be found using beam tables, or from code specified approximate design factors. Flat slab two-way systems have drop panels (for shear), while flat plates do not.

Criteria for Column Design

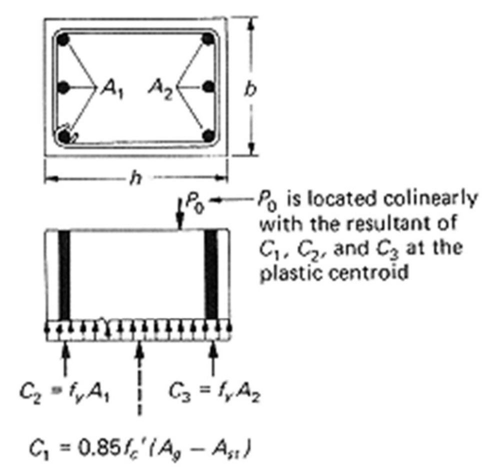
(American Concrete Institute) ACI 318-14 Code and Commentary:

$P_u \le \phi_c P_n$ where

P_u is a factored load
ϕ is a resistance factor
P_n is the nominal load capacity (strength)

Load combinations, ex: 1.4D (D is dead load)
1.2D + 1.6L (L is live load)

For compression, ϕ_c = 0.75 and $P_n = 0.85P_o$ for spirally reinforced, ϕ_c = 0.65 and $P_n = 0.8P_o$ for tied columns where $P_o = 0.85f'_c(A_g - A_{st}) + f_yA_{st}$ and P_o is the name of the maximum axial force with no concurrent bending moment.



Columns which have reinforcement ratios, $\rho_g = \dfrac{A_{st}}{A_g}$, in the range of 1% to 2% will usually be the most economical, with 1% as a minimum and 8% as a maximum by code.

Bars are symmetrically placed, typically.

Spiral ties are harder to construct.

Columns with Bending (Beam-Columns)

Concrete columns rarely see only axial force and must be designed for the combined effects of axial load and bending moment. The **interaction** diagram shows the reduction in axial load a column can carry with a bending moment.

Design aids commonly present the interaction diagrams in the form of load vs. equivalent eccentricity for standard column sizes and bars used.

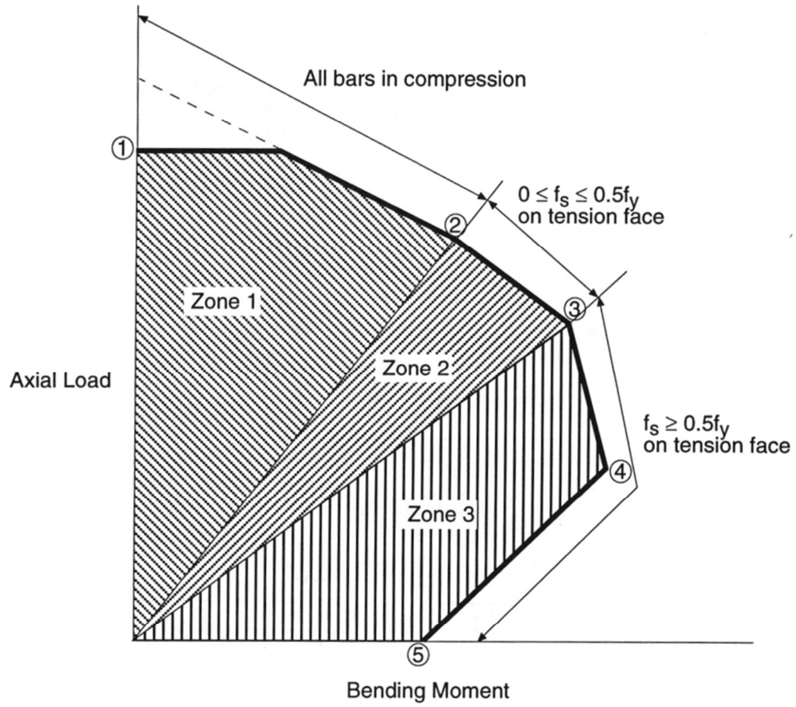


Figure 5-3 Transition Stages on Interaction Diagram

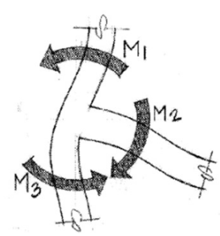
Rigid Frames

Monolithically cast frames with beams and column elements will have members with shear, bending and axial loads. Because the joints can rotate, the effective length must be determined from methods like that presented in the handout on Rigid Frames. The charts for evaluating k for non-sway and sway frames can be found in the ACI code.

Frame Columns

Because joints can rotate in frames, the effective length of the column in a frame is harder to determine. The stiffness (EI/L) of each member in a joint determines how rigid or flexible it is. To find k, the relative stiffness, G or Ψ, must be found for both ends, plotted on the alignment charts, and connected by a line for braced and unbraced fames.

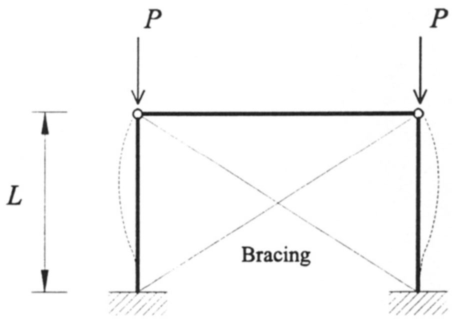
$$G = \Psi = \frac{\Sigma EI/l_c}{\Sigma EI/l_b}$$



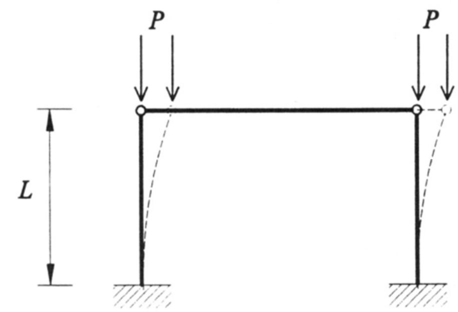
where

E = modulus of elasticity for a member
I = moment of inertia of for a member
l_c = length of the column from center to center
l_b = length of the beam from center to center

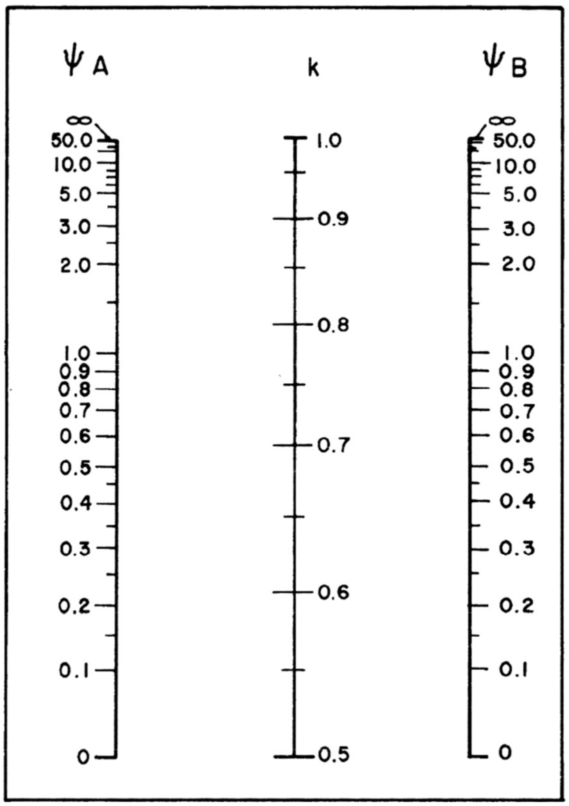
- For pinned connections we typically use a value of 10 for Ψ.
- For fixed connections we typically use a value of 1 for Ψ.



Braced – non-sway frame

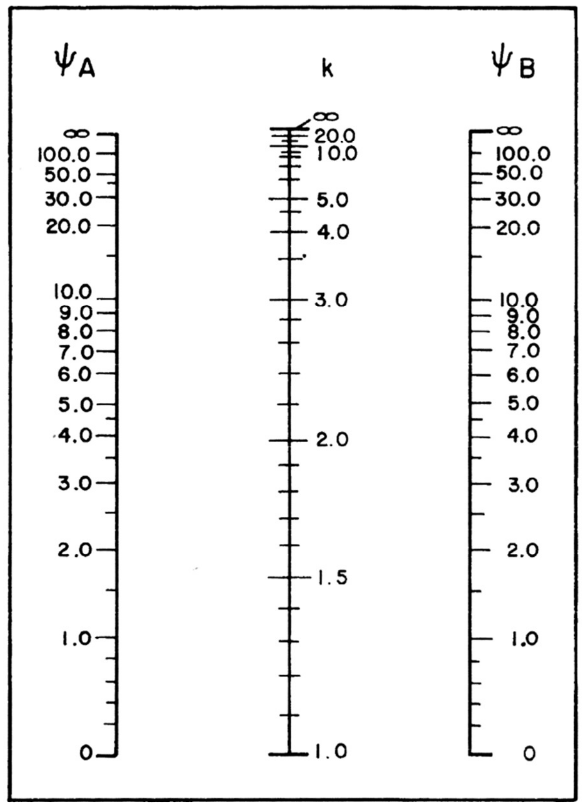


Unbraced – sway frame



(a)

Nonsway Frames



(b)

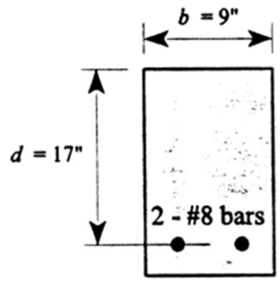
Sway Frames

<u>Slenderness</u>

Slenderness effects can be neglected if $kl/r \leq 22$ for columns not braced against sidesway, $kl/r \leq 34 + 12(M_1/M_2)$ and less than 40 for columns braced against sidesway where M_1/M_2 is negative if the column is bent in single curvature, and positive for double curvature.

Example 1

Determine the design moment capacity for the reinforced concrete cross section shown Assume f'_c = 3000 psi and Grade 60 reinforcing steel.



Example 2

(a) Determine the ultimate moment capacity of a beam with dimensions b = 10 in. and $d_{effective}$ = 15 in. and that has three No. 9 bars (3.0 in.^{2}) of tension-reinforcing steel. Assume that h = 18 in., F_y = 40 ksi, and f'_c = 5 ksi. (b) Assume also that the section is used as a cantilever beam 10 ft long, where the service loads are dead load = 400 lb/ft and live load = 300 lb/ft. Is the beam adequate in bending? Calculate the ultimate moment capacity of the beam first.

Solution:

(a) $a = A_sF_y/0.85f'_cb = (3)(40{,}000)/(0.85)(5000)(10) = 2.82$ in.

$\phi M_n = \phi A_sF_y[d - a/2] = 0.9(3)(40{,}000)[15 - (2.82)/(2)] = 1{,}466{,}640$ in.-lb

Check for overreinforcement, $c = 0.375 \cdot 15 = 5.625$. Depth of stress block $a = 0.80 \cdot 5.625$ in. = 4.5 in. $A_{s,max}$ = (0.85)(5ksi)(4.5in.)(10in.)/(40ksi) = 4.78 in.^{2} The beam is not over reinforced Check for minimum steel: $A_{s,min} = \dfrac{3\sqrt{f'_c}}{F_y}bd$ =*0.80 in^2*, so beam is sufficiently reinforced.

(b) $U = 1.2D + 1.6L = 1.2(400) + 1.6(300) = 960$ lb/ft

$M_u = w_uL^2/2 = (960)(10^2)/2 = 48{,}000$ ft-lb = 576,000 in.-lb

lb-in lb-in

Since $M_u = 576{,}000 < \phi M_n = 1{,}466{,}640$, the beam is adequate in bending.

EXAMPLE

Determine the ultimate moment capacity of a beam of dimensions b = 250 mm and d = 350 mm and that has 300 mm^2 of reinforcing steel. Assume that F_y = 400 MPa and f'_c = 25 MPa.

