Mechanics and the Foundations of Modern Physics

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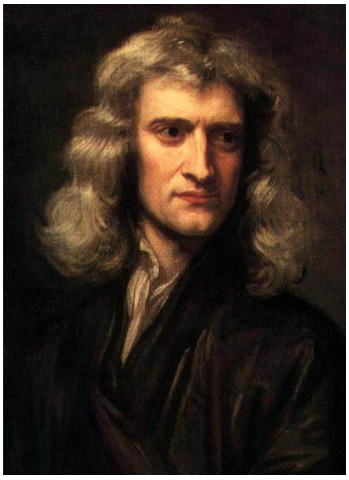
Isaac Newton (1642-1727)



Isaac Newton was arguably the most brilliant intellect of all time, enhanced by his curiosity, ambition, and an ability to concentrate on a problem for days at a time without food or sleep.

Born in Woolsthorpe, England, Newton was admitted to Cambridge University in 1661, where his mathematics professor was Isaac Barrow, the first holder of the Lucasian Chair in Mathematics. The university closed during the plague years 1664-65, and students were sent home. During that time Newton invented calculus, discovered the law of universal gravitation, and showed by experiment that white light consists of light of all colors. Returning to Cambridge, Newton continued to excel, so that in 1669 Barrow resigned his chair in favor of Newton.

Many of his discoveries were unknown to his contemporaries, until his friend Sir Edmund Halley finally convinced him to publish. Newton spent the years 1684-86 writing his masterpiece, the *Principia*, a task he made especially challenging by reproving all his results without calculus, so readers could understand them. Newton presented his laws of motion, derived Kepler's laws, calculated the mass of the Sun, explained the precession of the equinoxes, and much else.

Newton ultimately became Warden of the Mint, President of the Royal Society, a member of Parliament, and was knighted. Toward the end of his life he wrote: "I do not know what I may appear to the world, but to myself I seem to have been only like a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me." Nevertheless, Newton changed the world. He died in 1727 and is buried in Westminster Abbey.

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