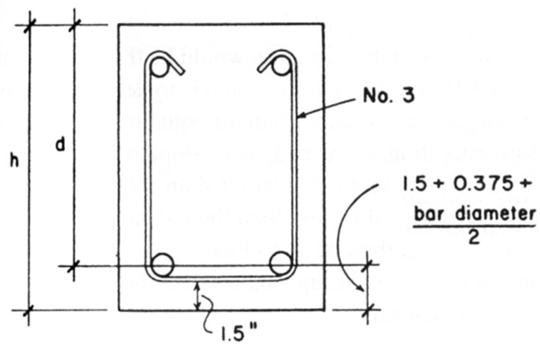
Solution:

$$a = \frac{A_sF_y}{0.85f'_cb} = \frac{(300)(400)}{(0.85)(25)(250)} = 22.6 \text{ mm}$$

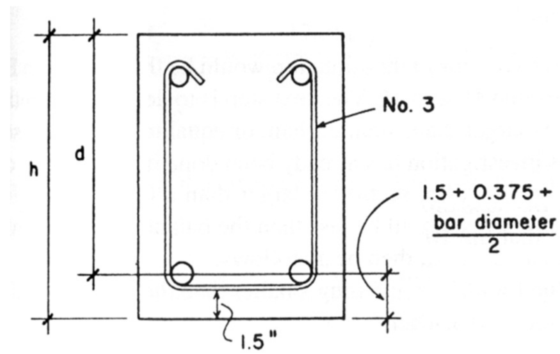
$$\phi M_n = \phi A_sF_y\left(d - \frac{a}{2}\right) = 0.9(300)(400)\left(350 - \frac{22.6}{2}\right) = 36.5 \text{ kN}\cdot\text{m}$$

Example 3

Example 1. The service load bending moments on a beam are 58 kip-ft [78.6 kN-m] for dead load and 38 kip-ft [51.5 kN-m] for live load. The beam is 10 in. [254 mm] wide, f'_c is 3000 psi [27.6 MPa], and f_y is 60 ksi [414 MPa]. Determine the depth of the beam and the tensile reinforcing required.



Example 3 *(continued)*



Example 4

A simply supported beam 20 ft long carries a service dead load of 300 lb/ft and a live load of 500 lb/ft. Design an appropriate beam (for flexure only). Use grade 40 steel and concrete strength of 5000 psi.

SOLUTION:

Find the design moment, M_u, from the factored load combination of 1.2D + 1.6L. It is good practice to guess a beam size to include self weight in the dead load, because "service" means dead load of everything except the beam itself.

Guess a size of 10 in x 12 in. Self weight for normal weight concrete is the density of 150 lb/ft^3 multiplied by the cross section area: self weight = $150\ {}^{lb}\!/_{ft^3}(10in)(12in)\cdot(\frac{1ft}{12in})^2$ = 125 lb/ft

w_u = 1.2(300 lb/ft + 125 lb/ft) + 1.6(500 lb/ft) = 1310 lb/ft

The maximum moment for a simply supported beam is $\frac{wl^2}{8}$: $M_u = \frac{w_u l^2}{8} = \frac{1310\ {}^{lb}\!/_{ft}(20ft)^2}{8}$ 65,500 lb-ft

M_n required = M_u/ϕ = $\frac{65{,}500^{lb-ft}}{0.9}$ = 72,778 lb-ft

To use the design chart aid, find $R_n = \frac{M_n}{bd^2}$, estimating that d is about 1.75 inches less than h:

d = 12in – 1.75 in – ~~(0.375)~~ = 10.25 in (NOTE: If there are stirrups, you must also subtract the diameter of the stirrup bar.)

$R_n = \frac{72{,}778^{lb-ft}}{(10in)(1025in)^2}\cdot(12\ {}^{in}\!/_{ft})$ = 831 psi

ρ corresponds to approximately 0.023 (which is less than that for 0.005 strain of 0.0319) , so the estimated area required, A_s, can be found:

$A_s = \rho bd$ = (0.023)(10in)(10.25in) = 2.36 in^2

The number of bars for this area can be found from handy charts.

(Whether the number of bars actually fit for the width with cover and space between bars <u>must also be considered</u>. If you are at ρ_{max} <u>do not</u> choose an area bigger than the maximum!)

Try A_s = 2.37 in^2 from 3#8 bars

d = 12 in – 1.5 in (cover) – ½ (8/8in diameter bar) = 10 in

Check ρ = 2.37 in^2/(10 in)(10 in) = 0.0237 which is less than $\rho_{max-0.005}$ = 0.0319 OK (We cannot have an over reinforced beam!!)

Find the moment capacity of the beam as designed, ϕM_n

$a = A_s f_y/0.85f'_c b$ = 2.37 in^2 (40 ksi)/[0.85(5 ksi)10 in] = 2.23 in

$\phi M_n = \phi A_s f_y(d-a/2) = 0.9(2.37in^2)(40ksi)(10in - \frac{2.23in}{2})\cdot(\frac{1}{12\ {}^{in}\!/_{ft}}) =$ 63.2 k-ft $\not>$ 65.5 k-ft needed (not OK)

So, we can increase d to 13 in, and ϕM_n = 70.3 k-ft (OK). Or increase A_s to 2 # 10's (2.54 in^2), for a = 2.39 in and ϕM_n of 67.1 k-ft (OK). <u>*Don't exceed ρ_{max} or $\rho_{max-0.005}$ if you want to use ϕ=0.9*</u>

Example 5

A simply supported beam 20 ft long carries a service dead load of 425 lb/ft (including self weight) and a live load of 500 lb/ft. Design an appropriate beam (for flexure only). Use grade 40 steel and concrete strength of 5000 psi.

SOLUTION:

Find the design moment, M_u, from the factored load combination of 1.2D + 1.6L. *If self weight is not included in the service loads,* you need to guess a beam size to include self weight in the dead load, because "service" means dead load of everything except the beam itself.

w_u = 1.2(425 lb/ft) + 1.6(500 lb/ft) = 1310 lb/ft

The maximum moment for a simply supported beam is $\frac{wl^2}{8}$: $M_u = \frac{w_u l^2}{8} = \frac{1310^{lb}/_{ft}(20ft)^2}{8}$ 65,500 lb-ft

M_n required = $M_u/\phi = \frac{65{,}500^{lb-ft}}{0.9}$ = 72,778 lb-ft

To use the design chart aid, we can find $R_n = {M_n}/{bd^2}$, and estimate that h is roughly 1.5-2 times the size of b, and h = 1.1d (rule of thumb): d = h/1.1 = (2b)/1.1, so d ≈ 1.8b or b ≈ 0.55d.

We can find R_n at the maximum reinforcement ratio for our materials, keeping in mind ρ_{max} at a strain = 0.005 is 0.0319 off of the chart at about 1070 psi, with ρ_{max} = 0.037. Let's substitute b for a function of d:

$R_n = 1070\text{ psi} = \frac{72{,}778^{lb-ft}}{(0.55d)(d)^2}\cdot(12^{in}/_{ft})$ Rearranging and solving for d = 11.4 inches

That would make b a little over 6 inches, which is impractical. 10 in is commonly the smallest width.

So if h is commonly 1.5 to 2 times the width, b, h ranges from 14 to 20 inches. (10x1.5=15 and 10x2 = 20)

Choosing a depth of 14 inches, d ≅ 14 - 1.5 (clear cover) - ½(1" diameter bar guess) -3/8 in (stirrup diameter) = 11.625 in.

Now calculating an updated $R_n = \frac{72{,}778\ \text{lb}-\text{ft}}{(10\text{in})(11.625\text{in})^2}\cdot(12\ \text{in}/_{\text{ft}}) = 646.2\text{psi}$

ρ now is 0.020 (under the limit at 0.005 strain of 0.0319), so the estimated area required, A_s, can be found:

$A_s = \rho bd$ = (0.020)(10in)(11.625in) = 1.98 in^2

The number of bars for this area can be found from handy charts.
(Whether the number of bars actually fit for the width with cover and space between bars <u>must also be considered</u>. If you are at $\rho_{max-0.005}$ <u>do not</u> choose an area bigger than the maximum!)

Try A_s = 2.37 in^2 from 3#8 bars. (or 2.0 in^2 from 2 #9 bars. 4#7 bars don't fit...)

d(actually) = 14 in. – 1.5 in (cover) – ½ (8/8 in bar diameter) – 3/8 in. (stirrup diameter) = 11.625 in.

Check ρ = 2.37 in^2/(10 in)(11.625 in) = 0.0203 which is less than $\rho_{max-0.005}$ = 0.0319 OK (We cannot have an over reinforced beam!!)

Find the moment capacity of the beam as designed, ϕM_n

$a = A_s f_y/0.85f'_c b$ = 2.37 in^2 (40 ksi)/[0.85(5 ksi)10 in] = 2.23 in

$\phi M_n = \phi A_s f_y(d-a/2) = 0.9(2.37\text{in}^2)(40\text{ksi})(11.625\text{in} - \frac{2.23\text{in}}{2})\cdot(\frac{1}{12^{in}/_{ft}}) =$ 74.7 k-ft > 65.5 k-ft needed

OK! <u>Note</u>**:** *If the section doesn't work, you need to increase d or A_s as long as you don't exceed $\rho_{max-0.005}$*

Example 6

A simply supported beam 25 ft long carries a service dead load of 2 k/ft, an estimated self weight of 500 lb/ft and a live load of 3 k/ft. Design an appropriate beam (for flexure only). Use grade 60 steel and concrete strength of 3000 psi.

SOLUTION:

Find the design moment, M_u, from the factored load combination of 1.2D + 1.6L. If self weight is estimated, and the selected size has a larger self weight, the design moment must be adjusted for the extra load.

w_u = 1.2(2 k/ft + 0.5 k/ft) + 1.6(3 k/ft) = 7.8 k/ft So, $M_u = \frac{w_u l^2}{8} = \frac{7.8 \,{}^{k}\!/\!_{ft} (25ft)^2}{8}$ 609.4 k-ft

M_n required = $M_u/\phi = \frac{609.4^{k-ft}}{0.9}$ = 677.1 k-ft

To use the design chart aid, we can find $R_n = \frac{M_n}{bd^2}$, and estimate that h is roughly 1.5-2 times the size of b, and h = 1.1d (rule of thumb): d = h/1.1 = (2b)/1.1, so d ≈ 1.8b or b ≈ 0.55d.

We can find R_n at the maximum reinforcement ratio for our materials off of the chart at about 700 psi with $\rho_{max-0.005}$ = 0.0135. Let's substitute b for a function of d:

R_n = 700 psi = $\frac{677.1^{k-ft}(1000^{lb/k})}{(0.55d)(d)^2} \cdot (12 \,{}^{in}\!/\!_{ft})$ Rearranging and solving for d = 27.6 inches

That would make b 15.2 in. (from 0.55d). Let's try 15. So,
h ≅ d + 1.5 (clear cover) +½(1" diameter bar guess) +3/8 in (stirrup diameter) = 27.6 +2.375 = 29.975 in.

Choosing a depth of 30 inches, d ≅ 30 - 1.5 (clear cover) - ½(1" diameter bar guess) -3/8 in (stirrup diameter) = 27.625 in.

Now calculating an updated $R_n = \frac{677{,}100 \text{ lb}-\text{ft}}{(15in)(27.625in)^2} \cdot (12 \,{}^{in}\!/\!_{ft}) = 710 psi$ This is larger than R_n for the 0.005 strain limit!

We can't just use $\rho_{max-.005}$. The way to reduce R_n is to increase b or d or both. Let's try increasing h to 31 in., then R_n = 661 psi with d = 28.625 in.. That puts us under $\rho_{max-0.005}$. We'd have to remember to keep UNDER the area of steel calculated, which is hard to do.

From the chart, ρ ≈ 0.013, less than the $\rho_{max-0.005}$ of 0.0135, so the estimated area required, A_s, can be found:
$A_s = \rho bd$ = (0.013)(15in)(29.625in) = 5.8 in^2

The number of bars for this area can be found from handy charts. Our charts say there can be 3 – 6 bars that fit when ¾" aggregate is used. We'll assume 1 inch spacing between bars. The actual limit is the maximum of 1 in, the bar diameter or 1.33 times the maximum aggregate size.

Try A_s = 6.0 in^2 from 6#9 bars. Check the width: 15 – 3 (1.5 in cover each side) – 0.75 (two #3 stirrup legs) – 6*1.128 – 5*1.128 in. = -1.16 in NOT OK.
Try A_s = 5.08 in^2 from 4#10 bars. Check the width: 15 – 3 (1.5 in cover each side) – 0.75 (two #3 stirrup legs) – 4*1.27 – 3*1.27 in. = 2.36 OK.
d(actually) = 31 in. – 1.5 in (cover) – ½ (1.27 in bar diameter) – 3/8 in. (stirrup diameter) = 28.49 in.

Find the moment capacity of the beam as designed, ϕM_n

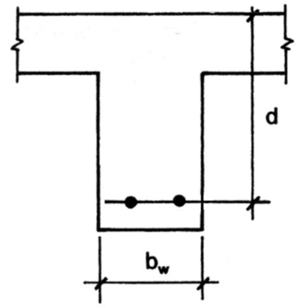
$a = A_s f_y / 0.85 f'_c b$ = 5.08 in^2 (60 ksi)/[0.85(3 ksi)15 in] = 8.0 in

$\phi M_n = \phi A_s f_y (d-a/2) = 0.9(5.08in^2)(60ksi)(28.49in - \frac{8.0in}{2}) \cdot (\frac{1}{12 \,{}^{in}\!/\!_{ft}}) =$ 559.8 k-ft < 609 k-ft needed!! (NO GOOD)

More steel isn't likely to increase the capacity much unless we are close. It looks like we need more steel **and** lever arm. Try h = 32 in. AND b = 16 in., then M_u^* (with the added self weight of 33.3 lb/ft) = 680.2 k-ft, ρ ≈ 0.012, As = 0.012(16in)(29.42in)=5.66 in^2. 6#9's won't fit, but 4#11's will: ρ = 0.0132 ✓, a = 9.18 in, and ϕM_n = 697.2 k-ft which is finally larger than 680.2 k-ft **OK**

Example 7

Example 3. A T-section is to be used for a beam to resist positive moment. The following data are given: beam span is 18 ft [5.49 m], beams are 9 ft [2.74 m] center to center, slab thickness is 4 in. [0.102 m], beam stem dimensions are b_w = 15 in. [0.381 m] and d = 22 in. [0.559 m], f'_c = 4 ksi [27.6 MPa], f_y = 60 ksi [414 MPa]. Find the required area of steel and select the reinforcing bars for a dead load moment of 125 kip-ft [170 kN-m] plus a live load moment of 100 kip-ft [136 kN-m].



Example 8

Design a T-beam for a floor with a 4 in slab supported by 22-ft-span-length beams cast monolithically with the slab. The beams are 8 ft on center and have a web width of 12 in. and a total depth of 22 in.; f'_c = 3000 psi and f_y = 60 ksi. Service loads are 125 psf and 200 psf dead load which does not include the weight of the floor system.

SOLUTION:

1. Establish the design moment:

$$\text{slab weight} = \frac{96(4)}{144}(0.150) = 0.400 \text{ kip/ft}$$

$$\text{stem weight} = \frac{12(18)}{144}(0.150) = \underline{0.225}$$

$$\text{total} = 0.625 \text{ kip/ft}$$

$$\text{service DL} = 8(0.200) = 1.60 \text{ kips/ft}$$

$$\text{service LL} = 8(0.125) = 1.00 \text{ kip/ft}$$

Calculate the factored load and moment:

$$w_u = 1.2(0.625 + 1.60) + 1.6(1.00) = 4.27 \text{ kip/ft}$$

$$M_u = \frac{w_u \ell^2}{8} = \frac{4.27(22)^2}{8} = 258 \text{ ft-kips}$$

2. Assume an effective depth $d = h - 3$ in.:

$$d = 22 - 3 = 19 \text{ in.}$$

3. Determine the effective flange width:

$$\tfrac{1}{4} \text{ span length} = 0.25(22)(12) = 66 \text{ in.}$$

$$b_w + 16h_f = 12 + 16(4) = 76 \text{ in.}$$

$$\text{beam spacing} = 96 \text{ in.}$$

Use an effective flange width b = 66 in.

4. Determine whether the beam behaves as a true T-beam or as a rectangular beam by computing the practical moment strength ϕM_{nf} with the full effective flange assumed to be in compression. This assumes that the bottom of the compressive stress block coincides with the bottom of the flange, as shown in Figure 3-10. Thus

$$\phi M_{nf} = \phi(0.85f'_c)bh_f\left(d - \frac{h_f}{2}\right)$$

$$= 0.9(0.85)(3)(66)\frac{4(19 - 4/2)}{12} = 858 \text{ ft-kips}$$

5. Since 858 ft-kips > 258 ft-kips, the total effective flange need not be completely utilized in compression (i.e., $a < h_f$), and the T-beam behaves as a wide rectangular beam with a width b of 66 in.

6. Design as a rectangular beam with b and d as known values (see Section 2-15):

$$\text{required } R_n = \frac{M_u}{\phi bd^2} = \frac{258(12)}{0.9(66)(19)^2} = 0.1444 \text{ ksi}$$

7. From Table A-8, select the required steel ratio to provide a R_n of 0.1444 ksi

required ρ =0.0024

8. Calculate the required steel area:

$$\text{required } A_s = \rho bd$$

$$= 0.0024(66)(19) = 3.01 \text{ in.}^2$$

9. Select the steel bars. Use 3#9 (A_s = 3.00 in.^{2})

minimum b_w = 7.125 in **(O.K.)**

Check the effective depth d:

$$d = 22 - 1.5 - 0.38 - \frac{1.125}{2} = 19.56 \text{ in.}$$

19.49 in. > 19 in. (O.K.)

10. Check $A_{s,\min}$. From Table A-5:

$$A_{s,\min} = 0.0033b_w d$$

$$= 0.0033(12)(19) = 0.75 \text{ in.}^2$$

0.75 in.^{2} < 3.00 in^2

11. Check $A_{s,\max}$:

$$A_{s,\max} = 0.0135(66)(19)$$

$$= 16.93 \text{ in.}^2 > 3.00 \text{ in.}^2$$ (O.K)

12. Verify the moment capacity:
(Is $M_u \le \phi M_n$)

a = (3.00)(60)/[0.85(3)(66)] = 1.07 in.

$$\phi M_n = 0.9(3.00)(60)(19.56 - \frac{1.07}{2})\ \tfrac{1}{12}$$

= 256.9 ft-kips (Not O.K)

Choose more steel, A_s = 3.16 in^2 from 4-#8's

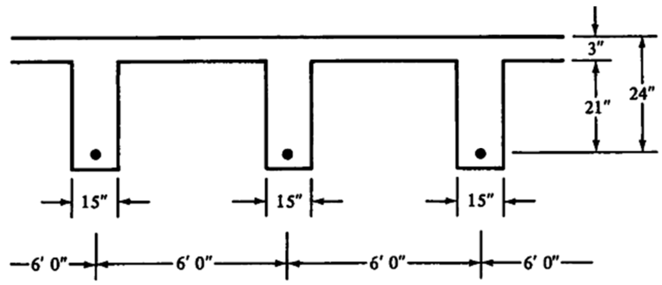
d = 19.62 in, a = 1.13 in

ϕM_n = 271.0 ft-kips, which is OK

13. Sketch the design

Example 9

Design a T-beam for the floor system shown for which b_w and d are given. M_D = 200 ft-k, M_L = 425 ft-k, f'_c = 3000 psi and f_y = 60 ksi, and simple span = 18 ft.



SOLUTION

Effective Flange Width

(a) $\frac{1}{4} \times 18' = 4'6'' = \underline{\underline{54''}}$

(b) $15'' + (2)(8)(3) = 63''$

(c) $6'0'' = 72''$

Moments Assuming $\phi = 0.90$

$$M_u = (1.2)(200) + (1.6)(425) = 920 \text{ ft-k}$$

$$M_n = \frac{M_u}{0.90} = \frac{920}{0.90} = 1022 \text{ ft-k}$$

First assume $a \leq h_f$ (which is very often the case. Then the design would proceed like that of a rectangular beam with a width equal to the effective width of the T beam flange.

$$\frac{M_u}{\phi bd^2} = \frac{920(12{,}000)}{(0.9)(54)(24)^2} = 394.4 \text{ psi}$$

from Table A.12, $\rho = 0.0072$

$$a = \frac{\rho f_y d}{0.85 f'_c} = \frac{0.0072(60)(24)}{(0.85)(3)} = 4.06 \text{ in.} > h_f = 3 \text{ in.}$$

The beam acts like a T beam, not a rectangular beam, and if the values for ρ and a above are not correct. If the value of a had been $\leq h_f$, the value of A_s would have been simply $\rho bd = 0.0072(54)(24) = 9.33 \text{ in}^2$. Now break the beam up into two parts (Figure 5.7) and design it as a T beam.

Assuming $\phi = 0.90$

$$A_{sf} = \frac{(0.85)(3)(54-15)(3)}{60} = 4.97 \text{ in.}^2$$

$$M_{uf} = (0.9)(4.97)(60)(24 - \tfrac{3}{2}) = 6039 \text{ in.-k} = 503 \text{ ft-k}$$

$$M_{uw} = 920 - 503 = 417 \text{ ft-k}$$

Designing a rectangular beam with b_w = 15 in. and d = 24 in. to resist 417 k-ft

$$\frac{M_{uw}}{\phi b_w d^2} = \frac{(12)(417)(1000)}{(0.9)(15)(24)^2} = 643.5$$

$\rho_w = 0.0126$ from Appendix Table A.12

$$A_{sw} = (0.0126)(15)(24) = 4.54 \text{ in.}^2$$

$$A_s = 4.97 + 4.54 = \underline{\underline{9.51 \text{ in.}^2}}$$

Check minimum reinforcing:

$$A_{s\ \min} = \frac{3\sqrt{f'_c}}{f_y} b_w d = \frac{3\sqrt{3000}(15)(24)}{60{,}000} = 0.986 \text{ in}^2$$

but not less than

$$A_{s\ \min} = \frac{200 b_w d}{f_y} = \frac{200(15)(24)}{60{,}000} = \underline{\underline{1.2 in^2}}$$

Only 2 rows fit, so try 8-#10 bars, A_s = 10.16 in^2

for equilibrium: $T = C_w + C_f$

$T = A_s f_y$ = (10.16)(60) = 609.6 k

$C_f = 0.85f'_c(b-b_w)h_f$ and $C_w = 0.85f'_c a b_w$

$C_w = T - C_f$ = 609.6 k – (0.85)(3)(54-15)3 = 311.25 k

a = 311.25/(0.85*3*15) = 8.14 in

Check strain (ε_t) and ϕ:

$c = a/\beta_1$ = 8.14 in/0.85 = 9.58

$$\varepsilon_t = \left(\frac{d-c}{c}\right)(0.003) = \left(\frac{24-9.58}{9.58}\right)(0.003) = 0.0045 \not> 0.005!$$

We could try 10-#9 bars at 10 in^2, T =600 k, C_w = 301.65 k, a = 7.89, ε_t = 0.0061; ϕ = 0.9!

Finally check the capacity:

$$M_n = C_w(d - \frac{a}{2}) + C_f(d - \frac{h_f}{2})$$

= [301.65(24-7.89/2) + 298.35(24-3/2)]1ft/12in

=1063.5 k-ft

So: ϕM_n = 0.9(1063.5) = 957.2 k-ft ≥ <u>920 k-ft</u> (OK)

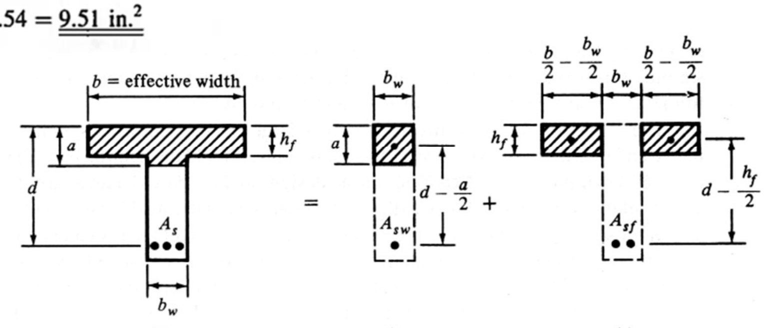


Figure 5.7 Separation of T beam into rectangular parts.

Example 10

Example 6. A one-way solid concrete slab is to be used for a simple span of 14 ft [4.27 m]. In addition to its own weight, the slab carries a superimposed dead load of 30 psf [1.44 kPa] plus a live load of 100 psf [4.79 kPa]. Using f'_c = 3 ksi [20.7 MPa] and f_y = 40 ksi [276 MPa], design the slab for minimum overall thickness.

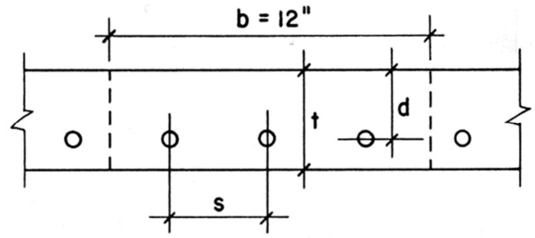


TABLE 13.6 Areas Provided By Spaced Reinforcement

Bar Spacing (in.)	Area Provided (in.^{2}/ft width)								
	No. 3	No. 4	No. 5	No. 6	No. 7	No. 8	No. 9	No. 10	No. 11
3	0.44	0.80	1.24	1.76	2.40	3.16	4.00		
3.5	0.38	0.69	1.06	1.51	2.06	2.71	3.43	4.35	
4	0.33	0.60	0.93	1.32	1.80	2.37	3.00	3.81	4.68
4.5	0.29	0.53	0.83	1.17	1.60	2.11	2.67	3.39	4.16
5	0.26	0.48	0.74	1.06	1.44	1.89	2.40	3.05	3.74
5.5	0.24	0.44	0.68	0.96	1.31	1.72	2.18	2.77	3.40
6	0.22	0.40	0.62	0.88	1.20	1.58	2.00	2.54	3.12
7	0.19	0.34	0.53	0.75	1.03	1.35	1.71	2.18	2.67
8	0.16	0.30	0.46	0.66	0.90	1.18	1.50	1.90	2.34
9	0.15	0.27	0.41	0.59	0.80	1.05	1.33	1.69	2.08
10	0.13	0.24	0.37	0.53	0.72	0.95	1.20	1.52	1.87
11	0.12	0.22	0.34	0.48	0.65	0.86	1.09	1.38	1.70
12	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56
13	0.10	0.18	0.29	0.40	0.55	0.73	0.92	1.17	1.44
14	0.09	0.17	0.27	0.38	0.51	0.68	0.86	1.09	1.34
15	0.09	0.16	0.25	0.35	0.48	0.63	0.80	1.01	1.25
16	0.08	0.15	0.23	0.33	0.45	0.59	0.75	0.95	1.17
18	0.07	0.13	0.21	0.29	0.40	0.53	0.67	0.85	1.04
24	0.05	0.10	0.15	0.22	0.30	0.39	0.50	0.63	0.78

Example 11

nple 2-9

Design a simple-span one-way slab to carry a uniformly distributed live load of 400 psf. The span is 10 ft (center to center of supports). Use $f'_c = 4000$ psi and $f_y = 60{,}000$ psi. Select the thickness to be not less than the ACI minimum thickness requirement.

Solution:

Determine the required minimum h and use this to estimate the slab dead weight.

1. From ACI Table 9.5(a), for a simply supported, solid, one-way slab,

$$\text{minimum } h = \frac{\ell}{20} = \frac{10(12)}{20} = 6.0 \text{ in.}$$

Try $h = 6$ in. and design a 12-in.-wide segment.

2. Determine the slab weight dead load:

$$\frac{6(12)}{144}(0.150) = 0.075 \text{ kip/ft}$$

The total design load is

$$w_u = 1.2w_{DL} + 1.6w_{LLL}$$
$$= 1.2(0.075) + 1.6(0.400)$$
$$= 0.730 \text{ kip/ft}$$

3. Determine the design moment:

$$M_u = \frac{w_u \ell^2}{8} = \frac{0.73(10)^2}{8} = 9.125 \text{ ft-kips}$$

4. Establish the approximate d. Assuming No. 6 bars and minimum concrete cover on the bars of ¾ in.,

$$\text{assumed } d = 6.0 - 0.75 - 0.375 = 4.88 \text{ in.}$$

5. Determine the required R_n:

$$\text{required } R_n = \frac{M_u}{\phi bd^2}$$
$$= \frac{9.125(12)}{0.9(12)(4.88)^2} = 0.4257 \text{ ksi}$$

6. From Table A-10, for a required $R_n = 0.4257$, the required $\rho = 0.0077$. (Note that the required ρ selected is the next *higher* value from Table A-10.) Thus

$$\rho_{max} = 0.0181 > 0.0077 \qquad \text{(O.K.)}$$

Use $\rho = 0.0077$.

7. $$\text{required } A_s = \rho bd = 0.0077(12)(4.88) = 0.45 \text{ in.}^2\text{/ft}$$

8. Select the main steel (from Table A-4). Select No. 5 bars at 7½ in. o.c. ($A_s = 0.50$ in.^{2}). The assumption on bar size was satisfactory. The code requirements for maximum spacing have been discussed in Section 2-13. Minimum spacing of bars in slabs, practically, should not be less than 4 in., although the ACI Code allows bars to be placed closer together, as discussed in Example 2-7. Check the maximum spacing (ACI Code, Section 7.6.5):

$$\text{maximum spacing} = 3h \quad \text{or} \quad 18 \text{ in.}$$
$$3h = 3(6) = 18 \text{ in.}$$
$$7\tfrac{1}{2} \text{ in.} < 18 \text{ in.} \qquad \text{(O.K.)}$$

Therefore use No. 5 bars at 7½ in. o.c.

9. Select shrinkage and temperature reinforcement (ACI Code, Section 7.12):

$$\text{required } A_s = 0.0018bh$$
$$= 0.0018(12)(6) = 0.13 \text{ in.}^2\text{/ft}$$

Select No. 3 bars at 10 in. o.c. ($A_s = 0.13$ in.^{2}) or No. 4 bars at 18 in. o.c. ($A_s = 0.13$ in.^{2}):

$$\text{maximum spacing} = 5h \quad \text{or} \quad 18 \text{ in.}$$

Use No. 3 bars at 10 in. o.c.

10. The main steel area must exceed the area required for shrinkage and temperature steel (ACI Code, Section 10.5.4):

$$0.50 \text{ in.}^2 > 0.13 \text{ in.}^2 \qquad \text{(O.K.)}$$

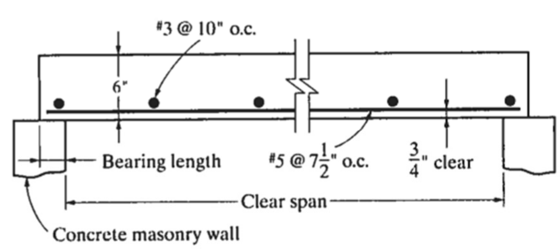
11. Verify the moment capacity:
(Is $M_u \le \phi M_n$)

$$a = \frac{(0.50)(60)}{0.85(4)(12)} = 0.74in$$

$$\phi M_n = 0.9(0.50)(60)(5.0625 - {}^{0.74}\!/_{2})\, {}^{1}\!/_{12}$$

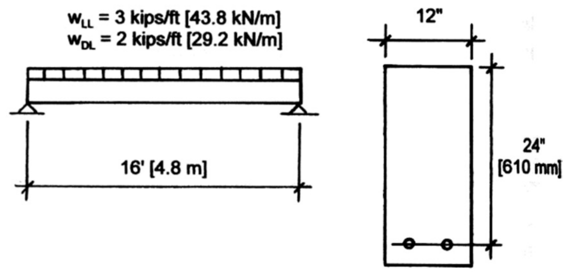
$$= 10.6 \text{ ft-kips} \qquad \text{OK)}$$

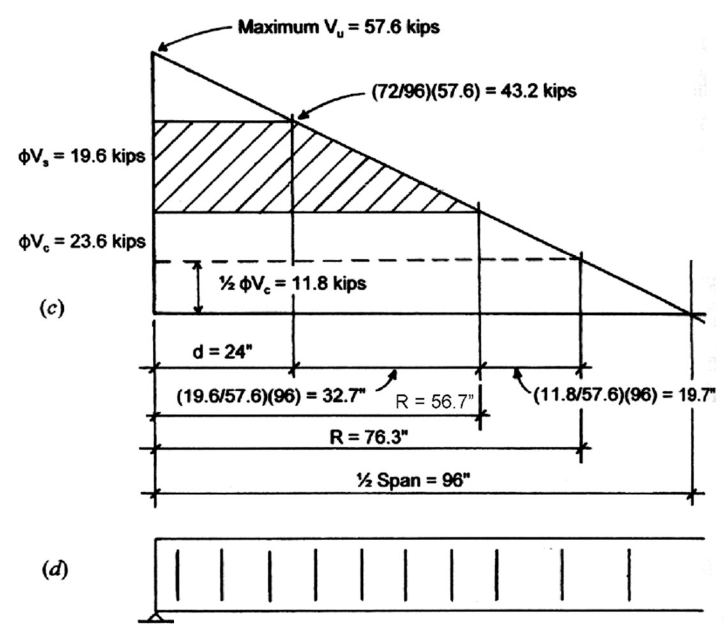
12. A design sketch is drawn:



Example 12

Example 7. Design the required shear reinforcement for the simple beam shown in Figure 13.18. Use $f'_c = 3$ ksi [20.7 MPa] and $f_y = 40$ ksi [276 MPa] and single U-shaped stirrups.





Example 13

For the simply supported concrete beam shown in Figure 5-61, determine the stirrup spacing (if required) using No. 3 U stirrups of Grade 60 (f_y = 60 ksi). Assume f'_c = 3000 psi.

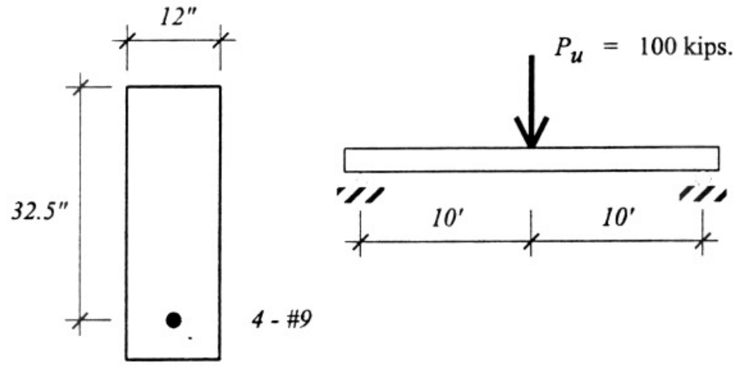


Figure 5-61: Simply supported concrete beam for Example 5-15.

f'_c = 3000 psi. For #3 bars, A_s = 0.11 in.^{2},
F_y = 60 ksi. with 2 legs, then A_v = 0.22 in.^{2}

Solution:

V_u = 50 kips (neglecting weight of the beam)

$$\phi V_c = \phi\lambda 2\sqrt{f'_c}\, b_w d$$

$$= \frac{(0.75)(1)2\sqrt{3000}(12)(32.5)}{1000} = 32.0 \text{ kips} < V_u \quad \therefore \text{Need Stirrups}$$

Note: If $V_u = \frac{1}{2}\varphi V_c$, minimum stirrups would still be required.

$$V_u \le \phi V_c + \phi V_s$$

$$\therefore \phi V_s = V_u - \phi V_c = 50 - 32.0 = 18.0 \text{ kips} \qquad (< \phi 4\sqrt{f'_c}\, b_w d = 64.1 \text{ kips})$$

$$s_{req'd} \le \frac{\phi A_v F_y d}{\phi V_s} = \frac{(0.75)(0.22in^2)(60ksi)(32.5in)}{18.0k}$$

$$= 17.875 \text{ in.}$$

$$s_{max} = \frac{d}{2} = \frac{32.5}{2} = 16.2 \text{ in.} \quad \text{☜ controls}$$

$$= 24 \text{ in.}$$

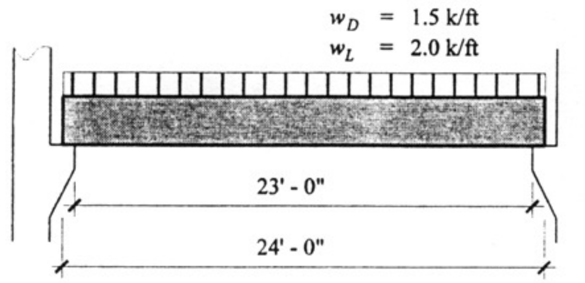
$$s_{req'd} \le \frac{A_v F_y}{50 b_w} = \frac{(0.22)(60{,}000)}{50(12)} = 22.0 \text{ in.}$$, but 16" (d/2) would be the maximum as well.

when $\phi V_c > V_u > \dfrac{\phi V_c}{2}$

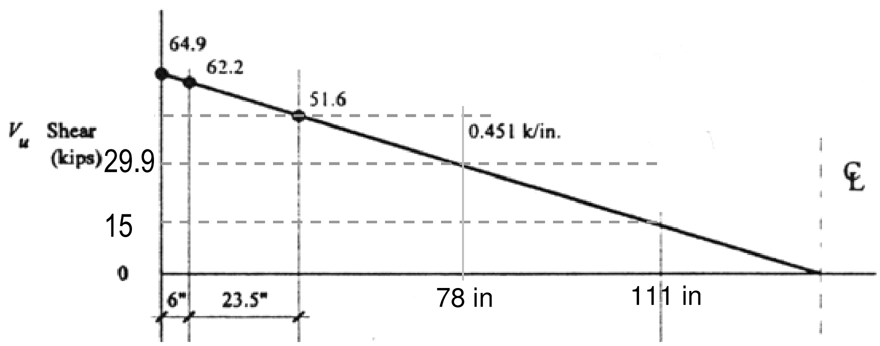
$\therefore$ Use #3 U @ 16" max spacing

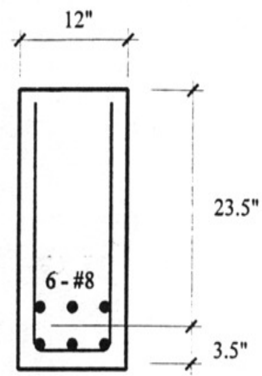
Example 14

Design the shear reinforcement for the simply supported reinforced concrete beam shown with a dead load of 1.5 k/ft and a live load of 2.0 k/ft. Use 5000 psi concrete and Grade 60 steel. Assume that the point of reaction is at the end of the beam.



SOLUTION:





Shear diagram:

Find self weight = 1 ft x (27/12 ft) x 150 lb/ft^3 = 338 lb/ft = 0.338 k/ft

w_u = 1.2 (1.5 k/ft + 0.338 k/ft) + 1.6 (2 k/ft) = 5.41 k/ft (= 0.451 k/in)

$V_{u\,(max)}$ is at the ends = $w_uL/2$ = 5.41 k/ft (24 ft)/2 = 64.9 k

$V_{u\,(support)} = V_{u\,(max)} - w_u$(distance) = 64.9 k – 5.4 1k/ft (6/12 ft) = 62.2 k

V_u for design is d away from the support = $V_{u\,(support)} - w_u(d)$ = 62.2 k – 5.41 k/ft (23.5/12 ft) = 51.6 k

Concrete capacity:

We need to see if the concrete needs stirrups for strength or by requirement because $V_u \le \phi V_c + \phi V_s$ (design requirement)

$\phi V_c = \phi 2\lambda \sqrt{f'_c}\, b_w d$ = 0.75 (2)(1.0)$\sqrt{5000}$psi (12 in) (23.5 in) = 299106 lb = 29.9 kips (< 51.6 k!)

Locating end points:

29.9 k = 64.9k – 0.451 k/in x (a)
a = 78 in

15 k = 64.9k – 0.451 k/in x (b)
b = 111 in.

Stirrup design and spacing

We need stirrups: $A_v = V_s s/f_y d$

$\phi V_s \ge V_u - \phi V_c$ = 51.6 k – 29.9 k = 21.7 k

Spacing requirements are in Table 3-8 and depend on $\phi V_c/2$ = 15.0 k and $2\phi V_c$ = 59.8 k

2 legs for a #3 is 0.22 in^2, so $s_{req'd} \le \phi A_v f_{yt} d/\phi V_s$ = 0.75(0.22 in^2)(60 ksi)(23.5 in)/21.7 k = 10.72 in Use s = 10"

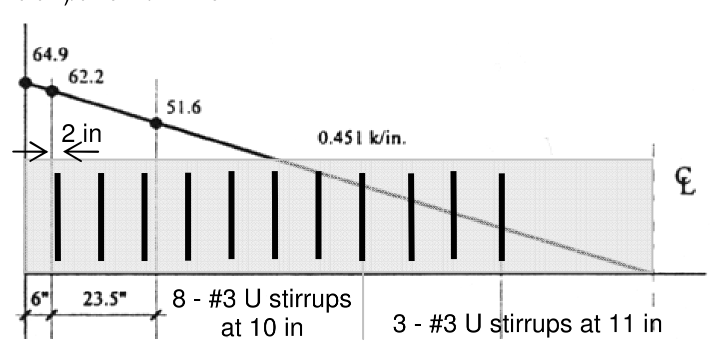
our maximum falls into the d/2 or 24", so d/2 governs with 11.75 in Our 10" is ok.

This spacing is valid until $V_u = \phi V_c$ and that happens at (64.9 k – 29.9 k)/0.451 k/in = 78 in

We can put the first stirrup at a minimum of 2 in from the support face, so we need 10" spaces for (78 – 2 - 6 in)/10 in = 7 even (8 stirrups altogether ending at 78 in)

After 78" we can change the spacing to the required (but not more than the maximum of d/2 = 11.75 in ≤ 24in);

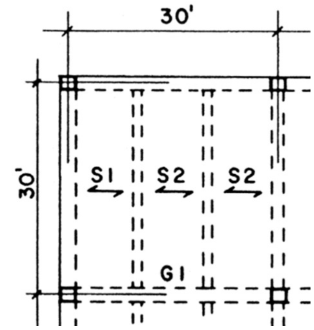
$s = A_v f_{yt} / 50 b_w$ = 0.22 in^2 (60,000 psi)/50 (12 in) =
22 in $\le A_v f_{yt}/0.75\sqrt{f'_c}\, b_w$ =
0.22 in^2 (60,000 psi)/[0.75$\sqrt{5000}$psi(12 in)] = 20.74 in

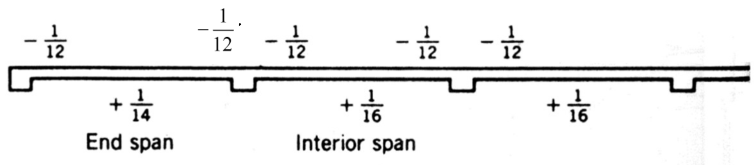


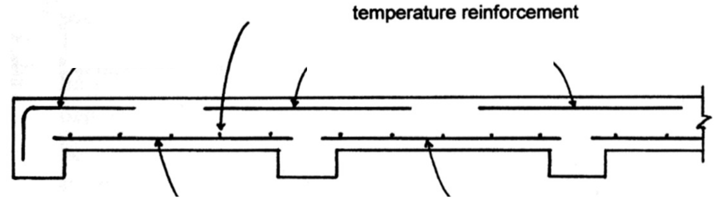
We need to continue to 111 in, so (111 – 78 in)/ 11 in = 3 even

Example 15

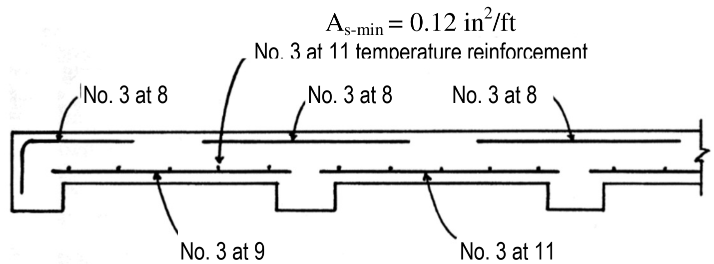
Example 1. A solid one-way slab is to be used for a framing system similar to that shown in Figure 14.1. Column spacing is 30 ft. with evenly spaced beams occurring at 10 ft. center to center. Superimposed loads on the structure (floor live load plus other construction dead load) are a dead load of 38 psf [1.82 kPa] and a live load of 100 psf [4.79 kPa]. Use $f'_c = 3$ ksi [20.7 MPa] and $f_y = 40$ ksi [275 MPa]. Determine the thickness for the slab and select its reinforcement.





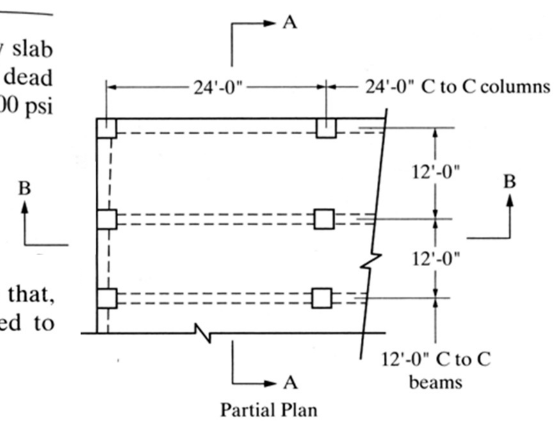


Example 15 (continued)



Example 16

Example 6-1



The floor system shown in Figure 6-4 consists of a continuous one-way slab supported by continuous beams. The service loads on the floor are 25 psf dead load (does not include weight of slab) and 250 psf live load. Use $f'_c = 3000$ psi (normal-weight concrete) and $f_y = 60{,}000$ psi. The bars are uncoated.

Design the continuous one-way floor slab.

Solution:

The primary difference in this design from previous flexural designs is that, because of continuity, the ACI coefficients and equations will be used to determine design shears and moments.

A. Continuous one-way floor slab

1. Determine the slab thickness. The slab will be designed to satisfy the ACI minimum thickness requirements from Table 9.5(a) of the code and this thickness will be used to estimate slab weight.

   With both ends continuous,

$$\text{minimum } h = \frac{1}{28}\ell_n = \frac{1}{28}(11)(12) = 4.71 \text{ in.}$$

   With one end continuous,

$$\text{minimum } h = \frac{1}{24}\ell_n = \frac{1}{24}(11)(12) = 5.5 \text{ in.}$$

   Try a $5\frac{1}{2}$-in.-thick slab. Design a 12-in.-wide segment ($b = 12$ in.).

2. Determine the load:

$$\text{slab dead load} = \frac{5.5}{12}(150) = 68.8 \text{ psf}$$

$$\text{total dead load} = 25.0 + 68.8 = 93.8 \text{ psf}$$

$$w_u = 1.2w_{DL} + 1.6w_{LL} = 1.2(93.8) + 1.6(250) = 112.6 + 400.0 = 516.2 \text{ psf} \quad \text{(design load)}$$

Because we are designing a slab segment that is 12 in. wide, the foregoing loading is the same as 512.6 lb/ft or 0.513 kip/ft.

Example 16 (continued)

3. Determine the moments and shears. Moments are determined using the ACI moment equations. Refer to Figures 6-1 and 6-4. Thus

$$+M_u = \frac{1}{14} w_u \ell_n^2 = \frac{1}{14}(0.513)(11)^2 = 4.43 \text{ ft-kips} \qquad \textit{(end span)}$$

$$+M_u = \frac{1}{16} w_u \ell_n^2 = \frac{1}{16}(0.513)(11)^2 = 3.88 \text{ ft-kips} \qquad \textit{(interior span)}$$

$$-M_u = \frac{1}{10} w_u \ell_n^2 = \frac{1}{10}(0.513)(11)^2 = 6.20 \text{ ft-kips} \qquad \textit{(end span - first interior support)}$$

$$-M_u = \frac{1}{11} w_u \ell_n^2 = \frac{1}{11}(0.513)(11)^2 = 5.64 \text{ ft-kips} \qquad \textit{(interior span – both supports)}$$

$$-M_u = \frac{1}{24} w_u \ell_n^2 = \frac{1}{24}(0.513)(11)^2 = 2.58 \text{ ft-kips} \qquad \textit{(end span – exterior support)}$$

Similarly, the shears are determined using the ACI shear equations. In the end span at the face of the first interior support,

$$V_u = 1.15\frac{w_u \ell_n}{2} = 1.15(0.513)\left(\frac{11}{2}\right) = 3.24 \text{ kips} \qquad \textit{(end span – first interior support)}$$

whereas at all other supports,

$$V_u = \frac{w_u \ell_n}{2} = (0.513)\left(\frac{11}{2}\right) = 2.82 \text{ kips}$$

4. Design the slab. Assume #4 bars for main steel with ¾ in. cover: $d = 5.5 – 0.75 – ½(0.5) = 4.5$ in.

5. Design the steel. (All moments must be considered.) For example, the negative moment in the end span at the first interior support:

$$R_n = \frac{M_u}{\phi bd^2} = \frac{6.20(12)(1000)}{0.9(12)(4.5)^2} = 340^{ft-kips} \qquad \text{so } \rho \cong 0.006$$

$A_s = \rho bd = 0.006(12)(4.5) = 0.325$ in^2 per ft. width of slab $\therefore$ Use #4 at 7 in. (16.5 in. max. spacing)

The minimum reinforcement required for flexure is the same as the shrinkage and temperature steel.

(Verify the moment capacity is achieved: a 0.67 in. and $\phi M_n = 6.38$ ft-kips > 6.20 ft-kips)

For grade 60 the minimum for shrinkage and temperature steel is:

$A_{s\text{-}min} = 0.0018bt = 0.0018\,(12)(5.5) = 0.12$ in^2 per ft. width of slab $\therefore$ Use #3 at 11 in. (18 in. max spacing)

6. Check the shear strength.

$$\phi V_c = \phi 2\sqrt{f_c'}bd = 0.75(2)\sqrt{3000}(12)(4.5) = 4436.6lb = 4.44 \text{ kips}$$

$V_u \leq \phi V_c$ Therefore the thickness is O.K.

7. Development length for the flexure reinforcement is required. (Hooks are required at the spandrel beam.) For example, #6 bars:

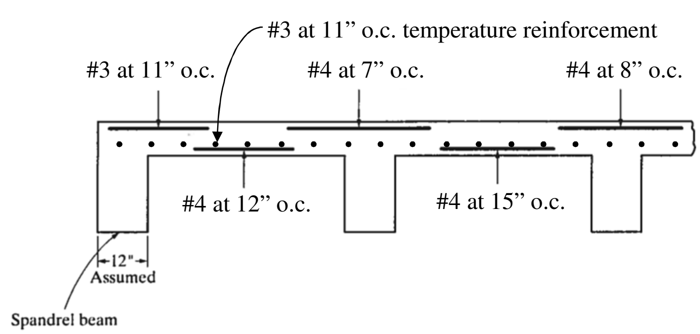
$$l_d = \frac{d_b F_y}{25\sqrt{f_c'}} \qquad \textit{or 12 in. minimum}$$

With grade 40 steel and 3000 psi concrete:

$$l_d = \frac{\tfrac{6}{8}in(40{,}000psi)}{25\sqrt{3000}psi} = 21.9in$$

(which is larger than 12 in.)

8. Sketch:



Example 17

A building is supported on a grid of columns that is spaced at 30 ft on center in both the north-south and east-west directions. Hollow core planks with a 2 in. topping span 30 ft in the east-west direction and are supported on precast L and inverted T beams. Size the hollow core planks assuming a live load of 100 lb/ft^2. Choose the shallowest plank with the least reinforcement that will span the 30 ft while supporting the live load.

SOLUTION:

The shallowest that works is an 8 in. deep hollow core plank.

The one with the least reinforcing has a strand pattern of 68-S, which contains 6 strands of diameter 8/16 in. = ½ in. The S indicates that the strands are straight. The plank supports a superimposed service load of 124 lb/ft^2 at a span of 30 ft with an estimated camber at erection of 0.8 in. and an estimated long-time camber of 0.2 in.

The weight of the plank is 81 lb/ft^2.

3.6 Hollow-Core Load Tables (cont.)

Strand Pattern Designation
76-S

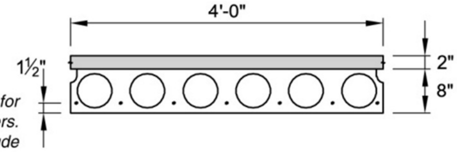
S = straight
Diameter of strand in 16ths
Number of strand (7)

Safe loads shown include dead load of 10 lb/ft² for untopped members and 15 lb/ft² for topped members. Remainder is live load. Long-time cambers include superimposed dead load but do not include live load.

Capacity of sections of other configurations are similar. For precise values, see local hollow-core manufacturer.

Key
385– Safe superimposed service load, lb/ft²
0.1 – Estimated camber at erection, in.
0.2 – Estimated long-time camber, in.

4'-0" × 8"
Normalweight Concrete



f'_c = 5000 psi

f_{pu} = 270,000 psi

Section Properties

		No Topping	2 in. topping
A	=	215 in.²	-
I	=	1666 in.⁴	3071 in.⁴
y_b	=	4.00 in.	5.29 in.
y_t	=	4.00 in.	4.71 in.
S_b	=	417 in.³	581 in.³
S_t	=	417 in.³	652 in.³
wt	=	224 lb/ft	324 lb/ft
DL	=	56 lb/ft²	81 lb/ft²
V/S	=	1.92 in.	

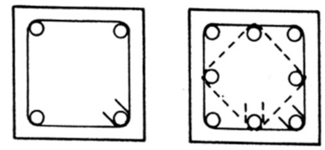
4HC8 + 2

Table of safe superimposed service load, lb/ft², and cambers, in. **2 in. Normalweight Topping**

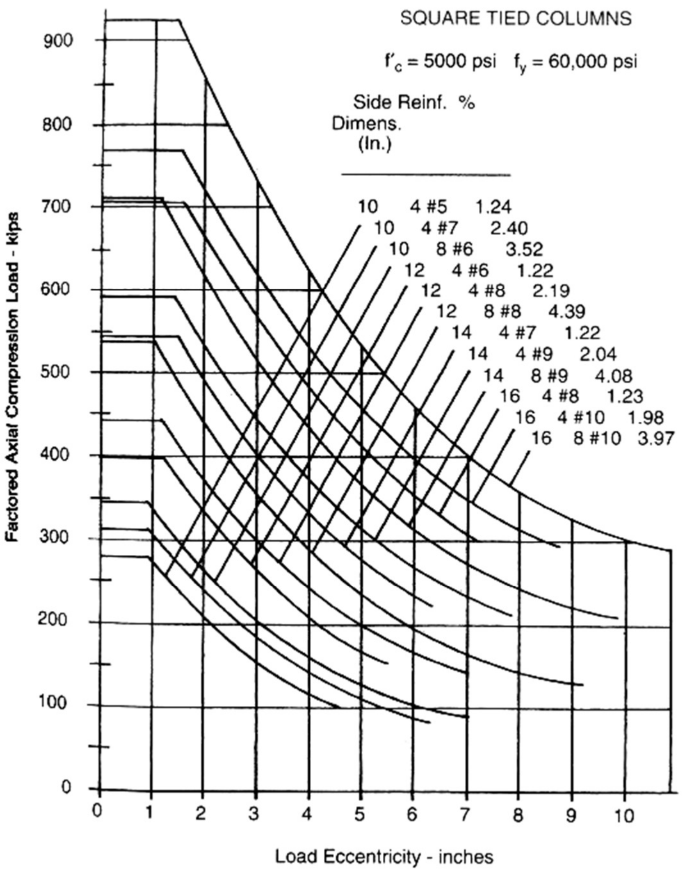
Strand designation code	Span, ft 13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
66-S	400	365	333	308	282	256	224	197	173	153	135	119	105	93	82	68	56	45	36	26								
	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.2	0.2	0.2	0.2	0.1	0.0	–0.0	–0.1	–0.2	–0.3								
	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.1	0.1	0.0	–0.1	–0.2	–0.3	–0.4	–0.6	–0.7	–0.9	–1.2	–1.4								
76-S	474	435	396	366	340	304	267	235	208	184	164	146	130	116	103	88	74	62	51	41	31							
	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.3	0.4	0.4	0.4	0.3	0.3	0.3	0.3	0.2	0.2	0.1	–0.0	–0.1	–0.2							
	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.3	0.2	0.2	0.2	0.1	0.0	–0.1	–0.2	–0.4	–0.5	–0.7	–0.9	–1.2	–1.4							
58-S	445	405	374	342	318	298	275	260	243	228	217	196	177	159	143	126	110	95	82	70	59	49	40	32				
	0.3	0.3	0.3	0.4	0.4	0.5	0.5	0.5	0.5	0.6	0.6	0.6	0.6	0.6	0.6	0.5	0.5	0.5	0.1	0.3	0.2	0.1	0.0	–0.1				
	0.3	0.3	0.4	0.4	0.4	0.4	0.5	0.5	0.5	0.5	0.4	0.3	0.3	0.3	0.2	0.1	–0.1	–0.2	–0.4	–0.6	–0.9	–1.2	–1.5	–1.8				
68-S		463	426	393	366	342	319	299	282	267	251	239	216	195	177	158	140	124	110	97	84	73	62	53	44	36	28	
		0.4	0.4	0.5	0.5	0.6	0.6	0.7	0.7	0.7	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.7	0.7	0.6	0.5	0.4	0.2	0.1	–0.1	
		0.4	0.5	0.5	0.6	0.6	0.6	0.6	0.7	0.7	0.7	0.6	0.6	0.6	0.5	0.4	0.3	0.2	0.0	–0.2	–0.4	–0.6	–0.9	–1.2	–1.6	–2.0	–2.4	
78-S		472	435	402	375	348	325	305	288	273	257	245	232	220	207	186	167	149	133	119	106	94	83	73	64	55	46	38
		0.5	0.5	0.6	0.6	0.7	0.7	0.8	0.9	0.9	1.0	1.0	1.0	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.0	0.9	0.9	0.7	0.6	0.5	0.3
		0.5	0.6	0.6	0.7	0.7	0.8	0.8	0.8	0.9	0.9	0.9	0.9	0.8	0.8	0.7	0.7	0.6	0.4	0.3	0.1	–0.1	–0.3	–0.6	–0.9	–1.3	–1.7	–2.2

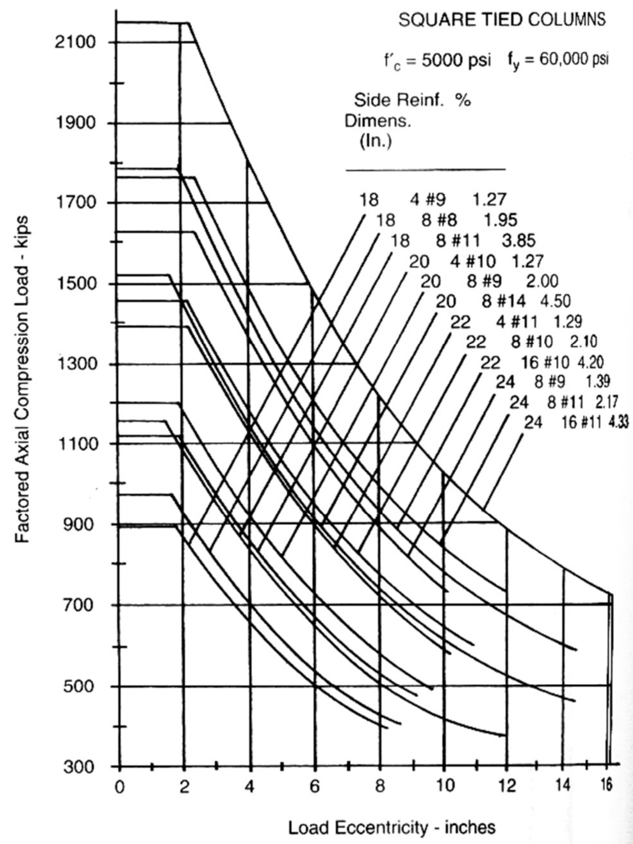
Strength is based on strain compatibility; bottom tension is limited to $7.5\sqrt{f'_c}$; see pages 3–8 through 3–11 for explanation.
See item 3, note 4, Section 3.3.2 for explanation of vertical line.

Example 18



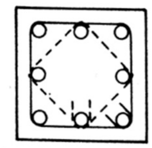
Example 1. A square tied column with $f'_c = 5$ ksi and steel with $f_y = 60$ ksi sustains an axial compression load of 150 kips dead load and 250 kips live load with no computed bending moment. Find the minimum practical column size if reinforcing is a maximum of 4% and the maximum size if reinforcing is a minimum of 1%. Also, design for e = 6 in.





Example 19

Determine the capacity of a 16" x 16" column with 8- #10 bars, tied. Grade 40 steel and 4000 psi concrete.



SOLUTION:

Find ϕP_n, with ϕ=0.65 and $P_n = 0.80P_o$ for tied columns and

$$P_o = 0.85 f_c'(A_g - A_{st}) + f_y A_{st}$$

Steel area (found from reinforcing bar table for the bar size):

A_{st} = 8 bars $\times$ (1.27 in^2) = 10.16 in^2

Concrete area (gross):

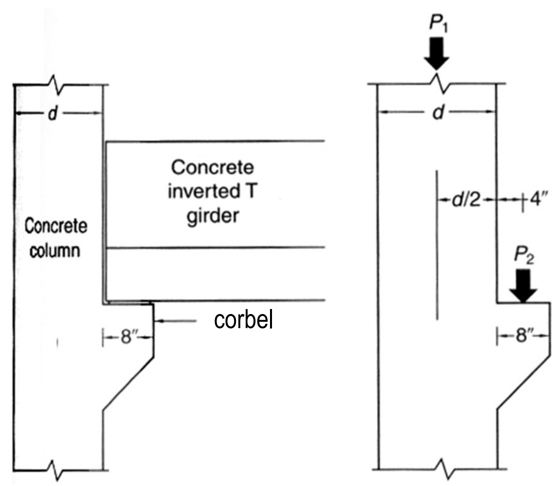
A_g = 16 in $\times$ 16 in = 256 in^2

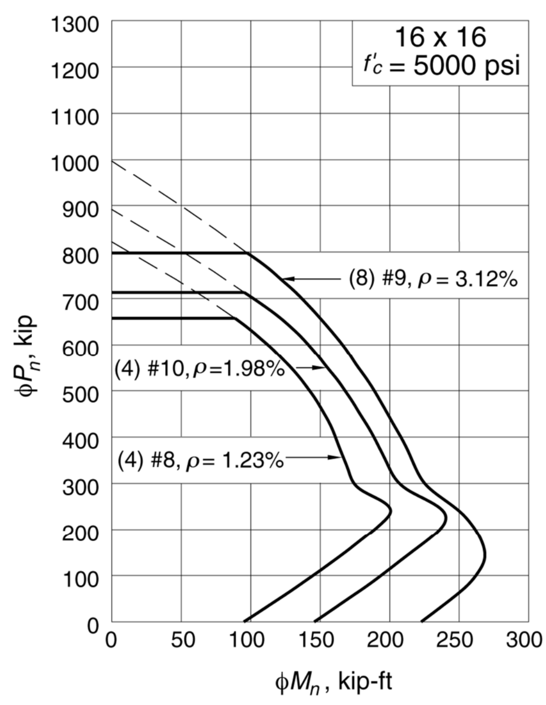
Grade 40 reinforcement has f_y = 40,000 psi and f_c' = 4000psi

ϕP_n = (0.65)(0.80)[0.85(4000 psi)(256 in^2 – 10.16 in^2) + (40,000 psi)(10.16 in^2)] = 646,026 lb = 646 kips

Example 20

16" x 16" precast reinforced columns support inverted T girders on corbels as shown. The unfactored loads on the corbel are 81 k dead, and 72 k live. The unfactored loads on the column are 170 k dead and 150 k live. Determine the reinforcement required using the interaction diagram provided. Assume that half the moment is resisted by the column above the corbel and the other half is resisted by the column below. Use grade 60 steel and 5000 psi concrete.





Example 21

EXAMPLE 5–4

Design a short square tied column to carry an axial dead load of 300 kip and a live load of 200 kip. Assume that the applied moments on the column are negligible. Use $f'_c = 4{,}000$ psi and $f_y = 60{,}000$ psi.

Solution

Step 1 The factored load, P_u, is:

$$P_u = 1.2P_D + 1.6P_L$$
$$P_u = 1.2(300) + 1.6(200)$$
$$P_u = 680 \text{ kip}$$

Assume $\rho_g = 0.03$.

Step 2 The required area of the column, A_g, is:

$$A_g = \frac{P_u}{0.8\phi[0.85f'_c(1 - \rho_g) + f_y\rho_g]}$$
$$A_g = \frac{680}{0.80\,(0.65)[0.85\,(4)(1 - 0.03) + 60(0.03)]}$$
$$A_g = 257 \text{ in}^2$$

Step 3 For a square column, the size, h, is:

$$h = \sqrt{A_g} = \sqrt{257}$$
$$\therefore h = 16.0 \text{ in.}$$

Try a 16 in. × 16 in. column:

$$A_g = (16)(16) = 256 \text{ in}^2$$

Step 4 The required amount of steel, A_{st}, is:

$$A_{st} = \frac{P_u - 0.8\phi(0.85f'_c\,A_g)}{0.8\phi(f_y - 0.85f'_c)}$$
$$A_{st} = \frac{680 - 0.8 \times 0.65(0.85 \times 4 \times 256)}{0.8 \times 0.65(60 - 0.85 \times 4)} = 7.73 \text{ in}^2$$

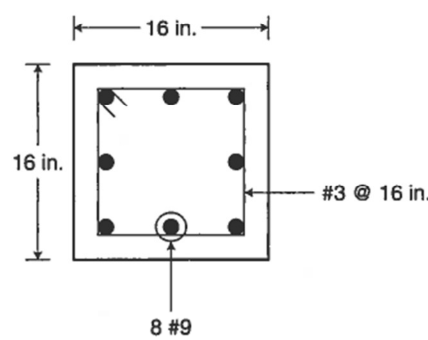
Step 5 Select the size and number of bars. For a square column with bars uniformly distributed along the edges, we keep the number of bars as multiples of four. Using Table A2–9, 8 #9 bars ($A_s = 8 \text{ in}^2$) are selected.

From Table A5–1 $\longrightarrow$ Maximum of 12 #9 bars $\therefore$ ok

Step 6 Because the longitudinal bars are #9, select #3 bars for the ties. The maximum spacing of the ties (s_{max}) is:

$$s_{max} = \min\{16d_b, 48d_t, b_{min}\}$$
$$s_{max} = \min\{16(1.128), 48(3/8), 16\}$$
$$s_{max} = \min\{18.0, 18.0, 16.0\}$$
$$\therefore s_{max} = 16 \text{ in.}$$

The selected ties are #3 @ 16 in.



Example 22

Design a 10 ft long circular spiral column for a braced system to support the service dead and live loads of 300 k and 460 k, respectively, and the service dead and live moments of 100 ft-k each. The moment at one end is zero. Use $f_c' = 4{,}000$ psi and $f_y = 60{,}000$ psi.

Solution

1. $P_u = 1.2(300) + 1.6(460) = 1096$ k
   $M_u = 1.2(100) + 1.6(100) = 280$ ft-k
2. Assume $\rho_g = 0.01$, from Equation 16.10:

$$A_g = \frac{P_u}{0.60[0.85f_c'(1-\rho_g) + f_y\rho_g]}$$

$$= \frac{1096}{0.60[0.85(4)(1-0.01) + 60(0.01)]}$$

$$= 460.58\,\text{in.}^2$$

$$\frac{\pi h^2}{4} = 460.58$$

or $h = 24.22$ in.
Use $h = 24$ in., $A_g = 452$ in.^{2}

3. Assume #9 size of bar and 3/8 in. spiral
   center-to-center distance
   $= 24 - 2(\text{cover}) - 2(\text{spiral diameter}) - 1\ (\text{bar diameter})$
   $= 24 - 2(1.5) - 2(3/8) - 1.128 = 19.12$ in.

ACI 7.7: Concrete exposed to earth or weather:
No. 6 through No. 18 bars....... 2 in. minimum

$$\gamma = \frac{19.12}{24} = 0.8$$

Use the interaction diagram Appendix D.21

4. $K_n = \dfrac{P_u}{\phi f_c' A_g} = \dfrac{1096}{(0.75)(4)(452)} = 0.808$

$$R_n = \frac{M_u}{\phi f_c' A_g h} = \frac{3360}{(0.75)(4)(452)(24)} = 0.103$$

5. At the intersection point of K_n and R_n, $\rho_g = 0.02$
6. The point is above the strain line = 1, hence $\phi = 0.75$ **OK**
7. $A_{st} = (0.02)(452) = 9.04$ in.^{2}
   From Appendix D.2, select 12 bars of #8, $A_{st} = 9.48$ in.^{2}
   From Appendix D.14 for a core diameter of $24 - 3 = 21$ in., 17 bars of #8 can be arranged in a row
8. Selection of spirals
   From Appendix D.13, size = 3/8 in.
   pitch = 2¼ in.
   Clear distance = 2.25 − 3/8 = 1.875 > 1 in. **OK**
9. $K = 1$, $l = 10 \times 12 = 120$ in., $r = 0.25(24) = 6$ in.

$$\frac{Kl}{r} = \frac{1(120)}{6} = 20$$

$$\left(\frac{M_1}{M_2}\right) = 0$$

$$34 - 12\left(\frac{M_1}{M_2}\right) = 34$$

ACI 10.12: In nonsway frames it shall be permitted to ignore slenderness effects for compression members that satisfy: $\frac{kl_u}{r} \le 34 - 12\left(M_1/M_2\right)$

since $(Kl/r) < 34$, short column.

Factored Moment Resistance of Concrete Beams, ϕM_n (k-ft) with f'_c = 4 ksi, f_y = 60 ksi[a]

	Approximate Values for a/d		
	0.1	0.2	0.3
	Approximate Values for ρ		
b x d (in)	0.0057	0.01133	0.017
10 x 14	2 #6	2 #8	3 #8
	53	90	127
10 x 18	3 #5	2 #9	3 #9
	72	146	207
10 x 22	2 #7	3 #8	(3 #10)
	113	211	321
12 x 16	2 #7	3 #8	4 #8
	82	154	193
12 x 20	2 #8	3 #9	4 #9
	135	243	306
12 x 24	2 #8	3 #9	(4 #10)
	162	292	466
15 x 20	3 #7	4 #8	5 #9
	154	256	383
15 x 25	3 #8	4 #9	4 #11
	253	405	597
15 x 30	3 #8	5 #9	(5 #11)
	304	608	895
18 x 24	3 #8	5 #9	6 #10
	243	486	700
18 x 30	3 #9	6 #9	(6 #11)
	385	729	1074
18 x 36	3 #10	6 #10	(7 #11)
	586	1111	1504
20 x 30	3 # 10	7 # 9	6 # 11
	489	851	1074
20 x 35	4 #9	5 #11	(7 #11)
	599	1106	1462
20 x 40	6 #8	6 #11	(9 #11)
	811	1516	2148
24 x 32	6 #8	7 #10	(8 #11)
	648	1152	1528
24 x 40	6 #9	7 #11	(10 #11)
	1026	1769	2387
24 x 48	5 #10	(8 #11)	(13 #11)
	1303	2426	3723

[a]Table yields values of factored moment resistance in kip-ft with reinforcement indicated. Reinforcement choices shown in parentheses require greater width of beam or use of two stack layers of bars. (*Adapted and corrected from Simplified Engineering for Architects and Builders, 11th ed, Ambrose and Tripeny, 2010.)*

Column Interaction Diagrams

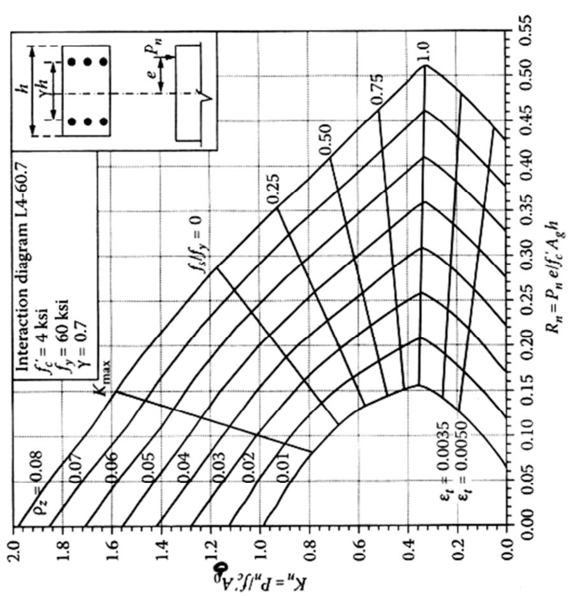


FIGURE D.15 Column interaction diagram for tied column with bars on end faces only. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

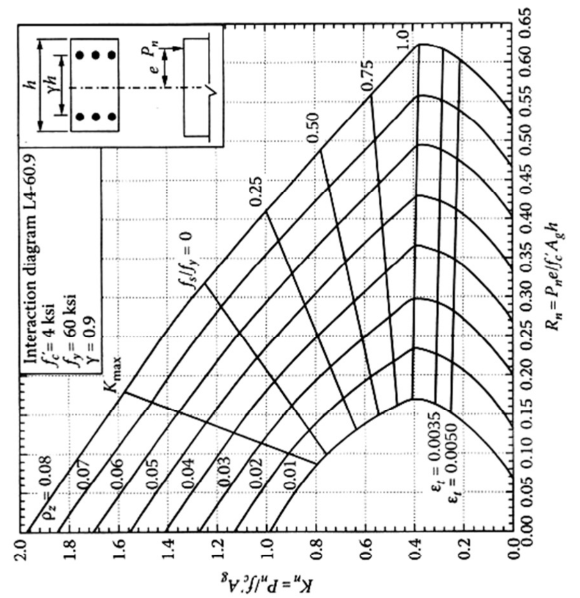


FIGURE D.16 Column interaction diagram for tied column with bars on end faces only. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

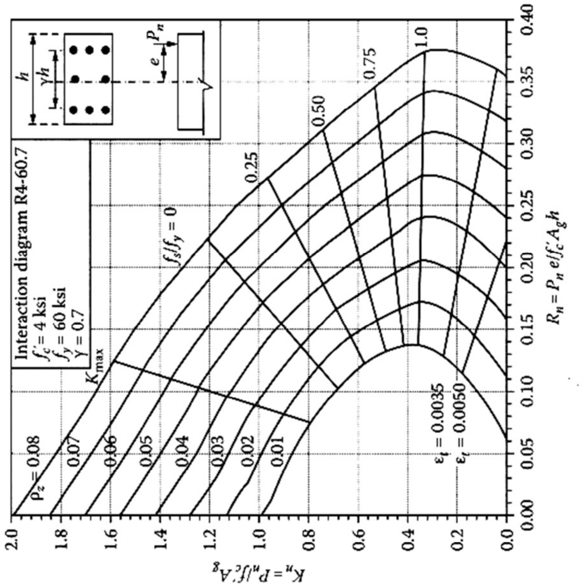


FIGURE D.17 Column interaction diagram for tied column with bars on all faces. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

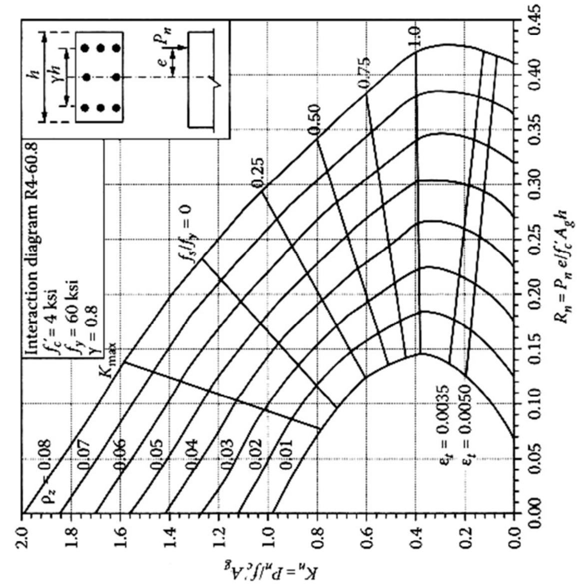


FIGURE D.18 Column interaction diagram for tied column with bars on all faces. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

Column Interaction Diagrams

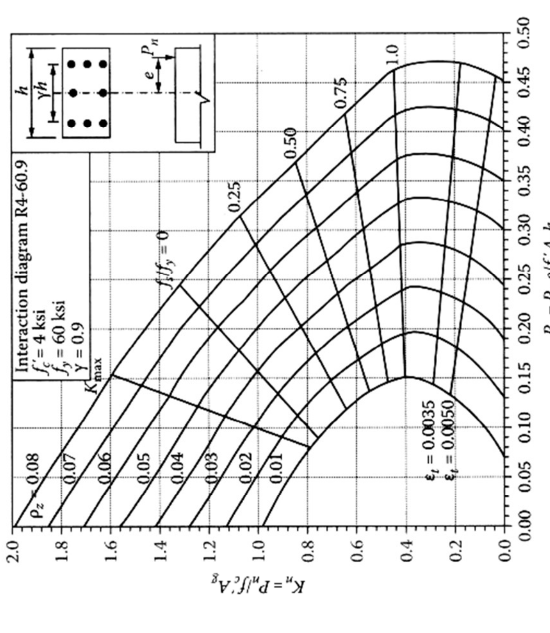


FIGURE D.19 Column interaction diagram for tied column with bars on all faces. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

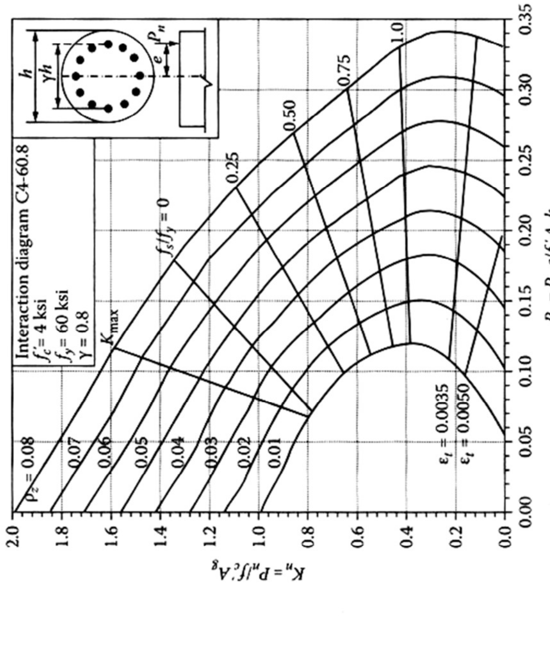


FIGURE D.21 Column interaction diagram for circular spiral column. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

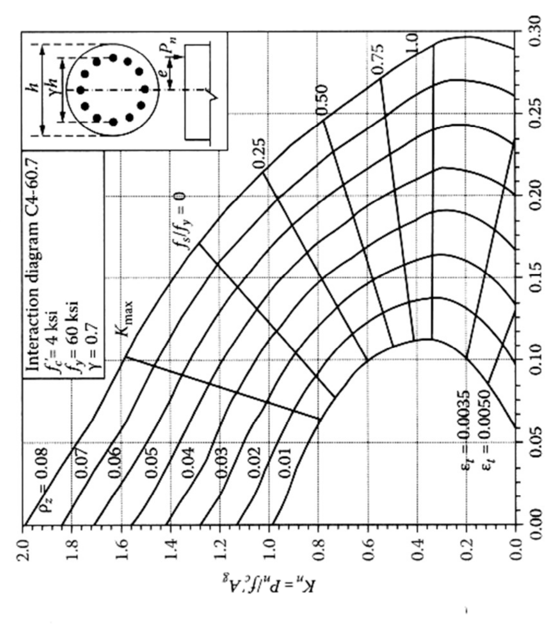


FIGURE D.20 Column interaction diagram for circular spiral column. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

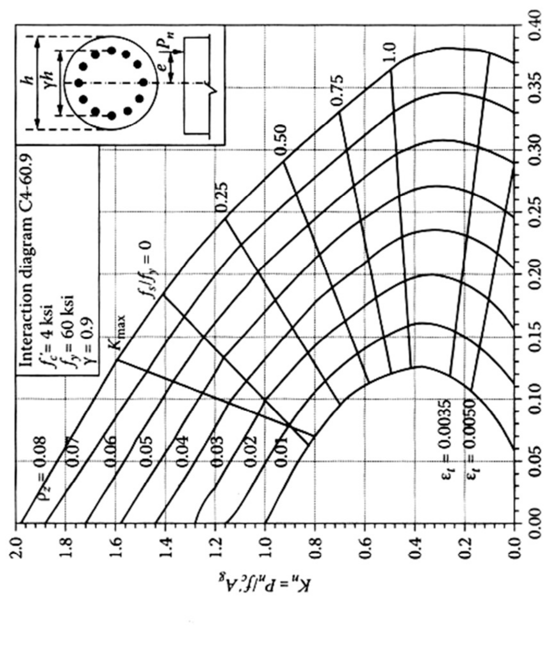
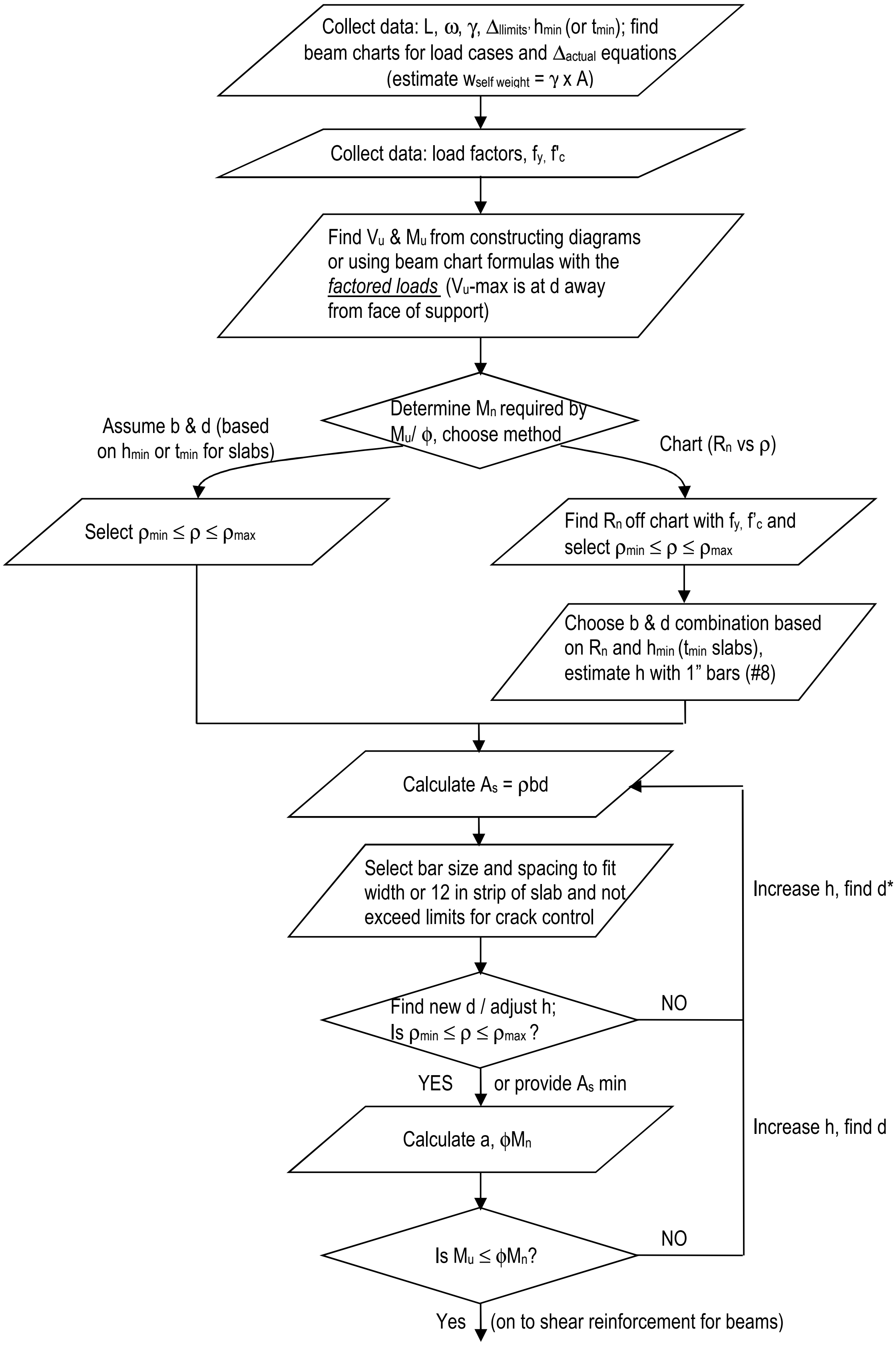


FIGURE D.22 Column interaction diagram for circular spiral column. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

Beam / One-Way Slab Design Flow Chart



Beam / One-Way Slab Design Flow Chart - continued